On the General Solution to Einstein's Vacuum Field and Its Implications for Relativistic Degeneracy

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The general solution to Einstein's vacuum field equations for the point-mass in all its configurations must be determined in such a way as to provide a means by which an infinite sequence of particular solutions can be readily constructed. It is from such a solution that the underlying geometry of Einstein's universe can be rightly explored. I report here on the determination of the general solution and its consequences for the theoretical basis of relativistic degeneracy, i. e. gravitational collapse and the black hole.

1 Introduction

A serious misconception prevails that the so-called "Schwarzschild solution" is a solution for the vacuum field. Not only is this incorrect, it is not even Schwarzschild's solution. The aforesaid solution was obtained by David Hilbert [1], a full year after Karl Schwarzschild [2] obtained his original solution. Moreover, Hilbert's metric is a corruption of the solution first found by Johannes Droste [3], and subsequently by Hermann Weyl [4] by a different method.

The orthodox concepts of gravitational collapse and the black hole owe their existence to a confusion as to the true nature of the r-parameter in the metric tensor for the gravitational field.

The error in the conventional analysis of Hilbert's solution is twofold in that two tacit and invalid assumptions are made:

(a) r is a coordinate and radius (of some kind) in the gravitational field;

(b) The regions $0 < r < \alpha = 2m$ and $\alpha < r < \infty$ are valid.

Contrary to the conventional analysis the nature and range or the r-parameter must be determined by rigorous mathematical means, *not* by mere assumption, tacit or otherwise. When the required mathematical rigour is applied it is revealed that $r_0 = \alpha$ denotes a point, not a 2-sphere, and that $0 < r < \alpha$ is undefined on the Hilbert metric. The consequence of this is that gravitational collapse, if it occurs in Nature at all, cannot produce a relativistic black hole under any circumstances. Since the Michell-Laplace dark body is not a black hole either, there is no theoretical basis for it whatsoever. Furthermore, the conventional conception of gravitational collapse is demonstrably false.

The sought for general solution must not only result in a means for construction of an infinite sequence of particular solutions, it must also naturally produce the solutions due to Schwarzschild, Droste and Weyl, and M. Brillouin [5]. To obtain the general solution the general conditions that the required solution must satisfy must be established. Abrams [9] has determined these conditions. I obtain them by other arguments, and therefrom construct the general solution, from which the original Schwarzschild solution, the Droste/Weyl solution, and the Brillouin solution all arise quite naturally. It will be evident that the black hole is theoretically unsound. Indeed, it never arose in the solutions of Schwarzschild, Droste and Weyl, and Brillouin. It comes solely from the mathematically inadmissible assumptions conventionally imposed upon the Hilbert metric.

I provide herein a derivation of the general solution for the simple point-mass and briefly discuss its geometry. Although I have obtained the complete solution up to the rotating point-charge I reserve its derivation to a subsequent paper and similarly a full discussion of the geometry to a third paper. However, I include the expression for the overall general solution as a prelude to my following papers.

2 The general solution for the simple point-mass and its basic geometry

A general metric for the static, time-symmetric, cento-symmetric configuration of energy or matter in quasi-Cartesian coordinates is,

$$\begin{aligned} ds^2 &= L(r)dt^2 - M(r)(dx^2 + dy^2 + dz^2) - \\ &\quad - N(r)(xdx + ydy + zdz)^2, \\ r &= \sqrt{x^2 + y^2 + z^2}, \end{aligned} \tag{1}$$

where, $\forall\, t$, L, M, N are analytic functions such that,

$$L, M, N > 0\,. \tag{2}$$

In polar coordinates (1) becomes,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2), \tag{3}$$

where analytic $A, B, C > 0$ owing to (2).

Transform (3) by setting

$$r^* = \sqrt{C(r)}\,, \tag{4}$$

then

$$ds^2 = A^*(r^*)dt^2 - B^*(r^*)dr^{*2} - \\ - r^{*2}(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{5}$$

from which one obtains in the usual way,

$$ds^2 = \left(\frac{r^*-\alpha}{r^*}\right)dt^2 - \left(\frac{r^*}{r^*-\alpha}\right)dr^{*2} - \\ - r^{*2}(d\theta^2 + \sin^2\theta d\varphi^2)\,. \tag{6}$$

Substituting (4) gives

$$ds^2 = \left(\frac{\sqrt{C}-\alpha}{\sqrt{C}}\right)dt^2 - \left(\frac{\sqrt{C}}{\sqrt{C}-\alpha}\right)\frac{C'^2}{4C}dr^2 - \\ - C(d\theta^2 + \sin^2\theta d\varphi^2)\,. \tag{7}$$

Thus, (7) is a general metric in terms of one unknown function $C(r)$. The following arguments are coordinate independent since $C(r)$ in (7) is an arbitrary function.

The general metric for Special Relativity is,

$$ds^2 = dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2 d\varphi^2\right), \tag{8}$$

and the radial distance (the *proper distance*) between two *points* is,

$$d = \int_{r_0}^{r} dr = r - r_0\,. \tag{9}$$

Let a test particle be located at each of the points r_0 and $r > r_0$ (owing to the isotropy of space there is no loss of generality in taking $r \geqslant r_0 \geqslant 0$). Then by (9) the distance between them is given by

$$d = r - r_0\,,$$

and if $r_0 = 0$, $d \equiv r$ in which case the distance from $r_0 = 0$ is the same as the radius (the *curvature radius*) of a great circle, the circumference χ of which is from (8),

$$\chi = 2\pi\sqrt{r^2} = 2\pi r\,. \tag{10}$$

In other words, the curvature radius and the proper radius are identical, owing to the pseudo-Euclidean nature of (8). Furthermore, d gives the radius of a sphere centred at the *point* r_0. Let the test particle at r_0 acquire mass. This produces a gravitational field centred at the *point* $r_0 \geqslant 0$. The geometrical relations between the components of the metric tensor of General Relativity must be precisely the same in the metric of Special Relativity. Therefore the distance between r_0 and $r > r_0$ is no longer given by (9) and the curvature radius no longer by (10). Indeed, the proper radius R_p, in keeping with the geometrical relations on (8), is now given by,

$$R_p = \int_{r_0}^{r} \sqrt{-g_{11}}dr\,, \tag{11}$$

where from (7),

$$-g_{11} = \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)^{-1} \frac{[C'(r)]^2}{4C(r)}\,. \tag{12}$$

Equation (11) with (12) gives the mapping of d from the flat spacetime of Special Relativity into the curved spacetime of General Relativity, thus,

$$R_p(r) = \int \sqrt{\frac{\sqrt{C}}{\sqrt{C}-\alpha}\,\frac{C'}{2\sqrt{C}}}\,dr = \\ = \sqrt{\sqrt{C(r)}\left(\sqrt{C(r)}-\alpha\right)} + \\ + \alpha \ln\left|\frac{\sqrt{\sqrt{C(r)}} + \sqrt{\sqrt{C(r)}-\alpha}}{K}\right|, \\ K = \text{const}\,. \tag{13}$$

The relationship between r and R_p is

$$r \to r_0 \Rightarrow R_p \to 0\,,$$

so from (13) it follows,

$$r \to r_0 \;\Rightarrow C(r_0) = \alpha^2,\; K = \sqrt{\alpha}\,.$$

So (13) becomes,

$$R_p(r) = \sqrt{\sqrt{C(r)}\left(\sqrt{C(r)}-\alpha\right)} + \\ + \alpha \ln\left|\frac{\sqrt{\sqrt{C(r)}} + \sqrt{\sqrt{C(r)}-\alpha}}{\sqrt{\alpha}}\right|. \tag{14}$$

Therefore (7) is singular only at $r = r_0$, where $C(r_0) = \alpha^2$ and $g_{00} = 0 \;\forall\; r_0$, irrespective of the value of r_0. $C(r_0) = \alpha^2$ emphasizes the true meaning of α, viz., α is a scalar invariant which fixes the spacetime for the point-mass from an infinite number of mathematically possible forms, as pointed out by Abrams. Moreover, α embodies the effective gravitational mass of the source of the field, and fixes a boundary to an otherwise incomplete spacetime. Furthermore, one can see from (13) and (14) that r_0 is arbitrary, i.e. the point-mass can be located at any point and its location has no intrinsic meaning. Furthermore, the condition $g_{00} = 0$ is clearly equivalent to the boundary condition $r \to r_0 \Rightarrow R_p \to 0$, from which it follows that $g_{00} = 0$ is the *end result* of gravitational collapse. There exists no value of r making $g_{11} = 0$.

If $C' = 0$ for $r > r_0$ the structure of (7) is destroyed: $g_{11} = 0$ for $r > r_0 \Rightarrow B(r) = 0$ for $r > r_0$ in violation of (3). Therefore $C' \neq 0$. For (7) to be spatially asymptotically flat,

$$\lim_{r\to\infty} \frac{C(r)}{\left(r - r_0\right)^2} = 1\,. \tag{15}$$

Since $C(r)$ must behave like $(r - r_o)^2$ and make (7) singular only at $r = r_0$, $C(r)$ must be a strictly monotonically increasing function. Then by virtue of (15) and the fact that $C' \neq 0$, it follows that $C' > 0$ for $r > r_0$. Thus the necessary conditions that must be imposed upon $C(r)$ to render a solution to (3) are:

1. $C'(r) > 0$ for $r > r_0$;
2. $\lim\limits_{r\to\infty} \dfrac{C(r)}{\left(r - r_0\right)^2} = 1$;
3. $C(r_0) = \alpha^2$.

I call the foregoing the Metric Conditions of Abrams for the point-mass (MCA) since when $r_0 = 0$ they are precisely the conditions he determined by his use of (3) and the field equations. In addition to MCA any admissible function $C(r)$ must reduce (7) to the metric of Special Relativity when $\alpha = 2m = 0$.

The invalid conventional assumptions that $0 < r < \alpha$ and that r is a radius of sorts in the gravitational field lead to the incorrect conclusion that $r = \alpha$ is a 2-sphere in the gravitational field of the point-mass. The quantity $r = \alpha$ does not describe a 2-sphere; it does not yield a Schwarzschild sphere; it is actually a *point*. Stavroulakis [10, 8, 9] has also remarked upon the true nature of the r-parameter (coordinate radius). Since MCA must be satisfied, admissible systems of coordinates are restricted to a particular (infinite) class. To satisfy MCA, and therefore (3), and (7), the form that $C(r)$ can take must be restricted to,

$$C_n(r) = \left[(r - r_0)^n + \alpha^n\right]^{\frac{2}{n}}\,, \tag{16}$$

$$r_0 \in (\Re - \Re^-),\ n \in \Re^+,$$

where n and r_0 are arbitrary. I call equations (16) Schwarzschild forms. The value of n in (16) fixes a set of coordinates, and the infinitude of such reflects the fact that no set of coordinates is privileged in General Relativity.

The general solution for the simple point-mass is therefore,

$$ds^2 = \left(\frac{\sqrt{C_n} - \alpha}{\sqrt{C_n}}\right) dt^2 - \left(\frac{\sqrt{C_n}}{\sqrt{C_n} - \alpha}\right) \frac{C_n'^{\,2}}{4C_n} dr^2 - C_n(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{17}$$

$$C_n(r) = \left[(r - r_0)^n + \alpha^n\right]^{\frac{2}{n}}\,,\ n \in \Re^+,$$

$$r_0 \in (\Re - \Re^-)\,,$$

$$r_0 < r < \infty\,,$$

where n and r_0 are arbitrary. Therefore with r_0 arbitrary, (17) reduces to the metric of Special Relativity when $\alpha = 2m = 0$.

From (17), with $r_0 = 0$ and n taking integer values, the following infinite sequence obtains:

$C_1(r) = (r + \alpha)^2$ (Brillouin's solution)

$C_2(r) = (r^2 + \alpha^2)$

$C_3(r) = (r^3 + \alpha^3)^{\frac{2}{3}}$ (Schwarzschild's solution)

$C_4(r) = (r^4 + \alpha^4)^{\frac{1}{2}}$, etc.

Hilbert's solution is rightly obtained when $r_0 = \alpha$, i. e. when $r_0 = \alpha$ and the values of n take integers, the infinite sequence of particular solutions is then given by,

$C_1(r) = r^2$ [Droste/Weyl/(Hilbert) solution]

$C_2(r) = (r - \alpha)^2 + \alpha^2$,

$C_3(r) = \left[(r - \alpha)^3 + \alpha^3\right]^{\frac{2}{3}}$,

$C_4(r) = \left[(r - \alpha)^4 + \alpha^4\right]^{\frac{1}{2}}$, etc.

The curvature $f = R^{ijkm}R_{ijkm}$ is finite everywhere, including $r = r_0$. Indeed, for metric (17) the Kretschmann scalar is,

$$f = \frac{12\alpha^2}{C_n^3} = \frac{12\alpha^2}{\left[\left(r - r_0\right)^n + \alpha^n\right]^{\frac{6}{n}}}\,. \tag{18}$$

Gravitational collapse does not produce a curvature singularity in the gravitational field of the point-mass. The scalar invariance of $f(r_0) = \frac{12}{\alpha^4}$ is evident from (18).

All the particular solutions of (17) are inextendible, since the singularity when $r = r_0$ is quasiregular, irrespective of the values of n and r_0. Indeed, the circumference χ of a great circle becomes,

$$\chi = 2\pi\sqrt{C(r)}. \tag{19}$$

Then the ratio

$$\lim_{r\to r_0} \frac{\chi}{R_p} \to \infty, \tag{20}$$

shows that $R_p(r_0) \equiv 0$ is a quasiregular singularity and cannot be extended.

Equation (19) shows that $\chi = 2\pi\alpha$ is also a scalar invariant for the point-mass.

It is plain from the foregoing that the Kruskal-Szekeres extension is meaningless, that the "Schwarzschild radius" is meaningless, that the orthodox conception of gravitational collapse is incorrect, and that the black hole is not consistent at all with General Relativity. All arise wholely from a bungled analysis of Hilbert's solution.

3 Implications for gravitational collapse

As is well known the gravitational potential Φ for an arbitrary metric is

$$g_{00} = (1 - \Phi)^2\,, \tag{21}$$

from which it is concluded that gravitational collapse occurs at $\Phi = 1$. Physically, the conventional process of collapse involves Newtonian gravitation down to the so-called "gravitational radius". Far from the source, the alleged weak field potential is,

$$\Phi = \frac{m}{r} ,$$

and so

$$g_{00} = 1 - \frac{\alpha}{r} , \qquad (22)$$

$$\alpha = 2m .$$

The scalar α is conventionally called the "gravitational radius", or the "Schwarzschild radius", or the "event horizon". However, as I have shown, neither α nor the coordinate radius r are radii in the gravitational field. In the case of the Hilbert metric, $r_0 = \alpha$ is a *point*, not a 2-sphere. It is the location of the point-mass. In consequence of this $g_{00} = 0$ is the end result of gravitational collapse. It therefore follows that in the vacuum field,

$$0 < g_{00} < 1 , \qquad 1 < |g_{11}| < \infty ,$$

$$\alpha < \sqrt{C(r)} .$$

In the case of the Hilbert metric, $C(r) = r^2$, so

$$0 < g_{00} < 1, \qquad 1 < |g_{11}| < \infty ,$$

$$\alpha < r .$$

In the case of Schwarzschild's metric we have $C(r) = \left(r^3 + \alpha^3\right)^{\frac{2}{3}}$, so

$$0 < g_{00} < 1, \qquad 1 < |g_{11}| < \infty ,$$

$$0 < r .$$

It is unreasonable to expect the weak field potential function to be strictly Newtonian. Only in the infinitely far field is Newton's potential function to be recovered. Consequently, the conventional weak field expression (22) cannot be admitted with the conventional interpretation thereof. The correct potential function must contain the arbitrary location of the point-mass. From (21),

$$\Phi = 1 - \sqrt{g_{00}} = 1 - \sqrt{1 - \frac{\alpha}{\sqrt{C(r)}}} ,$$

so in the weak far field,

$$\Phi \approx 1 - \left(1 - \frac{\alpha}{2\sqrt{C}}\right) = \frac{m}{\sqrt{C}} ,$$

and so

$$g_{00} = 1 - \frac{\alpha}{\sqrt{C(r)}} = 1 - \frac{\alpha}{\left[(r - r_0)^n + \alpha^n\right]^{\frac{1}{n}}} , \qquad (23)$$

$$r_0 \in (\Re - \Re^-),\ n \in \Re^+ .$$

Then

$$\text{as } r \to \infty,\ g_{00} \to 1 - \frac{\alpha}{r - r_0} ,$$

and Newton is recovered at infinity.

According to (23), at $r = r_0$, $g_{00} = 0$ and $\Phi = \frac{1}{2}$. The weak field potential approaches a finite maximum of $\frac{1}{2}$ (i. e. $\frac{1}{2}c^2$), in contrast to Newton's potential. The conventional concept of gravitational collapse at $r_s = \alpha$ is therefore meaningless.

Similarly, it is unreasonable to expect Kepler's 3rd Law to be unaffected by general relativity, contrary to the conventional analysis. Consider the Lagrangian,

$$\begin{aligned} L &= \frac{1}{2}\left[\left(1 - \frac{\alpha}{\sqrt{C_n}}\right)\left(\frac{dt}{d\tau}\right)^2\right] - \\ &- \frac{1}{2}\left[\left(1 - \frac{\alpha}{\sqrt{C_n}}\right)^{-1}\left(\frac{d\sqrt{C_n}}{d\tau}\right)^2\right] - \\ &- \frac{1}{2}\left[C_n\left(\left(\frac{d\theta}{d\tau}\right)^2 + \sin^2\theta\left(\frac{d\varphi}{d\tau}\right)^2\right)\right] , \\ C_n(r) &= \left[\left(r - r_0\right)^n + \alpha^n\right]^{\frac{2}{n}} ,\ n \in \Re^+ , \\ &r_0 \in (\Re - \Re^-),\ r_0 < r < \infty , \end{aligned} \qquad (24)$$

where τ is the proper time.

Restricting motion, without loss of generality, to the equatorial plane, $\theta = \frac{\pi}{2}$, the Euler-Lagrange equations for (24) are,

$$\begin{aligned} &\left(1 - \frac{\alpha}{\sqrt{C_n}}\right)^{-1}\frac{d^2\sqrt{C_n}}{d\tau^2} + \frac{\alpha}{2C_n}\left(\frac{dt}{d\tau}\right)^2 - \\ &-\left(1 - \frac{\alpha}{\sqrt{C_n}}\right)^{-2}\frac{\alpha}{2C_n}\left(\frac{d\sqrt{C_n}}{d\tau}\right)^2 - \sqrt{C_n}\left(\frac{d\varphi}{d\tau}\right)^2 = 0 , \end{aligned} \qquad (25)$$

$$\left(1 - \frac{\alpha}{\sqrt{C_n}}\right)\frac{dt}{d\tau} = \text{const} = k , \qquad (26)$$

$$C_n\frac{d\varphi}{d\tau} = \text{const} = h , \qquad (27)$$

and $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ becomes,

$$\begin{aligned} &\left(1 - \frac{\alpha}{\sqrt{C_n}}\right)\left(\frac{dt}{d\tau}\right)^2 - \\ &-\left(1 - \frac{\alpha}{\sqrt{C_n}}\right)^{-1}\left(\frac{d\sqrt{C_n}}{d\tau}\right)^2 - C_n\left(\frac{d\varphi}{d\tau}\right)^2 = 1 . \end{aligned} \qquad (28)$$

Using the foregoing equations it readily follows that the angular velocity is,

$$\omega = \sqrt{\frac{\alpha}{2C_n^{\frac{3}{2}}}} . \qquad (29)$$

Then,

$$\lim_{r \to r_0} \omega = \frac{1}{\alpha\sqrt{2}} \tag{30}$$

is a scalar invariant which shows that the angular velocity approaches a finite limit, in contrast to Newton's theory where it becomes unbounded. Schwarzschild obtained this result for his particular solution. Equation (29) is the General Relativistic modification of Kepler's 3rd Law.

For a falling particle in a true Schwarzschild field,

$$d\tau = \sqrt{g_{00}}\, dt = \sqrt{1 - \frac{\alpha}{\sqrt{C(r)}}}\, dt\,.$$

Therefore, as a neutral test particle approaches the field source at r_0 along a radial geodesic, $d\tau \to 0$. Thus, according to an external observer, it takes an infinite amount of coordinate time for a test particle to reach the source. Time stops at the Schwarzschild point-mass. The conventional concepts of the Schwarzschild sphere and its interior are meaningless.

Doughty [10] has shown that the acceleration of a test particle approaching the point-mass along a radial geodesic is given by,

$$a = \frac{\sqrt{-g_{11}}\left(-g^{11}\right)|g_{00,1}|}{2g_{00}}. \tag{31}$$

By (17),

$$a = \frac{\alpha}{2C^{\frac{3}{4}}\left(\sqrt{C} - \alpha\right)^{\frac{1}{2}}}\,.$$

Clearly, as $r \to r_0$, $a \to \infty$, independently of the value of r_0. In the case of $C(r) = r^2$, where $r_0 = \alpha$,

$$a = \frac{\alpha}{2r^{\frac{3}{2}}\sqrt{r - \alpha}}\,, \tag{32}$$

so $a \to \infty$ as $r \to r_0 = \alpha$.

Applying (31) to the Kruskal-Szekeres extension gives rise to the absurdity of an infinite acceleration at $r = \alpha$ where it is conventionally claimed that there is no matter and no singularity. It is plainly evident that gravitational collapse terminates at a Schwarzschild simple point-mass, not in a black hole. Also, one can readily see that the alleged interchange of the spatial and time coordinates "inside" the "Schwarzschild sphere" is nonsensical. To amplify this, in (17), suppose $\sqrt{C(r)} < \alpha$, then

$$ds^2 = -\left(\frac{\alpha}{\sqrt{C}} - 1\right) dt^2 + \left(\frac{\alpha}{\sqrt{C}} - 1\right)^{-1} \frac{C'^2}{4C} dr^2 - C\left(d\theta^2 + \sin^2 d\varphi^2\right). \tag{33}$$

Let $r = \tilde{t}$ and $t = \tilde{r}$, then

$$ds^2 = \left(\frac{\alpha - \sqrt{C}}{\sqrt{C}}\right)^{-1} \frac{\dot{C}^2}{4C} d\tilde{t}^2 - \left(\frac{\alpha - \sqrt{C}}{\sqrt{C}}\right) d\tilde{r}^2 - C(\tilde{t})\left(d\theta^2 + \sin^2 d\varphi^2\right). \tag{34}$$

This is a time dependent metric which does not have any relationship to the original static problem. It does not extend (17) at all, as also noted by Brillouin in the particular solution given by him. Equation (34) is meaningless.

It is noteworthy that Hagihara [11] has shown that all geodesics that do not run into the Hilbert boundary at $r_0 = \alpha$ are complete. His result is easily extended to any $r_0 \geqslant 0$ in (17).

The correct conclusion is that gravitational collapse terminates at the point-mass without the formation of a black hole in all general relativistic circumstances.

4 Generalization of the vacuum solution for charge and angular momentum

The foregoing analysis can be readily extended to include the charged and rotating point-mass. In similar fashion it follows that the Reissner-Nordstrom, Carter, Graves-Brill, Kerr, and Kerr-Newman black holes are all inconsistent with General Relativity.

In a subsequent paper I shall derive the following overall general solution for the point-mass when $\Lambda = 0$,

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\varphi\right)^2 - \frac{\sin^2\theta}{\rho^2}\left[\left(C_n + a^2\right)d\varphi - a dt\right]^2 - \frac{\rho^2}{\Delta}\frac{{C'_n}^2}{4C_n}dr^2 - \rho^2 d\theta^2\,,$$

$$C_n(r) = \left[\left(r - r_0\right)^n + \beta^n\right]^{\frac{2}{n}}, \; r_0 \in (\Re - \Re^-)\,,$$

$$n \in \Re^+, \; a = \frac{L}{m}, \; \rho^2 = C_n + a^2\cos^2\theta\,,$$

$$\Delta = C_n - \alpha\sqrt{C_n} + q^2 + a^2\,,$$

$$\beta = m + \sqrt{m^2 - q^2 - a^2\cos^2\theta}, \; a^2 + q^2 < m^2,$$

$$r_0 < r < \infty\,.$$

The different configurations for the point-mass are easily extracted from this set of equations by the setting of the values of the parameters in the obvious way.

Dedication

I dedicate this paper to the memory of Dr. Leonard S. Abrams: (27 Nov. 1924 — 28 Dec. 2001).

Epilogue

My interest in the problem of the black hole was aroused by coming across the papers of the American physicist Leonard S. Abrams, and subsequently to the original papers of Schwarzschild, Droste, Weyl, Hilbert, and Brillouin. I was drawn to the logic of Abrams' approach in his determination of the required metric in terms of a single generalised function and the conditions that this function must satisfy to render a solution for the point-mass. It was not until I read Abrams that I became aware of the startling facts that the "Schwarzschild solution" is not due to Schwarzschild, that Schwarzschild did not predict the black hole and made none of the claims about black holes that are invariably attributed to him in the textbooks and almost invariably in the literature. These facts alone give cause for disquiet and reading of the original papers gives cause for serious concern about how modern science is reported.

Dr. Leonard S. Abrams was born in Chicago in 1924 and died on December 28, 2001, in Los Angeles at the age of 77. He received a B. S. in Mathematics from the California Institute of Technology and a Ph. D. in physics from the University of California at Los Angeles at the age of 45. He spent almost all of his career working in the private sector, although he taught at a variety of institutions including California State University at Dominguez Hills and at the University of Southern California. He was a pioneer in applying game theory to business problems and was an expert in noise theory, but his first love always was general relativity. His principle theoretical contributions focused on non-black hole solutions to Einstein's equations and on the inextendability of the "Schwarzschild" solution. Dr. Abrams is survived by his wife and two children.

Dr. Abrams encountered great resistance to publication of his work on General Relativity. Nonetheless he continued with his work and managed to publish several important papers despite the obstacles placed in his way by the mainstream authorities.

I extend my thanks to Diana Abrams for providing me with information about her late husband.

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