On the General Solution to Einstein's Vacuum Field for the Point-Mass when $\lambda \neq 0$ and Its Consequences for Relativistic Cosmology

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It is generally alleged that Einstein's theory leads to a finite but unbounded universe. This allegation stems from an incorrect analysis of the metric for the point-mass when $\lambda \neq 0$. The standard analysis has incorrectly assumed that the variable r denotes a radius in the gravitational field. Since r is in fact nothing more than a real-valued parameter for the actual radial quantities in the gravitational field, the standard interpretation is erroneous. Moreover, the true radial quantities lead inescapably to $\lambda = 0$ so that, cosmologically, Einstein's theory predicts an infinite, static, empty universe.

1 Introduction

It has been shown [1, 2, 3] that the variable r which appears in the metric for the gravitational field is neither a radius nor a coordinate in the gravitational field, and further [3], that it is merely a real-valued parameter in the pseudo-Euclidean spacetime (M_s, g_s) of Special Relativity, by which the Euclidean distance $D = |r - r_0| \in (M_s,\, g_s)$ is mapped into the non-Euclidean distance $R_p \in (M_g, g_g)$, where (M_g, g_g) denotes the pseudo-Riemannian spacetime of General Relativity. Owing to their invalid assumptions about the variable r, the relativists claim that $r = \sqrt{\frac{3}{\lambda}}$ defines a "horizon" for the universe (e.g. [4]), by which the universe is supposed to have a finite volume. Thus, they have claimed a finite but unbounded universe. This claim is demonstrably false.

The standard metric for the simple point-mass when $\lambda \neq 0$ is,

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\lambda}{3}\, r^2\right) dt^2 - \left(1 - \frac{2m}{r} - \frac{\lambda}{3}\, r^2\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{1}$$

The relativists simply look at (1) and make the following assumptions.

(a) *The variable r is a radial coordinate in the gravitational field*;

(b) *r can go down to 0*;

(c) *A singularity in the gravitational field can occur only where the Riemann tensor scalar curvature invariant (or Kretschmann scalar) $f = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$ is unbounded*.

The standard analysis has never proved these assumptions, but nonetheless simply takes them as given. I have demonstrated elsewhere [3] that when $\lambda = 0$, these assumptions are false. I shall demonstrate herein that when $\lambda \neq 0$ these assumptions are still false, and further, that λ can only take the value of zero in Einstein's theory.

2 Definitions

As is well-known, the basic spacetime of the General Theory of Relativity is a metric space of the Riemannian geometry family, namely — the four-dimensional pseudo-Riemannian space with Minkowski signature. Such a space, like any Riemannian metric space, is strictly negative non-degenerate, i. e. the fundamental metric tensor $g_{\alpha\beta}$ of such a space has a determinant which is strictly negative: $g = \det \| g_{\alpha\beta} \| < 0$.

Space metrics obtained from Einstein's equations can be very different. This splits General Relativity's spaces into numerous families. The two main families are derived from the fact that the energy-momentum tensor of matter $T_{\alpha\beta}$, contained in the Einstein equations, can (1) be linearly proportional to the fundamental metric tensor $g_{\alpha\beta}$ or (2) have a more compound functional dependence. The first case is much more attractive to scientists, because in this case one can use $g_{\alpha\beta}$, taken with a constant numerical coefficient, instead of the usual $T_{\alpha\beta}$, in the Einstein equations. Spaces of the first family are known as *Einstein spaces*.

From the purely geometrical perspective, an Einstein space [5] is described by any metric obtained from

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = \kappa T_{\alpha\beta} - \lambda g_{\alpha\beta}\,,$$

where κ is a constant and $T_{\alpha\beta} \propto g_{\alpha\beta}$, and therefore includes all partially degenerate metrics. Accordingly, such spaces become non-Einstein only when the determinant g of the metric becomes

$$g = \det \| g_{\alpha\beta} \| = 0\,.$$

In terms of the required physical meaning of General Relativity I shall call a spacetime associated with a non-degenerate metric, an Einstein universe, and the associated metric an Einstein metric.

Cosmological models involving either $\lambda \neq 0$ or $\lambda = 0$, which do not result in a degenerate metric, I shall call relativistic cosmological models, which are necessarily Einstein universes, with associated Einstein metrics.

Thus, any "partially" degenerate metric where $g \neq 0$ is not an Einstein metric, and the associated space is not an Einstein universe. Any cosmological model resulting in a "partially" degenerate metric where $g \neq 0$ is neither a relativistic cosmological model nor an Einstein universe.

3 The general solution when $\lambda \neq 0$

The general solution for the simple point-mass [3] is,

$$ds^2 = \left(\frac{\sqrt{C_n}-\alpha}{\sqrt{C_n}}\right)dt^2 - \left(\frac{\sqrt{C_n}}{\sqrt{C_n}-\alpha}\right)\frac{C_n'^{\,2}}{4C_n}dr^2 - C_n(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{2}$$

$$C_n(r) = \left[|r - r_0|^n + \alpha^n\right]^{\frac{2}{n}}, \; n \in \Re^+,$$

$$\alpha = 2m, \quad r_0 \in \Re\,,$$

where n and r_0 are arbitrary and r is a real-valued parameter in (M_s, g_s).

The most general static metric for the gravitational field [3] is,

$$ds^2 = A(D)dt^2 - B(D)dr^2 - C(D)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\,, \tag{3}$$

$$D = |r - r_0|, \quad r_0 \in \Re\,,$$

where analytic $A, B, C > 0 \;\forall\; r \neq r_0$.

In relation to (3) I identify the coordinate radius D, the r-parameter, the radius of curvature R_c, and the proper radius (proper distance) R_p.

1. *The coordinate radius is* $D = |r - r_0|$.
2. *The* r*-parameter is the variable* r.
3. *The radius of curvature is* $R_c = \sqrt{C(D(r))}$.
4. *The proper radius is* $R_p = \int \sqrt{B(D(r))}dr$.

I remark that $R_p(D(r))$ gives the mapping of the Euclidean distance $D = |r - r_0| \in (M_s, g_s)$ into the non-Euclidean distance $R_p \in (M_g, g_g)$ [3]. Furthermore, the geometrical relations between the components of the metric tensor are inviolable and therefore hold for all metrics with the form of (3).

Thus, on the metric (2),

$$R_c = \sqrt{C_n(D(r))}\,,$$

$$R_p = \int \sqrt{\frac{\sqrt{C_n}}{\sqrt{C_n}-\alpha}\,\frac{C_n'}{2\sqrt{C_n}}}\;dr\,.$$

Transform (3) by setting,

$$r^* = \sqrt{C(D(r))}\,, \tag{4}$$

to carry (3) into,

$$ds^2 = A^*(r^*)dt^2 - B^*(r^*)dr^{*2} - r^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{5}$$

For $\lambda \neq 0$, one finds in the usual way that the solution to (5) is,

$$ds^2 = \left(1 - \frac{\alpha}{r^*} - \frac{\lambda}{3}r^{*2}\right)dt^2 - \left(1 - \frac{\alpha}{r^*} - \frac{\lambda}{3}r^{*2}\right)^{-1}dr^{*2} - r^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{6}$$

$$\alpha = \text{const.}$$

Then by (4),

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C}} - \frac{\lambda}{3}C\right)dt^2 - \left(1 - \frac{\alpha}{\sqrt{C}} - \frac{\lambda}{3}C\right)^{-1}\frac{C'^2}{4C}dr^2 - C\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{7}$$

$$C = C(D(r)), \quad D = D(r) = |r - r_0|, \quad r_0 \in \Re\,,$$

$$\alpha = \text{const}\,,$$

where $r \in (M_s, g_s)$ is a real-valued parameter and also $r_0 \in (M_s, g_s)$ is an arbitrary constant which specifies the position of the point-mass in parameter space.

When $\alpha = 0$, (7) reduces to the empty de Sitter metric, which I write generally, in view of (7), as

$$ds^2 = \left(1 - \frac{\lambda}{3}F\right)dt^2 - \left(1 - \frac{\lambda}{3}F\right)^{-1}d\sqrt{F}^{\,2} - F\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{8}$$

$$F = F(D(r)), \quad D = D(r) = |r - r_0|, \quad r_0 \in \Re\,.$$

If $F(D(r)) = r^2$, $r_0 = 0$, and $r \geqslant r_0$, then the usual form of (8) is obtained,

$$ds^2 = \left(1 - \frac{\lambda}{3}r^2\right)dt^2 - \left(1 - \frac{\lambda}{3}r^2\right)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{9}$$

The admissible forms for $C(D(r))$ and $F(D(r))$ must now be generally ascertained.

If $C' \equiv 0$, then $B(D(r)) = 0 \;\forall\; r$, in violation of (3). Therefore $C' \neq 0 \;\forall\; r \neq r_0$.

Now $C(D(r))$ must be such that when $r \to \pm\infty$, equation (7) must reduce to (8) asymptotically. So,

$$\text{as } r \to \pm\infty, \; \frac{C(D(r))}{F(D(r))} \to 1.$$

I have previously shown [3] that the condition for singularity on a metric describing the gravitational field of the point-mass is,

$$g_{00}(r_0) = 0\,. \tag{10}$$

Thus, by (7), it is required that,

$$1 - \frac{\alpha}{\sqrt{C(D(r_0))}} - \frac{\lambda}{3}C(D(r_0)) = 1 - \frac{\alpha}{\beta} - \frac{\lambda}{3}\beta^2 = 0\,, \tag{11}$$

having set $\sqrt{C(D(r_0))} = \beta$. Thus, β is a scalar invariant for (7) that must contain the independent factors contributing to the gravitational field, i.e. $\beta = \beta(\alpha, \lambda)$. Consequently it is required that when $\lambda = 0$, $\beta = \alpha = 2m$ to recover (2), when $\alpha = 0$, $\beta = \sqrt{\frac{3}{\lambda}}$ to recover (8), and when $\alpha = \lambda = 0$, and $\beta = 0$, $C(D(r)) = |r - r_0|^2$ to recover the flat spacetime of Special Relativity. Also, when $\alpha = 0$, $C(D(r))$ must reduce to $F(D(r))$. The value of $\beta = \beta(\lambda) = \sqrt{F(D(r_0))}$ in (8) is also obtained from,

$$g_{00}(r_0) = 0 = 1 - \frac{\lambda}{3}F(D(r_0)) = 1 - \frac{\lambda}{3}\beta^2\,.$$

Therefore,

$$\beta = \sqrt{\frac{3}{\lambda}}\,. \tag{12}$$

Thus, to render a solution to (7), $C(D(r))$ must at least satisfy the following conditions.

1. $C'(D(r)) \neq 0 \;\forall\; r \neq r_0$.
2. As $r \to \pm\infty$, $\frac{C(D(r))}{F(D(r))} \to 1$.
3. $C(D(r_0)) = \beta^2$, $\beta = \beta(\alpha, \lambda)$.
4. $\lambda = 0 \Rightarrow \beta = \alpha = 2m$ and $C = \left(|r - r_0|^n + \alpha^n\right)^{\frac{2}{n}}$.
5. $\alpha = 0 \Rightarrow \beta = \sqrt{\frac{3}{\lambda}}$ and $C(D(r)) = F(D(r))$.
6. $\alpha = \lambda = 0 \Rightarrow \beta = 0$ and $C(D(r)) = |r - r_0|^2$.

Both α and $\beta(\alpha, \lambda)$ must also be determined.

Since (11) is a cubic, it cannot be solved exactly for β. However, I note that the two positive roots of (11) are approximately α and $\sqrt{\frac{3}{\lambda}}$. Let $P(\beta) = 1 - \frac{\alpha}{\beta} - \frac{\lambda}{3}\beta^2$. Then according to Newton's method,

$$\beta_{m+1} = \beta_m - \frac{P(\beta_m)}{P'(\beta_m)} = \beta_m - \frac{\left(1 - \frac{\alpha}{\beta_m} - \frac{\lambda}{3}\beta_m^2\right)}{\left(\frac{\alpha}{\beta_m^2} - \frac{2\lambda}{3}\beta_m\right)}\,. \tag{13}$$

Taking $\beta_1 = \alpha$ into (13) gives,

$$\beta \approx \beta_2 = \frac{3\alpha - \lambda\alpha^3}{3 - 2\lambda\alpha^2}\,, \tag{14a}$$

and

$$\beta \approx \beta_3 = \frac{3\alpha - \lambda\alpha^3}{3 - 2\lambda\alpha^2} - \left[\frac{1 - \frac{\alpha(3 - 2\lambda\alpha^2)}{(3\alpha - \lambda\alpha^3)} - \frac{\lambda}{3}\left(\frac{3\alpha - \lambda\alpha^3}{3 - 2\lambda\alpha^2}\right)^2}{\alpha\left(\frac{3 - 2\lambda\alpha^2}{3\alpha - \lambda\alpha^3}\right)^2 - \frac{2\lambda}{3}\left(\frac{3\alpha - \lambda\alpha^3}{3 - 2\lambda\alpha^2}\right)}\right], \tag{14b}$$

etc., which satisfy the requirement that $\beta = \beta(\alpha, \lambda)$.

Taking $\beta_1 = \sqrt{\frac{3}{\lambda}}$ into (13) gives,

$$\beta \approx \beta_2 = \sqrt{\frac{3}{\lambda}} + \frac{\alpha}{\alpha\sqrt{\frac{\lambda}{3}} - 2}\,, \tag{15a}$$

and

$$\beta \approx \beta_3 = \sqrt{\frac{3}{\lambda}} + \frac{\alpha}{\alpha\sqrt{\frac{\lambda}{3}} - 2} - \left[\frac{1 - \frac{\alpha}{\left(\sqrt{\frac{3}{\lambda}} + \frac{\alpha}{\alpha\sqrt{\frac{\lambda}{3}} - 2}\right)} - \frac{\lambda}{3}\left(\sqrt{\frac{3}{\lambda}} + \frac{\alpha}{\alpha\sqrt{\frac{\lambda}{3}} - 2}\right)^2}{\frac{\alpha}{\left(\sqrt{\frac{3}{\lambda}} + \frac{\alpha}{\alpha\sqrt{\frac{\lambda}{3}} - 2}\right)^2} - \frac{2\lambda}{3}\left(\sqrt{\frac{3}{\lambda}} + \frac{\alpha}{\alpha\sqrt{\frac{\lambda}{3}} - 2}\right)}\right], \tag{15b}$$

etc., which satisfy the requirement that $\beta = \beta(\alpha, \lambda)$.

However, according to (14a) and (14b), when $\lambda = 0$, $\beta = \alpha = 2m$, and when $\alpha = 0$, $\beta \neq \sqrt{\frac{3}{\lambda}}$. According to (15a), (15b), when $\lambda = 0$, $\beta \neq \alpha = 2m$, and when $\alpha = 0$, $\beta = \sqrt{\frac{3}{\lambda}}$. The required form for β, and therefore the required form for $C(D(r))$, *cannot be constructed*, i.e. *it does not exist*. There is no way $C(D(r))$ can be constructed to satisfy all the required conditions to render an admissible solution to (7) in the form of (3). Therefore, the assumption that $\lambda \neq 0$ is incorrect, and so $\lambda = 0$. This can be confirmed in the following way.

The proper radius $R_p(r)$ of (8) is given by,

$$R_p(r) = \int \frac{d\sqrt{F}}{\sqrt{1 - \frac{\lambda}{3}F}} = \sqrt{\frac{3}{\lambda}}\arcsin\sqrt{\frac{\lambda}{3}F(r)} + K\,,$$

where K is a constant. Now, the following condition must be satisfied,

$$\text{as } r \to r_0^{\pm}, \; R_p \to 0^{+}\,,$$

and therefore,

$$R_p(r_0) = 0 = \sqrt{\frac{3}{\lambda}}\arcsin\sqrt{\frac{\lambda}{3}F(r_0)} + K\,,$$

and so,

$$R_p(r)=\sqrt{\frac{3}{\lambda}}\left[\arcsin\sqrt{\frac{\lambda}{3}F(r)}-\arcsin\sqrt{\frac{\lambda}{3}F(r_0)}\right]. \quad (16)$$

According to (8),

$$g_{00}(r_0)=0\Rightarrow F(r_0)=\frac{3}{\lambda}.$$

But then, by (16),

$$\sqrt{\frac{\lambda}{3}F(r)}\equiv 1,$$

$$R_p(r)\equiv 0.$$

Indeed, by (16),

$$\sqrt{\frac{\lambda}{3}F(r_0)}\leqslant\sqrt{\frac{\lambda}{3}F(r)}\leqslant 1,$$

or

$$\sqrt{\frac{3}{\lambda}}\leqslant\sqrt{F(r)}\leqslant\sqrt{\frac{3}{\lambda}},$$

and so

$$F(r)\equiv\frac{3}{\lambda}, \quad (17)$$

and

$$R_p(r)\equiv 0. \quad (18)$$

Then $F'(D(r))\equiv 0$, and so there exists no function $F(r)$ which renders a solution to (8) in the form of (3) when $\lambda\neq 0$ and therefore there exists no function $C(D(r))$ which renders a solution to (7) in the form of (3) when $\lambda\neq 0$. Consequently, $\lambda=0$.

Owing to their erroneous assumptions about the r-parameter, the relativists have disregarded the requirement that $A, B, C>0$ in (3) must be met. If the required form (3) is relaxed, in which case the resulting metric is *non-Einstein*, and cannot therefore describe an Einstein universe, (8) can be written as,

$$ds^2=-\frac{3}{\lambda}\left(d\theta^2+\sin^2\theta d\varphi^2\right). \quad (8b)$$

This means that metric (8)$\equiv$(8b) maps the whole of (M_s, g_s) into the *point* $R_p(D(r))\equiv 0$ of the de Sitter "space" (M_{ds}, g_{ds}).

Einstein, de Sitter, Eddington, Friedmann, and the modern relativists all, have incorrectly *assumed* that r is a radial coordinate in (8), and consequently think of the "space" associated with (8) as extended in the sense of having a volume greater than zero. This is incorrect.

The radius of curvature of the point $R_p(D(r))\equiv 0$ is,

$$R_c(D(r))\equiv\sqrt{\frac{3}{\lambda}}.$$

The "surface area" of the point is,

$$A=\frac{12\pi}{\lambda}.$$

De Sitter's empty spherical universe has zero volume. Indeed, by (8) and (8b),

$$V=\lim_{r\to\pm\infty}\frac{3}{\lambda}\int_{r_0}^{r}0\,dr\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\varphi=0,$$

consequently, de Sitter's empty spherical universe is indeed "empty"; and meaningless. It is *not* an Einstein universe.

On (8) and (8b) the ratio,

$$\frac{2\pi\sqrt{F(r)}}{R_p(r)}=\infty\ \forall\ r.$$

Therefore, the lone point which consitutes the empty de Sitter "universe" (M_{ds}, g_{ds}) is a quasiregular singularity and consequently cannot be extended.

It is the unproven and invalid assumptions about the variable r which have lead the relativists astray. They have carried this error through all their work and consequently have completely lost sight of legitimate scientific theory, producing all manner of nonsense along the way. Eddington [4], for instance, writes in relation to (1), $\gamma=1-\frac{2m}{r}-\frac{\alpha r^2}{3}$ for his equation (45.3), and said,

> *At a place where γ vanishes there is an impassable barrier, since any change dr corresponds to an infinite distance ids surveyed by measuring rods. The two positive roots of the cubic (45.3) are approximately*
>
> $$r=2m \quad and \quad r=\sqrt{\left(\tfrac{3}{\alpha}\right)}.$$
>
> *The first root would represent the boundary of the particle — if a genuine particle could exist — and give it the appearance of impenetrability. The second barrier is at a very great distance and may be described as the* horizon *of the world.*

Note that Eddington, despite these erroneous claims, did not admit the sacred black hole. His arguments however, clearly betray his assumption that r is a radius on (1). I also note that he has set the constant numerator of the middle term of his γ to $2m$, as is usual, however, like all the modern relativists, he did not indicate how this identity is to be achieved. This is just another assumption. As Abrams [6] has pointed out in regard to (1), one cannot appeal to far-field Keplerian orbits to fix the constant to $2m$ — but the issue is moot, since $\lambda=0$.

There is no black hole associated with (1). The Lake-Roeder black hole is inconsistent with Einstein's theory.

4 The homogeneous static models

It is routinely alleged by the relativists that the static homogeneous cosmological models are exhausted by the line-elements of Einstein's cylindrical model, de Sitter's spherical model, and that of Special Relativity. This is not correct, as I shall now demonstrate that the only homogeneous universe admitted by Einstein's theory is that of his Special Theory of Relativity, which is a static, infinite, pseudo-Euclidean, empty world.

The cosmological models of Einstein and de Sitter are composed of a single world line and a single point respectively, neither of which can be extended. Their line-elements therefore *cannot* describe any Einstein universe.

If the Universe is considered as a continuous distribution of matter of proper macroscopic density ρ_{00} and pressure P_0, the stress-energy tensor is,

$$T_1^1 = T_2^2 = T_3^3 = -P_0\,, \quad T_4^4 = \rho_{00}\,,$$

$$T_\nu^\mu = 0, \quad \mu \neq \nu\,.$$

Rewrite (5) by setting,

$$A^*(r^*) = e^\nu, \quad \nu = \nu(r^*)\,,$$

$$B^*(r^*) = e^\sigma, \quad \sigma = \sigma(r^*)\,. \tag{19}$$

Then (5) becomes,

$$ds^2 = e^\nu dt^2 - e^\sigma dr^{*2} - r^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\,. \tag{20}$$

It then follows in the usual way that,

$$8\pi P_0 = e^{-\sigma}\left(\frac{\bar\nu}{r^*} + \frac{1}{r^{*2}}\right) - \frac{1}{r^{*2}} + \lambda\,, \tag{21}$$

$$8\pi\rho_{00} = e^{-\sigma}\left(\frac{\bar\sigma}{r^*} - \frac{1}{r^{*2}}\right) + \frac{1}{r^{*2}} - \lambda\,, \tag{22}$$

$$\frac{dP_0}{dr^*} = -\frac{\rho_{00}+P_0}{2}\bar\nu\,, \tag{23}$$

where

$$\bar\nu = \frac{d\nu}{dr^*}, \quad \bar\sigma = \frac{d\sigma}{dr^*}\,.$$

Since P_0 is to be the same everywhere, (23) becomes,

$$\frac{\rho_{00}+P_0}{2}\bar\nu = 0\,.$$

Therefore, the following three possibilities arise,

1. $\dfrac{d\nu}{dr^*} = 0$;
2. $\rho_{00} + P_0 = 0$;
3. $\dfrac{d\nu}{dr^*} = 0$ and $\rho_{00} + P_0 = 0$.

The 1st possibility yields Einstein's so-called cylindrical model, the 2nd yields de Sitter's so-called spherical model, and the 3rd yields Special Relativity.

5 Einstein's cylindrical cosmological model

In this case, to reduce to Special Relativity,

$$\nu = \text{const} = 0.$$

Therefore, by (21),

$$8\pi P_0 = \frac{e^{-\sigma}}{r^{*2}} - \frac{1}{r^{*2}} + \lambda\,,$$

and by (19),

$$8\pi P_0 = \frac{1}{B^*(r^*)r^{*2}} - \frac{1}{r^{*2}} + \lambda\,,$$

and by (4),

$$8\pi P_0 = \frac{1}{BC} - \frac{1}{C} + \lambda\,,$$

so

$$\frac{1}{B} = 1 - \left(\lambda - 8\pi P_0\right)C\,,$$

$$C = C(D(r)), \quad D(r) = |r - r_0|, \quad B = B(D(r))\,,$$

$$r_0 \in \Re\,.$$

Consequently, Einstein's line-element can be written as,

$$\begin{aligned} ds^2 = dt^2 - \left[1 - \left(\lambda - 8\pi P_0\right)C\right]^{-1} d\sqrt{C}^{\,2} - \\ - C\left(d\theta^2 + \sin^2 d\varphi^2\right) = \\ = dt^2 - \left[1 - \left(\lambda - 8\pi P_0\right)C\right]^{-1}\frac{C'^2}{4C}dr^2 - \\ - C\left(d\theta^2 + \sin^2 d\varphi^2\right)\,, \end{aligned} \tag{24}$$

$$C = C(D(r)), \quad D(r) = |r - r_0|, \quad r_0 \in \Re\,,$$

where r_0 is arbitrary.

It is now required to determine the admissible form of $C(D(r))$.

Clearly, if $C' \equiv 0$, then $B = 0\ \forall\ r$, in violation of (3). Therefore, $C' \neq 0\ \forall\ r \neq r_0$.

When $P_0 = \lambda = 0$, (24) must reduce to Special Relativity, in which case,

$$P_0 = \lambda = 0 \Rightarrow C(D(r)) = |r - r_0|^2\,.$$

The metric (24) is singular when $g_{11}^{-1}(r_0) = 0$, i.e. when,

$$1 - \left(\lambda - 8\pi P_0\right)C(r_0) = 0\,,$$

$$\Rightarrow\ C(r_0) = \frac{1}{\lambda - 8\pi P_0}\,. \tag{25}$$

Therefore, for $C(D(r))$ to render an admissible solution to (24) in the form of (3), it must at least satisfy the following conditions:

1. $C' \neq 0 \ \forall \ r \neq r_0$;
2. $P_0 = \lambda = 0 \Rightarrow C(D(r)) = |r - r_0|^2$;
3. $C(r_0) = \frac{1}{\lambda - 8\pi P_0}$.

Now the proper radius on (24) is,

$$R_p(r) = \int \frac{d\sqrt{C}}{\sqrt{1 - \left(\lambda - 8\pi P_0\right) C}} = \\ = \frac{1}{\sqrt{\lambda - 8\pi P_0}} \arcsin \sqrt{(\lambda - 8\pi P_0) C(r)} + K ,$$

$$K = \text{const.} ,$$

which must satisfy the condition,

$$\text{as } r \to r_0^{\pm}, \ R_p \to 0^+ .$$

Therefore,

$$R_p(r_0) = 0 = \frac{1}{\sqrt{\lambda - 8\pi P_0}} \times \\ \times \arcsin \sqrt{(\lambda - 8\pi P_0) C(r_0)} + K ,$$

so

$$R_p(r) = \frac{1}{\sqrt{\lambda - 8\pi P_0}} \left[\arcsin \sqrt{(\lambda - 8\pi P_0) C(r)} - \right. \\ \left. - \arcsin \sqrt{(\lambda - 8\pi P_0) C(r_0)} \right] . \tag{26}$$

Now if follows from (26) that,

$$\sqrt{(\lambda - 8\pi P_0) C(r_0)} \leqslant \sqrt{(\lambda - 8\pi P_0) C(r)} \leqslant 1 ,$$

so

$$C(r_0) \leqslant C(r) \leqslant \frac{1}{\left(\lambda - 8\pi P_0\right)} ,$$

and therefore by (25),

$$\frac{1}{\left(\lambda - 8\pi P_0\right)} \leqslant C(r) \leqslant \frac{1}{\left(\lambda - 8\pi P_0\right)} .$$

Thus,

$$C(r) \equiv \frac{1}{\left(\lambda - 8\pi P_0\right)} ,$$

and so $C'(r) \equiv 0 \Rightarrow B(r) \equiv 0$, in violation of (3). Therefore there exists no $C(D(r))$ to satisfy (24) in the form of (3) when $\lambda \neq 0$, $P_0 \neq 0$. Consequently, $\lambda = P_0 = 0$, and (24) reduces to,

$$ds^2 = dt^2 - \frac{C'^2}{4C} dr^2 - C \left(d\theta^2 + \sin^2 d\varphi^2\right) . \tag{27}$$

The form of $C(D(r))$ must still be determined.

Clearly, if $C' \equiv 0$, $B(D(r)) = 0 \ \forall \ r$, in violation of (3). Therefore, $C' \neq 0 \ \forall \ r \neq r_0$.

Since there is no matter present, it is required that,

$$C(r_0) = 0 \quad \text{and} \quad \frac{C(D(r))}{|r - r_0|^2} = 1 .$$

This requires trivially that,

$$C(D(r)) = |r - r_0|^2 .$$

Therefore (27) becomes,

$$ds^2 = dt^2 - \frac{(r - r_0)^2}{|r - r_0|^2} dr^2 - |r - r_0|^2 \left(d\theta^2 + \sin^2 d\varphi^2\right) = \\ = dt^2 - dr^2 - |r - r_0|^2 \left(d\theta^2 + \sin^2 d\varphi^2\right) ,$$

which is precisely the metric of Special Relativity, according to the natural reduction on (2).

If the required form (3) is relaxed, in which case the resulting metric is *not* an Einstein metric, Einstein's cylindrical line-element is,

$$ds^2 = dt^2 - \frac{1}{\left(\lambda - 8\pi P_0\right)} \left(d\theta^2 + \sin^2 \theta d\varphi^2\right) . \tag{28}$$

This is a line-element which cannot describe an Einstein universe. The Einstein space described by (28) consists of only one "world line", through the point,

$$R_p(r) \equiv 0 .$$

The spatial extent of (28) is a single point. The radius of curvature of this point space is,

$$R_c(r) \equiv \frac{1}{\sqrt{\lambda - 8\pi P_0}} .$$

For all r, the ratio $\frac{2\pi R_c}{R_p}$ is,

$$\frac{\frac{2\pi}{\sqrt{\lambda - 8\pi P_0}}}{R_p(r)} = \infty .$$

Therefore $R_p(r) \equiv 0$ is a quasiregular singular point and consequently cannot be extended.

The "surface area" of this point space is,

$$A = \frac{4\pi}{\lambda - 8\pi P_0} .$$

The volume of the point space is,

$$V = \lim_{r \to \pm\infty} \frac{1}{\left(\lambda - 8\pi P_0\right)} \int_{r_0}^{r} 0 \ dr \int_0^{\pi} \sin\theta \ d\theta \int_0^{2\pi} d\varphi = 0 .$$

Equation (28) maps the whole of (M_s, g_s) into a quasiregular singular "world line".

Einstein's so-called "cylindrical universe" is meaningless. It does not contain a black hole.

6 De Sitter's spherical cosmological model

In this case,

$$\rho_{00} + P_0 = 0\,.$$

Adding (21) to (22) and setting to zero gives,

$$8\pi\left(\rho_{00} + P_0\right) = e^{-\sigma}\left(\frac{\bar{\sigma}}{r^*} + \frac{\bar{\nu}}{r^*}\right) = 0\,,$$

or

$$\bar{\nu} = -\bar{\sigma}\,.$$

Therefore,

$$\nu(r^*) = -\sigma(r^*) + \ln K_1\,, \qquad (29)$$

$$K_1 = \text{const.}$$

Since ρ_{00} is required to be a constant independent of position, equation (22) can be immediately integrated to give,

$$e^{-\sigma} = 1 - \frac{\lambda + 8\pi\rho_{00}}{3}\,r^{*2} + \frac{K_2}{r^*}\,, \qquad (30)$$

$$K_2 = \text{const.}$$

According to (30),

$$-\sigma = \ln\left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}\,r^{*2} + \frac{K_2}{r^*}\right),$$

and therefore, by (29),

$$\nu = \ln\left[\left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}\,r^{*2} + \frac{K_2}{r^*}\right)K_1\right].$$

Substituting into (20) gives,

$$ds^2 = \left[\left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}\,r^{*2} + \frac{K_2}{r^*}\right)K_1\right]dt^2 - \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}\,r^{*2} + \frac{K_2}{r^*}\right)^{-1}dr^{*2} - r^{*2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

which is, by (4),

$$ds^2 = \left[\left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}C + \frac{K_2}{\sqrt{C}}\right)K_1\right]dt^2 - \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}C + \frac{K_2}{\sqrt{C}}\right)^{-1}\frac{C'^2}{4C}dr^2 - C\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \qquad (31)$$

Now, when $\lambda = \rho_{00} = 0$, equation (31) must reduce to the metric for Special Relativity. Therefore,

$$K_1 = 1, \quad K_2 = 0\,,$$

and so de Sitter's line-element is,

$$ds^2 = \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}C\right)dt^2 - \left(1 - \frac{\lambda + 8\pi\rho_{00}}{3}C\right)^{-1}\frac{C'^2}{4C}\,dr^2 - C\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (32)$$

$$C = C(D(r)), \quad D(r) = |r - r_0|, \quad r_0 \in \Re\,,$$

where r_0 is arbitrary.

It remains now to determine the admissible form of $C(D(r))$ to render a solution to equation (32) in the form of equation (3).

If $C' \equiv 0$, then $B(D(r)) = 0\ \forall\ r$, in violation of (3). Therefore $C' \neq 0\ \forall\ r \neq r_0$.

When $\lambda = \rho_{00} = 0$, (32) must reduce to that for Special Relativity. Therefore,

$$\lambda = \rho_{00} = 0 \Rightarrow C(D(r)) = |r - r_0|^2\,.$$

Metric (32) is singular when $g_{00}(r_0) = 0$, i.e. when

$$1 - \frac{\lambda + 8\pi\rho_{00}}{3}C(r_0) = 0\,,$$

$$\Rightarrow C(r_0) = \frac{3}{\lambda + 8\pi\rho_{00}}\,. \qquad (33)$$

Therefore, to render a solution to (32) in the form of (3), $C(D(r))$ must at least satisfy the following conditions:

1. $C' \neq 0\ \forall\ r \neq r_0$;
2. $\lambda = \rho_{00} = 0 \Rightarrow C(D(r)) = |r - r_0|^2$;
3. $C(r_0) = \frac{3}{\lambda + 8\pi\rho_{00}}$.

The proper radius on (32) is,

$$R_p(r) = \int \frac{d\sqrt{C}}{\sqrt{1 - \left(\frac{\lambda + 8\pi\rho_{00}}{3}\right)C}} = \sqrt{\frac{3}{\lambda + 8\pi\rho_{00}}}\arcsin\sqrt{\left(\frac{\lambda + 8\pi\rho_{00}}{3}\right)C(r)} + K\,, \qquad (34)$$

$$K = \text{const}\,,$$

which must satisfy the condition,

$$\text{as } r \to r_0^{\pm},\ R_p(r) \to 0^{+}\,.$$

Therefore,

$$R_p(r_0) = 0 = \sqrt{\frac{3}{\lambda + 8\pi\rho_{00}}}\arcsin\sqrt{\left(\frac{\lambda + 8\pi\rho_{00}}{3}\right)C(r_0)} + K\,,$$

so (34) becomes,

$$R_p(r) = \sqrt{\frac{3}{\lambda+8\pi\rho_{00}}}\left[\arcsin\sqrt{\left(\frac{\lambda+8\pi\rho_{00}}{3}\right)C(r)} - \right.$$
$$\left. - \arcsin\sqrt{\left(\frac{\lambda+8\pi\rho_{00}}{3}\right)C(r_0)}\,\right]. \tag{35}$$

It then follows from (35) that,

$$\sqrt{\left(\frac{\lambda+8\pi\rho_{00}}{3}\right)C(r_0)} \leqslant \sqrt{\left(\frac{\lambda+8\pi\rho_{00}}{3}\right)C(r)} \leqslant 1\,,$$

or

$$C(r_0) \leqslant C(r) \leqslant \frac{3}{\lambda+8\pi\rho_{00}}\,.$$

Then, by (33),

$$\frac{3}{\lambda+8\pi\rho_{00}} \leqslant C(r) \leqslant \frac{3}{\lambda+8\pi\rho_{00}}\,.$$

Therefore, $C(r)$ is a constant function for all r,

$$C(r) \equiv \frac{3}{\lambda+8\pi\rho_{00}}\,, \tag{36}$$

and so,

$$C'(r) \equiv 0\,,$$

which implies that $B(D(r)) \equiv 0$, in violation of (3). Consequently, there exists no function $C(D(r))$ to render a solution to (32) in the form of (3). Therefore, $\lambda = \rho_{00} = 0$, and (32) reduces to the metric of Special Relativity in the same way as does (24).

If the required form (3) is relaxed, in which case the resulting metric is *not* an Einstein metric, de Sitter's line-element is,

$$ds^2 = -\frac{3}{\lambda+8\pi\rho_{00}}\left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{37}$$

This line-element cannot describe an Einstein universe. The Einstein space described by (37) consists of only one point:

$$R_p(r) \equiv 0\,.$$

The radius of curvature of this point is,

$$R_c(r) \equiv \sqrt{\frac{3}{\lambda+8\pi\rho_{00}}}\,,$$

and the "surface area" of the point is,

$$A = \frac{12\pi}{\lambda+8\pi\rho_{00}}\,.$$

The volume of de Sitter's "spherical universe" is,

$$V = \left(\frac{3}{\lambda+8\pi\rho_{00}}\right)\lim_{r\to\pm\infty}\int_{r_0}^{r} 0\,dr\int_0^{\pi}\sin\theta\,d\theta\int_0^{2\pi}d\varphi = 0\,.$$

For all values of r, the ratio,

$$\frac{2\pi\sqrt{\frac{3}{\lambda+8\pi\rho_{00}}}}{R_p(r)} = \infty\,.$$

Therefore, $R_p(r) \equiv 0$ is a quasiregular singular point and consequently cannot be extended.

According to (32), metric (37) maps the whole of (M_s, g_s) into a quasiregular singular point.

Thus, de Sitter's spherical universe is meaningless. It does not contain a black hole.

When $\rho_{00} = 0$ and $\lambda \neq 0$, de Sitter's empty universe is obtained from (37). I have already dealt with this case in section 3.

7 The infinite static homogeneous universe of special relativity

In this case, by possibility 3 in section **4**,

$$\bar{\nu} = \frac{d\nu}{dr^*} = 0, \quad \text{and} \quad \rho_{00} + P_0 = 0\,.$$

Therefore,

$$\nu = \text{const} = 0 \quad \text{by section } \mathbf{5}$$

and

$$\bar{\sigma} = -\bar{\nu} \quad \text{by section } \mathbf{6}\,.$$

Hence, also by section **6**,

$$\sigma = -\nu = 0\,.$$

Therefore, (20) becomes,

$$ds^2 = dt^2 - dr^{*2} - r^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

which becomes, by using (4),

$$ds^2 = dt^2 - \frac{C'^2}{4C}dr^2 - C\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

$$C = C(D(r)), \quad D(r) = |r - r_0|, \quad r_0 \in \Re\,,$$

which, by the analyses in sections 5 and 6, becomes,

$$ds^2 = dt^2 - \frac{(r-r_0)^2}{|r-r_0|^2}dr^2 - |r-r_0|^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \tag{38}$$
$$r_0 \in \Re\,,$$

which is the flat, empty, and infinite spacetime of Special Relativity, obtained from (2) by natural reduction.

When $r_0 = 0$ and $r \geqslant r_0$, (38) reduces to the usual form used by the relativists,

$$ds^2 = dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) .$$

The radius of curvature of (38) is,

$$D(r) = |r - r_0| .$$

The proper radius of (38) is,

$$R_p(r) = \int_0^{|r-r_0|} d|r - r_0| = \int_{r_0}^{r} \frac{(r - r_0)}{|r - r_0|} dr = |r - r_0| \equiv D .$$

The ratio,

$$\frac{2\pi D(r)}{R_p(r)} = \frac{2\pi|r - r_0|}{|r - r_0|} = 2\pi \;\forall\; r .$$

Thus, only (38) can represent a static homogeneous universe in Einstein's theory, contrary to the claims of the modern relativists. However, since (38) contains no matter it cannot model the universe other than locally.

8 Cosmological models of expansion

In view of the foregoing it is now evident that the models proposed by the relativists purporting an expanding universe are also untenable in the framework of Einstein's theory. The line-element obtained by the Abbé Lemaître and by Robertson, for instance, is inadmissible. Under the false assumption that r is a radius in de Sitter's spherical universe, they proposed the following transformation of coordinates on the metric (32) (with $\rho_{00} \neq 0$ in the misleading form given in formula 9),

$$\bar{r} = \frac{r}{\sqrt{1 - \frac{r^2}{W^2}}} e^{-\frac{t}{W}} , \quad \bar{t} = t + \frac{1}{2} W \ln\left(1 - \frac{r^2}{W^2}\right) , \tag{39}$$

$$W^2 = \frac{\lambda + 8\pi\rho_{00}}{3} ,$$

to get

$$ds^2 = d\bar{t}^2 - e^{\frac{2\bar{t}}{W}} \left(d\bar{r}^2 + \bar{r}^2 d\theta^2 + \bar{r}^2 \sin^2\theta d\varphi^2\right) ,$$

or, by dropping the bar and setting $k = \frac{1}{W}$,

$$ds^2 = dt^2 - e^{2kt} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right) . \tag{40}$$

Now, as I have shown, (32) has no solution in $C(D(r))$ in the form (3), so transformations (39) and metric (40) are meaningless concoctions of mathematical symbols. Owing to their false assumptions about the parameter r, the relativists mistakenly think that $C(D(r)) \equiv r^2$ in (32). Furthermore, if the required form (3) is relaxed, thereby producing *non-Einstein metrics*, de Sitter's "spherical universe" is given by (37), and so, by (35), (36), and (40),

$$C(D(r)) = r^2 \equiv \frac{\lambda + 8\pi\rho_{00}}{3} ,$$

and the transformations (39) and metric (40) are again utter nonsense. The Lemaître-Robertson line-element is inevitably, unmitigated claptrap. This can be proved generally as follows.

The most general non-static line-element is

$$\begin{aligned} ds^2 = A(D,t)dt^2 - B(D,t)dD^2 - \\ - C(D,t)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \end{aligned} \tag{41}$$

$$D = |r - r_0|, \quad r_0 \in \Re$$

where analytic $A, B, C > 0 \;\forall\; r \neq r_0$ and $\forall\; t$.

Rewrite (41) by setting,

$$A(D,t) = e^{\nu}, \quad \nu = \nu(G(D),t) ,$$

$$B(D,t) = e^{\sigma}, \quad \sigma = \sigma(G(D),t) ,$$

$$C(D,t) = e^{\mu}G^2(D), \quad \mu = \mu(G(D),t) ,$$

to get

$$ds^2 = e^{\nu}dt^2 - e^{\sigma}dG^2 - e^{\mu}G^2(D)\left(d\theta^2 + \sin^2\theta d\varphi^2\right) . \tag{42}$$

Now set,

$$r^* = G(D(r)) , \tag{43}$$

to get

$$ds^2 = e^{\nu}dt^2 - e^{\sigma}dr^{*2} - e^{\mu}r^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \tag{44}$$

$$\nu = \nu(r^*,t) , \quad \sigma = \sigma(r^*,t) , \quad \mu = \mu(r^*,t) .$$

One then finds in the usual way that the solution to (44) is,

$$\begin{aligned} ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4}r^{*2}\right)^2} \times \\ \times \left[dr^{*2} + r^{*2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \end{aligned} \tag{45}$$

where k is a constant.

Then by (43) this becomes,

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4}G^2\right)^2}\left[dG^2 + G^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] ,$$

or,

$$\begin{aligned} ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4}G^2\right)^2} \times \\ \times \left[G'^2 dr^2 + G^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] , \end{aligned} \tag{46}$$

$$G' = \frac{dG}{dr}\,,$$

$$G = G(D(r))\,, \quad D(r) = |r - r_0|\,, \quad r_0 \in \Re\,.$$

The admissible form of $G(D(r))$ must now be determined.

If $G' \equiv 0$, then $B(D,t) = 0\ \forall\ r$ and $\forall\ t$, in violation of (41). Therefore $G' \neq 0\ \forall\ r \neq r_0$.

Metric (46) is singular when,

$$1 + \frac{k}{4} G^2(r_0) = 0\,,$$

$$\Rightarrow\ G(r_0) = \frac{2}{\sqrt{-k}} \Rightarrow k < 0\,. \tag{47}$$

The proper radius on (46) is,

$$R_p(r,t) = e^{\frac{1}{2}g(t)} \int \frac{dG}{1 + \frac{k}{4}G^2} =$$

$$= e^{\frac{1}{2}g(t)} \left(\frac{2}{\sqrt{k}} \arctan \frac{\sqrt{k}}{2} G(r) + K \right),$$

$$K = \text{const}\,,$$

which must satisfy the condition,

$$\text{as } r \to r_0^{\pm},\ R_p \to 0^+\,.$$

Therefore,

$$R_p(r_0,t) = e^{\frac{1}{2}g(t)} \left(\frac{2}{\sqrt{k}} \arctan \frac{\sqrt{k}}{2} G(r_0) + K \right) = 0\,,$$

and so

$$R_p(r,t) = e^{\frac{1}{2}g(t)} \frac{2}{\sqrt{k}} \left[\arctan \frac{\sqrt{k}}{2} G(r) - \arctan \frac{\sqrt{k}}{2} G(r_0) \right]. \tag{48}$$

Then by (47),

$$R_p(r,t) = e^{\frac{1}{2}g(t)} \frac{2}{\sqrt{k}} \left[\arctan \frac{\sqrt{k}}{2} G(r) - \arctan \sqrt{-1} \right], \tag{49}$$

$$k < 0\,.$$

Therefore, there exists no function $G(D(r))$ rendering a solution to (46) in the required form of (41).

The relativists however, owing to their invalid assumptions about the parameter r, write equation (46) as,

$$ds^2 = dt^2 - \frac{e^{g(t)}}{\left(1 + \frac{k}{4}r^2\right)^2} \left[dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right], \tag{50}$$

having assumed that $G(D(r)) \equiv r$, and erroneously take r as a radius on the metric (50), valid down to 0. Metric (50) is a meaningless concoction of mathematical symbols. Nevertheless, the relativists transform this meaningless expression with a meaningless change of "coordinates" to obtain the Robertson-Walker line-element, as follows.

Transform (46) by setting,

$$\bar{G}(\bar{r}) = \frac{G(r)}{1 + \frac{k}{4}G^2}\,.$$

This carries (46) into,

$$ds^2 = dt^2 - e^{g(t)} \left[\frac{d\bar{G}^2}{\left(1 - \kappa \bar{G}^2\right)} + \bar{G}^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right]. \tag{51}$$

This is easily seen to be the familiar Robertson-Walker line-element if, following the relativists, one incorrectly assumes $\bar{G} \equiv \bar{r}$, disregarding the fact that the admissible form of $\bar{G}$ *must be ascertained*. In any event (51) is meaningless, owing to the meaninglessness of (50), which I confirm as follows.

$\bar{G}' \equiv 0 \Rightarrow \bar{B} = 0\ \ \forall\ \bar{r}$, in violation of (41). Therefore $\bar{G}' \neq 0\ \forall\ \bar{r} \neq \bar{r}_0$.

Equation (51) is singular when,

$$1 - k\bar{G}^2(\bar{r}_0) = 0\ \Rightarrow \bar{G}(\bar{r}_0) = \frac{1}{\sqrt{k}}\ \Rightarrow k > 0\,. \tag{52}$$

The proper radius on (51) is,

$$\bar{R}_p = e^{\frac{1}{2}g(t)} \int \frac{d\bar{G}}{\sqrt{1 - k\bar{G}^2}}$$

$$= e^{\frac{1}{2}g(t)} \left(\frac{1}{\sqrt{k}} \arcsin \sqrt{k}\bar{G}(\bar{r}) + K \right),$$

$$K = \text{const.}\,,$$

which must satisfy the condition,

$$\text{as } \bar{r} \to \bar{r}_0^{\pm},\ \bar{R}_p \to 0^+\,,$$

so

$$\bar{R}_p(\bar{r}_0,t) = 0 = e^{\frac{1}{2}g(t)} \left(\frac{1}{\sqrt{k}} \arcsin \sqrt{k}\bar{G}(\bar{r}_0) + K \right).$$

Therefore,

$$\bar{R}_p(\bar{r},t) = e^{\frac{1}{2}g(t)} \frac{1}{\sqrt{k}} \times \left[\arcsin \sqrt{k}\bar{G}(\bar{r}) - \arcsin \sqrt{k}\bar{G}(\bar{r}_0) \right]. \tag{53}$$

Then

$$\sqrt{k}\bar{G}(\bar{r}_0) \leqslant \sqrt{k}\bar{G}(\bar{r}) \leqslant 1\,,$$

or

$$\bar{G}(\bar{r}_0) \leqslant \bar{G}(\bar{r}) \leqslant \frac{1}{\sqrt{k}} .$$

Then by (52),

$$\frac{1}{\sqrt{k}} \leqslant \bar{G}(\bar{r}) \leqslant \frac{1}{\sqrt{k}} ,$$

so

$$\bar{G}(\bar{r}) \equiv \frac{1}{\sqrt{k}} .$$

Consequently, $\bar{G}'(\bar{r}) = 0 \ \forall \ \bar{r}$ and $\forall \ t$, in violation of (41). Therefore, there exists no function $\bar{G}(\bar{D}(\bar{r}))$ to render a solution to (51) in the required form of (41).

If the conditions on (41) are relaxed in the fashion of the relativists, non-Einstein metrics with expanding radii of curvature are obtained. Nonetheless the associated spaces have zero volume. Indeed, equation (40) becomes,

$$ds^2 = dt^2 - e^{2kt}\frac{(\lambda + 8\pi\rho_{00})}{3}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) . \quad (54)$$

This is not an Einstein universe. The radius of curvature of (54) is,

$$R_c(r,t) = e^{kt}\sqrt{\frac{\lambda + 8\pi\rho_{00}}{3}} ,$$

which expands or contracts with the sign of the constant k. Even so, the proper radius of the "space" of (54) is,

$$R_p(r,t) = \lim_{r \to \pm\infty} \int_{r_0}^{r} 0 \ dr \equiv 0 .$$

The volume of this point-space is,

$$V = \lim_{r \to \pm\infty} e^{2kt}\frac{(\lambda + 8\pi\rho_{00})}{3} \int_{r_0}^{r} 0 \ dr \int_{0}^{\pi} \sin\theta \ d\theta \int_{0}^{2\pi} \equiv 0 .$$

Metric (54) consists of a single "world line" through the point $R_p(r,t) \equiv 0$. Furthermore, $R_p(r,t) \equiv 0$ is a quasi-regular singular point-space since the ratio,

$$\frac{2\pi e^{kt}\sqrt{\lambda + 8\pi\rho_{00}}}{\sqrt{3}R_p(r,t)} \equiv \infty .$$

Therefore, $R_p(r,t) \equiv 0$ cannot be extended.

Similarly, equation (51) becomes,

$$ds^2 = dt^2 - \frac{e^{g(t)}}{k}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) , \quad (55)$$

which is not an Einstein metric. The radius of curvature of (55) is,

$$R_c(r,t) = \frac{e^{\frac{1}{2}g(t)}}{\sqrt{k}} ,$$

which changes with time. The proper radius is,

$$R_p(r,t) = \lim_{r \to \pm\infty} \int_{r_0}^{r} 0 \ dr \equiv 0 ,$$

and the volume of the point-space is

$$V = \lim_{r \to \pm\infty} \frac{e^{g(t)}}{k} \int_{r_0}^{r} 0 \ dr \int_{0}^{\pi} \sin\theta \ d\theta \int_{0}^{2\pi} \equiv 0 .$$

Metric (55) consists of a single "world line" through the point $R_p(r,t) \equiv 0$. Furthermore, $R_p(r,t) \equiv 0$ is a quasi-regular singular point-space since the ratio,

$$\frac{2\pi e^{\frac{1}{2}g(t)}}{\sqrt{k}R_p(r,t)} \equiv \infty .$$

Therefore, $R_p(r,t) \equiv 0$ cannot be extended.

It immediately follows that the Friedmann models are all invalid, because the so-called Friedmann equation, with its associated equation of continuity, $T^{\mu\nu}_{;\mu} = 0$, is based upon metric (51), which, as I have proven, has *no solution* in $\overline{G}(\overline{r})$ in the required form of (41). Furthermore, metric (55) cannot represent an Einstein universe and therefore has no cosmological meaning. Consequently, the Friedmann equation is also nothing more than a meaningless concoction of mathematical symbols, destitute of any physical significance whatsoever. Friedmann incorrectly assumed, just as the relativists have done all along, that the parameter r is a radius in the gravitational field. Owing to this erroneous assumption, his treatment of the metric for the gravitational field violates the inherent geometry of the metric and therefore violates the geometrical form of the pseudo-Riemannian spacetime manifold. The same can be said of Einstein himself, who did not understand the geometry of his own creation, and by making the same mistakes, failed to understand the implications of his theory.

Thus, the Friedmann models are all invalid, as is the Einstein-de Sitter model, and all other general relativistic cosmological models purporting an expansion of the universe. Furthermore, there is no general relativistic substantiation of the Big Bang hypothesis. Since the Big Bang hypothesis rests solely upon an invalid interpretation of General Relativity, it is abject nonsense. The standard interpretations of the Hubble-Humason relation and the cosmic microwave background are not consistent with Einstein's theory. Einstein's theory cannot form the basis of a cosmology.

9 Singular points in Einstein's universe

It has been pointed out before [7, 8, 3] that singular points in Einstein's universe are quasiregular. No curvature type singularities arise in Einstein's universe. The oddity of a point being associated with a non-zero radius of curvature is an inevitable consequence of Einstein's geometry. There is *nothing* more pointlike in Einstein's universe, and nothing more pointlike in the de Sitter point world or the Einstein cylindrical world line. A point as it is usually conceived of in Minkowski space *does not exist* in Einstein's universe. The modern relativists have not understood this inescapable fact.

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Dedication

I dedicate this paper to the memory of Dr. Leonard S. Abrams: (27 Nov. 1924 — 28 Dec. 2001).

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