A Theory of Gravity Like Electrodynamics

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This study looks at the field of inhomogeneities of time coordinates. Equations of motion, expressed through the field tensor, show that particles move along time lines because of rotation of the space itself. Maxwell-like equations of the field display its sources, which are derived from gravitation, rotations, and inhomogeneity of the space. The energy-momentum tensor of the field sets up an inhomogeneous viscous media, which is in the state of an ultrarelativistic gas. Waves of the field are transverse, and the wave pressure is derived from mainly sub-atomic processes — excitation/relaxation of atoms produces the positive/negative wave pressures, which leads to a test of the whole theory.

Contents

1. Inhomogeneity of observable time. Defining the field . . 15
2. The field tensor. Its observable components: gravitational inertial force and the space rotation tensor 17
3. Equations of free motion. Putting the acting force into a form like Lorentz's force 18
4. The field equations like electrodynamics 19
5. Waves of the field 20
6. Energy-momentum tensor of the field 21
7. Physical properties of the field 22
8. Action of the field without sources 23
9. Plane waves of the field under gravitation is neglected. The wave pressure 24
10. Physical conditions in atoms 26
11. Conclusions 28

1 Inhomogeneity of observable time. Defining the field

The meaning of Einstein's General Theory of Relativity consists of his idea that all properties of the world are derived from the geometrical structure of space-time, from the world-geometry, in other words. This is a way to geometrize physics. The introduction of his artificial postulates became only of historical concern subsequent to his setting up of the meaning of the theory — all the postulates are naturally contained in the geometry of a four-dimensional pseudo-Riemannian space with the sign-alternating signature $(-+++)$ or $(+---)$ he assigned to the basic space-time of the theory.

Verification of the theory by experiments has shown that the four-dimensional pseudo-Riemannian space satisfies our observable world in most cases. In general we can say that all that everything we can obtain theoretically in this space geometry must have a physical interpretation.

Here we take a pseudo-Riemannian space with the signature $(+---)$, where time is real and spatial coordinates are imaginary, because the observable projection of a four-dimensional impulse on the spatial section of any given observer is positive in this case. We also assign to space-time Greek indices, while spatial indices are Latin*.

As it is well-known [1], $dS = m_0 c ds$ is an elementary action to displace a free mass-bearing particle of rest-mass m_0 through a four-dimensional interval of length ds. What happens to matter during this action? To answer this question let us substitute the square of the interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ into the action. As a result we see that

$$dS = m_0 c ds = m_0 c \sqrt{g_{\alpha\beta}\, dx^\alpha dx^\beta}\,, \tag{1}$$

so the particle moves in space-time along geodesic lines (free motion), because the field carries the fundamental metric tensor $g_{\alpha\beta}$. At the same time Einstein's equations link the metric tensor $g_{\alpha\beta}$ to the energy-momentum tensor of matter through the four-dimensional curvature of space-time. This implies that the gravitational field is linked to the field of the space-time metric in the frames of the General Theory of Relativity. For this reason one regularly concludes that the action (1) displacing free mass-bearing particles is produced by the gravitational field.

Let us find which field will manifest by the action (1) as a source of free motion, if the space-time interval ds therein is written with quantities which would be observable by a real observer located in the four-dimensional pseudo-Riemannian space.

A formal basis here is the mathematical apparatus of physically observable quantities (the theory of chronometric invariants), developed by Zelmanov in the 1940's [2, 3]. Its essence is that if an observer accompanies his reference body,

*Alternatively, Landau and Lifshitz in their *The Classical Theory of Fields* [1] use the space signature $(-+++)$, which gives an advantage in certain cases. They also use other notations for tensor indices: in their book space-time indices are Latin, while spatial indices are Greek.

his observable quantities are projections of four-dimensional quantities on his time line and the spatial section — *chronometrically invariant quantities*, made by projecting operators $b^\alpha = \frac{dx^\alpha}{ds}$ and $h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta$ which fully define his real reference space (here b^α is his velocity with respect to his real references). Thus, the chr.inv.-projections of a world-vector Q^α are $b_\alpha Q^\alpha = \frac{Q_0}{\sqrt{g_{00}}}$ and $h^i_\alpha Q^\alpha = Q^i$, while chr.inv.-projections of a world-tensor of the 2nd rank $Q^{\alpha\beta}$ are $b^\alpha b^\beta Q_{\alpha\beta} = \frac{Q_{00}}{g_{00}}$, $h^{i\alpha} b^\beta Q_{\alpha\beta} = \frac{Q^i_0}{\sqrt{g_{00}}}$, $h^i_\alpha h^k_\beta Q^{\alpha\beta} = Q^{ik}$. Physically observable properties of the space are derived from the fact that chr.inv.-differential operators $\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{{}^*\partial}{\partial t}$ are non-commutative $\frac{{}^*\partial^2}{\partial x^i \partial t} - \frac{{}^*\partial^2}{\partial t\, \partial x^i} = \frac{1}{c^2} F_i \frac{{}^*\partial}{\partial t}$ and $\frac{{}^*\partial^2}{\partial x^i \partial x^k} - \frac{{}^*\partial^2}{\partial x^k \partial x^i} = \frac{2}{c^2} A_{ik} \frac{{}^*\partial}{\partial t}$, and also from the fact that the chr.inv.-metric tensor h_{ik} may not be stationary. The observable characteristics are the chr.inv.-vector of gravitational inertial force F_i, the chr.inv.-tensor of angular velocities of the space rotation A_{ik}, and the chr.inv.-tensor of rates of the space deformations D_{ik}, namely

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \quad \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2} \tag{2}$$

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right), \tag{3}$$

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}, \quad D^{ik} = -\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}, \quad D^k_k = \frac{{}^*\partial \ln\sqrt{h}}{\partial t}, \tag{4}$$

where w is gravitational potential, $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation, $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$ is the chr.inv.-metric tensor, and also $h = \det\|h_{ik}\|$, $h g_{00} = -g$, $g = \det\|g_{\alpha\beta}\|$. Observable inhomogeneity of the space is set up by the chr.inv.-Christoffel symbols $\Delta^i_{jk} = h^{im}\Delta_{jk,m}$, which are built just like Christoffel's usual symbols $\Gamma^\alpha_{\mu\nu} = g^{\alpha\sigma}\Gamma_{\mu\nu,\sigma}$ using h_{ik} instead of $g_{\alpha\beta}$.

A four-dimensional generalization of the main chr.inv.-quantities F_i, A_{ik}, and D_{ik} (by Zelmanov, the 1960's [4]) is: $F_\alpha = -2c^2 b^\beta a_{\beta\alpha}$, $A_{\alpha\beta} = c h^\mu_\alpha h^\nu_\beta a_{\mu\nu}$, $D_{\alpha\beta} = c h^\mu_\alpha h^\nu_\beta d_{\mu\nu}$, where $a_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha b_\beta - \nabla_\beta b_\alpha\right)$, $d_{\alpha\beta} = \frac{1}{2}\left(\nabla_\alpha b_\beta + \nabla_\beta b_\alpha\right)$.

In this way, for any equations obtained using general covariant methods, we can calculate their physically observable projections on the time line and the spatial section of any particular reference body and formulate the projections in terms of their real physically observable properties, from which we obtain equations containing only quantities measurable in practice.

Expressing $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ through the observable time interval

$$d\tau = \frac{1}{c}\, b_\alpha dx^\alpha = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i \tag{5}$$

and also the observable spatial interval $d\sigma^2 = h_{\alpha\beta}\, dx^\alpha dx^\beta = h_{ik}\, dx^i dx^k$ (note, $b^i = 0$ for an observer who accompanies his reference body), we come to the formula

$$ds^2 = c^2 d\tau^2 - d\sigma^2. \tag{6}$$

Using this formula, we can write down the action (1) to displace a free mass-bearing particle in the form

$$dS = m_0 c \sqrt{b_\alpha b_\beta\, dx^\alpha dx^\beta - h_{\alpha\beta}\, dx^\alpha dx^\beta}\,. \tag{7}$$

If the particle is at rest with respect to the observer's reference body, then its observable displacement along his spatial section is $dx^i = 0$, so its observable chr.inv.-velocity vector equals zero; $\mathrm{v}^i = \frac{dx^i}{d\tau} = 0$. Such a particle moves only along time lines. In this case, in the accompanying reference frame, we have $h_{\alpha\beta}\, dx^\alpha dx^\beta = h_{ik}\, dx^i dx^k = 0$ hence the action is

$$dS = m_0 c\, b_\alpha dx^\alpha, \tag{8}$$

so the mass-bearing particle moves freely along time lines because it is carried solely by the vector field b^α.

What is the physical meaning of this field? The vector b^α is the operator of projection on time lines (non-uniform, in general case) of a real observer, who accompanies his reference body. This implies that the vector field b^α defines the geometrical structure of the real space-time along time lines. Projecting an interval of four-dimensional coordinates dx^α onto the time line of a real observer in his accompanying reference frame, we obtain the interval of real physical time $d\tau = \frac{1}{c} b_\alpha dx^\alpha = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2} v_i dx^i$ he observes. For his measurements in the same spatial point, in other words, along the same time line, $d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt$. This formula and the previous one lead us to the conclusion that the components of the observer's vector b^α define a "density" of physically observable time in his accompanying reference frame. As it is easy to see, the observable time density depends on the gravitational potential and, in the general case, on the rotation of the space. Hence, the vector field b^α in the accompanying reference frame is the field of inhomogeneity of observable time references. For this reason we will call it the *field of density of observable time*.

In the same way, a field of the tensor $h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta$ projecting four-dimensional quantities on the observer's spatial section is the *field of density of the spatial section*.

From the geometric viewpoint, we can illustrate the conclusions in this way. The vector field b^α and the tensor field $h_{\alpha\beta}$ of the accompanying reference frame of an observer, located in a four-dimensional pseudo-Riemannian space, "split" the space into time lines and a spatial section, properties of which (such as inhomogeneity, anisotropy, curvature, etc.) depend on the physical properties of the observer's reference body. Owing to this "splitting" process, the field of the fundamental metric tensor $g_{\alpha\beta}$, containing the geometrical structure of this space, "splits" as well (7). Its "transverse component" is the time density field, a four-dimensional potential of which is the monad vector b^α. The "longitudinal component" of this splitting is the field of density of the spatial section.

2 The field tensor. Its observable components: gravitational inertial force and the space rotation tensor

Chr.inv.-projections of the four-dimensional vector potential b^α of a time density field are, respectively

$$\varphi = \frac{b_0}{\sqrt{g_{00}}} = 1\,, \qquad q^i = b^i = 0\,. \tag{9}$$

Emulating the way that Maxwell's electromagnetic field tensor is introduced, we introduce the *tensor of a time density field* as the rotor of its four-dimensional vector potential

$$F_{\alpha\beta} = \nabla_\alpha\, b_\beta - \nabla_\beta\, b_\alpha = \frac{\partial b_\beta}{\partial x^\alpha} - \frac{\partial b_\alpha}{\partial x^\beta}\,. \tag{10}$$

Taking into account that $F_{00} = F^{00} = 0$, as for any antisymmetric tensor of the 2nd rank, after some algebra we obtain the other components of the field tensor $F_{\alpha\beta}$

$$F_{0i} = \frac{1}{c^2}\sqrt{g_{00}}\,F_i\,, \qquad F_{ik} = \frac{1}{c}\left(\frac{\partial v_i}{\partial x^k} - \frac{\partial v_k}{\partial x^i}\right), \tag{11}$$

$$F_{0\cdot}^{\cdot 0} = -\frac{1}{c^3}\,v_k F^k, \qquad F_{0\cdot}^{\cdot i} = -\frac{1}{c^2}\sqrt{g_{00}}\,F^i, \tag{12}$$

$$F_{k\cdot}^{\cdot 0} = -\frac{1}{\sqrt{g_{00}}}\left(\frac{1}{c^2}\,F_k + \frac{2}{c^2}\,v^m A_{mk} - \frac{1}{c^4}\,v_k v_m F^m\right), \tag{13}$$

$$F_{k\cdot}^{\cdot i} = \frac{1}{c^3}\,v_k F^i + \frac{2}{c}\,A_{k\cdot}^{\cdot i}\,, \qquad F^{ik} = -\frac{2}{c}\,A^{ik}, \tag{14}$$

$$F^{0k} = -\frac{1}{\sqrt{g_{00}}}\left(\frac{1}{c^2}\,F^k + \frac{2}{c^2}\,v_m A^{mk}\right). \tag{15}$$

We denote chr.inv.-projections of the field tensor just like the chr.inv.-projections of the Maxwell tensor [5], to display their physical sense. We will refer to the time projection

$$E^i = \frac{F_{0\cdot}^{\cdot i}}{\sqrt{g_{00}}} = -\frac{1}{c^2}\,F^i, \qquad E_i = h_{ik}E^k = -\frac{1}{c^2}\,F_i \tag{16}$$

of the field tensor $F_{\alpha\beta}$ as "electric". The spatial projection

$$H^{ik} = F^{ik} = -\frac{2}{c}\,A^{ik}, \quad H_{ik} = h_{im}h_{kn}F^{mn} = -\frac{2}{c}\,A_{ik} \tag{17}$$

of the field tensor will be referred to as "magnetic". So, we arrive at physical definitions of the components:

> The "electric" observable component of a time density field manifests as the gravitational inertial force F_i. The "magnetic" observable component of a time density field manifests as the angular velocity A_{ik} of the space rotation.

In accordance with the above, two particular cases of time density fields are possible. These are:

1. If a time density field has $H_{ik} = 0$ and $E^i \neq 0$, then the field is strictly of the "electric" kind. This particular case corresponds to a holonomic (non-rotating) space filled with gravitational force fields;
2. A time density field is of the "magnetic" kind, if therein $E^i = 0$ and $H_{ik} \neq 0$. This is a non-holonomic space, where fields of gravitational inertial forces are homogeneous or absent. This case is possible also if, according to the chr.inv.-definition of the force

   $$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \quad \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,, \tag{18}$$

   where the first term — a force of gravity would be reduced by the second term — is a centrifugal force of inertia.

In addition to the field tensor $F_{\alpha\beta}$, we introduce the field pseudotensor $F^{*\alpha\beta}$ dual and in the usual way [1]

$$F^{*\alpha\beta} = \frac{1}{2}\,E^{\alpha\beta\mu\nu}F_{\mu\nu}\,, \quad F_{*\alpha\beta} = \frac{1}{2}\,E_{\alpha\beta\mu\nu}F^{\mu\nu}, \tag{19}$$

where the four-dimensional completely antisymmetric discriminant tensors $E^{\alpha\beta\mu\nu} = \frac{e^{\alpha\beta\mu\nu}}{\sqrt{-g}}$ and $E_{\alpha\beta\mu\nu} = e_{\alpha\beta\mu\nu}\sqrt{-g}$, transforming regular tensors into pseudotensors in inhomogeneous anisotropic pseudo-Riemannian spaces, are not physically observable quantities. The completely antisymmetric unit tensor $e^{\alpha\beta\mu\nu}$, being defined in a Galilean reference frame in Minkowski space [1], does not have this quality either. Therefore we employ Zelmanov's chr.inv.-discriminant tensors $\varepsilon^{\alpha\beta\gamma} = b_\sigma E^{\sigma\alpha\beta\gamma}$ and $\varepsilon_{\alpha\beta\gamma} = b^\sigma E_{\sigma\alpha\beta\gamma}$ [2], which in the accompanying reference frame are

$$\varepsilon^{ikm} = \frac{e^{ikm}}{\sqrt{h}}\,, \qquad \varepsilon_{ikm} = e_{ikm}\sqrt{h}\,. \tag{20}$$

Using components of the field tensor $F_{\alpha\beta}$, we obtain chr.inv.-projections of the field pseudotensor, which are

$$H^{*i} = \frac{F_{0\cdot}^{*\cdot i}}{\sqrt{g_{00}}} = -\frac{1}{c}\,\varepsilon^{ikm}A_{km} = -\frac{2}{c}\,\Omega^{*i}, \tag{21}$$

$$E^{*ik} = F^{*ik} = \frac{1}{c^2}\,\varepsilon^{ikm}F_m\,, \tag{22}$$

where $\Omega^{*i} = \frac{1}{2}\,\varepsilon^{ikm}A_{km}$ is the chr.inv.-pseudovector of angular velocities of the space rotation. Their relations to the field tensor chr.inv.-projections express themselves just like any chr.inv.-pseudotensors [5, 6], by the formulae

$$H^{*i} = \frac{1}{2}\,\varepsilon^{imn}H_{mn}\,, \qquad H_{*i} = \frac{1}{2}\,\varepsilon_{imn}H^{mn}, \tag{23}$$

$$\varepsilon^{ipq}H_{*i} = \frac{1}{2}\,\varepsilon^{ipq}\varepsilon_{imn}H^{mn} = H^{pq}, \tag{24}$$

$$\varepsilon_{ikp}H^{*p} = E_{ik}\,, \qquad E^{*ik} = -\varepsilon^{ikm}E_m\,, \tag{25}$$

where $\varepsilon^{ipq}\varepsilon_{imn} = \delta^p_m\delta^q_n - \delta^q_m\delta^p_n$, see [1, 5, 6] for details.

We introduce the invariants $J_1 = F_{\alpha\beta}F^{\alpha\beta}$ and $J_2 = F_{\alpha\beta}F^{*\alpha\beta}$ for a time density field. Their formulae are

$$J_1 = F_{\alpha\beta}F^{\alpha\beta} = \frac{4}{c^2}A_{ik}A^{ik} - \frac{2}{c^4}F_iF^i, \qquad (26)$$

$$J_2 = F_{\alpha\beta}F^{*\alpha\beta} = -\frac{8}{c^3}F_i\Omega^{*i}, \qquad (27)$$

so the time density field can be *spatially isotropic* (one of the invariants becomes zero) under the conditions:

- the invariant $A_{ik}A^{ik}$ of the space rotation field and the invariant F_iF^i of the gravitational inertial force field are proportional one to another $A_{ik}A^{ik} = \frac{1}{2c^2}F_iF^i$;
- $F_i\Omega^{*i} = 0$, so the acting gravitational inertial force F_i is orthogonal to the space rotation pseudovector Ω^{*i};
- both of the conditions are realized together.

3 Equations of free motion. Putting the acting force into a form like Lorentz's force

Time lines are geodesics by definition. In accordance with the *least action principle*, an action replacing a particle along a geodesic line is minimum. Actually, the least action principle implies that geodesic lines are also lines of the least action. This is the physical viewpoint.

We are going to consider first a free mass-bearing particle, which is at rest with respect to an observer and his reference body. Such a particle moves only along a time line, so it moves solely because of the action of the inhomogeneity of time coordinates along the time line — a time density field.

The action that a time density field expends in displacing a free mass-bearing particle of rest-mass m_0 at dx^α has the value $dS = m_0c\,b_\alpha dx^\alpha$ (8). Because of the least action, variation of the action integral along geodesic lines equals zero

$$\delta\int_a^b dS = 0\,, \qquad (28)$$

which, after substituting $dS = m_0cb_\alpha dx^\alpha$, becomes $\delta\int_a^b dS = m_0c\,\delta\int_a^b b_\alpha dx^\alpha = m_0c\int_a^b \delta b_\alpha dx^\alpha + m_0c\int_a^b b_\alpha d\delta x^\alpha$ where $\int_a^b b_\alpha d\delta x^\alpha = b_\alpha\delta x^\alpha\big|_a^b - \int_a^b db_\alpha\delta x^\alpha = -\int_a^b db_\alpha\delta x^\alpha$. Because $\delta b_\alpha = \frac{\partial b_\alpha}{\partial x^\beta}\delta x^\beta$ and $db_\alpha = \frac{\partial b_\alpha}{\partial x^\beta}dx^\beta$,

$$\delta\int_a^b dS = m_0c\,\delta\int_a^b\left(\frac{\partial b_\beta}{\partial x^\alpha} - \frac{\partial b_\alpha}{\partial x^\beta}\right)dx^\beta\delta x^\alpha. \qquad (29)$$

This variation is zero, so along time lines we have

$$m_0c\left(\frac{\partial b_\beta}{\partial x^\alpha} - \frac{\partial b_\alpha}{\partial x^\beta}\right)dx^\beta = 0\,. \qquad (30)$$

This condition, being divided by the interval ds, gives general covariant equations of motion of the particle

$$m_0c\,F_{\alpha\beta}U^\beta = 0\,, \qquad (31)$$

wherein $F_{\alpha\beta}$ is the time density field tensor and U^β is the particle's four-dimensional velocity*.

Taking chr.inv.-projections of (31) multiplied by c^2, we obtain chr.inv.-equations of motion of the particle

$$m_0c^3\frac{F_{0\sigma}U^\sigma}{\sqrt{g_{00}}} = 0\,, \qquad m_0c^2F^{i}_{\cdot\sigma}U^\sigma = 0\,, \qquad (32)$$

where the scalar equation gives the work to displace the particle, and the vector equations its observable acceleration.

It is interesting to note that the left side of the equations, which is the acting force, both in the general covariant form and its chr.inv.-projections we have obtained, has the same form as Lorentz's force, which displaces charged particles in electromagnetic fields [5]. From the mathematical viewpoint this fact implies that the time density field acts on mass-bearing particles as the electromagnetic field moves electric charge.

Taking $ds^2 = c^2d\tau^2 - d\sigma^2 = c^2d\tau^2\left(1 - \frac{\mathrm{v}^2}{c^2}\right)$, that is formula (4) into account, we obtain

$$U^\alpha = \frac{dx^\alpha}{ds} = \frac{1}{c\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\frac{dx^\alpha}{d\tau}\,, \qquad (33)$$

$$U^0 = \frac{\frac{1}{c^2}v_k\mathrm{v}^k + 1}{\sqrt{g_{00}}\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\,, \qquad U^i = \frac{1}{c\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\mathrm{v}^i. \qquad (34)$$

Using the components obtained for the field tensor $F_{\alpha\beta}$ (11–15) and taking into account that the observable velocity of the particle we are considering is $\mathrm{v}^i = 0$, we transform the chr.inv.-equations of motion (32) into the final form. The scalar equation becomes zero, while the vector equations become $m_0F^i = 0$ or, substituting $E^i = -\frac{1}{c^2}F^i$ (16),

$$m_0c^2E^i = 0\,, \qquad (35)$$

leading us to the following conclusions:

1. The "electric" and the "magnetic" components of a time density field do not produce work to displace a free mass-bearing particle along time lines. Such a particle falls freely along its own time line under the time density field;
2. In this case $E^i = 0$, so the particle falls freely along its own time line, being carryied solely by the "magnetic" component $H_{ik} = -\frac{2}{c}A_{ik} \neq 0$ of the time density field;
3. Inhomogeneity of the spatial section (the chr.inv.-Christoffel symbols Δ^i_{jk}) or its deformations (the chr.inv.-deformation rate tensor D_{ik}) do not have an effect on free motion along time lines.

*Do not confound this vector $U^\alpha = \frac{dx^\alpha}{ds}$ with the vector $b^\alpha = \frac{dx^\alpha}{ds}$: they are built on different dx^α. The vector b^α contains displacement of the observer with respect to his reference body, while the vector U^α contains displacement of the particle.

In other words, the "magnetic" component $H_{ik} = -\frac{2}{c}A_{ik}$ of a time density field "screws" particles into time lines (a very rough analogy). There are no other sources which could cause particles to move along time lines, because observable particles with the whole spatial section move from past into future, hence $H_{ik} \neq 0$ everywhere in our real world. So, our real space is strictly non-holonomic, $A_{ik} \neq 0$.

This purely mathematical result brings us to the very important conclusion that under any conditions a real space is non-holonomic at the "start", that is, a "primordial non-orthogonality" of the real spatial section to time lines. Conditions such as three-dimensional rotations of the reference body, are only additions, intensifying or reducing this start-rotation of the space, depending on their relative directions*.

We are now going to consider the second case of free motion — the general case, where a free mass-bearing particle moves freely not only along time lines, but also along the spatial section with respect to the observer and his reference body. Chr.inv.-equations of motion in this general case had been deduced by Zelmanov [2]. They have the form

$$\begin{gathered} \frac{dE}{d\tau} - mF_i \mathrm{v}^i + mD_{ik}\mathrm{v}^i\mathrm{v}^k = 0, \qquad E = mc^2 \\ \frac{d(m\mathrm{v}^i)}{d\tau} - mF^i + 2m\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k + m\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k = 0. \end{gathered} \tag{36}$$

Let us express the equations through the "electric" and the "magnetic" observable components of the acting field of time density. Substituting $E^i = -\frac{1}{c^2}F^i$ and $H_{ik} = -\frac{2}{c}A_{ik}$ into the Zelmanov equations (36), we obtain

$$\begin{gathered} \frac{dE}{d\tau} + mc^2 E_i \mathrm{v}^i + mD_{ik}\mathrm{v}^i\mathrm{v}^k = 0, \\ \frac{d(m\mathrm{v}^i)}{d\tau} + mc^2\left(E^i + \frac{1}{c}H^{ik}\mathrm{v}_k\right) + \\ + 2mD^i_k\mathrm{v}^k + m\Delta^i_{nk}\mathrm{v}^n\mathrm{v}^k = 0. \end{gathered} \tag{37}$$

From this we see that a free mass-bearing particle moves freely along the spatial section because of the factors:

1. The particle is carried with a time density field by its "electric" $E^i \neq 0$ and "magnetic" $H_{ik} \neq 0$ components;
2. The particle is also moved by forces which manifest as an effect of inhomogeneity Δ^i_{nk} and deformations D_{ik} of the spatial section. As we can see from the scalar equation, the field of the space inhomogeneities does not produce any work to displace free mass-bearing particles, only the space deformation field produces the work.

In particular, a mass-bearing particle can be moved freely along the spatial section, solely because of the field of time density. As it easy to see from equations (37), this is possible under the following conditions

$$D_{ik}\mathrm{v}^i\mathrm{v}^k = 0\,, \qquad D^i_k = -\frac{1}{2}\,\Delta^i_{nk}\mathrm{v}^n, \tag{38}$$

so it is possible in the following particular cases:

- if the spatial section has no deformations, $D_{ik} = 0$;
- if, besides the absence of the deformations ($D_{ik} = 0$), the spatial section is homogeneous, $\Delta^i_{nk} = 0$.†

The scalar equations of motion (37) also show that, under the particular conditions (38), the energy dE to displace the particle at dx^i equals the work

$$dE = -mc^2 E_i dx^i \tag{39}$$

the "electric" field component E_i expends for this displacement. The vector equations of motion in this particular case show that the "electric" and the "magnetic" components of the acting field of time density accelerate the particle just like external forces‡

$$\frac{dp^i}{d\tau} = -mc^2\left(E^i + \frac{1}{c}H^{ik}\mathrm{v}_k\right). \tag{40}$$

Looking at the right sides of equations (39, 40), we see that they have a form identical to the right sides of the chr.inv.-equations of motion of a charged particle in the electromagnetic field [5]. This implies also that the field of time density acts on mass-bearing particles as an electromagnetic field moves electric charge.

4 The field equations like electrodynamics

As is well-known, the theory of the electromagnetic field, in a pseudo-Riemannian space, characterizes the field by a system of equations known also as the *field equations*:

- Lorentz's condition stipulates that the four-dimensional vector potential A^α of the field remains unchanged just like any four-dimensional vector in a four-dimensional pseudo-Riemannian space

$$\nabla_\sigma A^\sigma = 0\,; \tag{41}$$

- the charge conservation law (the continuity equation) shows that the field-inducing charge cannot be destroyed, but merely re-distributed in the space

$$\nabla_\sigma j^\sigma = 0\,, \tag{42}$$

*A similar conclusion had also been given by the astronomer Kozyrev [7], from his studies of the interior of stars. In particular, besides the "start" self-rotation of the space, he had come to the conclusion that additional rotations will produce an inhomogeneity of observable time around rotating bulky bodies like stars or planets. The consequences should be more pronounced in the interaction of the components of bulky double stars [8]. He was the first to used the term "time density field". It is interesting that his arguments, derived from a purely phenomenological analysis of astronomical observations, did not link to Riemannian geometry and the mathematical apparatus of the General Theory of Relativity.

†However the first condition $D_{ik} = 0$ would be sufficient.

‡Here the chr.inv.-vector $p^i = m\mathrm{v}^i$ is the particle's observable impulse.

where j^α is the four-dimensional current vector; its observable projections are the chr.inv.-charge density scalar $\rho = \frac{1}{c\sqrt{g_{00}}}\,j_0$ and the chr.inv.-current density vector j^i, which are sources inducing the field;

- Maxwell's equations show properties of the field, expressed by components of the field tensor $F_{\alpha\beta}$ and its dual pseudotensor $F^{*\alpha\beta}$. The first group of the Maxwell equations contains the field sources ρ and j^i, the second group does not contain the sources

$$\nabla_\sigma F^{\alpha\sigma} = \frac{4\pi}{c}\,j^\alpha\,, \qquad \nabla_\sigma F^{*\alpha\sigma} = 0\,. \tag{43}$$

We can put all the equations into chr.inv.-form, employing Zelmanov's formula [2] for the divergence of a vector Q^α, where he expressed the divergence through chr.inv.-projections $\varphi = \frac{Q_0}{\sqrt{g_{00}}}$ and $q^i = Q^i$ of this vector

$$\nabla_\sigma Q^\sigma = \frac{1}{c}\left(\frac{^*\partial\varphi}{\partial t} + \varphi D\right) + {^*\nabla_i}\, q^i - \frac{1}{c^2}F_i q^i\,, \tag{44}$$

where we use his notation for *chr.inv.-divergence*

$${^*\nabla_i}\, q^i = \frac{^*\partial q^i}{\partial x^i} + q^i\,\frac{^*\partial \ln\sqrt{h}}{\partial x^i} = \frac{^*\partial q^i}{\partial x^i} + q^i \Delta^j_{ji}\,. \tag{45}$$

In particular, the chr.inv.-Maxwell equations, which are chr.inv.-projections of the Maxwell general covariant equations (43), had first been obtained for an arbitrary field potential by del Prado and Pavlov [9], Zelmanov's students, at Zelmanov's request. The equations are

$$\left.\begin{aligned} &{^*\nabla_i}\, E^i - \frac{1}{c}\,H^{ik}A_{ik} = 4\pi\rho \\ &{^*\nabla_k}\, H^{ik} - \frac{1}{c^2}F_k H^{ik} - \frac{1}{c}\left(\frac{^*\partial E^i}{\partial t} + E^i D\right) = \frac{4\pi}{c}\,j^i \end{aligned}\right\}\ \text{I}, \tag{46}$$

$$\left.\begin{aligned} &{^*\nabla_i}\, H^{*i} - \frac{1}{c}\,E^{*ik}A_{ik} = 0 \\ &{^*\nabla_k}\, E^{*ik} - \frac{1}{c^2}F_k E^{*ik} - \frac{1}{c}\left(\frac{^*\partial H^{*i}}{\partial t} + H^{*i}D\right) = 0 \end{aligned}\right\}\ \text{II}, \tag{47}$$

From the mathematical viewpoint, equations of the field are a system of 10 equations in 10 unknowns (the Lorentz condition, the charge conservation law, and two groups of the Maxwell equations), which define the given vector field A^α and its inducing sources in a pseudo-Riemannian space. Actually, equations like these should exist for any four-dimensional vector field, a time density field included. The only difference should be that the equations should be changed according to a formula for the specific vector potential.

We are going to deduce such equations for the field b^α we are considering — *equations of a time density field*.

Because $\varphi = 1$ and $q^i = 0$ are chr.inv.-projections of the potential b^α of a time density field, the *Lorentz condition* $\nabla_\sigma b^\sigma = 0$ for a time density field b^α becomes the equality

$$D = 0\,, \tag{48}$$

where $D = h^{ik}D_{ik}$, being the spur of the deformation rate tensor, is the rate of expansion of an elementary volume. Actually, the obtained Lorentz condition (48) implies that the value of an elementary volume filled with a time density field remains unchanged under its deformations.

We now collect chr.inv.-projections of the tensor of a time density field $F_{\alpha\beta}$ and of the field pseudotensor $F^{*\alpha\beta}$ together: $E_i = -\frac{1}{c^2}\,F_i$, $H^{ik} = -\frac{2}{c}\,A^{ik}$, $H^{*i} = -\frac{2}{c}\,\Omega^{*i}$, $E^{*ik} = \frac{1}{c^2}\,\varepsilon^{ikm}F_m$. We also take Zelmanov's identities for the chr.inv.-discriminant tensors [2] into account

$$\frac{^*\partial\varepsilon_{imn}}{\partial t} = \varepsilon_{imn}\,D\,, \qquad \frac{^*\partial\varepsilon^{imn}}{\partial t} = -\varepsilon^{imn}D\,, \tag{49}$$

$${^*\nabla_k}\;\varepsilon_{imn} = 0\,, \qquad {^*\nabla_k}\;\varepsilon^{imn} = 0\,. \tag{50}$$

Substituting the chr.inv.-projections into (46, 47) along with the obtained Lorentz condition $D = 0$ (48), we arrive at *Maxwell-like chr.inv.-equations* for a time density field

$$\left.\begin{aligned} &\frac{1}{c^2}\,{^*\nabla_i}\, F^i - \frac{2}{c^2}A_{ik}A^{ik} = -4\pi\rho \\ &\frac{2}{c}\,{^*\nabla_k}\, A^{ik} - \frac{2}{c^3}F_k A^{ik} - \frac{1}{c^3}\frac{^*\partial F^i}{\partial t} = -\frac{4\pi}{c}\,j^i \end{aligned}\right\}\ \text{I}, \tag{51}$$

$$\left.\begin{aligned} &{^*\nabla_i}\,\Omega^{*i} + \frac{1}{c^2}\,F_i\,\Omega^{*i} = 0 \\ &{^*\nabla_k}\left(\varepsilon^{ikm}F_m\right) - \frac{1}{c^2}\,\varepsilon^{ikm}F_kF_m + 2\,\frac{^*\partial\Omega^{*i}}{\partial t} = 0 \end{aligned}\right\}\ \text{II}, \tag{52}$$

so that the field-inducing sources ρ and j^i are

$$\rho = -\frac{1}{4\pi c^2}\left({^*\nabla_i}\, F^i - 2A_{ik}A^{ik}\right), \tag{53}$$

$$j^i = -\frac{1}{2\pi}\,{^*\nabla_k}\, A^{ik} - \frac{1}{2\pi c^2}F_k A^{ik} - \frac{1}{4\pi c^2}\frac{^*\partial F^i}{\partial t}\,. \tag{54}$$

The "*charge*" *conservation law* $\nabla_\sigma j^\sigma = 0$ (the continuity equation), after substituting chr.inv.-projections $\varphi = c\rho$ and $q^i = j^i$ of the "current" vector j^α, takes the chr.inv.-form

$$\begin{aligned} &\frac{1}{c^2}\frac{^*\partial}{\partial t}\left(A_{ik}A^{ik}\right) + \frac{1}{c^2}F_i\frac{^*\partial A^{ik}}{\partial x^k} - \frac{^*\partial^2 A^{ik}}{\partial x^i\partial x^k} + \\ &\quad + \left(\frac{1}{c^2}F_i\Delta^j_{jk} + \frac{^*\partial\Delta^j_{jk}}{\partial x^i} + \Delta^j_{ji}\Delta^l_{lk}\right)A^{ik} - \\ &\quad - \frac{1}{2c^2}\,F^i\frac{^*\partial\Delta^j_{ji}}{\partial t} - \frac{1}{c^4}F_iF_kA^{ik} = 0\,, \end{aligned} \tag{55}$$

The Lorentz condition (48), the Maxwell-like equations (51, 52), and the continuity equation (55) we have obtained are *chr.inv.-equations of a time density field*.

5 Waves of the field

Let us turn now to d'Alembert's equations. We are going to obtain the equations for a time density field.

d'Alembert's operator $\Box = g^{\alpha\beta}\,\nabla_\alpha\nabla_\beta$, being applied to a field, may or may not be zero. The second case is known as the d'Alembert equations with field-inducing sources, while the first case is known as the d'Alembert equations without sources. If the field has no sources, then the field is free. This is a wave. So, the d'Alembert equations without sources are equations of propagation of waves of the field.

From this reason, the d'Alembert equations for the vector potential b^α of a time density field without the sources

$$\Box b^\alpha = 0 \tag{56}$$

are the equations of propagation of waves of the time density field. Chr.inv.-projections of the equations are

$$b_\sigma \Box b^\sigma = 0\,, \qquad h^i_\sigma \Box b^\sigma = 0\,. \tag{57}$$

We substitute chr.inv.-projections $\varphi = 1$ and $q^i = 0$ of the field potential b^α into this. Then, taking into account that the Lorentz condition for the field b^α is $D = 0$ (48), after some algebra we obtain the *chr.inv.-d'Alembert equations* for the time density field without sources

$$\begin{gathered} \frac{1}{c^2}\,F_i F^i - D_{ik}D^{ik} = 0\,, \\ \frac{1}{c^2}\frac{{}^*\partial F^i}{\partial t} + h^{km}\left\{\frac{{}^*\partial D^i_m}{\partial x^k} + \frac{{}^*\partial A^{\cdot i}_{m\cdot}}{\partial x^k} + \right. \\ \left. + \,\Delta^i_{kn}\left(D^n_m - A^{\cdot n}_{m\cdot}\right) - \Delta^n_{km}\left(D^i_n - A^{\cdot i}_{n\cdot}\right)\right\} = 0\,. \end{gathered} \tag{58}$$

Unfortunately, a term like $\frac{1}{a^2}\frac{\partial^2 q^i}{\partial t^2}$ containing the linear speed a of the waves is not present, because of $q^i = 0$. For this reason we have no possibility of saying anything about the speed of waves traveling in time density fields. At the same time the obtained equations (58) display numerous specific peculiarities of a space filled with the waves:

1. The rate of deformations of a surface element in waves of a time density field is powered by the value of the acting gravitational inertial force F_i. If $F_i = 0$, the observable spatial metric h_{ik} is stationary;
2. If a space, filled with waves of a time density field, is homogeneous $\Delta^i_{kn} = 0$ and also the acting force field is stationary $F_i = \text{const}$, the spatial structure of the space deformations is the same as that of the space rotation field.

6 Energy-momentum tensor of the field

Proceeding from the general covariant equations of motion along only time lines, we are going to deduce the energy-momentum tensor for time density fields. It is possible to do this in the following way.

The aforementioned equations $m_0 c\,F_{\alpha\beta}\,U^\beta = 0$ (31), being taken in contravariant (upper-index) form, are

$$m_0 c F^{\alpha\cdot}_{\cdot\sigma}\,U^\sigma = 0\,, \tag{59}$$

where U^σ is the four-dimensional velocity of the particle. The left side of the equations has the dimensions $[\text{gramme}/\text{sec}]$ as well as a four-dimensional force. Because of motion along only time lines, such particle moves solely under the action of a time density field whose tensor is $F_{\alpha\beta}$.

If this free-moving particle is not a point-mass, then it can be represented by a current j^α of the time density field. On the other hand, such currents are defined by the 1st group $\nabla_\sigma F^{\alpha\sigma} = \frac{4\pi}{c}\,j^\alpha$ of the Maxwell-like equations of the field. In this case equations of motion (59), drawing an analogy with an electromagnetic field current, take the form

$$\mu F^{\alpha\cdot}_{\cdot\sigma}\,j^\sigma = 0\,. \tag{60}$$

The numerical coefficient μ here is a new fundamental constant. This new constant having the dimension $[\text{gramme}/\text{sec}]$ gives the dimensions $[\text{gramme}/\text{cm}^2\times\text{sec}^2]$ to the left side of the equations, making the left side a current of the acting four-dimensional force (59) through $1\,\text{cm}^2$ per 1 second. The numerical value of this constant μ can be found from measurements of the wave pressure of a time density field, see formula (101) below. However it does not exclude that future studies of the problem will yield an analytic formula for μ, linking it to other fundamental constants.

Chr.inv.-projections of the equations (60)

$$\frac{\mu F_{0\sigma}\,j^\sigma}{\sqrt{g_{00}}} = 0\,, \qquad \mu F^{i\cdot}_{\cdot\sigma}\,j^\sigma = 0\,, \tag{61}$$

after substituting the $F_{\alpha\beta}$ components (11–15) take the form

$$\mu E_k\,j^k = 0\,, \qquad \mu c\left(\rho E^i - \frac{1}{c}\,H^{i\cdot}_{\cdot k}\,j^k\right) = 0\,, \tag{62}$$

where E^i is the "electric" observable component and H_{ik} is the "magnetic" observable component of the time density field. Sources ρ and j^i inducing the field are defined by the 1st group of the Maxwell-like chr.inv.-equations (51).

Actually, the term*

$$f^\alpha = \mu F^{\alpha\cdot}_{\cdot\sigma}\,j^\sigma \tag{63}$$

on the left side of the general covariant equations of motion (60) can be transformed with the 1st Maxwell-like group $\nabla_\beta F^{\sigma\beta} = \frac{4\pi}{c}\,j^\sigma$ to the form $f_\alpha = \frac{\mu c}{4\pi}\,F_{\alpha\sigma}\nabla_\beta F^{\sigma\beta}$ which is

$$f_\alpha = \frac{\mu c}{4\pi}\left[\nabla_\beta\left(F_{\alpha\sigma}F^{\sigma\beta}\right) - F^{\sigma\beta}\,\nabla_\beta F_{\alpha\sigma}\right], \tag{64}$$

where we express the second term in the form $F^{\sigma\beta}\nabla_\beta F_{\alpha\sigma} = \frac{1}{2}\,F^{\sigma\beta}\left(\nabla_\beta F_{\alpha\sigma} + \nabla_\sigma F_{\beta\alpha}\right) = -\frac{1}{2}\,F^{\sigma\beta}\left(\nabla_\beta F_{\sigma\alpha} + \nabla_\sigma F_{\alpha\beta}\right) = -\frac{1}{2}\,F^{\sigma\beta}\nabla_\sigma F_{\alpha\beta} = \frac{1}{2}\,F^{\sigma\beta}\nabla_\alpha F_{\sigma\beta}$. Using this formula, we transform the current f^α (63) to the form

$$f_\alpha = \frac{\mu c}{4\pi}\,\nabla_\beta\left(-F_{\alpha\sigma}F^{\beta\sigma} + \frac{1}{4}\,\delta^\beta_\alpha F_{pq}F^{pq}\right), \tag{65}$$

*From the physical viewpoint, this term is a current of the acting four-dimensional force, produced by the time density field.

so we write the current f^{α} in the form

$$f^{\alpha} = \nabla_{\beta} T^{\alpha\beta} \tag{66}$$

just as electrodynamics does to deduce the energy-momentum tensor $T^{\alpha\beta}$ of electromagnetic fields. In this way, we obtain the *energy-momentum tensor* of a time density field, which is

$$T^{\alpha\beta} = \frac{\mu c}{4\pi}\left(-F^{\alpha\cdot}_{\cdot\sigma}F^{\beta\sigma} + \frac{1}{4}\,g^{\alpha\beta}F_{pq}F^{pq}\right), \tag{67}$$

the form of which is the same as the energy-momentum tensor of electromagnetic fields [1, 5] to within the coefficient of its dimension. It is easy to see that the tensor is symmetric, so its spur is zero, $T^{\sigma}_{\sigma} = g_{\alpha\beta}\,T^{\alpha\beta} = 0$.

So forth we deduce the chr.inv.-projections of the energy-momentum tensor of a time density field

$$q = \frac{T_{00}}{g_{00}}\,, \qquad J^i = \frac{c\,T^i_0}{\sqrt{g_{00}}}\,, \qquad U^{ik} = c^2 T^{ik}. \tag{68}$$

After substituting the required components of the field tensor $F_{\alpha\beta}$ (11–15), we obtain

$$q = \frac{\mu}{4\pi c}\left(A_{ik}A^{ik} + \frac{1}{2c^2}F_kF^k\right), \tag{69}$$

$$J^i = -\frac{\mu}{2\pi c}F_kA^{ik}, \tag{70}$$

$$U^{ik} = -\frac{\mu c}{4\pi}\Bigg(4A^{i\cdot}_{\cdot m}A^{mk} + \frac{1}{c^2}F^iF^k + + A_{pq}A^{pq}h^{ik} - \frac{1}{2c^2}F_pF^ph^{ik}\Bigg). \tag{71}$$

In accordance with dimensions, the chr.inv.-projections have the following physical meanings:

- the time observable projection q [gramme/cm×sec²] is the energy [gm×cm²/sec²] this time density field contains in 1 cm³. Actually, the chr.inv.-scalar q is the *observable density of the field*;
- the mixed observable projection J^i [gramme/sec³] is the energy the time density field transfers through 1 cm² per second, in other words, this is the *observable density of the field momentum*;
- the spatial observable projection U^{ik} [gm×cm/sec⁴] is the tensor of the field momentum flux observable density, in other words, the *field strength tensor*.

7 Physical properties of the field

It has been proven by Zelmanov [10], that the chr.inv.-field strength tensor U^{ik}, can be written in covariant (lower index) form as follows

$$U_{ik} = p_0 h_{ik} - \alpha_{ik} = p h_{ik} - \beta_{ik}\,, \tag{72}$$

where $\alpha_{ik} = \beta_{ik} + \frac{1}{3}\,\alpha h_{ik}$ is the viscous strength tensor of the field. Zelmanov called α_{ik} the *viscosity of the 2nd kind* (here $\alpha = h^{ik}\alpha_{ik} = \alpha^n_n$ is its spur). Its anisotropic part β_{ik}, called the *viscosity of the 1st kind*, manifests as anisotropic deformations of the space. The quantity p_0 is that pressure inside the medium, which equalizes its density in the absence of viscosity, p is the true pressure of the medium*. It is easy to see that the viscous strength tensors α_{ik} and β_{ik} are chr.inv.-quantities by their definitions.

By extracting the viscous strength tensors α_{ik} and β_{ik} from the formula of the strength tensor U_{ik} of a time density field, we are going to deduce the equation of state of the field.

Transforming U^{ik} (71) into covariant form and also keeping the formula for q (69) in the mind, we write

$$U_{ik} = -qc^2h_{ik} - - \frac{\mu c}{4\pi}\left(4A_{im}A^{m\cdot}_{\cdot k} + \frac{1}{c^2}F_iF_k - \frac{1}{c^2}F_mF^mh_{ik}\right), \tag{73}$$

which, after equating to $U_{ik} = p_0 h_{ik} - \alpha_{ik}$ (72), gives the equilibrium pressure in the field

$$p_0 = -qc^2, \tag{74}$$

while the *viscous strength tensor* of the field is

$$\alpha_{ik} = \frac{\mu c}{4\pi}\left(4A_{im}A^{m\cdot}_{\cdot k} + \frac{1}{c^2}F_iF_k - \frac{1}{c^2}F_mF^mh_{ik}\right). \tag{75}$$

Because the spur of this tensor α_{ik}, as it is easy to see, is not zero, $\alpha = h^{ik}\alpha_{ik} = -\frac{\mu c}{\pi}\left(A_{ik}A^{ik} + \frac{1}{2c^2}F_kF^k\right) \neq 0$, the tensor $\alpha_{ik} = \beta_{ik} + \frac{1}{3}\,\alpha h_{ik}$ has the non-zero anisotropic part

$$\beta_{ik} = \frac{\mu c}{4\pi}\Bigg(4A_{im}A^{m\cdot}_{\cdot k} + \frac{1}{c^2}F_iF_k - - \frac{1}{3c^2}F_mF^mh_{ik} + \frac{4}{3}A_{mn}A^{mn}h_{ik}\Bigg), \tag{76}$$

so viscous strengths of time density fields are anisotropic. It is also easy to see that this anisotropy increases with the value $A_{pq}A^{pq}$ of the space rotation.

Because the viscous strengths α_{ik} are anisotropic, the equilibrium pressure $p_0 = -qc^2$ and the true pressure p inside the medium are different. The true pressure is

$$p = \frac{\mu c}{12\pi}\left(A_{ik}A^{ik} + \frac{1}{2c^2}F_kF^k\right), \tag{77}$$

*The equation of state of a medium is the relation between the pressure p inside the medium and its density q. In a non-viscous medium or where the viscous strengths are isotropic, the true pressure p is the same as the equilibrium pressure p_0. The equation of state of a dust medium has the form $p = 0$. Ultra-relativistic gases have the equation of state $p = \frac{1}{3}qc^2$. The equation of state of matter inside atomic nuclei is $p = qc^2$. Vacuum and μ-vacuum have the equation of state $p = -qc^2$, see [5].

which gives the *equation of state* for time density fields

$$p = \frac{1}{3}\,qc^2. \tag{78}$$

Finally, we write the strength tensor $U_{ik} = ph_{ik} - \beta_{ik}$ of a time density field in the form

$$U_{ik} = \frac{1}{3}\,qc^2 h_{ik} - \beta_{ik}\,. \tag{79}$$

So, we can conclude for the physical properties of time density fields:

1. In general, a time density field is a non-stationary distributed medium, because its density may be $q \neq \text{const}$. The field becomes stationary $q = \text{const}$ under stationary space rotation $A_{ik} = \text{const}$, and stationary gravitational inertial force $F_i = \text{const}$;
2. A time density field bears momentum, because $J^i = -\frac{\mu}{2\pi c}\,F_k A^{ik} \neq 0$. So, the field can transfer impulse. The field does not transfer impulse $J^i = 0$, if the space does not rotate $A_{ik} = 0$. The absence of gravitation does not affect the field's transfer of impulse, because the "inertial" part of the force F_i remains unchanged even in the absence of gravitational fields;
3. A time density field is an emitting medium $J^i \neq 0$ in a non-holonomic (rotating) space. In a holonomic (non-rotating) space the field does not produce radiations;
4. A time density field is a viscous medium. The viscosity α_{ik} (75), derived from non-zero rotation of the space or from gravitational inertial force, is anisotropic. The anisotropy β_{ik} increases with the space rotation speed. The field is viscous anisotropic anyhow, because its viscous strengths would be $\alpha_{ik} = 0$ and $\beta_{ik} = 0$ only if both $A_{ik} = 0$ and $F_i = 0$. But in this case the field density would be $q = 0$, so the field itself is not there;
5. Therefore the equilibrium pressure p_0 does not possess a physical sense for time density fields; only the true pressure is real $p = p_0 - \frac{1}{3}\alpha$;
6. The equation of state for time density fields is $p = \frac{1}{3}qc^2$ (78) indicating that such fields are in the *state of an ultrarelativistic gas* — at positive density of the medium its inner pressure becomes positive, the medium is compressed.

8 Action of the field without sources

According to §27 of *The Classical Theory of Fields* [1], an elementary action for a whole system consisting of an electromagnetic field and a single charged particle, which are located in a pseudo-Riemannian space, contains three parts*

$$\begin{aligned} dS &= dS_{\mathrm{m}} + dS_{\mathrm{mf}} + dS_{\mathrm{f}} = \\ &= m_0 c\,ds + \frac{e}{c}\,\mathcal{A}_\alpha dx^\alpha + a\,\mathcal{F}_{\alpha\beta}\,\mathcal{F}^{\alpha\beta} dV dt\,, \end{aligned} \tag{80}$$

where $\mathcal{A}^\alpha$ is the four-dimensional electromagnetic field potential, $\mathcal{F}_{\alpha\beta} = \nabla_\alpha \mathcal{A}_\beta - \nabla_\beta \mathcal{A}_\alpha$ is the electromagnetic field tensor, $dV = dx\,dy\,dz$ is an elementary thee-dimensional volume filled with this field.

The first term S_{m} is "that part of the action which depends only on the properties of the particles, that is, just the action for free particles. ... The quantity S_{mf} is that part of the action which depends on the interaction between the particles and the field. ... Finally S_{f} is that part of the action which depends only on the properties of the field itself, that is, S_{f} is the action for a field in the absence of charges".

Because the action S_{f} must depend only on the field properties, the action must be taken over the space volume, filled with the field. The action must be scalar: only the 1st field invariant $J_1 = \mathcal{F}_{\alpha\beta}\,\mathcal{F}^{\alpha\beta}$ has this property. The 2nd field invariant $J_2 = \mathcal{F}_{\alpha\beta}\,\mathcal{F}^{*\alpha\beta}$ is pseudoscalar, not scalar, leading to the detailed discussion in Landau and Lifshitz.

"The numerical value of a depends on the choice of units for measurement of the field. ... From now on we shall use the Gaussian system of units; in this system a is a dimensionless quantity equal to $\frac{1}{16\pi}$".

According to §27 of *The Classical Theory of Fields* we have $dS_{\mathrm{f}} = a\,\mathcal{F}_{\alpha\beta}\,\mathcal{F}^{\alpha\beta} dV dt = \frac{1}{16\pi c}\,\mathcal{F}_{\alpha\beta}\,\mathcal{F}^{\alpha\beta} d\Omega$, where $d\Omega = c\,dt\,dV = c\,dt\,dx\,dy\,dz$ is an elementary space (four-dimensional) volume. So the action (80) takes the final form

$$dS = m_0 c\,ds + \frac{e}{c}\,\mathcal{A}_\alpha dx^\alpha + \frac{1}{16\pi c}\,\mathcal{F}_{\alpha\beta}\,\mathcal{F}^{\alpha\beta} d\Omega\,. \tag{81}$$

According to this consideration, we write an elementary action for the whole system consisting of a time density field and a single mass-bearing particle, which falls freely along time lines in a pseudo-Riemannian space, as follows

$$\begin{aligned} dS = dS_{\mathrm{m}} + dS_{\mathrm{mt}} &= m_0 c\,ds + a_{\mathrm{mt}}\,F_{\alpha\beta}\,F^{\alpha\beta} d\Omega = \\ &= m_0 c\,b_\alpha dx^\alpha + a_{\mathrm{mt}}\,F_{\alpha\beta}\,F^{\alpha\beta} d\Omega\,, \end{aligned} \tag{82}$$

where $F_{\alpha\beta}$ is the time density field tensor, a_{mt} is a constant consisting of other fundamental constants.

The first term S_{m} is that part of the action for the interaction between the particle and the time density field carrying it into motion along time lines. The second term

*In accordance with the least action principle, this action must have a minimum, so the integral of the action between a pair of world-points and the action itself must be positive. A negative action could give rise to a quantity with arbitrarily "large" negative values, which cannot have a minimum. Because in *The Classical Theory of Fields* Landau and Lifshitz take a pseudo-Riemannian space with the signature $(-+++)$, they write in §3 that "... the clock at rest always indicates a greater time interval than the moving one". Therefore they put "minus" before the action. To the contrary, we stick to a pseudo-Riemannian space with Zelmanov's signature $(+---)$, because in this case three-dimensional observable impulse is positive. In a space with such a signature, a regular observer takes his own flow of observable time positive always, $d\tau > 0$. Any particle, moving from past into future, has also a positive count of its own time coordinate $dt > 0$ with respect to the observer's clock. Therefore we put "plus" before the action.

$S_{\rm mt}$, depending only on the field properties, is the action for the field in the absence of its sources. In the absence of time density fields the second term $S_{\rm mt}$ is zero, so only $S_{\rm m} = m_0 cds$ remains here. A time density field is absent if the space is free of rotation $A_{ik} = 0$ and gravitational inertial forces $F_i = 0$, therefore if the conditions $g_{0i} = 0$ and $g_{00} = 1$ are true. This situation is possible in a pseudo-Riemannian space with a unit diagonal metric, which is the Minkowski space of the Special Theory of Relativity, where there is no gravitational field and no rotation. But in considering real space, we are forced to take a time density field into account. So we need to consider the terms $S_{\rm m}$ and $S_{\rm mt}$ together.

The constant $a_{\rm mt}$, according to its dimension, is the same as the constant μ in the energy-momentum tensor of time density fields, taken with the numerical coefficient $a = \frac{1}{16\pi}$, in the Gaussian system of units.

As a result, we obtain the action (82) in the final form

$$dS = dS_{\rm m} + dS_{\rm mt} = m_0 c\, b_\alpha dx^\alpha + \frac{\mu}{16\pi} F_{\alpha\beta} F^{\alpha\beta} d\Omega\,. \quad (83)$$

Because an action for a system is expressed through Lagrange's function L of the system as $dS = Ldt$, we take the action $dS_{\rm mt}$ in the form $dS_{\rm mt} = \frac{\mu c}{16\pi} F_{\alpha\beta} F^{\alpha\beta} dV dt = Ldt$, for the Lagrangian of an elementary volume $dV = dx dy dz$ of the field. We therefore obtain the *Lagrangian density* in time density fields

$$\Lambda = \frac{\mu c}{16\pi} F_{\alpha\beta} F^{\alpha\beta} = \frac{\mu}{4\pi c}\left(A_{ik}A^{ik} - \frac{1}{2c^2} F_i F^i\right). \quad (84)$$

The term $A_{ik}A^{ik}$ here, being expressed through the space rotation angular velocity pseudovector Ω^{*i}, is

$$A_{km}A^{km} = \varepsilon_{kmn}\Omega^{*n}A^{km} = 2\Omega_{*n}\Omega^{*n}, \quad (85)$$

because $\varepsilon_{nkm}\Omega^{*n} = \frac{1}{2}\varepsilon^{npq}\varepsilon_{nkm}A_{pq} = \frac{1}{2}\left(\delta^p_k\delta^q_m - \delta^q_k\delta^p_m\right)A_{pq} = A_{km}$ and $\Omega_{*n} = \frac{1}{2}\varepsilon_{nkm}A^{km}$. So the space rotation plays the first violin, defining the Lagrangian density in time density fields. Rotation velocities in macro-processes are incommensurably small in comparison with rotations of atoms and particles. For instance, in the 1st Bohr orbit in an atom of hydrogen, measuring the value of Λ in the units of the energy-momentum constant μ, we have $\Lambda \simeq 9.1\times10^{21}\mu$. On the Earth's surface near the equator the value is $\Lambda \simeq 2.8\times10^{-20}\mu$, so it is in order of 10^{42} less than in atoms. Therefore, because the Lagrangian of a system is the difference between its kinetic and potential energies, we conclude that time density fields produce their main energy flux in atoms and sub-atomic interactions, while the energy flux produced by the fields of macro-processes is negligible.

9 Plane waves of the field under gravitation is neglected. The wave pressure

In general, because the electric and the magnetic strengths of a time density field are $E_i = -\frac{1}{c^2} F_i$ and $H^{ik} = -\frac{1}{c} A^{ik}$, the chr.inv.-vector of its momentum density J^i (70) can be written as follows

$$J^i = -\frac{\mu}{2\pi c} F_k A^{ik} = -\frac{\mu c}{4\pi} E_k H^{ik}. \quad (86)$$

We are going to consider a particular case, where the field depends on only one coordinate. Waves of such a field traveling in one direction are known as *plane waves*.

We assume the field depends only on the axis $x^1 = x$, so only the component $J^1 = -\frac{\mu}{2\pi c} F_k A^{1k}$ of the field's chr.inv.-momentum density vector is non-zero. Then a plane wave of the field travels along the axis $x^1 = x$. Assuming the space rotating in xy plane (only the components $A^{12} = -A^{21}$ are non-zeroes) and replacing the tensor A^{ik} with the space rotation angular velocity pseudovector Ω_{*m} in the form $\varepsilon^{mik}\Omega_{*m} = \frac{1}{2}\varepsilon^{mik}\varepsilon_{mpq}A^{pq} = \frac{1}{2}\left(\delta^i_p\delta^k_q - \delta^k_p\delta^i_q\right)A^{pq} = A^{ik}$, we obtain

$$J^1 = -\frac{\mu}{2\pi c} F_2 A^{12} = -\frac{\mu}{2\pi c} F_2\, \varepsilon^{123}\Omega_{*3}\,. \quad (87)$$

It is easy to see that while a plane wave of the field travels along the axis $x^1 = x$, the field's "electric" and "magnetic" strengths are directed along the axes $x^2 = y$ and $x^3 = z$, i. e. orthogonal to the direction the wave travels. Therefore waves travelling in time density fields are *transverse waves*.

Following the arguments of Landau and Lifshitz in §47 of *The Classical Theory of Fields* [1], we define the *wave pressure* of a field as the total flux of the field energy-momentum, passing through a unit area of a wall. So the pressure $\mathfrak{F}_i$ is the sum

$$\mathfrak{F}_i = T_{ik} n^k + T'_{ik} n^k \quad (88)$$

of the spatial components of the energy-momentum tensor $T_{\alpha\beta}$ in a wave, falling on the wall, and of the energy-momentum tensor $T'_{\alpha\beta}$ in the reflected wave, projected onto the unit spatial vector $\vec{n}_{(k)}$ orthogonal to the wall surface.

Because the chr.inv.-strength tensor of a field is $U_{ik} = c^2 h_{i\alpha} h_{k\beta} T^{\alpha\beta} = c^2 T_{ik}$ [2], we obtain

$$\mathfrak{F}_i = \frac{1}{c^2}\left(U_{ik}n^k + U'_{ik}n^k\right), \quad (89)$$

where $U_{ik} = c^2 T_{ik}$ and $U'_{ik} = c^2 T'_{ik}$ are the chr.inv.-strength tensors in the falling wave and in the reflected wave. So the three-dimensional wave pressure vector $\mathfrak{F}_i$ has the property of chronometric invariance.

Using our formulae for the density q (68) and the strength tensor U_{ik} (71) obtained for time density fields, we are going to find the pressure a wave of such field exerts on a wall.

We consider the problem in a weak gravitational field, assuming its potential w and the attracting force of gravity negligible. We can do this because formulae (68) and (71) contain gravitation in only higher order terms. So the space rotation plays the first violin in the wave pressure $\mathfrak{F}_i$ in time density fields, gravitational inertial forces act there only because of their inertial part.

A plane wave travels along a single spatial direction: we assume axis $x^1 = x$. In this case the chr.inv.-field strength tensor U_{ik} has the sole non-zero component U_{11}. All the other components of the strength tensor U_{ik} are zero, which simplifies this consideration.

We assume the space rotating around the axis $x^3 = z$ (the rotation is in the xy-plane) at a constant angular velocity Ω. In this case $A_{12} = -A_{21} = -\Omega$, $A_{13} = 0$, $A_{23} = 0$, so the components of the rotation linear velocity $v_i = A_{ik}x^k$ are $v_1 = -\Omega y$, $v_2 = \Omega x$, $v_3 = 0$. Then the components of the acting gravitational inertial force will be $F_1 = -\frac{\partial v_1}{\partial t} = \Omega \frac{\partial y}{\partial t} = \Omega v_2 = \Omega^2 x$, $F_2 = -\frac{\partial v_2}{\partial t} = \Omega^2 y$, $F_3 = 0$. Because in this case $A_{ik}A^{ik} = 2A_{12}A^{12} = 2\Omega^2$ and $A_{1m}A^{m\cdot}_{\cdot 1} = A_{1m}A^{mn}h_{1n} = A_{12}A^{21}h_{11} = -\Omega^2 h_{11}$, we obtain

$$q = \frac{\mu}{4\pi c}\left[2\Omega^2 + \frac{1}{2c^2}\,\Omega^4\left(x^2+y^2\right)\right], \tag{90}$$

$$U_{11} = \frac{\mu c}{4\pi}\left[2\Omega^2 h_{11} - \frac{1}{c^2}\,\Omega^4 x^2 + \frac{1}{2c^2}\,\Omega^4\left(x^2+y^2\right)h_{11}\right]. \tag{91}$$

We assume a coefficient of the reflection $\Re$ as the ratio between the density of the field energy q' in the reflected wave to the energy density q in the falling wave. Actually, because $q' = \Re q$, the reflection coefficient $\Re$ is the energy loss of the field after the reflection.

We assume $x = x_0 = 0$ at the reflection point on the surface of the wall. Then we have $U_{11} = qc^2 h_{11}$, which, after substituting into (89), gives the pressure

$$\mathfrak{F}_1 = (1+\Re)\, q\, h_{11} n^1 \tag{92}$$

that a plane wave of a time density field exerts on the wall.

To bring this formula into final form in a Riemannian space becomes a problem, because the coordinate axes are curved there, and inhomogeneous. For this reason we cannot define the angles between directions in a Riemannian space itself, the angle of incidence and the angle of reflection of a wave for instance. At the same time, to consider this problem in the Minkowski space of the Special Theory of Relativity, as done by Landau and Lifshitz for the pressure of plane electromagnetic waves [1], would be senseless — because in Minkowski space we have $g_{00} = 1$ and $g_{0i} = 0$, then $F_i = 0$ and $A_{ik} = 0$, which implies no time density fields there.

To solve this problem correctly for a Riemannian space, let us introduce a *locally geodesic reference frame*, following Zelmanov. We therefore introduce a locally geodesic reference frame at the point of reflection of a wave on the surface of a wall. Within infinitesimal vicinities of any point of such a reference frame the fundamental metric tensor is

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\alpha\beta}}{\partial \tilde{x}^\mu \partial \tilde{x}^\nu}\right)(\tilde{x}^\mu - x^\mu)(\tilde{x}^\nu - x^\nu) + \ldots, \tag{93}$$

i. e. its components at a point, located in the vicinities, are different from those at the point of reflection to within only the higher order terms, values of which can be neglected. Therefore, at any point of a locally geodesic reference frame the fundamental metric tensor can be considered constant, while the first derivatives of the metric (the Christoffel symbols) are zero.

As a matter of fact, within infinitesimal vicinities of any point located in a Riemannian space, a locally geodesic reference frame can be set up. At the same time, at any point of this locally geodesic reference frame a tangential flat Euclidean space can be set up so that this reference frame, being locally geodesic for the Riemannian space, is the global geodesic for that tangential flat space.

The fundamental metric tensor of a flat Euclidean space is constant, so the values of $\tilde{g}_{\mu\nu}$, taken in the vicinities of a point of the Riemannian space, converge to the values of the tensor $g_{\mu\nu}$ in the flat space tangential at this point. Actually, this means that we can build a system of basis vectors $\vec{e}_{(\alpha)}$, located in this flat space, tangential to curved coordinate lines of the Riemannian space.

In general, coordinate lines in Riemannian spaces are curved, inhomogeneous, and are not orthogonal to each other (if the space is non-holonomic). So the lengths of the basis vectors may sometimes be very different from unity.

We denote a four-dimensional vector of infinitesimal displacement by $d\vec{r} = (dx^0, dx^1, dx^2, dx^3)$, so that $d\vec{r} = \vec{e}_{(\alpha)}dx^\alpha$, where components of the basis vectors $\vec{e}_{(\alpha)}$ tangential to the coordinate lines are $\vec{e}_{(0)} = \{e^0_{(0)}, 0, 0, 0\}$, $\vec{e}_{(1)} = \{0, e^1_{(1)}, 0, 0\}$, $\vec{e}_{(2)} = \{0, 0, e^2_{(2)}, 0\}$, $\vec{e}_{(3)} = \{0, 0, 0, e^2_{(3)}\}$. The scalar product of the vector $d\vec{r}$ with itself is $d\vec{r}d\vec{r} = ds^2$. On the other hand, the same quantity is $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$. As a result we have $g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)}e_{(\beta)}\cos\left(x^\alpha; x^\beta\right)$. So we obtain

$$g_{00} = e^2_{(0)}\,, \qquad g_{0i} = e_{(0)}e_{(i)}\cos\left(x^0; x^i\right), \tag{94}$$

$$g_{ik} = e_{(i)}e_{(k)}\cos\left(x^i; x^k\right). \tag{95}$$

The gravitational potential is $\mathrm{w} = c^2(1-\sqrt{g_{00}})$. So the time basis vector $\vec{e}_{(0)}$ tangential to the time line $x^0 = ct$, having the length $e_{(0)} = \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}$, is smaller than unity the greater is the gravitational potential w.

The space rotation linear velocity v_i and, according to it, the chr.inv.-metric tensor h_{ik} are

$$v_i = -c\, e_{(i)}\cos\left(x^0; x^i\right), \tag{96}$$

$$h_{ik} = e_{(i)}e_{(k)}\Big[\cos(x^0; x^i)\cos(x^0; x^k) - \cos(x^i; x^k)\Big]. \tag{97}$$

Harking back to the formula for the pressure $\mathfrak{F}_1$ (92), that a plane wave of a time density field traveling along the axis $x^1 = x$ exerts on a wall, we have

$$\mathfrak{F}_1 = (1+\Re)\, q\Big[\cos^2(x^0; x^1) + 1\Big]\, n_{(1)}e^2_{(1)}\cos(x^1; n^1), \tag{98}$$

because according to the signature $(+---)$, the spatial coordinate axes in the pseudo-Riemannian space are directed opposite to the same axes x^i in the tangential flat Euclidean space.

We denote $\cos(x^1; n^1) = \cos\theta$, where θ is the angle of reflection. Assuming $e_{(1)} = 1$, $n_{(1)} = 1$, $v_{(1)} = v$ we obtain the field density $q = \frac{\mu}{2\pi c}\Omega^2\left(1+\frac{v^2}{4c^2}\right)$, so that the wave pressure $\mathfrak{F}_{\text{N}} = \mathfrak{F}_1 \cos\theta$ normal to the wall surface is

$$\mathfrak{F}_{\text{N}} = (1+\Re)\left(1+\frac{v^2}{c^2}\right) q\cos^2\theta\,, \tag{99}$$

which, for low rotational velocities gives*

$$\mathfrak{F}_{\text{N}} = (1+\Re)\, q\cos^2\theta\,, \qquad q = \frac{\mu}{2\pi c}\,\Omega^2. \tag{100}$$

Most of rotations we observe are slow. The maximum of the known velocities is that for an electron in the 1st Bohr orbit ($v_{\text{b}} = 2.18\times10^8$ cm/sec). Therefore the ratio $\frac{v^2}{c^2}$, taking reaches a maximum numerical value of only 5.3×10^{-5}.

The presence of wave pressure in time density fields provides a way of measuring the numerical value of the energy-momentum constant μ, specific for such fields. For instance, a gyroscope, rotating around the axis $x^3 = z$, will be a source of circular waves of the field of time density propagating in the xy-plane. In this case the chr.inv.-field strength tensor U_{ik} has the non-zero components U_{11}, U_{12}, U_{21}. It is easy to calculate that the normal wave pressure of a circular wave will be different from the pressure of a plane wave (99) in only higher order terms. The same situation applies for spherical waves†. Therefore the normal pressure exerted by the waves on a wall orthogonal to the direction $x^1 = x$, shall be

$$\mathfrak{F}_{\text{N}} = \frac{\mu}{2\pi c}\,(1+\Re)\,\Omega^2 \tag{101}$$

to within the higher order terms withheld. Rotations at 6×10^3 rpm ($\Omega = 100$ rps) are achievable in modern gyroscopes, rotations in atoms are much greater, taking their maximum angular velocity to 4.1×10^{16} rps in the 1st Bohr orbit. A torsion balance registers forces, values of which are about 10^{-5} dynes. Then in accordance with the formula (101), if the wave pressure in an experiment is $\mathfrak{F}_{\text{N}} \approx 10^{-5}$ din/cm², derived from atomic transformations, the constant's numerical value will be in the order of $\mu \approx 10^{-28}$ gramme/sec.

Of course this is a crude supposition, based on the precision limits of measurement. Anyhow, the exact numerical value of the energy-momentum constant μ will be ascertained from special measurements with a torsion balance.

*Formula (100) is the same as $\mathfrak{F}_{\text{N}} = (1+\Re)\,q\cos^2\theta$ — the normal pressure exerted by a plane electromagnetic wave in Minkowski space, (see §47 in *The Classical Theory of Fields* [1]). So the wave pressure of a time density field depends on the reflection coefficient $0 \leqslant \Re \leqslant 1$ in the same way as the pressure of electromagnetic waves.

†In a real experiment such a gyroscope, being an arbitrarily thin disc, will be a source of spherical waves of a time density field which propagates in all spatial directions. The waves will merely have a maximum amplitude in the gyroscope's rotation plane xy.

10 Physical conditions in atoms

So we have obtained formulae for chr.inv.-projections of the energy-momentum tensor of time density fields, which are physically observable characteristics of such fields — the energy density q (69), the momentum density J^i (70), and the strength tensor U_{ik} (71).

The formulae must be valid everywhere, the inside of atoms included. At the same time, physical conditions in atoms are subject to Bohr's quantum postulates. For an external observer, an atom can be represented as a tiny gyroscope, the rotations of which are ruled by the quantum laws. The quantised rotations of electrons are sources of a time density field, which shall be perceptible, because of the super-rapid angular velocities up to the maximum value in the 1st Bohr orbit $\Omega_{\text{b}} = 4.1\times10^{16}$ rps. This is a way of formulating the physical conditions under which a time density field exists in atoms.

Taking the above into account, we formulate the physical conditions with postulates, which result from the application of Bohr's postulates to a time density field in atoms.

POSTULATE I *A time density field in an atom remains unchanged in the absence of external influences. The atom radiates or absorbs waves of the time density field only in transitions of the electrons between their stationary orbits.*

Naturally, when an atom is in a stable state, all its electrons are located in their orbits. Such a stable atom, having a set of quantum orbital angular velocities, must possess numerous quantum states of the time density field. The quantum states are set up with the second postulate‡.

POSTULATE II *A time density field is quantised in atoms. Its energy density and the momentum density take quantum numerical values which, in accordance with the quantization of electron orbits, in n-th stationary orbit are*

$$q_{\text{n}} = \frac{\mu}{2\pi c}\left(1+\frac{v_{\text{n}}^2}{4c^2}\right)\frac{v_{\text{n}}^2}{R_{\text{n}}^2}\,, \tag{102}$$

‡To introduce the second postulate we assume a reference frame in an atom, where an electron rotates around the nucleus at the angular velocity Ω in the xy-plane. Then $A_{12} = -A_{21} = -\Omega$, $A_{13} = 0$, $A_{23} = 0$. So out of all components of Ω^{*i} only Ω^{*3} is non-zero: $\Omega^{*3} = = \frac{1}{2}\varepsilon^{3mn}A_{mn} = \frac{1}{2}\left(\varepsilon^{312}A_{12} + \varepsilon^{321}A_{21}\right) = \varepsilon^{312}A_{12} = \frac{e^{312}}{\sqrt{h}}A_{12} = -\frac{\Omega}{\sqrt{h}}$ and $\Omega_{*3} = \frac{1}{2}\varepsilon_{3mn}A^{mn} = \varepsilon_{312}A^{12} = e_{312}\sqrt{h}\,A_{12} = -\sqrt{h}\,\Omega$. In calculating $h = \det\|h_{ik}\|$, it should be noted that the components of the space rotation linear velocity $v_i = A_{ik}x^k$ in this reference frame are $v_1 = -\Omega y$, $v_2 = \Omega x$, $v_3 = 0$. We obtain $h_{11} = 1 + \frac{1}{c^2}\Omega^2 y^2$, $h_{22} = 1 + \frac{1}{c^2}\Omega^2 x^2$, $h_{12} = -\frac{1}{c^2}\Omega^2 xy$, $h_{33} = 1$. Then $h = \det\|h_{ik}\| = = h_{11}h_{22} - (h_{12})^2 = 1 + \frac{1}{c^2}\Omega^2(x^2+y^2)$. In the 1st Bohr orbit we have $\frac{1}{c^2}\Omega^2(x^2+y^2) = \frac{1}{c^2}\Omega^2 R^2 = 5.3\times10^{-5}$, so we can set $h \approx 1$ to within the higher order terms withheld. Harking back to the formulae for Ω^{*3} and Ω_{*3}, we see that the space rotates in atoms at a constant angular velocity $\Omega^{*3} = -\Omega$, $\Omega_{*3} = -\Omega$, then in the assumed reference frame we have $A_{ik}A^{ik} = 2A_{12}A^{12} = 2\Omega_{*3}\Omega^{*3} = 2\Omega^2$, and also $F_1 = -\frac{\partial v_1}{\partial t} = \Omega^2 x$, $F_2 = -\frac{\partial v_2}{\partial t} = \Omega^2 y$, $F_3 = 0$, which is taken into account in Postulate II.

$$J_{\rm n} = \sqrt{(J_i J^i)_{\rm n}} = \frac{\mu}{2\pi c}\,\Omega^3_{\rm n} R_{\rm n} = \frac{\mu}{2\pi c}\frac{v^3_{\rm n}}{R^2_{\rm n}}\,. \tag{103}$$

Calculating the field density in neighbouring levels n and n+1, we take into account that the n-th orbital radius relates to the 1st Bohr radius as $R_{\rm n} = {\rm n}^2 R_{\rm b}$. As a result we obtain

$$\bar{q} = q_{\rm n} - q_{\rm n+1} = = \frac{\mu}{2\pi c}\,\Omega^2_{\rm b}\left\{\left[\frac{1}{{\rm n}^6} - \frac{1}{({\rm n}+1)^6}\right] + \frac{v^2_{\rm b}}{4c^2}\left[\frac{1}{{\rm n}^8} - \frac{1}{({\rm n}+1)^8}\right]\right\}, \tag{104}$$

so the difference between the field density in the neighbour levels is inversely proportional to $\rm n^7$, and $\rm n \gg 1$ gives

$$\bar{q} = q_{\rm n} - q_{\rm n+1} \approx \frac{1}{{\rm n}^7}\frac{3\mu}{\pi c}\,\Omega^2_{\rm b}\,, \tag{105}$$

and $\bar{q} \to 0$ for quantum numbers $\rm n \to \infty$.

Theoretically, the non-zero field density, $q \neq 0$, must result in a flux of the field momentum (this flux is set up by the field strength tensor $U_{ik} = \frac{1}{3}\,qc^2 h_{ik} - \beta_{ik}$). So an electron, moving in its orbit, should be radiating a momentum flux of the time density field (waves of the field). Because of the momentum loss in the radiation, the electron's own angular velocity would decrease, contradicting the experimental facts on the stability of atoms in the absence of external influences. To obviate this contradiction the third postulate is,

POSTULATE III *An atom radiates a quantum portion of momentum flux of a time density field, when an electron transits from the n-th quantum level to the (n+1)-th level in the atom. When an electron transits from the (n+1)-th level to the n-th level, the atom absorbs the same portion of the momentum flux, which is*

$$\overline{U}_{11} = U^{\rm n}_{11} - U^{\rm n+1}_{11} = = \frac{\mu c}{2\pi}\,\Omega^2_{\rm b}\left\{\left[\frac{1}{{\rm n}^6} - \frac{1}{({\rm n}+1)^6}\right] - \frac{v^2_{\rm b}}{4c^2}\left[\frac{1}{{\rm n}^8} - \frac{1}{({\rm n}+1)^8}\right]\right\}. \tag{106}$$

We assume in this formula that the atom radiates/absorbs a plane wave of a time density field, which travels along the $x^1 = x$ axis. Taking this formula with $\rm n \gg 1$, we have

$$\overline{U}_{11} = U^{\rm n}_{11} - U^{\rm n+1}_{11} \approx \frac{1}{{\rm n}^7}\frac{3\mu c}{\pi}\,\Omega^2_{\rm b}\,, \tag{107}$$

which, for quantum numbers $\rm n \to \infty$, gives $\overline{U}_{11} \to 0$. So for quantum numbers $\rm n \gg 1$ we have the ratio

$$\overline{U}_{11} = \bar{q}c^2. \tag{108}$$

In accordance with the correspondence principle, any result of quantum theory at high quantum numbers must coincide with the relevant classical results; any difference being imperceptible. We therefore take into consideration the formulae for q (69) and U_{ik} (71) in atoms, obtained by the methods of the classical theory of fields, under the condition $h \approx 1$. As a result we get the formulae $q = \frac{\mu}{2\pi c}\,\Omega^2\left(1 + \frac{v^2}{4c^2}\right) \approx \frac{\mu}{2\pi c}\,\Omega^2$ and $U_{ik} = \frac{\mu c}{2\pi}\,\Omega^2\left(h_{11} - \frac{v^2}{2c^2} + \frac{v^2}{4c^2}h_{11}\right) \approx \frac{\mu c}{2\pi}\,\Omega^2$, leading to the same relationship $U_{11} = qc^2$ that quantum theory has given (108). So the correspondence principle is valid for time density fields in atoms.

Postulate III has two consequences:

CONSEQUENCE I *An atom undergoing excitation radiates the momentum flux of a time density field, producing a positive wave pressure in the field.*

Calculating this positive pressure, orthogonal to the surface of a wall (here θ is the angle of reflection, $\mathfrak{R}$ is the reflection coefficient) for quantum numbers $\rm n \gg 1$, we obtain

$$\overline{\mathfrak{F}}_{\rm N} = (1 + \mathfrak{R})\,\bar{q}\cos^2\theta\,. \tag{109}$$

CONSEQUENCE II *An atom undergoing relaxation absorbs the momentum flux of a time density field. In this case the wave pressure in a time density field near the atom becomes negative.*

As a matter of fact, this negative pressure around a relaxing atom should be

$$\overline{\mathfrak{F}}_{\rm N} = -(1 + \mathfrak{R})\,\bar{q}\cos^2\theta\,. \tag{110}$$

That is, in accordance with this theory, excitation of atoms causes radiation of waves of the time density field. An effect derived from the radiation should be the positive pressure of the waves. On the other hand, relaxing atoms, absorbing waves of the time density field, should be sources of negative wave pressure.

It is interesting that this effect is opposite to that which atoms produce in an electromagnetic field — it is well-known that relaxing atoms radiate electromagnetic waves, so they produce a positive wave pressure in an electromagnetic field.

The predicted repulsion/attraction produced by atomic processes, being outside the actions of electromagnetic or gravitational fields, are peculiarities of only the theory of the time density field herein. So the given conclusions open up wide possibilities for checking the whole theory in practice.

In particular, for instance, if a torsion balance registered the repulsing/attracting wave pressure $\overline{\mathfrak{F}}_{\rm N}$ derived from subatomic excitation/relaxation processes, we will have obtained the numerical value of the energy-momentum constant μ for time density fields. After substituting $\bar{q}$ (105) into the wave pressure $\overline{\mathfrak{F}}_{\rm N}$, assuming $\cos\theta = 1$, we arrive at a formula for experimental calculations

$$\mu = \frac{\pi c {\rm n}^7}{3\Omega^2_{\rm b}}\frac{\overline{\mathfrak{F}}_{\rm N}}{(1 + \mathfrak{R})}\,. \tag{111}$$

A torsion balance registered such forces at $\sim 10^{-5}$ dynes in prior experiments. The torsion balance had a 2-in long nylon thread, 15 μm in diameter, and a 3-in long wooden balance suspended in the ratio 8:1 of the length. The balance had a reflecting shield at the end of the long arm and a lead load on the short arm. The torsion balance was located inside a box isolated from air convection and light radiation. Chemical reactions of the opposite directions, processes of crystallization and dissolution were sources of a time density field acting on the torsion balance. Prof. Kyril Stanyukovich and Dr. Larissa Borissova assisted me in the experiments that were repeated a number of time during a period of 2 years in Moscow (Russia). The balance underwent deviations of up to 90° in directions predicted by this theory.

Even heating up bodies and cooling down bodies gave the same thermal influence, moving the balance in opposite directions, according to the theory, so the discovered phenomenon is outside thermal influences on torsion balance.

The techniques and measurements are very simply, and could therefore be reproduced in any physical laboratory. Anyway the experiments should be continued, with the aim of determining the exact numerical value of the energy-momentum constant μ for time density fields through formula (111).

11 Conclusions

Let us collect the main results of this analysis.

By projecting an interval of four-dimensional coordinates dx^α onto the time line of an observer, who accompanies his references ($b^i = 0$), we obtain an interval of physical time $d\tau = \frac{1}{c}\, b_\alpha dx^\alpha$ he observes. Observations at the same spatial point give $d\tau = \sqrt{g_{00}}\, dt$, so the operator of projection on time lines b^α defines observable inhomogeneity of time references in the accompanying reference frame.

So, observable inhomogeneities of time references can be represented as a field of "density" of observable time τ. The projecting operator b^α is the field "potential", chr.inv.-projections of which are $\varphi = 1$ and $q^i = 0$.

The field tensor $F_{\alpha\beta} = \nabla_\alpha b_\beta - \nabla_\beta b_\alpha$ for time density fields was introduced as well as Maxwell's electromagnetic field tensor. Its chr.inv.-projections $E^i = -\frac{1}{c^2} F^i$ and $H_{ik} = -\frac{2}{c} A_{ik}$ are derived from the gravitational inertial force and rotation of the space. We referred to the E^i and H_{ik} as the "electric" and "magnetic" observable components of the time density field, respectively. We also introduced the field pseudotensor $F^{*\alpha\beta}$, dual of the $F_{\alpha\beta}$, and also the field invariants.

Equations of motion of a free mass-bearing particle, being expressed through the E^i and H_{ik}, group them into an acting force of a form similar to the Lorentz force. In particular if the particle moves only along time lines, it moves solely because of the "magnetic" component $H_{ik} \neq 0$ of a time density field. In other words, the space rotation A_{ik} effectively "screws" particles into the time lines. Because observable particles with the whole spatial section move from past into future, a "starting" non-holonomity, $A_{ik} \neq 0$, will exist in our real space that is a "primordial non-orthogonality" of the real spatial section to the time lines. Other physical conditions (gravitation, rotation, etc.) are only augmentations that intensify or reduce this starting-rotation of the space.

A system of equations of a time density field consists of Lorentz's condition $\nabla_\sigma b^\sigma = 0$, two groups of Maxwell-like equations, $\nabla_\sigma F^{\alpha\sigma} = \frac{4\pi}{c} j^\alpha$ and $\nabla_\sigma F^{*\alpha\sigma} = 0$, and the continuity equation $\nabla_\sigma j^\sigma = 0$, which define the main properties of the field and its-inducing sources. All the equations have been deduced here in chr.inv.-form.

The energy-momentum tensor $T^{\alpha\beta}$ we have deduced for time density fields has the following observable projections: chr.inv.-scalar q of the field density; chr.inv.-vector J^i of the field momentum density, and chr.inv.-tensor U^{ik} of the field strengths. Their specific formulas define physical properties of such fields:

1. A time density field is non-stationary distributed medium $q \neq \text{const}$, it becomes stationary, $q = \text{const}$, under stationary rotation, $A_{ik} = \text{const}$, of the space and stationary gravitational inertial force $F_i = \text{const}$;
2. The field bears momentum ($J^i \neq 0$ in the general case), so it can transfer impulse;
3. In a rotating space, $A_{ik} \neq 0$, the field is an emitting medium;
4. The field is viscous. The viscosity α_{ik} is anisotropic. The anisotropy increases with the space rotation speed;
5. The equation of state of the field is $p = \frac{1}{3} q c^2$, so the field is like an ultrarelativistic gas: at positive density the pressure is positive — the medium compresses.

For a plane wave of the field considered, we have concluded that waves of the time density fields are transverse. The wave pressure in the fields is derived from atomic and sub-atomic transformations mainly, because of huge rotational velocities. Exciting atoms produces a positive wave pressure in the time density field, while the wave pressure resulting from relaxing atoms is negative. This effect is opposite to that of the electromagnetic field — relaxing atoms radiate γ-quanta, producing a positive pressure of light waves.

Experimental tests have a basis in the predicted repulsion/attraction, produced by sub-atomic processes, being outside of known effects of electromagnetic or gravitational fields, which are peculiarities only of this theory. A torsion balance registered such forces at $\sim 10^{-5}$ dynes in prior experiments. The registered repulsion/attraction is outside thermal effects on the torsion balance.

The results we have obtained in this study imply that even if inhomogeneity of time references is a tiny correction to ideal time, a field of the inhomogeneities that is a time density field, manifest as gravitational and inertial forces, has a more fundamental effect on observable phenomena, than those previouly supposed.

Acknowledgements

I began this research in 1984 when I, a young scientist in those years, started my scientific studies under the direction of Prof. Kyril Stanyukovich, who in the 1940's was already a well-known expert in the General Theory of Relativity. At that time we found an article of 1971, wherein Prof. Nikolai Kozyrev (1908–1983) reported on his experiments with a torsion balance [7].

His high precision torsion balance, having unequal arms, registered weak forces of attraction/repulsion at 10^{-5}–10^{-6} dynes; the forces derived from creative/destructive processes in his laboratory. Proceeding from his considerations, redistribution of energy should produce a non-uniformity of time that generates a force field of attraction or repulsion depending on the creative/destructive direction of the redistributing energy process.

Kozyrev did not put this idea into any mathematical form. So his propositions, having a purely phenomenological a basis, remained without a theory. Kozyrev was the famous experimental physicist and astronomer of the 20th century who discovered volcanic activity in the Moon (1958), the atmosphere of Mercury (1963), and many other phenomena. His authority in physical experiment was beyond any doubt.

In those years Stanyukovich was head of the Department of Fundamental Theoretical Metrology, Surface and Vacuum Scientific Centre, State Committee for Standards (Moscow, Russia). Dr. Larissa Borissova worked in his Department. We were all close friends, despite our age difference. We had a good time working with Stanyukovich, who was friendly in his conversations on different scientific problems.

I was interested by Kozyrev's experiments with the torsion balance, therefore Stanyukovich proposed me that I investigate them, meaning that it would be helpful for me to build the required theory. During 1984–1985 I twice visited the laboratory of late Kozyrev in Pulkovo Astronomical Observatory, near Leningrad. Then, in Moscow, we made a copy of his torsion balance, and modified it to make it more sensitive.

Stanyukovich was right — the experiments were approbated with strictly positive result. We repeated the experiments for some of his colleagues, in particular for Dr. Vitaly Schelest.

More than 15 years were required for the development of this theory. I finished the whole theory only in 2004. Stanyukovich had died; only Larissa Borissova and I remain. Our years of friendly conversations with Stanyukovich, and his patient personal instructions reached us by all his experience in theoretical physics. Actually, we are beholden to him. Today I would like to do only one thing — satisfy the hopes of my teacher.

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