Sources of Stellar Energy and the Theory of the Internal Constitution of Stars

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This is a presentation of research into the inductive solution to the problem on the internal constitution of stars. The solution is given in terms of the analytic study of regularities in observational astrophysics. Conditions under which matter exists in stars are not the subject of a priori suppositions, they are the objects of research.

In the first part of this research we consider two main correlations derived from observations: "mass-luminosity" and "period — average density of Cepheids". Results we have obtained from the analysis of the correlations are different to the standard theoretical reasoning about the internal constitution of stars. The main results are: (1) in any stars, including even super-giants, the radiant pressure plays no essential part — it is negligible in comparison to the gaseous pressure; (2) inner regions of stars are filled mainly by hydrogen (the average molecular weight is close to $1/2$); (3) absorption of light is derived from Thomson dispersion in free electrons; (4) stars have an internal constitution close to polytropic structures of the class $3/2$.

The results obtained, taken altogether, permit calculation of the physical conditions in the internal constitution of stars, proceeding from their observational characteristics L, M, and R. For instance, the temperature obtained for the centre of the Sun is about 6 million degrees. This is not enough for nuclear reactions.

In the second part, the Russell-Hertzsprung diagram, transformed according to physical conditions inside stars shows: the energy output inside stars is a simple function of the physical conditions. Instead of the transection line given by the heat output surface and the heat radiation surface, stars fill an area in the plane of density and temperature. The surfaces coincide, being proof of the fact that there is only one condition — the radiation condition. Hence stars generate their energy not in any reactions. Stars are machines, directly generating radiations. The observed diagram of the heat radiation, the relation "mass-luminosity-radius", cannot be explained by standard physical laws. Stars exist in just those conditions where classical laws are broken, and a special mechanism for the generation of energy becomes possible. Those conditions are determined by the main direction on the diagram and the main point located in the direction. Physical coordinates of the main point have been found using observational data. The constants (physical coordinates) should be included in the theory of the internal constitution of stars which pretend to adequately account for observational data. There in detail manifests the inconsistency of the explanations of stellar energy as given by nuclear reactions, and also calculations as to the percentage of hydrogen and helium in stars.

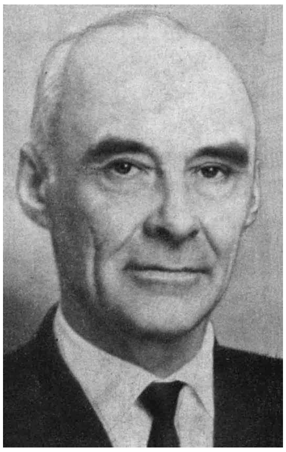
Also considered are peculiarities of some sequences in the Russell-Hertzsprung diagram, which are interesting from the theoretical viewpoint.

*Editor's remark: This is the doctoral thesis of Nikolai Aleksandrovich Kozyrev (1908–1983), the famous astronomer and experimental physicist — one of the founders of astrophysics in the 1930's, the discoverer of lunar volcanism (1958), and the atmosphere of Mercury (1963) (see the article *Kozyrev* in the *Encyclopaedia Britannica*). Besides his studies in astronomy, Kozyrev contributed many original experimental and theoretical works in physics, where he introduced the "causal or asymmetrical mechanics" which takes the physical properties of time into account. See his articles reporting on his many years of experimental research into the physical properties of time, *Time in Science and Philosophy* (Prague, 1971) and *On the Evolution of Double Stars*, *Comptes rendus* (Bruxelles, 1967). Throughout his scientific career Kozyrev worked at the Pulkovo Astronomical Observatory near St. Petersburg (except for the years 1946–1957 when he worked at the Crimean branch of the Observatory). In 1936 he was imprisoned for 10 years without judicial interdiction, by the communist regime in the USSR. Set free in 1946, he completed the draft of this doctoral thesis and published it in Russian in the local bulletin of the Crimean branch of the Observatory (*Proc. Crimean Astron. Obs.*, 1948, v. 2, and 1951, v. 6). Throughout the subsequent years he continued to expand upon his thesis. Although this research was started in the 1940's, it remains relevant today, because the basis here is observational data on stars of regular classes. This data has not changed substantially during the intervening decades. (Translated from the final Russian text by D. Rabounski and S. J. Crothers.)

The author dedicates this paper to the blessed memory of Prof. Aristarch A. Belopolski

Contents

Introduction



Prof. Nikolai Kozyrev, 1970's

Energy, radiated by the Sun and stars into space, is maintained by special sources which should keep stars radiating light during at least a few billion years. The energy sources should be dependent upon the physical conditions of matter inside stars. It follows from this fact that stars are stable space bodies. During the last decade, nuclear physics discovered thermonuclear reactions that could be the energy source satisfying the above requirements. The reactions between protons and numerous light nuclei, which result in transformations of hydrogen into helium, can be initiated under temperatures close to the possible temperature of the inner regions of stars — about 20 million degrees. Comparing different thermonuclear reactions, Bethe concluded that the energy of the Sun and other stars of the main sequence is generated in cyclic reactions where the main part is played by nitrogen and carbon nuclei, which capture protons and then produce helium nuclei [1]. This theory, developed by Bethe and widely regarded in recent years, has had no direct astrophysical verification until now. Stars produce various amounts of energy, e. g. stars of the giants sequence have temperatures much lower than that which is necessary for thermonuclear reactions, and the presence of bulk convection in upper shells of stars, supernova explosions, peculiar ultra-violet spectra lead to the conclusion that energy is generated even in the upper shells of stars and, sometimes, it is explosive. It is quite natural to inquire as to a general reason for all the phenomena. Therefore we should be more accurate in our attempts to apply the nuclear reaction theory to stars. It is possible to say (without exaggeration), that during the last century, beginning with Helmholtz's contraction hypothesis, every substantial discovery in physics led to new attempts to explain stellar energy. Moreover, after every attempt it was claimed that this problem was finally solved, despite the fact that there was no verification in astrophysical data. It is probable that there is an energy generation mechanism of a particular kind, unknown in an Earthly laboratory. At the same time, this circumstance cannot be related to a hypothesis that some exclusive conditions occur inside stars. Conditions inside many stars (e. g. the infrared satellite of ε Aurigae) are close to those that can be realized in the laboratory. The reason that such an energy generation mechanism remained elusive in experiments is due to peculiarities in the experiment statement and, possibly, in the necessity for large-scale considerations in the experiment. Considering physical theories, it is possible that their inconsistency in the stellar energy problem arises for the reason that the main principles of interaction between matter and radiant energy need to be developed further.

Much of the phenomena and empirical correlations discovered by observational astrophysics are linked to the problem of the origin of stellar energy, hence the observational data have no satisfying theoretical interpretation. First, it is related to behaviour of a star as a whole, i. e. to problems associated with the theory of the internal constitution of stars. Today's theories of the internal constitution of stars are built upon a priori assumptions about the behaviour of matter and energy in stars. One tests the truth or falsity of the theories by comparing the results of the theoretical analysis to observational data. This is one way to build various models of stars, which is very popular nowadays. But such an approach cannot be very productive, because the laws of Nature are sometimes so unexpected that many such trials, in order to guess them, cannot establish the correct solution. Because empirical correlations, characterizing a star as a whole, are surely obtained from observations, we have therein a possibility of changing the whole statement of the problem, formulating it in another way — considering the world of stars as a giant laboratory, where matter and radiant energy can be in enormously different scales of states, and proceeding from our analysis of observed empirical correlations obtained in the stellar laboratory, having made no arbitrary assumptions, we can find conditions governing the behaviour of matter and energy in stars as some unknown terms in the correlations, formulated as mathematical equations. Such a problem can seems hopelessly intractable, owing to so many unknown terms. Naturally, we do not know: (1) the phase state of matter — Boltzmann gas, Fermi gas, or something else; (2) the manner of energy transfer — radiation or convection — possible under some mechanism of energy generation; (3) the rôle of the radiant pressure inside stars, and other factors linked to the radiant pressure, namely — (4) the value of the absorption coefficient; (5) chemical composition of stars, i. e. the average numerical value of the molecular weight inside stars, and finally, (6) the mechanism generating stellar energy. To our good fortune is the fact that the main correlation of observational astrophysics, that between mass and luminosity of stars, although giving no answer as to the origin of stellar energy, gives data about the other unknowns. Therefore, employing the relation "period — average density of Cepheids", we make more precise our conclusions about the internal constitution of stars. As a result there is a possibility, even without knowledge of the origin of stellar energy, to calculate the physical conditions inside stars by proceeding from their observable characteristics: luminosity L, mass M, and radius R. On this basis we can interpret another correlation of observational astrophysics, the Russell-Hertzsprung diagram — the correlation between temperature and luminosity of stars, which depends almost exclusively on the last unknown (the mechanism generating stellar energy). The formulae obtained are completely unexpected from the viewpoint of theoretical physics. At the same time they are so typical that we have in them a possibility of studying the physical process which generates stellar energy.

This gives us an inductive method for determining a solution to the problem of the origin of stellar energy. Following this method we use some standard physical laws in subsequent steps of this research, laws which may be violated by phenomenology . However this circumstance cannot invalidate this purely astrophysical method. It only leads to the successive approximations so characteristic of the phenomenological method. Consequently, the results we have obtained in Part I can be considered as the first order of approximation.

The problem of the internal constitution of stars has been very much complicated by many previous theoretical studies. Therefore, it is necessary to consider this problem from the outset with the utmost clarity. Observations show that a star, in its regular duration, is in a balanced or quasi-balanced state. Hence matter inside stars should satisfy conditions of mechanical equilibrium and heat equilibrium. From this we obtain two main equations, by which we give a mathematical formulation of our problem. Considering the simplest case, we neglect the rotation of a star and suppose it spherically symmetric.

PART I

Chapter 1

Deducing the Main Equations of Equilibrium in Stars

1.1 Equation of mechanical equilibrium

Let us denote by P the total pressure, i. e. the sum of the gaseous pressure p and the radiant energy pressure B, taken at a distance r from the centre of a star. The mechanical equilibrium condition requires that the change of P in a unit of distance along the star's radius must be kept in equilibrium by the weight of a unit of the gas volume

$$\frac{dP}{dr} = -g\rho\,, \tag{1.1}$$

where ρ is the gas density, g is the gravity force acceleration. If φ is the gravitational potential

$$g = -\operatorname{grad}\varphi\,, \tag{1.2}$$

and the potential satisfies Poisson equation

$$\nabla^2\varphi = -4\pi G\rho\,,$$

where $G = 6.67{\times}10^{-8}$ is the gravitational constant. For spherical symmetry,

$$\nabla^2\varphi = \operatorname{div}\operatorname{grad}\varphi = \frac{1}{r^2}\frac{dr^2\operatorname{grad}\varphi}{dr}\,. \tag{1.4}$$

Comparing the equalities, we obtain the equation of mechanical equilibrium for a star

$$\frac{1}{\rho r^2}\frac{d}{dr}\left[\frac{r^2 dP}{\rho dr}\right] = -4\pi G\,, \tag{1.5}$$

where

$$P = p + B\,. \tag{1.6}$$

Radiations are almost isotropic inside stars. For this reason B equals one third of the radiant energy density. As we show in the next paragraph, we can put the radiant energy density, determined by the Stephan-Boltzmann law, in a precise form. Therefore,

$$B = \frac{1}{3}\,\alpha T^4, \tag{1.7}$$

where $\alpha = 7.59{\times}10^{-15}$ is Stephan's constant, T is the absolute temperature. The pressure P depends, generally speaking, upon the matter density and the temperature. This correlation is given by the matter phase state. If the gas is ideal, it is

$$p = nkT = \frac{\Re T}{\mu}\,\rho\,. \tag{1.8}$$

Here n is the number of particles in a unit volume of the gas, $k{=}1.372{\times}10^{-16}$ is Boltzmann's constant, $\Re{=}8.313{\times}10^{7}$ is Clapeyron's constant, μ is the average molecular weight.

For example, in a regular Fermi gas the pressure depends only on the density

$$p = K\rho^{5/3}, \quad K = \mu_e^{5/3} K_{\text{H}}\,, \quad K_{\text{H}} = 9.89{\times}10^{12}, \tag{1.9}$$

where μ_e is the number of the molecular weight units for each free electron.

We see that the pressure distribution inside a star can be obtained from (1.5) only if we know the temperature distribution. The latter is determined by the heat equilibrium condition.

1.2 Equation of heat equilibrium

Let us denote by ε the quantity of energy produced per second by a unit mass of stellar matter. The quantity ε is dependent upon the physical conditions of the matter in a star, so ε is a function of the radius r of a star. To study ε is the main task of this research. The heat equilibrium condition (known also as the energy balance condition) can be written as follows

$$\operatorname{div} F = \varepsilon\,\rho\,, \tag{1.10}$$

where F is the total flow of energy, being the sum of the radiant energy flow F_R, the energy flow F_c dragged by convection currents, and the heat conductivity flow F_T

$$F = F_R + F_c + F_T\,. \tag{1.11}$$

First we determine F_R. Radiations, being transferred through a layer of thickness ds, change their intensity I through the layer of thickness ds, according to Kirchhoff's law

$$\frac{dI}{ds} = -\kappa\rho\big(I - E\big)\,, \tag{1.12}$$

where κ is the absorption coefficient per unit mass, E is the radiant productivity of an absolute black body (calculated per unit of solid angle ω). In polar coordinates this equation is

$$\cos\theta\,\frac{\partial I}{\partial r} - \frac{\sin\theta}{r}\frac{\partial I}{\partial\theta} = -\kappa\rho\big(I - E\big)\,, \tag{1.12a}$$

where θ is the angle between the direction of the normal to the layer (the direction along the radius r) and the radiation direction (the direction of the intensity I). The flow F_R and the radiant pressure B are connected to the radiation intensity by the relations

$$F_R = \int I\cos\theta d\omega\,, \qquad Bc = \int I\cos^2\theta\, d\omega\,, \tag{1.13}$$

where c is the velocity of light, while the integration is taken over all solid angles. We denote

$$\int I d\omega = J\,. \tag{1.14}$$

Multiplying (1.12a) by $\cos\theta$ and taking the integral over all solid angles $d\omega$, we have

$$c\,\frac{dB}{dr} - \frac{1}{r}\left(J - 3Bc\right) = -\kappa\rho F_R\,.$$

In order to obtain F_R we next apply Eddington's approximation

$$3Bc = J = 4\pi E\,, \tag{1.15}$$

thereby taking F_R to within high order terms. Then

$$F_R = -\frac{c}{\kappa\rho}\frac{dB}{dr}\,. \tag{1.16}$$

Let us consider the convective energy flow F_c. Everyday we see huge convection currents in the surface of the Sun (it is possible this convection is forced by sudden production of energy). To make the convective energy flow F_c substantial, convection currents of matter should be rapid and cause transfer of energy over long distances in a star. Such conditions can be in regions of unstable convection of matter, where free convection can be initiated. Schwarzschild's pioneering research [2], and subsequent works by other astrophysicists (Unsöld, Cowling, Bierman and others) showed that although a star is in the state of stable mechanical and heat equilibrium as a whole, free convection can start in regions where (1) stellar energy sources rapidly increase their power, or (2) the ionization energy is of the same order as the heat energy of the gas.

We assume convection currents flowing along the radius of star. We denote by Q the total energy per unit of convection current mass. Hence, Q is the sum of the inner energy of the gas, the heat function, the potential and kinetic energies. We regularly assume that a convection current retains its own energy along its path, i. e. it changes adiabatically, and dissipation of its energy occurs only when the current stops. Then the energy flow transferred by the convection, according to Schmidt [3], is

$$F_c = -A\rho\frac{dQ}{dr}, \qquad A = \bar{v}\bar{\lambda}. \tag{1.17}$$

The quantity A is the convection coefficient, $\bar{\lambda}$ is the average length travelled by the convection current, $\bar{v}$ is the average velocity of the current. If the radiant pressure is negligible in comparison to the gaseous pressure, in an ideal gas (according to the 1st law of thermodynamics) we have

$$\frac{dQ}{dr} = c_v\frac{dT}{dr} + p\frac{d\,\frac{1}{\rho}}{dr}, \tag{1.18}$$

or, in another form,

$$\frac{dQ}{dr} = c_p\frac{dT}{dr} - \frac{1}{\rho}\frac{dp}{dr}, \tag{1.18}$$

where c_v is the heat capacity of the gas under constant volume, $c_p$is the heat capacity under constant pressure

$$c_p = c_v + \frac{\Re}{\mu}.$$

Denoting

$$\frac{c_p}{c_v} = \Gamma,$$

we have

$$c_p = \frac{\Gamma}{\Gamma - 1}\frac{\Re}{\mu}. \tag{1.20}$$

After an obvious transformation we arrive at the formulae

$$\frac{dQ}{dr} = -\frac{1}{\rho}\frac{dp}{dr}u, \qquad u = 1 - \frac{\Gamma}{4(\Gamma-1)}\frac{pdB}{Bdp}, \tag{1.21}$$

(for a monatomic gas $\Gamma = 5/3$).

The heat conductivity flow has a formula analogous to (1.17). Because particles move in any direction in a gas, in the formula for A we have one third of the average velocity of particles instead of $\bar{v}$. In this case dQ/dr is equal to only the first term of equation (1.18), and so dQ/dr has the same-order numerical value that it has in the energy convective flow F_c. Therefore, taking A from F_c (1.17) into account, we see that F_c is much more that F_T. In only very rare exceptions, like a degenerate gas, can the heat conductivity flow F_T be essential for energy transfer.

Using formulae (1.10), (1.16), (1.17), (1.21), we obtain the heat equilibrium equation

$$\frac{1}{\rho r^2}\frac{1}{dr}\left[\frac{r^2 db}{\kappa\rho dr}\right] - \frac{1}{c\rho r^2}\frac{1}{dr}\left[r^2 Au\frac{dp}{dr}\right] = -\frac{\varepsilon}{c}. \tag{1.22}$$

We finally note that, because ε is tiny value in comparison to the radiation per mass unit, even tiny changes in the state of matter should break the equalities. Therefore even for large regions in stars the heat equilibrium condition (1.10) can be locally broken. The same can be said about the equation for the convective energy flow, because huge convections in stars can be statistically interpreted in only large surfaces like that of a whole star. Therefore the equations we have obtained can be supposed as the average along the whole radius of a star, and taken over a long time. Then the equations are true.

The aforementioned limitations do not matter in our analysis because we are interested in understanding the behaviour of a star as a whole.

1.3 The main system of the equations. Transformation of the variables

In order to focus our attention on the main task of this research, we begin by considering the equations obtained for equilibrium in the simplest case: (1) in the mechanical equilibrium equation we assume the radiant pressure B negligible in comparison to the gaseous pressure p, while (2) in the heat equilibrium equation we assume the convection term negligible. Then we obtain the main system of the equations in the form

$$\left.\begin{aligned} \frac{1}{\rho r^2}\frac{d}{dr}\left[\frac{r^2 dp}{\rho dr}\right] &= -4\pi G, \\ \frac{1}{\rho r^2}\frac{d}{dr}\left[\frac{r^2 dB}{\kappa\rho dr}\right] &= -\frac{\varepsilon}{c}. \end{aligned}\right\} \tag{I}$$

The radiant pressure depends only on the gas temperature T, according to formula (1.7). The absorption coefficient κ (taken per unit mass) depends p and B. This correlation is unknown. Also unknown is the energy ε produced by a unit mass of gas. Let us suppose the functions known. Then in order to solve the system we need to have the state equation of matter, connecting ρ, p, and B. In this case only two functions remain unknown: for instance p and B, whose dependence on the radius r is fully determined by equations (I). These functions should satisfy the following boundary conditions. In the surface of a star the total energy flow is $F_0 = F_{R_0}$ ($F_c = F_T = 0$). According formula (1.13),

$$F_{R_0} = \frac{1}{2}J_0 = \frac{3}{2}cB_0,$$

so, taking formula (1.16) into account, we obtain the condition in the surface of a star

$$\text{under } p = 0 \text{ we have } B = -\frac{2}{3}\frac{dB}{\kappa\rho dr}, \tag{1.23}$$

From equations (I) we see that the finite solution condition under $r = 0$ is the same as

$$\text{under } r = 0 \text{ we have } \frac{dp}{dr} = 0\,, \quad \frac{dB}{dr} = 0\,. \qquad (1.24)$$

The boundary conditions are absolutely necessary, they are true at the centre of any real star. The theory of the inner constitution of stars by Milne [4], built on solutions which do not satisfy these boundary conditions, does not mean that the boundary conditions are absolutely violated by the theory. In layers located far from the centre the boundary solutions can be realized, if derivatives of physical characteristics of matter are not continuous functions of the radius, but have breaks. Hence, Milne's theory permits a break a priori in the state equation of matter, so the theory permits stellar matter to exist in at least two different states. Following this hypothetical approach as to the properties of stellar matter, we can deduce conclusions about high temerpatures and pressures in stars. Avoiding the view that "peculiar" conditions exist in stars, we obtain a natural way of starting our research into the problem by considering the phase state equations of matter.

Hence we carry out very important transformations of the variables in the system (I). Instead of r and other variables we introduce dimensionless quantities bearing the same physical conditions. We denote by index c the values of the functions in the centre of a star ($r = 0$). Instead of r we introduce a dimensionless quantity x according to the formula

$$x = ar\,, \qquad a = \rho_c \sqrt{\frac{4\pi G}{p_c}}\,, \qquad (1.25)$$

and we introduce functions

$$\rho_1 = \frac{\rho}{\rho_c}\,, \quad p_1 = \frac{p}{p_c}\,, \quad B_1 = \frac{B}{B_c}\,, \quad \ldots \qquad (1.26)$$

Then, as it is easy to check, the system (I) transforms to the form

$$\left.\begin{aligned} &\frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dp_1}{\rho_1 dx}\right] = -1\,, \\ &\frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\kappa_1 \rho_1 dx}\right] = -\lambda\varepsilon_1\,, \end{aligned}\right\} \qquad \text{(Ia)}$$

where

$$\lambda = \frac{\varepsilon_c \kappa_c}{4\pi G c \gamma_c}\,, \qquad \gamma_c = \frac{B_c}{p_c}\,. \qquad (1.27)$$

Numerical values of all functions in the system (Ia) are between 0 and 1. Then the conditions at in the centre of a star ($x = 0$) take the form

$$p_1 = 1\,, \quad \frac{dp_1}{dx} = 0\,, \quad B_1 = 1\,, \quad \frac{dB_1}{dx} = 0\,. \qquad (1.28)$$

In the surface of a star ($x = x_0$), instead of (1.23), we can use the simple conditions

$$B_1 = 0\,, \qquad p_1 = 0\,. \qquad (1.29)$$

Here we can write the main system of the equations in terms of the new variables (Ia), taking convection into account. Because of (1.22), we obtain

$$\left.\begin{aligned} &\frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dp_1}{\rho_1 dx}\right] = -1\,, \\ &\frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\kappa_1 \rho_1 dx}\right] - \frac{\kappa_c \rho_c}{c\gamma_c}\frac{1}{\rho_1 x^2}\left[x^2 A u \frac{dp_1}{dx}\right] = -\lambda\varepsilon_1\,. \end{aligned}\right\} \qquad \text{(II)}$$

For an ideal gas, equation (1.21) leads to a very simple formula for u

$$u = 1 - \frac{\Gamma}{4(\Gamma - 1)}\frac{p_1\,dB_1}{B_1\,dp_1}\,. \qquad (1.30)$$

Owing to (1.5) and (1.6) it follows at last that the main system of the equations, taking the radiant pressure into account in the absence of convection, takes the form

$$\left.\begin{aligned} &\frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 d(p_1 + \gamma_c B_1)}{\rho_1 dx}\right] = -1\,, \\ &\frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\kappa_1 \rho_1 dx}\right] = -\lambda\varepsilon_1\,. \end{aligned}\right\} \qquad \text{(III)}$$

Chapter 2

Analysis of the Main Equations and the Relation "Mass-Luminosity"

2.1 Observed characteristics of stars

Astronomical observations give the following quantities characterizing star: radius R, mass M, and luminosity L (the total energy radiated by a star per second). We are going to consider correlations between the quantities and parameters of the main system of the star equilibrium equations. As a result, the main system of the equations considered under any phase state of stellar matter includes only two parameters characterizing matter and radiation inside a star: B_c and p_c.

Because of formula (1.25), we obtain

$$R = \frac{1}{\rho_c}\sqrt{\frac{p_c}{4\pi G}}\,x_0\,, \qquad (2.1)$$

where x_0 is the value of x at the surface of a star, where $p_1 = B_1 = 0$. With this formula, and introducing a state equation of matter, we can easily obtain the correlation $R = f(B_c, p_c)$. It should be noted that in the general case the value of x_0 in formula (2.1) is dependent on B_c and p_c. At the same time, because the equation system consists of functions variable between 0 and 1, the value of x_0 should be of the same order (i. e. close to 1). Therefore the first multiplier in (2.1) plays the main rôle.

Because of

$$M = 4\pi \int_0^R \rho r^2 dr\,,$$

we have

$$M = \frac{p_c^{3/2}}{G^{3/2}\sqrt{4\pi}\,\rho_c^2}\,M_{x_0}\,, \tag{2.2}$$

where

$$M_{x_0} = \int_0^{x_0} \rho_1 x^2 dx\,.$$

At last, the total luminosity of star is

$$L = 4\pi \int_0^R \varepsilon\,\rho r^2 dr\,,$$

and we obtain

$$\frac{L}{M} = \varepsilon_c \frac{L_{x_0}}{M_{x_0}}\,, \qquad L_{x_0} = \int_0^{x_0} \varepsilon_c\,\rho_1 x^2 dx\,. \tag{2.3}$$

Values of the quantities M_{x_0} and L_{x_0} should change a little under changes of p_c and B_c, remaining close to 1. If x_0, M_{x_0}, and L_{x_0} are the same for numerous stars, such stars are homological, so the stars actually have the *same structure*.

As it is easy to see, the average density $\bar{\rho}$ of star is connected to ρ_c by the formula

$$\bar{\rho} = \rho_c \frac{3M_{x_0}}{x_0^3}\,. \tag{2.4}$$

We find a formula for the total potential energy Ω of star thus

$$\Omega = -G \int_0^R \frac{M_r}{r}\,dM_r\,.$$

Multiplying the term under the integral by R, and dividing by M^2, we obtain

$$\Omega = -\frac{GM^2}{R}\,\Omega_{x_0} \tag{2.5}$$

and also

$$\Omega_{x_0} = \frac{x_0}{M_{x_0}^2} \int_0^{x_0} x\,\rho_1 M_x\,dx\,.$$

Under low radiant pressure, taking the equation of mechanical equilibrium into account, the system (I) gives

$$\int_0^{x_0} x\,\rho_1 M_x\,dx = -\int_0^{x_0} x^3 dp_1 = 3\int_0^{x_0} x^2 p_1 dx\,, \tag{2.5a}$$

from which we obtain

$$\Omega_{x_0} = \frac{3x_0 \displaystyle\int_0^{x_0} p_1 x^2 dx}{\left[\displaystyle\int_0^{x_0} \rho_1 x^2 dx\right]^2}\,. \tag{2.6}$$

Because all the functions included in the main system of equations can be expressed through B_1 and p_1, we can find the functions from the system of the differential equations with respect to two parameters B_c and p_c. Boundary conditions (1.28) are enough to find the solutions at the centre of a star. Hence, boundary conditions at the surface of a star (1.29) are true under only some relations between B_c and p_c. Therefore all quantities characterizing a star are functions of only one of two parameters, for instance B_c: $R = f_1(B_c)$, $M = f_2(B_c)$, $L = f_3(B_c)$. This circumstance, with the same chemical composition of stars, gives the relations: (1) "mass-luminosity" $L = \varphi_1(M)$ and (2) the Russell-Hertzsprung diagram $L = \varphi_2(R)$.

From the above we see that the equilibrium of stars has this necessary consequence: correlations between M, L, and R. Thus the correlations discovered by observational astrophysics can be predicted by the theory of the inner constitution of stars.

2.2 Stars of polytropic structure

Solutions to the main system of the equations give functions $p_1(x)$ and $B_1(x)$. Hence, solving the system we can as well obtain $B_1(p_1)$. If we set up a phase state, we can as well obtain the function $p_1(\rho_1)$.

Let us assume $p_1(\rho_1)$ as $p_1(\rho_1^{\Gamma})$, where Γ is a constant. Such a structure for a star is known as *polytropic*. Having stars of polytropic structure, we can easily find all the functions of x. Therefore, in order to obtain a representation of the solutions in the first instance, we are going to consider stars of polytropic structure. Emden's pioneering research on the internal constitution of stars was done in this way.

The aforementioned polytropic correlation can be used instead of the heat equilibrium equation, so only the first equation remains in the system. We introduce a new variable T_1 which, in an ideal gas, equals the reduced temperature

$$\frac{p_1}{\rho_1} = \rho_1^{\Gamma-1} = T_1\,, \tag{2.7}$$

or, in another form,

$$\rho_1 = T_1^n\,, \qquad n = \frac{1}{\Gamma - 1}\,, \qquad p_1 = T_1^{n+1}\,, \tag{2.7a}$$

so that we obtain

$$dp_1 = (n+1)\,T_1^n\,dT\,.$$

Substituting the formulae into the first equation of the main system (I), we obtain

$$E\left[T_1'\right] = \frac{1}{x_1^2}\frac{1}{dx_1}\left[x_1^2\frac{dT_1}{dx_1}\right] = -T_1^n\,, \tag{2.8}$$

where a new variable x_1 is introduced instead of x

$$x = \sqrt{n+1}\,x_1\,. \tag{2.9}$$

Emden's equation (2.8) can be integrated very easily if $n=0$ or $n=1$. Naturally, under $n=0$ (a star of constant density) we obtain

$$p_1 = T_1 = 1 - \frac{x_1^2}{6}\,, \tag{2.10}$$

so the remaining characteristics can be calculated just as easily. Under $n=1$ the substitution $n=T_1\,x_1$ reduces the differential equation (2.8) to the simple form $n''=-n$. Hence, under $n=1$, we have

$$T_1 = \frac{\sin x_1}{x_1}\,, \qquad p_1 = \frac{\sin^2 x_1}{x_1^2}\,. \tag{2.11}$$

With other polytropic indices n, we obtain solutions which are in series. All odd derivatives of the operator E should become zero under $x_1=0$. For even derivatives, we have

$$E_0^{(2i)}\left[T_1'\right] = \frac{2i+3}{2i+1}\,T_1^{(2i+2)}(0)\,. \tag{2.12}$$

Now, differentiating equation (2.8), we obtain derivatives in different orders of the function T_1 under $x_1=0$, so we obtain the coefficients of the series expansion. As a result we obtain the series

$$\begin{aligned} T_1 &= 1 - \frac{x_1^2}{3!} + \frac{n}{5!}\,x_1^4 - \frac{n\,(8n-5)}{3\times 7!}\,x_1^6 + \\ &+ \frac{n\,(122n^2-183n+70)}{9\times 9!}\,x_1^8 + \ldots \end{aligned} \tag{2.13}$$

Using (2.13), we move far away from the special point $x_1=0$. Subsequent solutions can be obtained by numerical integration. As a result we construct a table containing characteristics of stellar structures under different n (see Table 1).

The case of $n=3/2$ corresponds to an adiabatic change of the state of monatomic ideal gas ($\Gamma=5/3$) and also a regular Fermi gas (1.9). If $n=3$, we get a relativistic Fermi gas or an ideal gas under $B_1=p_1$ (the latter is known as Eddington's solution).

In polytropic structures we can calculate exact values of Ω_{x_0}. Naturally, the integral of the numerator of (2.6) can be transformed to

$$\int_0^{x_0} p_1 x^2 dx = \int_0^{x_0} T_1\, dM_x = -\int_0^{x_0} M_x \frac{dT_1}{dx}\, dx\,.$$

Emden's equation leads to

$$M_x = -(n+1)\,x^2\,\frac{dT_1}{dx}\,, \tag{2.14}$$

so we obtain

$$\begin{aligned} \int_0^{x_0} p_1 x^2 dx &= \frac{1}{n+1}\int_0^{x_0}\frac{M_x^2}{x^2}\,dx = \\ &= -\frac{M_{x_0}^2}{x_0\,(n+1)} + \frac{2}{n+1}\int_0^{M_{x_0}}\frac{M_x}{x}\,dM_x\,. \end{aligned}$$

Table 1

n	x_0	M_{x_0}	$\frac{x_0^2}{3M_{x_0}}$	Ω_{x_0}
0	2.45	4.90	1.0	3/5
1	4.52	9.04	3.4	3/4
3/2	5.81	11.1	5.9	6/7
2	7.65	12.7	11.4	1
2.5	10.2	14.4	24.1	6/5
3	13.8	16.1	54.4	3/2
3.25	17.0	17.5	88.2	12/7

Formula (2.5a) leads to another relation between the integrals. As a result we obtain

$$\left[1-\frac{6}{n+1}\right]\int_0^{x_0} p_1 x^2 dx = -\frac{M_{x_0}^2}{x_0\,(n+1)}\,,$$

and, substituting this into (2.6), we obtain Ritter's formula

$$\Omega_{x_0} = \frac{3}{5-n}\,. \tag{2.15}$$

This formula, in addition to other conclusions, leads to the fact that a star can have a finite radius only if $n<5$.

2.3 Solution to the simplest system of the equations

To begin, we consider the system (Ia), which is true in the absence of convection and if the radiant pressure is low. The absorption coefficient κ, the quantity of produced energy ε, and the phase state equation of matter, can be represented as products of different power functions p, B, ρ. Then the functions $\kappa_1=\kappa/\kappa_c$, $\varepsilon_1=\varepsilon/\varepsilon_c$, and the phase state equation, are dependent only on p_1, B_1, ρ_1; they have no parameters p_c, B_c, ρ_c. In this case the coefficient λ remains the sole parameter of the system. In this simplest case we study the system (Ia) under further limitations: we assume an ideal gas and κ independent of physical conditions. Thus, we have the correlations

$$\kappa=\text{const:}\;\; \kappa_1=1,\;\; p_1=B_1^{1/4}\rho_1\,,\;\; \varepsilon_1=f\,(p_1,B_1)\,, \tag{2.16}$$

$$\begin{aligned} \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dp_1}{\rho_1 dx}\right] &= -1\,, \\ \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\rho_1 dx}\right] &= -\lambda\varepsilon_1\,, \end{aligned} \tag{2.17}$$

where

$$\lambda = \frac{\varepsilon_c \kappa_c}{4\pi G c\,\gamma_c} \qquad \gamma_c = \frac{B_c}{p_c}\,. \tag{2.18}$$

Taking integrals on the both parts of (2.17), we obtain

$$\frac{x^2}{\rho_1}\frac{dB_1}{dx} = -\lambda L_x\,, \qquad \frac{x}{\rho_1}\frac{dp_1}{dx} = -M_x\,, \tag{2.19}$$

where we have introduced the notation

$$L_x = \int_0^x \varepsilon_1 \rho_1 x^2 dx\,, \qquad M_x = \int_0^x \rho_1 x^2 dx\,. \tag{2.20}$$

Integrating (2.19) using boundary conditions, we obtain

$$\lambda = \frac{l}{\displaystyle\int_0^{x_0} L_x \frac{\rho_1}{x^2}\, dx}\,, \qquad l = \int_0^{x_0} M_x \frac{\rho_1}{x^2}\, dx\,,$$

hence

$$\lambda = \frac{\displaystyle\int_0^{x_0} M_x \frac{\rho_1}{x^2}\, dx}{\displaystyle\int_0^{x_0} L_x \frac{\rho_1}{x^2}\, dx}\,. \tag{2.21}$$

From formulae (2.21) and (2.20) we conclude that the more concentrated are the sources of stellar energy, the greater is λ. If the source's productivity ε increases towards the centre of a star, $\lambda > 1$. If $\varepsilon = \text{const}$ along the radius, $\varepsilon_1 = 1$ and hence $\lambda = 1$. If stellar energy is generated mostly in the surface layers of a star, $\lambda < 1$. Equations (2.19) lead to

$$\frac{dB_1}{dp_1} = \frac{\lambda L_x}{M_x}\,. \tag{2.22}$$

Because of the boundary conditions $p_1 = 0$, $B_1 = 0$ and $p_1 = 1$, $B_1 = 1$, the derivative dB_1/dp_1 always takes the average value 1. Owing to

$$\left(\frac{dB_1}{dp_1}\right)_{x=0} = \lambda\,, \qquad \left(\frac{dB_1}{dp_1}\right)_{x=x_0} = \frac{\lambda L_{x_0}}{M_{x_0}}\,,$$

we come to the following conclusions: if energy sources are located at the centre of a star, $\lambda L_{x_0}/M_{x_0} < 1$; if energy sources are located on the surface, $\lambda L_{x_0}/M_{x_0} > 1$. If energy sources are homogeneously distributed inside a star, $\lambda L_{x_0}/M_{x_0} = 1$ and $B_1 = p_1$, so we have polytropic class 3, considered in the previous paragraph. This particular solution is the basis of Eddington's theory of the internal constitution of stars. If $n > 3$, $(dB_1/dp_1)_{x_0} \to \infty$ so we have $L_{x_0} \to \infty$. Therefore we conclude that polytropic classes $n > 3$ characterize stars where energy sources concentrate near the surface. Polytropic classes $n < 3$ correspond to stars where energy sources concentrate at the centre. Therefore the data of Table 1 characterize the most probable structures of stars. It should be noted that if $n < 3$, formulae (2.7) and (2.7a) lead to $(dB_1/dp_1)_{x_0} = 0$, and hence $L_{x_0} = 0$. So polytropic structures of stars where energy sources concentrate at the centre can exist only if there is an energy drainage in the upper layer of a star.

Differentiating formula (2.22) step-by-step and using the system (2.17) gives derivatives of $B_1(p_1)$ under $p_1 = 1$ and, hence, expansion of $B_1(p_1)$ into a Taylor series. The first terms of the expansion take the form

$$B_1 = 1 + \lambda(p_1 - 1) + \frac{3}{10}\lambda\left[\frac{\partial\varepsilon_1}{\partial p_1} + \lambda\frac{\partial\varepsilon_1}{\partial B_1}\right]_1 (p_1 - 1)^2 + \ldots$$

The surface condition $B_1 = 0$, being applied to this formula under $p_1 = 0$, gives an equation determining λ. This method gives a numerical value of λ which can be refined by numerical integration of the system (2.17). This integration can be done step-by-step.

The centre of a star, i. e. the point where $x = 0$, is the singular point of the differential equations (2.17). We can move far away from the singular point using series and then (as soon as their convergence becomes poor) we apply numerical integration. We re-write the system (2.7) as follows

$$\begin{aligned} E\left[\frac{B_1^{1/4}}{p_1}\frac{dp_1}{dx}\right] &= -p_1 B_1^{-1/4}\,, \\ E\left[\frac{B_1^{1/4}}{p_1}\frac{dB_1}{dx}\right] &= -\lambda\varepsilon_1 p_1 B_1^{-1/4}\,. \end{aligned} \tag{2.23}$$

Formula (2.12) gives

$$E_0^{(2i)}[u] = \frac{2i+3}{2i+1}\,[u]_0^{(2i+1)}\,. \tag{2.24}$$

Then, differentiating formula (2.23) step-by-step using (2.24), we obtain different order derivatives of the functions $p_1(x)$ and $B_1(x)$ under $x = 0$ that yields the possibility of expanding the functions into Laurent series. Here are the first few terms of the expansions

$$\begin{aligned} p_1 &= 1 - \frac{1}{3}\frac{x^2}{2!} + \frac{2}{15}\left[4 - \lambda\right]\frac{x^4}{4!} - \ldots \\ B_1 &= 1 - \frac{\lambda}{3}\frac{x^2}{2!} + \\ &+ \frac{2\lambda}{15}\left[(4-\lambda) + \frac{3}{2}\left(\frac{\partial\varepsilon_1}{\partial p_1} + \lambda\frac{\partial\varepsilon_1}{\partial B_1}\right)_0\right]\frac{x^4}{4!} - \ldots \end{aligned} \tag{2.25}$$

In order to carry out numerical integration we use formulae which can be easily obtained from the system (2.23), namely

$$\begin{aligned} p_1'' &= -p_1^2 B_1^{-1/2} + p_1'\left[\frac{p_1'}{p_1} - \frac{B_1'}{4B_1} - \frac{2}{x}\right], \\ B_1'' &= -\lambda\varepsilon_1 p_1^2 B_1^{-1/2} + B_1'\left[\frac{p_1'}{p_1} - \frac{B_1'}{4B_1} - \frac{2}{x}\right]. \end{aligned} \tag{2.23a}$$

In this system, we introduce the reduced temperature T_1 instead of B_1, and a new variable $u_1 = p_1^{1/4}$ instead of p_1

$$\begin{aligned} u_1'' &= -\frac{u_1^5}{4T_1^2} + u_1'\left[\left(\frac{u_1'}{u_1} - \frac{T_1'}{T_1}\right) - \frac{2}{x}\right], \\ T_1'' &= -\frac{\lambda\varepsilon_1 u_1^8}{4T_1^5} + T_1'\left[4\left(\frac{u_1'}{u_1} - \frac{T_1'}{T_1}\right) - \frac{2}{x}\right]. \end{aligned} \tag{2.23b}$$

This substitution gives a great advantage, because of small slow changes of the functions T_1 and u_1.

A numerical solution can be obtained close to the surface layer, but not in the surface itself, because the equations (2.23) can be integrated in the upper layers without problems. Naturally, assuming $M_x = M_{x_0} = \text{const}$ and $L_x = L_{x_0} = \text{const}$ in formula (2.19), we obtain

$$\frac{dp_1}{\rho_1} = -\frac{M_{x_0}}{x^2}\,dx\,, \qquad \frac{dB_1}{\rho_1} = -\frac{\lambda L_{x_0}}{x^2}\,dx\,, \qquad B_1 = \frac{\lambda L_{x_0}}{M_{x_0}}\,p_1\,. \tag{2.26}$$

The ideal gas equation and the last relation of (2.26) permit us to write down

$$\frac{dp_1}{\rho_1} = B_1^{1/4}\,\frac{dp_1}{p_1} = B_1^{-3/4} dB_1\,.$$

Integrating the first equation of (2.26), we obtain

$$4T_1 = M_{x_0}\,\frac{x_0 - x}{x_0 x}\,, \tag{2.27}$$

which gives a linear law for the temperature increase within the uppermost layers of a star.

To obtain λ by step-by-step integration, we need to have a criterion by which the resulting value is true. It is easy to see from (2.26) that such a criterion can be a constant value for the quotient B_1/p_1 starting from x located far away from the centre of a star. Solutions are dependent on changes of λ, therefore an exact numerical value of this parameter should be found. Performing the numerical integration, values of the functions near the surface of a star are not well determined. Therefore, in order to calculate L_{x_0} and M_{x_0} in would be better to use their integral formulae (2.20). If energy sources increase their productivity towards the centre of a star, we obtain an exact value for L_{x_0} even in a very rough solution for the system. The calculation of x_0 is not as good, but it can be obtained for fixed M_{x_0} and x far away from the centre through formula (2.27)

$$x_0 = \frac{x}{1 - \frac{4T_1}{M_{x_0}}\,x}\,. \tag{2.27a}$$

Using the above method, exact solutions to the system are obtained. Table 2 contains the characteristics of the solutions in comparison to the characteristics of Eddington's model*.

The last column contains a characteristic that is very important for the "mass-luminosity" relation (as we will see later).

Let us determine what changes are expected in the characteristics of the internal constitution of stars if the absorption coefficient κ is variable. If κ is dependent on the physical conditions, equation (2.22) takes the form

$$\frac{dB_1}{dp_1} = \frac{\kappa_1 \lambda L_x}{M_x}\,. \tag{2.22a}$$

*In his model $\varepsilon_1 = 1$, so the energy sources productivity is $\varepsilon = \text{const}$ along the radius (see the first row in the table). — Editor's remark.

Table 2

ε_1	λ	x_0	M_{x_0}	L_{x_0}	$\frac{\lambda L_{x_0}}{M_{x_0}^3}$
1	1	13.8	16.1	16.1	3.8×10^{-3}
B_1	1.76	10	12.4	2.01	1.8×10^{-3}
$B_1 p_1$	2.32	9	11.5	1.57	2.2×10^{-3}

The variability of κ can be determined by a function of the general form

$$\kappa_1 = \frac{p_1^{\alpha}}{B_1^{\beta}}\,.$$

At first we consider the simplest case where energy sources are homogeneously distributed inside a star. In this case $\varepsilon_1 = 1$, $L_x = M_x$, and equation (2.22a) can be integrated

$$B_1^{1+\beta} = \lambda\,\frac{1+\beta}{1+\alpha}\,p_1^{1+\alpha}\,.$$

Proceeding from the conditions at the centre of any star ($B_1 = p_1 = 1$), we obtain

$$\lambda = \frac{1+\alpha}{1+\beta}\,, \qquad B_1 = p_1^{\lambda}\,.$$

Hence the star has polytropic structure of class

$$n = \frac{4}{\lambda} - 1\,.$$

Looking from the physical viewpoint, the most probable effects are: decrease in the absorption coefficient of a star with depth, and also $\alpha \geqslant -1$. Because

$$\kappa_1 = p_1^{\frac{\alpha-\beta}{1+\beta}} = B_1^{\frac{\alpha-\beta}{1+\alpha}}\,,$$

κ_1 decreases with increase of p_1 and B_1 only if $\alpha < \beta$. Then it is evident that $\lambda < 1$ and $n > 3$. Hence, variability of κ results in an increase of polytropic class. According to the theory of photoelectric absorption,

$$\kappa_1 = \frac{\rho_1}{T_1^{3.5}}\,.$$

In this case $\alpha = 1$, $\beta = 1.125$ and hence $n = 3.25$. Table 1 gives respective numerical values of the characteristics x_0 and M_{x_0}. Other calculated characteristics are $\lambda = 0.94$ and $\lambda L_{x_0}/M_{x_0}^3 = \lambda/M_{x_0}^2 = 3.06\times10^{-3}$. All the numerical values are close to those calculated in Table 2. This is expected, if variability of κ leads to the same order effect for the other classes of energy source distribution inside stars.

Looking at Table 1 and Table 2 we see that the characteristics $x_0 \simeq M_{x_0} \simeq 10$ and $\lambda L_{x_0}/M_{x_0}^3 \simeq 2\times10^{-3}$ have tiny changes under different suppositions about the internal constitution of stars (the internal distribution of energy sources)*. There are three main cases: (1) sources of stellar energy, homogeneously distributed inside a star, (2) energy sources are so strongly concentrated at the centre of star that their productivity is proportional to the 8th order of the temperature, (3) polytropic structures where energy sources are concentrated at the surface — there is a drainage in the surface layer of a star. It should be noted that we did not consider other possible cases of distributed energy sources in a star, such as production of energy in only an "energetically active" layer at a middle distance from the centre. In such distributed energy sources, as it is easy to see from the second equation of the main system, there should be an isothermal core inside a star, and such a star is close to polytropic structures higher than class 3. In this case, instead of the former ε_1, we can build $\varepsilon/\varepsilon_{\max}=\varepsilon_1$, $0\leqslant\varepsilon_1\leqslant 1$, which will be subsumed into λ. However in such a case ε_1, and hence all characteristics obtained as solutions to the system, is dependent on p_c and B_c, and the possibility to solve the system everywhere inside a star sets up as well correlations between the parameters. At last we reach the very natural conclusion that energy is generated inside a star only under specific relations between B and p in that quantity which is required by the compatibility of the equilibrium equations. In order to continue this research and draw conclusions, it is very important to note the fact that the characteristic $\lambda L_{x_0}/M^3_{x_0}$ is actually the same for any stellar structure (see the last column in Table 2). This characteristic remains almost constant under even exotic distributions of energy sources in stars (exotic sources of stellar energy), because of a parallel increase/decrease of its numerator and denominator. Following a line of successive approximations, we have a right to accept the tables as the first order approximation which can be compared to observational data. All the above conclusions show that it is not necessary to solve the main system of the equilibrium equations (2.17) for more detailed cases of the aforementioned structures of stars. Therefore we did not prove the uniqueness of the parameter λ.

2.4 Physical conditions at the centre of stars

The average density of the Sun is $\bar{\rho}_\odot=1.411$. Using this numerical value in (2.4), we obtain a formula determining the central density of stars

$$\rho_c = 0.470\,\frac{x_0^3}{M_{x_0}}\,\frac{\frac{M}{M_\odot}}{\left(\frac{R}{R_\odot}\right)^3}\,. \qquad (2.28)$$

Taking this into account, formula (2.1) permits calculation of the gaseous pressure at the centre of a star

$$p_c = \frac{G}{4\pi}\left(\frac{M_\odot}{R_\odot^2}\right)^2\frac{x_0^4}{M^2_{x_0}}\,\frac{\left(\frac{M}{M_\odot}\right)^2}{\left(\frac{R}{R_\odot}\right)^4}\,. \qquad (2.29)$$

Because $M_\odot=1.985\times10^{33}$ and $R_\odot=6.95\times10^{10}$, we obtain

$$p_c = 8.9\times10^{14}\,\frac{x_0^4}{M^2_{x_0}}\,\frac{\left(\frac{M}{M_\odot}\right)^2}{\left(\frac{R}{R_\odot}\right)^4}\,. \qquad (2.30)$$

Thus the pressure at the centre of the Sun should be about 10^{16} dynes/cm^2 (ten billion atmospheres). It should be noted, as we see from the deductive method, the formulae for ρ_c and p_c are applicable to any phase state of matter.

Let us assume stars consisting of an ideal gas. Then taking the ratio of (2.30) to (2.28) and using the ideal gas equation (1.8), we obtain the temperature at the centre of a star

$$T_c = 2.29\times10^7\mu\,\frac{x_0}{M_{x_0}}\,\frac{\frac{M}{M_\odot}}{\frac{R}{R_\odot}}\,. \qquad (2.31)$$

Hence, the temperature at the centre of the Sun should be about 10 million degrees. As another example, consider the infrared satellite of ε Aurigae. For this star we have $M=24.6M_\odot$, $\log\left(L/L_\odot\right)=4.46$, $R=2{,}140R_\odot$ [5]. Calculating the central density and temperature by formulae (2.30) and (2.31), we obtain $T_c\simeq2\times10^5$ and $p_c\simeq2\times10^5$: thus the temperature is about two hundred thousand degrees and the pressure about one atmosphere. Because the star is finely located in the "mass-luminosity" diagram (Fig. 1) and the Russell-Hertzsprung diagram, we have reason to conclude: *the star has the internal constitution regular for all stars*. This conclusion can be the leading arrow pointing to the supposition that heat energy is generated in stars under physical conditions close to those which can be produced in an Earthly laboratory.

Let us prove that only inside white dwarfs (the stars of the very small radii — about one hundredth of $R_\odot$), the degenerate Fermi gas equation (1.9) can be valid. Naturally, if gas at the centre of a star satisfies the Fermi equation, we obtain $p_c=1\times10^{13}\rho_c^{5/3}\mu_e^{-5/3}$. Formulae (2.28) and (2.30) show that this condition is true only if

$$\frac{R}{R_\odot} = 3.16\times10^{-3}\,\frac{x_0 M^{1/3}_{x_0}}{\left(\frac{M}{M_\odot}\right)^{1/3}}\,\mu_e^{-5/3}\,. \qquad (2.32)$$

This formula remains true independently of the state of matter in other parts of the star. The last circumstance can affect only the numerical value of the factor $x_0 M^{1/3}_{x_0}$. At the same time, table 1 shows that we can assume the numerical value approximately equal to 10.† Formula (2.32) shows that

*It should be noted that the tables characterize the structure of stars only if the radiant pressure is low. In the opposite case all the characteristics x_0, M_{x_0}, and others are dependent on γ_c.

†For stars of absolutely different structure, including such boundary instances as the naturally impossible case of equally dense stars, and the cases where energy sources are located at the surface. — Editor's remark.

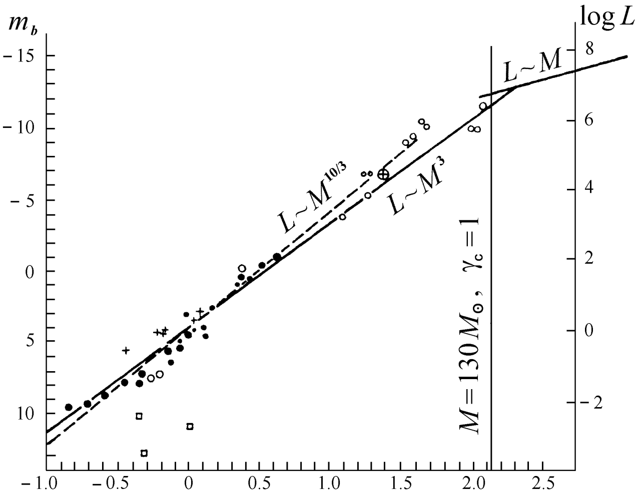


Fig. 1: The "mass-luminosity" relation. Here points are visual binaries, circles are spectral-binaries and eclipse variable stars, crosses are stars in Giades, squares are white dwarfs, the crossed circle is the satellite of ε Aurigae.

for regular degeneration of gas, stars (under $M = M_\odot$) should have approximately the same radius $R \simeq 2{\times}10^9$, i. e. about 20,000 km ($R = 0.03R_\odot$). Such dimensions are attributed to white dwarfs. For example, the satellite of Sirius has $M = 0.94M_\odot$ and $R = 0.035R_\odot$ [6]. If the density is more than the above mentioned (if the radius is less than $R = 0.03R_\odot$) and the mass of the star increases, formula (2.32) shows that regular degeneration can become relativistic degeneration

$$p = K\rho^{4/3}, \qquad K = K_{\rm H}\,\mu_e^{-4/3}, \qquad K_{\rm H} = 1.23{\times}10^{15}.$$

We apply these formulae to the centre of a star, and take equations (2.28) and (2.30) into account. As a result we see that the radius drops out of the formulae, so relativistic degeneration can be realized in a star solely in terms of the mass

$$\frac{M}{M_\odot} = 0.356 M_{x_0}\,, \qquad (\mu_e = 1). \tag{2.32a}$$

Because of Table 1, we see: $n = 0$ only if $M_{x_0} = 16.1$. Hence, the lower boundary of the mass of a non-degenerated gaseous star is $5.7M_\odot$. In order to study degenerated gaseous stars in detail, we should use the phase state equation that includes the regular state, the boundary state between the regular and degenerated states, and the degenerated state. Applying formulae (2.28) and (2.30) to the above ratio, we obtain a correlation between the radius and the mass of a star, which is unbounded for small radii. It should be noted that introduction of a mass-radius correlation is the essence of Chandrasekhar's theory of white dwarfs [7]. On the other hand, having observable sizes of white dwarfs, equation (2.32) taken under $x_0 M_{x_0}^{1/3} = 10$ gives the same numerical values for radii as Chandrasekhar's table (his well-known relation between the radius and mass of star). The exact numerical value of the ultimate mass calculated by him coincides with our $5.7M_\odot$. In Chandrasekhar's formula, as well as in our formula (2.32), radius is correlated opposite to mass. Today we surely know masses and radii of only three white dwarfs: the white dwarfs do not confirm the opposite correlation mass-radius. So, save for the radius of Sirius' satellite coinciding with our formula (2.32), we have no direct astrophysical confirmation about degeneration of gas inside white dwarfs.

Considering stars built on an ideal gas, we deduce a formula determining the mass of a star dependent on internal physical conditions. We can use formulae (2.30) and (2.31) or formula (2.2) directly. Applying the Boyle-Mariotte equation (1.8) to formula (2.2), and taking the Stephan-Boltzmann law (1.7) into account, we obtain

$$M = C\,\frac{\gamma_c^{1/2}}{\mu^2}\,M_{x_0}\,, \qquad C = \frac{\Re^2}{G^{3/2}\sqrt{\frac{4}{3}\pi\alpha}} = 2.251{\times}10^{33}. \tag{2.33}$$

Introducing the mass of the Sun $M_\odot = 1.985{\times}10^{33}$ into the equation, we obtain

$$M = 1.134 M_\odot\,\frac{\gamma_c^{1/2}}{\mu^2}\,M_{x_0}\,. \tag{2.34}$$

As we will see below, the "mass-luminosity" correlation shows γ_c is close to 1 for blue super-giants. Hence formula (2.34) gives the observed numerical values for masses of stars. The fact that we obtain true orders for numerical values of the masses of stars, proceeding only from numerical values of the fundamental constants G, $\Re$, α, is excellent confirmation of the theory.

2.5 The "mass-luminosity" relation

In deducing the "mass-luminosity" correlation, we assume: (1) the radiant pressure is negligible in comparison to the gaseous pressure everywhere inside a star; (2) stars consist of an ideal gas; (3) ε and κ can be approximated by functions like $p^\alpha B^\beta$. Then the main system of the equilibrium equations takes the form

$$\begin{aligned} \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dp_1}{\rho_1 dx}\right] &= -1\,, \\ \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\kappa_1\rho_1 dx}\right] &= -\lambda\varepsilon_1\,, \end{aligned} \tag{2.35}$$

where

$$\lambda = \frac{\varepsilon_c\kappa_c}{4\pi G c\gamma_c}\,, \qquad \gamma_c = \frac{B_c}{p_c}\,.$$

Solving the system, as we know, is possible under a numerical value of λ close to 1. Hence a star can be in equilibrium only if the energy generated inside it is determined by the formula

$$\varepsilon_c = \frac{\lambda 4\pi G c}{\kappa_c}\,\gamma_c\,. \tag{2.36}$$

If a star produces another quantity of energy, it will contract or expand until its new shape results in production of energy exactly by formula (2.36). Because γ_c determines the mass of a star (2.34) and ε_c determines the luminosity of a star, the "mass-luminosity" correlation should be contained in formula (2.36). In other words, the "mass-luminosity" correlation is the condition of equilibrium of stars.

Because of (2.3),

$$\varepsilon_c = \frac{L}{M}\frac{M_{x_0}}{L_{x_0}}\,.$$

Substituting this equation into (2.36), we obtain

$$L = \frac{4\pi G c}{\kappa_c}\frac{\lambda L_{x_0}}{M_{x_0}}\,M\gamma_c\,.$$

The quantity γ_c can be removed with the mass of a star by (2.33)

$$L = \frac{4\pi G^4 4\pi\alpha}{3\kappa_c\Re^4}\,\mu^4\left(\frac{\lambda L_{x_0}}{M^3_{x_0}}\right)M^3. \tag{2.37}$$

The luminosity of the Sun is $L_\odot = 3.78{\times}10^{33}$. Proceeding from formula (2.37), we obtain

$$\frac{L}{L_\odot} = 1.04{\times}10^4\,\frac{\mu^4}{\kappa_c}\left(\frac{\lambda L_{x_0}}{M^3_{x_0}}\right)\left(\frac{M}{M_\odot}\right)^3. \tag{2.38}$$

The formula (2.38) gives a very simple correlation: the luminosity of a star is proportional to the third order of its mass. In deducing this formula, we accepted that ε is determined by a function $\varepsilon \sim p^\alpha B^\alpha$, so ε_1 depends on p_1 and B_1. It is evident that rejection of this assumption cannot substantially change the obtained correlation (2.38). Naturally, under arbitrary ε, the quantity ε_1 depends on p_c and B_c. Thus the multiplier $\lambda L_{x_0}/M^3_{x_0}$ in formula (2.38) will have different numerical values for different stellar structures. At the same time Table 2 shows that this multiplier is approximately the same for absolutely different structures, including boundary structures which are exotic. Therefore the "mass-luminosity" correlation gives no information about sources of stellar energy — the correlation is imperceptible to their properties. However, other assumptions are very important. As we see from the deductive path to formula (2.33), the correlation between mass and luminosity can be deduced only if the pressure depends on temperature, so our formula (2.38) can be obtained only if the gas is ideal. It is also important to make the absorption coefficient κ constant for all stars. The rôle of the radiant pressure will be considered in the next paragraph.

And so forth we are going to compare formula (2.38) to observational data. Fig. 1 shows masses and luminosities of stars, according to today's data. The diagram has been built on masses of stars taken from Kuiper's data base [8], and the monograph by Russell and Moore [9]. We excluded Trumpler stars [10] from the Kuiper data, because their masses were measured uncertainly. Naturally, Trumpler calculated masses of such stars, located in stellar clusters, with the supposition that the K-term (the term for radiant velocities with respect to the whole cluster) is fully explained by Einstein's red shift. For this reason the calculated masses of Trumpler stars can be much more than their real masses. Instead of Trumpler stars, in order to fill the spaces of extremely bulky stars in the diagram, we used extremely bulky eclipse variable stars (VV Cephei, V 381 Scorpii) and data for Plasckett's spectral-variable star BD $+6^\circ$ 1309.

As we see in Fig. 1, our obtained correlation $L \sim M^3$ is in good accord with the observational data in all spectra of observed masses (having a small deviation inside $1.5m$). The dashed line $L \sim M^{10/3}$ is only a little different from our line. Parenago [11], Kuiper [8], Russell [9], and others accept this $L \sim M^{10/3}$ line as the best representation of observational data. Some researchers found the exponent of mass more than our's. For instance, Braize [12] obtained $L \sim M^{3.58}$. Even if such maximal deviation from our exponential index 3 is real, the theoretical result is excellent for most stars. The coefficient of proportionality in our formula (2.38) is very susceptible to μ. For this reason, coincidence of our theoretical correlation and observational data is evidence that the chemical composition of stars is the same on the average. The same should be said about the absorption coefficient κ: because physical conditions inside stars can be very different even under the same luminosity (for example, red giants and blue stars located in the main direction), it is an unavoidable conclusion that the absorption coefficient of stellar matter is independent of pressure and temperature. The conclusions justify our assumption in §1.3, when we solved the main system of equilibrium equations.

The fact that white dwarfs lie off the main sequence can be considered as a confirmation of degenerate gas inside them. Because a large increase of the absorption coefficient in white dwarfs in comparison to regular stars is not very plausible, another explanation can be given only if the structural multiplier $\lambda L_{x_0}/M_{x_0}$ in white dwarfs is $\sim$100 times more than in other stars. The location of white dwarfs in the Russell-Hertzsprung diagram can give a key to this problem.

At last, proceeding from observational data, we calculate the coefficient μ^4/κ_c in our theoretical formula (2.38). The line $L \sim M^3$, which is the best representation of observational data, lies a little above the point where the Sun is located. For this reason, under $M = M_\odot$, we should have $L = 1.8\,L_\odot$ in our formula (2.38). According to table 2, we assume $\lambda L_{x_0}/M^3_{x_0} = 2{\times}10^{-3}$. Then we obtain

$$\frac{\mu^4}{\kappa_c} = 0.08\,. \tag{2.39}$$

2.6 The radiant pressure inside stars

In the above we neglected the radiant pressure in comparison to the gaseous one in the equation of mechanical equilibrium of a star. Now we consider the main system of the equation (III), which takes the radiant pressure into account. If the absorption coefficient $\varkappa$ is constant ($\varkappa_1 = 1$), this system takes the form

$$\begin{aligned} \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dp_1}{\rho_1 dx}\right] &= -(1-\lambda\gamma_c\varepsilon_1)\,, \\ \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\rho_1 dx}\right] &= -\lambda\varepsilon_1\,. \end{aligned} \tag{2.40}$$

After calculations analogous to those carried out in deducing formula (2.21), we obtain

$$\lambda\,(1+\gamma_c) = \frac{\displaystyle\int_0^{x_0} M_x\,\frac{\rho_1}{x^2}\,dx}{\displaystyle\int_0^{x_0} L_x\,\frac{\rho_1}{x^2}\,dx}\,. \tag{2.41}$$

The ratio of integrals in this formula depends on the distribution of energy sources inside a star, i. e. on the structure of a star. This ratio maintains a numerical value close to 1 under any conditions. Thus $\lambda\,(1+\gamma_c)\sim 1$. If energy sources are distributed homogeneously throughout the volume of a star, we have $\varepsilon_1 = 1$, $L_x = M_x$ and hence the exact equality $\lambda\,(1+\gamma_c) = 1$. If energy sources are concentrated at the centre of a star, $\lambda\,(1+\gamma_c) > 1$. In this case, if the radiant pressure takes high values ($\gamma_c > 1$), the internal constitution of a star becomes very interesting, because in this case $\lambda\gamma_c > 1$ and the right side term in the first equation of (2.40) is positive at the centre of a star, our formula (2.41) leads to $p_1'' > 0$, and hence at the centre of such a star the gaseous pressure and the density have a minimum, while their maximum is located at a distance from the centre*.

From this we conclude that extremely bulky stars having high γ_c can be in equilibrium only if $\lambda\,(1+\gamma_c)\sim 1$, or, in other words, if the next condition is true

$$\varepsilon_c \sim \frac{4\pi G c}{\varkappa}\,. \tag{2.42}$$

Thus, starting from an extremely bulky stellar mass wherein $\gamma_c > 1$, the quantity of energy generated by a unit of the mass should be constant for all such extremely bulky stars. The luminosity of such stars, following formulae (2.3) and (2.2), should be directly proportional to their mass: $L\sim M$. This correlation is given by the straight line drawn in the upper right corner of Fig. 1. Original data due to Eddington [13] and others showed an inclination of the "mass-luminosity" line to this direction in the region of bulky stars (the upper right corner of the diagram). But further more exact data, as it was especially shown by Russell [9] and Baize [12], do not show the inclination for even extremely bulky stars (see our Fig. 2). Therefore we can conclude that there are no internal structures of stars for $\gamma_c > 1$; the ultimate case of possible masses of stars is the case where $\gamma_c = 1$. Having no suppositions about the origin of energy sources in stars†, it is very difficult to give an explanation of this fact proceeding from only the equilibrium of stars. The very exotic internal constitution of stars under $\gamma_c > 1$ suggests that if such stars really exist in nature, they are very rare exceptions.

In order to ascertain what influence γ_c has on the structure of a star, we consider the simplest (abstract) case where energy sources are distributed homogeneously throughout a star ($\varepsilon_1 = 1$). In this case, as we know,

$$\lambda = \frac{1}{1+\gamma_c}\,, \tag{2.43}$$

and the system (2.40) takes the form

$$\begin{aligned} \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dp_1}{\rho_1 dx}\right] &= -\frac{1}{1+\gamma_c}\,, \\ \frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\rho_1 dx}\right] &= -\frac{1}{1+\gamma_c}\,. \end{aligned} \tag{2.44}$$

Introducing a new variable $x_{\gamma_c=0}$ instead of x

$$x = \sqrt{1+\gamma_c}\,x_{\gamma_c=0}\,, \tag{2.45}$$

we obtain the main system in the same form as that in the absence of the radiant pressure. So, in this case the main characteristics of the internal constitution of star are

$$\begin{aligned} x_0 &= x_{0(\gamma_c=0)}\big(1+\gamma_c\big)^{1/2}, \\ M_{x_0} &= M_{x_0(\gamma_c=0)}\big(1+\gamma_c\big)^{3/2}, \\ L_{x_0} &= L_{x_0(\gamma_c=0)}\big(1+\gamma_c\big)^{3/2}, \qquad \lambda = \frac{\lambda_{\gamma_c=0}}{1+\gamma_c}\,. \end{aligned} \tag{2.46}$$

Characteristics indexed by $\gamma_c = 0$ are attributed to the structures of stars where $\gamma_c \ll 1$; their numerical values can be taken from our Table 2. Because Table 2 shows very small changes in M_{x_0} for very different structures of stars, formulae (2.46) should as well give an approximate picture for other structures of stars. Under high γ_c, the mass of a star (2.34) becomes

$$M \simeq 1.134\,M_\odot\,\frac{\gamma_c^{1/2}}{\mu^2}\big(1+\gamma_c\big)^{3/2} M_{x_0(\gamma_c=0)}\,. \tag{2.47}$$

*This amazing conclusion about the internal constitution of a star is true under only high values of the radiant pressure. In regular stars the radiant pressure is so low that we neglect it in comparison to the gaseous pressure (see previous paragraphs). — Editor's remark.

†That is the corner-stone of Kozyrev's research. — Editor's remark.

Astronomical observations show that maximum masses of stars reach $\sim 120 M_\odot$ — see Fig. 1, showing an inclination of the "mass-luminosity" correlation near $\log (M/M_\odot) = 2$. Assuming this mass in (2.47), and assuming $\gamma_c = 1$ and $M_{x_0} = 10$ for it, we obtain the average molecular weight $\mu = 0.51$.

Then in such stars, by formula (2.39), we obtain $\kappa = 0.8$. On the other hand, because the "mass-luminosity" correlation has a tendency to the line $L \sim M$ for extremely bulky masses (see Fig. 1), we obtain the ultimate value $\bar{\varepsilon} = 5\times 10^4$. For homogeneously distributed energy sources, formula (2.42) leads to $\kappa = 0.5$. If they are concentrated at the centre, $\varepsilon_c > \bar{\varepsilon} = \varepsilon_c (L_{x_0}/M_{x_0})$. Even in this case formula (2.42) leads to $\varepsilon_c > \bar{\varepsilon}$. There is some compensation, so the calculated numerical value of the absorption coefficient κ is true. An exact formula for $\bar{\varepsilon}$ can be easily obtained as

$$\frac{L}{M} = \bar{\varepsilon} = \frac{4\pi G c}{\kappa} \frac{L_{x_0}}{M_{x_0}} \frac{\displaystyle\int_0^{x_0} M_x \frac{\rho_1}{x^2}\, dx}{\displaystyle\int_0^{x_0} L_x \frac{\rho_1}{x^2}\, dx} \frac{\gamma_c}{1+\gamma_c}\,. \qquad (2.48)$$

So, having considered the "mass-luminosity" correlation, we draw the following important conclusions:

1. All stars (except possibly for white dwarfs) are built on an ideal gas;
2. In their inner regions, where stellar energy is generated, all stars have the same chemical composition, $\mu = \text{const} = 1/2$, so they are built on a mix of protons and electrons without substantial percentage of other nuclei;
3. The absorption coefficient per unit of mass κ is independent of the physical conditions inside stars, it is a little less than 1.

Thomson dispersion of light in free electrons has the same properties. Naturally, the Thomson dispersion coefficient per electron is

$$\sigma_0 = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.66\times 10^{-25}, \qquad (2.49)$$

where e and m_e are the charge and the mass of the electron. In the mix of protons and electrons we obtain

$$\kappa_{\text{T}} = \frac{\sigma_0}{m_{\text{H}}} = \frac{6.66\times 10^{-25}}{1.66\times 10^{-24}} = 0.40\,. \qquad (2.50)$$

The fact that our calculated approximate value of κ is close to $\kappa_{\text{T}} = 0.40$ shows that the interaction between light and matter inside stars is determined mainly by the Thomson process — acceleration of free electrons by the electric field of light waves.

Because μ stays in the "mass-luminosity" correlation (2.38) in fourth degree, the obtained theoretical value of μ is quite exact with respect to the real one. If $\kappa = \kappa_{\text{T}}$, as a result of (2.39) we have $\mu = 0.43$. Because μ cannot be less than $1/2$, the obtained ultimate value of $\kappa = 0.8$ is twice $\kappa_{\text{T}} = 0.40$. This fact can be explained by the circumstance that, in this case of extremely bulky masses, the structural coefficient in formula (2.38) should be twice as small. It is evident that we can accept $\mu = 1/2$ to within 0.05. If all heavy nuclei are ionized, their average molecular weight is 2. If we assume the average molecular weight in a star to be 0.55 instead of $1/2$, the percentage of ionized atoms of hydrogen χ_{H} becomes

$$2\chi_{\text{H}} + \frac{1}{2}(1-\chi_{\text{H}}) = \frac{1}{0.55}\,, \qquad \chi_{\text{H}} \simeq 90\%\,.$$

Thus the maximum admissible composition of heavy nuclei inside stars, permitted by the "mass-luminosity" correlation, is only a few percent. Under $\mu = 1/2$ the mass of a star, where $\gamma_c = 1$, is obtained as $130 M_\odot$. This value is indicated by the vertical line in Fig. 1.

At last we calculate the radiant pressure at the centre of the Sun. Formula (2.34) leads to $\gamma_{c\odot} \simeq 10^{-3}$. In this case the radiant pressure term in the equation of mechanical equilibrium can be neglected.

2.7 Comparing the obtained results to results obtained by other researchers

To deduce the "mass-luminosity" correlation by the explanation according to the regular theory of the internal constitution of stars, becomes very complicated because the theoreticians take a priori the absorption coefficient as dependent on the physical conditions. They supposed the absorption of light inside stars due to free-connected transitions of electrons (absorption outside spectral series) or transitions of electrons from one hyperbolic orbit to another in the field of positive charged nuclei. The theory of such absorption was first developed by Kramers, and subsequently by Gaunt, and especially, by Chandrasekhar [14]. According to Chandrasekhar, the absorption coefficient depends on physical conditions as

$$\kappa_{\text{Ch}} = 3.9\times 10^{25} \frac{\rho}{T^{3.5}} \left(1-\chi_{\text{H}}^2\right), \qquad (2.51)$$

where χ_{H}^2 is the percentage of hydrogen, the numerical factor is obtained for Russell's composition of elements. In order to clarify the possible rôle of such absorption in the "mass-luminosity" correlation, we assume (for simplicity)

$$\kappa_{\text{Ch}} = \frac{\kappa_0}{\gamma}\,. \qquad (2.52)$$

In this case, having small γ_c, formulae (2.38) and (2.33) show $L \sim M^5$. This exponent is large, so we cannot neglect γ_c in comparison to 1. If γ_c is large, the formulae show $L \sim M^{3/2}$. Thus, in order to coordinate theory and observations, we are forced to consider "middle" numerical values of

γ_c and reject the linear correlation between $\log L$ and $\log M$. Formulae (2.47) and (2.48) show

$$M^2 \sim \frac{\gamma_c\left(1+\gamma_c\right)^2}{\mu^4}\,, \qquad M^2 \sim \frac{1-\beta}{\mu^4\beta^4}\,,$$
$$L \sim M\,\frac{\gamma_c^2}{1+\gamma_c}\,, \qquad L \sim M^{3/2}\left(1-\beta\right)^{3/2}\mu\,. \tag{2.53}$$

Here are formulae where γ_c has been replaced with the constant β, one regularly uses in the theory of the internal constitution of stars

$$\beta = \frac{p_c}{p_0} = \frac{1}{1+\gamma_c}\,. \tag{2.54}$$

Thus the “mass-luminosity” correlation, described by the two formulae (2.53), becomes very complicated. The formulae are in approximate agreement with Eddington’s formulae [15] and others. The exact formula for (2.51) introduces the central temperature T_c into them. Under large γ_c, as we see from formulae (2.46), the formula for T_c (2.31) includes the multiplier β

$$T_c = 2.29{\times}10^7\,\mu\,\beta\left(\frac{x_0}{M_{x_0}}\right)_{\gamma_c=0}\frac{\frac{M}{M_\odot}}{\frac{R}{R_\odot}}\,. \tag{2.55}$$

Then, through T_c, the radius and the reduced temperature of a star can be introduced into the “mass-luminosity” correlation. This is the way to obtain the well-known Eddington temperature correction.

In order to coordinate the considered case of “middle” γ_c, we should accept $\gamma_c = 1$ starting from masses $M \simeq 10M_\odot$. So, for the Sun we obtain $\gamma_{c\odot} = 0.08$. As we see from formula (2.47), it is possible if $\mu \simeq 2$. Then formula (2.39), using the numerical value $\lambda L_{x_0}/M_{x_0}^3 = 3.8{\times}10^{-3}$ given by Eddington’s model, gives $\kappa_{c\odot} = 170$ and $\kappa_0 = 14$. The theoretical value of κ_0 can be obtained by comparing (2.52) and (2.51); it is

$$\kappa_0 = \frac{\alpha\mu}{3\Re\sqrt{T_{c\odot}}}\,3.9{\times}10^{25}\left(1-\chi_{\mathrm{H}}^2\right). \tag{2.56}$$

According to (2.55) we obtain $T_{c\odot} = 4{\times}10^7$. Then, by (2.56), we have $\kappa_0 = 0.4$. So, according to Eddington’s model, the theoretically obtained value of the absorption coefficient $\kappa_0 = 14$ is 30 times less than the $\kappa_0 = 0.4$ required, consistent with the observational data*. This divergence is the well-known “difficulty” associated with Eddington’s theory, already noted by Eddington himself. According to Strömgren [16], this difficulty can be removed if we accept the hypothesis that stars change their chemical composition with luminosity. Supposing the maximum hydrogen content, μ can vary within the boundaries $1/2 \leqslant \mu \leqslant 2$. Then, as we see from (2.56), the theoretical value of κ_0 decreases slightly. On the other hand, the previous paragraph showed that the value of κ_0, obtained from observations, decreases much more. As a result, the theoretical and observational values of κ can be matched (which is in accordance with Strömgren’s conclusion). All theoretical studies by Strömgren’s followers, who argued for evolutionary changes of relative amounts of hydrogen in stars, were born from the above hypothesis. The hypothesis became very popular, because it provided an explanation of stellar energy by means of thermonuclear reactions, as suggested by Bethe.

*As it was shown in the previous paragraph, Kozyrev’s theory gives $\kappa_0 = 0.5{-}0.8$ for stars having different internal constitutions, which corresponds well to observations. — Editor’s remark.

It is evident that the above theories are very strained. On the other hand, the simplicity of our theory and the general way it was obtained are evidence of its truth. It should be noted that our two main conclusions

$$(1)\;\; \mu = 1/2\,,\;\; \chi_{\mathrm{H}}^2 = 1\,; \qquad (2)\;\; \kappa = \kappa_{\mathrm{T}}\,,$$

obtained independently of each other, are physically connected. Naturally, if $\chi_{\mathrm{H}}^2 = 1$, Chandrasekhar’s formula (2.51) becomes inapplicable. Kramers absorption (free-connected transitions) becomes a few orders less; it scarcely reaches the Thomson process. At the same time, our main result is that $\gamma_c < 1$ for all stars, and this led to all the results of our theory. Therefore this result is so important that we mean to verify it by other astrophysical data. We will do it in the next chapter, analysing the correlation “period — average density of Cepheids”. In addition, according to our theory, the central regions of stars, where stellar energy is generated, consist almost entirely of hydrogen. This conclusion, despite its seemingly paradoxical nature, must be considered as an empirically established fact. We will see further that study of the problem of the origin of stellar energy will reconcile this result with spectroscopic data about the presence of heavy elements in the surface layers of stars.

2.8 The rôle of convection inside stars

In §1.3 we gave the equations of equilibrium of stars (II), which take convective transfer of energy into account. Assuming the convection coefficient $A = \mathrm{const}$, the second equation of the system (the heat equilibrium equation) can be written as

$$\frac{1}{\rho_1 x^2}\frac{d}{dx}\left[\frac{x^2 dB_1}{\rho_1 dx}\right] - \frac{\kappa_c\rho_c A}{c\gamma_c}\frac{1}{\rho_1 x^2}\left[x^2 u\frac{dp_1}{dx}\right] = -\lambda\varepsilon_1\,, \tag{2.57}$$

where

$$u = 1 - \frac{\Gamma}{4(\Gamma-1)}\frac{p_1}{B_1}\frac{dB_1}{dp_1}\,. \tag{2.58}$$

The convection term in (2.57) plays a substantial rôle only if

$$\frac{\kappa_c\rho_c A}{c\gamma_c} > 1\,, \qquad A > \frac{c\gamma_c}{\kappa_c\rho_c}\,. \tag{2.59}$$

Table 3

κ	x_1	M_{x_1}	λL_{x_0}	x_0	M_{x_0}	$\dfrac{x_0^3}{3M_{x_0}}$	$\dfrac{\lambda L_{x_0}}{M_{x_0}^3}$
const	2.4913	3.570	3.018	8.9	11.46	20.5	1.97×10^{-3}
$\kappa_{\rm Ch}$	1.88	1.25	1.25	11.2	12.4	37.0	0.65×10^{-3}

Hence the convection coefficient for the Sun should satisfy $A_\odot > 5\times10^7$. In super-giants, convection would be substantial only under $A > 10^{16}$. The convection coefficient A, as we see from formula (1.17), equals the product of the convective current velocity $\bar{v}$ and the average length $\bar{\lambda}$ travelled by the current. Thus convection can influence energy transfer inside super-giants if convection currents are about the size of the star (which seems improbable). At the same time, if a convection instability occurs in a star, the average length of travel of the current becomes the size of the whole convection zone. Then the coefficient A increases so much that it can reach values satisfying (2.59). If A is much more than the right side of (2.59), taking into account the fact that all terms of the equilibrium equation (2.57) are about 1, the term in square brackets is close to 0. Then, if A is large,

$$u = 0\,, \quad \text{hence} \quad B_1 = p_1^{\frac{4(\Gamma-1)}{\Gamma}}\,, \tag{2.60}$$

which is the equation of adiabatic changes of state. For a monatomic gas, $\Gamma = 5/3$ ($n = 3/2$) and hence

$$B_1 = p_1^{8/5}\,. \tag{2.61}$$

Because, according our conclusions, stars are built up almost entirely of hydrogen, Γ can be different from $5/3$ in only the upper layers of stars, which is insufficient in our consideration of a star as a whole. Therefore zones of free convection can appear because of an exotic distribution of energy sources.

Free convection can also start in another case, as soon as the temperature gradient of radiant equilibrium exceeds the temperature gradient of convective equilibrium. This is Schwarzschild's condition, and it can be written as

$$\left(\frac{d\log B_1}{d\log p_1}\right)_{\rm Rad} > \left(\frac{d\log B_1}{d\log p_1}\right)_{\rm Con},$$

which, taking (2.22) and (2.61) into account, leads to

$$\frac{\lambda L_x}{M_x} > 1.6\,\frac{B_1}{p_1}\,. \tag{2.62}$$

From this formula we see that free convection is impossible in the surface layers of stars. In central regions we obtain the next condition for free convection

$$\lambda > 1.6.$$

Table 2 shows that even when $\varepsilon_1 = B_1$, any star should contain a convective core. If ε_1 depends only on temperature and can be approximated by function $\varepsilon_1 = T^m$, the calculations show that λ reaches its critical value of 1.6 when $m = 3.5$. Thus a star has a convective core if $m > 3.5$. The radius x_1 of the convective core is determined by equality between the temperature gradients (see above). Writing (2.62) as an equality, we obtain

$$\lambda L_{x_1} = 1.6\,M_{x_1}\left(\frac{B_1}{p_1}\right)_{x_1} = 1.6\,M_{x_1}\rho_{x_1}\,. \tag{2.63}$$

It is evident that the size of the convective core increases if the energy sources become more concentrated at the centre of a star. In the case of a strong concentration, all energy sources become concentrated inside the convective core. Then inside the region of radiant equilibrium we have $\lambda L_x = \lambda L_{x_1} = \text{const}$. Because the border of the convective core is determined by equality of the physical characteristics' gradients in regions of radiant equilibrium and convective equilibrium, not only are p_1 and T_1 continuous inside such stars but so are their derivatives. Therefore such a structure for a star can be finely calculated by solving the main system of the equilibrium equations under $\varepsilon_1 = 0$ and boundary conditions: (1) under some values of $x = x_1$, quantities p_1, B_1 and their derivatives should have numerical values satisfying the solution to Emden's equation under $n = 3/2$; (2) under some value $x = x_0$ we should have $p_1 = B_1 = 0$. The four boundary conditions fully determine the solution. We can find x_1 by step-by-step calculations as done in §2.3 for λ.

The formulated problem, known as the problem of the internal constitution of a star having a point-source of energy and low radiant pressure, was first set up by Cowling [17]. In his calculations the absorption coefficient was taken as variable according to Chandrasekhar's formula (2.51): $\kappa = \kappa_{\rm Ch}$. However in §2.6 we showed that $\kappa = \kappa_{\rm T}$ inside all stars. Only in the surface layers of a star should κ increase to $\kappa_{\rm T}$. But, because of very slow changes of physical conditions along the radius of a star, κ remains $\kappa_{\rm T}$ in the greater part of the volume of a star. Therefore it is very interesting to calculate the internal structure of a star under given values of $\kappa = \text{const}$. We did this, differing thereby from Cowling's model, so that there are two alternatives: our model ($\kappa = \text{const}$) and Cowling's model ($\kappa_{\rm T}$). All the calculations were carried out by numerical integration of (2.23b) assuming there that $\varepsilon_1 = 0$.

Table 4

x	T_1	p_1	ρ_1
0.00	1.000	1.000	1.000
0.50	0.983	0.958	0.975
1.00	0.935	0.845	0.904
1.50	0.856	0.677	0.791
2.00	0.762	0.507	0.665
2.50	0.652	0.346	0.530
3.00	0.544	0.211	0.388
3.50	0.451	0.117	0.259
4.00	0.370	$0.598{\times}10^{-1}$	0.161
4.50	0.328	$0.284{\times}10^{-1}$	$0.936{\times}10^{-1}$
5.00	0.245	$0.125{\times}10^{-1}$	$0.510{\times}10^{-1}$
5.50	0.195	$0.52{\times}10^{-2}$	$0.266{\times}10^{-1}$
6.00	0.154	$0.20{\times}10^{-2}$	$0.129{\times}10^{-1}$
6.50	0.118	$0.67{\times}10^{-3}$	$0.57{\times}10^{-2}$
7.00	0.087	$0.20{\times}10^{-3}$	$0.23{\times}10^{-2}$
7.50	0.060	$0.49{\times}10^{-4}$	$0.82{\times}10^{-3}$
8.00	0.036	$0.64{\times}10^{-5}$	$0.18{\times}10^{-3}$
8.50	0.015	$0.19{\times}10^{-6}$	$0.79{\times}10^{-4}$
8.90	0.000	0.000	0.000

Table 3 gives the main characteristics of the "convective" model of a star under $\kappa = \text{const}$ and $\kappa = \kappa_{\text{Ch}}$. The κ_{Ch} are taken from Cowling's calculations. Values of λL_{x_0} were found by formula (2.62). In this model, distribution of energy sources inside the convective core does not matter. For this reason, the quantities λ and L_{x_0} are inseparable. If we would like to calculate them separately, we should set up the distribution function for them inside the convective core.

We see that the main characteristics of the structure of a star, the quantities x_0, M_{x_0}, and λL_{x_0}, are only a little different from those calculated in Table 2. The main difference between structures of stars under the two values $\kappa = \text{const}$ and $\kappa = \kappa_{\text{Ch}}$ is that under our $\kappa = \text{const}$ the convective core is larger, so such stars are close to polytropic structures of class 3/2, and there we obtain a lower concentration of matter at the centre: $\rho_c = 20.5\,\bar{\rho}$. Table 4 gives the full list of calculations for our convective model ($\kappa = \text{const}$).

Chapter 3

The Internal Constitution of Stars, Obtained from the Analysis of the Relation "Period — Average Density of Cepheids" and Other Observational Data

In the previous chapter we deduced numerous theoretical correlations, which give a possibility of calculating the physical characteristics of matter inside stars if their structural characteristics are known. In order to be sure of the calculations, besides our general theoretical considerations, it would be very important to obtain the structural characteristics proceeding from observational data, related at least to some classes of stars.

Properties of the internal structure of a star should manifest in its dynamical properties. Therefore we expect that the observed properties of variable stars would permit us to learn of their structures. For instance, the pulsation period of Cepheids should be dependent on both their physical characteristics and the distribution of the characteristics inside the stars. Theoretical deduction of this correlation can be done very strictly. Therefore we have a basis for this deduction in all its details.

Radiation of energy by an oscillating star must result in a dispersion of mechanical energy of its oscillations. It is most probable that the oscillation energy of variable stars is generated and supported by energy sources connected to the oscillation and radiation processes. In other words, such stars are self-inducing oscillating systems. Observable arcs of the oscillating luminosity and speed reveal a nonlinear nature for the oscillations, which is specific to self-inducing oscillating systems. The key point of a self-inducing oscillating system is a harmonic frequency equal to the natural frequency of the whole oscillating system. Therefore, making no attempt to understand the nature of the oscillations, we can deduce the oscillation period as the natural period of weak linear oscillations.

3.1 The main equation of pulsation

Typical Cepheids have masses less than 10 solar masses. For instance, δ Cephei has $M \simeq 9\,M_\odot$. In this case equation (2.34) leads to $\gamma_c < 0.1$, so Cepheids should satisfy $L \sim M^3$, i. e. the "mass-luminosity" relation. Therefore the radiant pressure plays no rôle in such stars, so considering their internal constitutions we should take into account only the gaseous pressure. In solving this problem we will consider linear oscillations, neglecting higher order terms. This problem becomes much simpler because temperature changes in such a star satisfy adiabatic oscillations in almost its whole volume, except only for the surface layer. Naturally, in order to obtain the ratio between observed temperature variations and adiabatic temperature variations close to 1, the average change of energy inside 1 gram in one second should be about $\bar{\varepsilon}$, i. e. $\sim 10^2$. This is 10^8 per half period. On the other hand, the heat energy of a unit of mass should be about Ω/M (according to the virial theorem), that is $\sim 10^{15}$ ergs by formula (2.5). Thus during the pulsation the relative change of the energy is only 10^7, so pulsations of stars are adiabatic, with high precision. We assume that the pulsation of a star can be determined by a simple standing wave with a frequency $n/2\pi$

$$V(r,t) = V(r)\sin nt, \quad a = \frac{\partial^2 V}{\partial t^2} = -n^2 V(r)\sin nt, \tag{3.1}$$

where $V(r)$ is the relative amplitude of the pulsation

$$V(r) = \frac{\delta r}{r}\,.$$

By making the above assumptions, Eddington had solved the problem of pulsation of a star.

Linking the coordinate r to the same particle inside a star, we have the continuity equation as follows

$$M_r = \text{const}\,, \qquad r^2 \rho\, dr = \text{const}\,. \tag{3.2}$$

Using the condition of adiabatic changes $\frac{\delta p}{p} = \Gamma \frac{\delta \rho}{\rho}$ and taking variation from the second equality, we obtain

$$\frac{\delta p}{p} = -\Gamma\left[3V + r\frac{dV}{dr}\right]. \tag{3.3}$$

It is evident that the equations of motion

$$\frac{dp}{\rho dr} = -(g+a)\,, \qquad g = \frac{GM_r}{r^2}$$

give, neglecting higher order terms,

$$\frac{d\delta p}{dr} = -a\rho + 4V\frac{dp}{dr}\,.$$

Substituting formula (3.3) into this equation, we obtain Eddington's equation of pulsation

$$\begin{aligned} &\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr}\left[4 + \frac{r}{p}\frac{dp}{dr}\right] + \\ &+ \frac{V}{r\Gamma}\frac{1}{p}\frac{dp}{dr}\left[(3\Gamma - 4) - \frac{n^2 r}{g}\right] = 0\,. \end{aligned} \tag{3.4}$$

We introduce a dimensionless variable x instead of r (we used this variable in our studies of the internal constitution of stars). As it is easy to see

$$\frac{g}{r} = 4\pi G\frac{\bar{\rho}_r}{3} = 4\pi G\rho_c\frac{M_x}{x^3}\,. \tag{3.5}$$

Substituting (3.5) into formula (3.4), we transform the pulsation equation to the form

$$\begin{aligned} &\frac{d^2V}{dr^2} + \frac{1}{x}\frac{dV}{dr}\left[4 + \frac{x}{p_1}\frac{dp_1}{dr}\right] - \\ &- \frac{V}{x\Gamma}\frac{1}{p_1}\frac{dp_1}{dr}\left[(4 - 3\Gamma) + \frac{n^2}{4\pi G\rho_c}\frac{x^3}{3M_x}\right] = 0\,. \end{aligned} \tag{3.6}$$

We transform this equation to self-conjugated form. Multiplying it by $x^4 p_1$, we obtain

$$\frac{d}{dx}\left[x^4 p_1\frac{dV}{dx}\right] - Vx^3\frac{dp_1}{dx}\frac{(4-3\Gamma)}{\Gamma}\left[1 - \lambda\frac{x^3}{3M_x}\right] = 0, \tag{3.7}$$

where

$$\lambda = \frac{n^2}{4\pi G\rho_c\left(\Gamma - \frac{4}{3}\right)}\,. \tag{3.8}$$

So the problem of finding the pulsation period has been reduced to a search for those numerical values of λ by which the differential equation (3.7) has a solution satisfying the "natural" boundary conditions

$$x^4 p_1\frac{dV}{dx}\bigg|_0^{x_0} = 0. \tag{3.9}$$

Formula (3.8) gives the correlation "period — average density of Cepheids" and, hence, the general correlation "period — average density of a star". It is evident that λ depends on the internal structure of a star. Its expected numerical value should be about 1. For a homogeneously dense star, $x^3/(3M_x) = 1$ everywhere inside it. In this case the differential equation (3.7) has the solution: $V = \text{const}$, $\lambda = 1$. This solution determines the main oscillation of such a star. In order to find the main oscillations of differently structured stars, we proceed from the solution by applying the method of perturbations.

3.2 Calculation of the mean values in the pulsation equation by the perturbation method

We write the pulsation equation in general form

$$\left(py'\right)' + qy\left(1 - \lambda\rho\right) = 0\,. \tag{3.10}$$

If we know a solution to this equation under another function $\rho = \rho_0$

$$\left(py_0'\right)' + qy_0\left(1 - \lambda_0\rho_0\right) = 0\,, \tag{3.11}$$

hence we know the function y_0 and the parameter λ_0. After multiplying (3.10) by y_0 and (3.11) by y, we subtract one from the other. Then we integrate the result, taking the limits 0 and x_0. So, we obtain

$$\int_0^{x_0} qyy_0\left[\lambda_0\rho_0 - \lambda\rho\right] dx = 0\,,$$

hence

$$\lambda = \lambda_0\frac{\int_0^{x_0} qyy_0\rho_0\, dx}{\int_0^{x_0} qyy_0\rho\, dx}\,. \tag{3.12}$$

If the oscillations are small, equation (3.10) is the same as (3.11) with only an infinitely small correction

$$\rho = \rho_0 + \delta\rho\,, \quad y = y_0 + \delta y\,, \quad \lambda = \lambda_0 + \delta\lambda\,.$$

Then the exact formula for $\delta\lambda$

$$\delta\lambda = -\lambda_0 \frac{\displaystyle\int_0^{x_0} q y y_0 \, \delta\rho \, dx}{\displaystyle\int_0^{x_0} q y y_0 \, \rho dx}$$

can be replaced by

$$\delta\lambda = -\lambda_0 \frac{\displaystyle\int_0^{x_0} q y_0^2 \, \delta\rho \, dx}{\displaystyle\int_0^{x_0} q y_0^2 \rho dx},$$

and thus we have

$$\lambda = \lambda_0 \frac{\displaystyle\int_0^{x_0} q y_0^2 \rho_0 \, dx}{\displaystyle\int_0^{x_0} q y_0^2 \rho dx}. \qquad (3.13)$$

In our case $y_0 = 1$ and $\lambda_0 = 1$. Comparing formulae (3.10) and (3.7), using (3.13), we obtain

$$\lambda = \lambda_0 \frac{3\displaystyle\int_0^{x_0} x \rho_1 M_x \, dx}{\displaystyle\int_0^{x_0} x^4 \rho_1 \, dx}. \qquad (3.14)$$

We re-write this equation, according to (2.5a), as follows

$$\lambda = \lambda_0 \frac{9\displaystyle\int_0^{x_0} p_1 x^2 dx}{\displaystyle\int_0^{x_0} \rho_1 x^4 dx}. \qquad (3.15)$$

If we introduce the average density $\bar{\rho}$ into formula (3.8) instead of the central one ρ_c, then according to (2.4),

$$\bar{\lambda} = \frac{n^2}{4\pi G \bar{\rho} \left(\Gamma - \frac{4}{3}\right)}, \qquad (3.16)$$

$$\bar{\lambda} = \lambda \frac{\rho_c}{\bar{\rho}} = \frac{x_0^3}{3M_{x_0}} \lambda . \qquad (3.17)$$

Using formulae (2.6) and (3.17) we re-write (3.15) as

$$\bar{\lambda} = \frac{x_0^2 \Omega_{x_0} M_{x_0}}{I_{x_0}}, \qquad (3.18)$$

where I_{x_0} is the dimensionless moment of inertia

$$I_{x_0} = \int_0^{x_0} \rho_1 x^4 dx . \qquad (3.19)$$

Formulae (3.16) and (3.18) determine the oscillation period of a star, $P = 2\pi/n$, independently of its average density $\bar{\rho}$. This result was obtained by Ledoux [18] by a completely different method. It is interesting that our equations (3.16) and (3.18) coincide with Ledoux's formulae.

We next calculate λ for stars of polytropic structures. In such cases I_{x_0} is

$$I_{x_0} = x_0^2 M_{x_0} - 6(n+1) \int_0^{x_0} T_1 \, x^2 dx , \qquad (3.20)$$

where n is the polytropic exponent. Thus

$$\frac{1}{\bar{\lambda}} = \frac{5-n}{3} \left[1 - 6(n+1) \frac{\displaystyle\int_0^{x_0} T_1 \, x^2 dx}{M_{x_0} x_0^2} \right]. \qquad (3.21)$$

Calculations of the numerical values of $\bar{\lambda}$ for cases of different polytropic exponents are given in Table 5.

Table 5

n	$\bar{\lambda}$
0	1.00
1	1.91
3/2	2.52
2	3.85
2.5	7.00
3	13.1

Under large $\bar{\lambda}$, much different from 1, the calculations for Table 5 are less precise. Therefore, in order to check the calculated results, it is interesting to compare the results for $n = 3$ to those obtained by Eddington via his exact solution of his adiabatic oscillation equation for his stellar model. For the stars we consider, he obtained, $\frac{n^2}{\pi G \rho_c \Gamma} = \frac{3}{10}(3 - 4/\Gamma)$. Hence, comparing his result to our formula (3.8), we obtain $\lambda = 9/40$ and $\bar{\lambda} = \frac{9}{40}\frac{\rho_c}{\bar{\rho}} = 12.23$. This is in good agreement with our result $\bar{\lambda} = 13.1$ given in Table 5.

3.3 Comparing the theoretical results to observational data

We represent the "period — average density" correlation in the next form

$$P\sqrt{\bar{\rho}_0} = c_1 , \qquad (3.22)$$

where P is the period (days), $\bar{\rho}_0$ is the average density expressed in the multiples of the average density of the Sun

$$n = \frac{2\pi}{86{,}400\,P}, \qquad \bar{\rho} = 1.411\,\bar{\rho}_0 .$$

Employing formula (3.16), it is easy to obtain a correlation between the coefficients $\bar{\lambda}$ and c_1

$$\bar{\lambda}\left(\Gamma - \frac{4}{3}\right) = 0.447\left(10c_1\right)^{-2}. \qquad (3.23)$$

By analysis of the "mass-luminosity" relation we have previously shown that the radiant pressure is much less than the gaseous pressure in a star. Therefore, because the inner regions of a star are primarily composed of hydrogen, the heat energy there is much more than the energy of ionization. So we have all grounds to assume $\Gamma = 5/3$, the ratio of the heat capacities for a monatomic gas. Hence

$$\bar{\lambda} = 1.34\left(10c_1\right)^{-2}. \tag{3.24}$$

In order to express c_1 in terms of the observed characteristics of a star, we replace $\bar{\rho}_0$ in formula (3.22) with the reduced temperature and the luminosity, via the "mass-luminosity" formula. The "mass-luminosity" correlation has a general form $L \sim M^{\alpha}$ for any star. We denote by $\bar{T}$ the reduced temperature of a star (with respect to the temperature of the Sun), and by M_b its reduced stellar magnitude. Then, by formula (3.22), we obtain

$$\left(0.30 - \frac{1}{5\alpha}\right)\left(M_b - M_{\odot}\right) + \log P + 3\log\bar{T} = \log c_1\,. \tag{3.25}$$

From this formula we see that, in order to find c_1, it is unnecessary to know the exact value of α (if, of course, α has a large numerical value). Eddington's formula for the "mass-luminosity" relation, taken for huge masses, gives $\alpha \sim 2$ (compare with 2.53). Therefore, Eddington's value of $c_1 = 0.100$ is overstated. Applying another correlation, $L \sim M^{10/3}$, Parenago [19] obtained $c_1 = 0.071$. Becker [20] carried out a precise analysis of observational data using Kuiper's empirical "mass-luminosity" arc. He obtained the average value of $c_1 = 0.076$ for Cepheids. Formula (2.4) gives $\bar{\lambda} = 2.7$ or $\bar{\lambda} = 2.3$, so that Table 5 leads us to conclude that Cepheids have structures close to the polytropic class $3/2$, like all other stars. Hence Cepheids have a low concentration of matter at the centre: $\rho_c = 6\bar{\rho}$.

This result is in qualitative agreement with the "natural viewpoint" that sources of stellar energy increase their productivity towards the centre of a star. However (as we saw in §2.8) the model for a point-source of energy and for a constant absorption coefficient, giving stars of minimal average densities, leads to a strong concentration at the centre, $\rho_c/\bar{\rho} = 20.5$. Thus $\bar{\lambda}$ for such a model should be more than an observable one. Really, having $\int_0^{x_0} p_1 x^2 dx = 6.06$ and $I_{x_0} = 140.0$ calculated by Table 4, formulae (3.15) and (3.17) give $\bar{\lambda} = 8.0$ for models with the ultimate concentration of energy sources. So, such stars are of the polytropic class $n = 2.5$. If the absorption coefficient is variable (Cowling's model), calculations give even more: $\bar{\lambda} = 8.4$.

Eddington and others, in their theoretical studies of the pulsation period within the framework of Eddington's model, explain the deviation between the theoretical and observed values of $\bar{\lambda}$ by an effect of the radiant pressure. Studies of pulsations under γ_c close to 1 show that the obtained formula for the period under low γ_c is true even if Γ is the reduced ratio of the heat capacities (which is, depending on the rôle of the radiant pressure, $4/3 \leqslant \Gamma \leqslant 5/3$).

Equation (3.23) shows that when $\bar{\lambda} = 12.23$ and the observable $c_1 = 0.075$ we have $\Gamma_{\text{eff}} = 1.40$. At the same time Γ_{eff} should undergo changes independently of γ_c, i. e. depending upon the rôle of the radiant pressure. For a monatomic gas, Eddington [21] and others obtained this correlation as

$$\Gamma_{\text{eff}} - \frac{4}{3} = \frac{1}{3}\,\frac{1 + 4\gamma_c}{(1+\gamma_c)(1+8\gamma_c)}\,. \tag{3.26}$$

Under $\Gamma_{\text{eff}} = 1.40$ we obtain $\gamma_c = 1.5$. We accept this numerical value in accordance with the average period of Cepheids, $P = 10^{\text{d}}$. Then, by the "mass-luminosity" relation, $M = 12\,M_{\odot}$. It is possible to think that this result is in good agreement with the conventional viewpoint on the rôle of the radiant pressure inside stars (see §2.7). However, because λ_c depends on the mass of a star, other periods give different Γ_{eff} (by formula 3.26) and hence other numerical values of c_1. Using formulae (3.26) and (3,23), we can calculate c_1 for variable stars having longer pulsations, with periods $20^{\text{d}} < P < 30^{\text{d}}$. Instead of the average value $\log c_1 = -1.12$ found by Becker for the stars, there should be $\log c_1 = -1.00$. Despite the small change, observations show no such increase of c_1 [20]. Therefore, our conclusion about the negligible rôle of the radiant pressure in stars, even inside super-giants, finds a new verification. This result verifies as well our results $\mu = 1/2$ and $\kappa = \kappa_{\text{T}}$, obtained in chapter 3.

3.4 Additional data about the internal constitution of stars

Some indications of the internal structure of stars can be obtained from analysis of the elliptic effect in the luminosity arcs of eclipse variable stars. Observations of such binaries gives the ratio of diameters at the equator of a star, which becomes elliptic because of the flow-deforming effect in such binary systems. For synchronous rotations of the whole system and each star in it, the compressed polar diameter of each star should be different (in the first order approximation) from the average equatorial one with a multiplier dependent on their masses. Thus, proceeding from the observed compression we can calculate the meridian compression ϵ. According to Clairaut's theory ϵ is proportional to φ, the ratio of the centrifugal force at the equator to the force of gravity

$$\epsilon = \alpha\varphi\,, \qquad \varphi = \frac{\omega^2}{3\pi G\bar{\rho}}\,,$$

where α is a constant dependent on the structure of the star. This constant was calculated for stars of polytropic structures by numerous researchers: Russell, Chandrasekhar and others. If $n = 0$ (homogeneous star), $\alpha = 1.25$. If $n = 1$, we have $\alpha = 15/(2\pi^2) = 0.755$. If $n = 5$ (the ultimate concentration, Roche's model), $\alpha = 0.50$. We see that the constant α is sensitive to changes in the structure of a star. Therefore determination of the numerical values of n in this way requires extremely precise observations. The values of n so obtained are very uncertain, despite the simplicity of the theory. Shapley first concluded that stars are almost homogeneous. This was verified by Luiten [22] who found the average value $\alpha = 0.57$ for a large number of stars like β Lyrae, and $\alpha = 0.71$ for stars like Algol. His results correspond to the polytropic structures $n = 3/2$ and $n = 1$ respectively.

The observed motion of the line of apsides in numerous eclipse binaries can be explained, in numerous cases, by their elliptic form. Because matter is more strongly concentrated in a binary system than in regular stars, the binary components interact like two point-masses, so there should be no motion of the line of apsides. Therefore the velocity of the line of apsides should be proportional (in the first order approximation) to $\alpha - 1/2$, where α is sensitive to changes in the structure of a star (as we showed above). Many theoretical studies on this theme give contradictory formulae for the velocity, depending on hypotheses about the properties of rotation in the pair. Russell, in his initial studies of this problem, supposed the rotating components solid bodies. This theory, being applied to the system Y Cygni by Russell and Dugan [23], gave $\alpha - 1/2 = 0.034$, which is the polytropic structure $1/2 < n < 2$. Other researchers, having made other suppositions, obtained larger n: $n \simeq 3$. It is probable that we can be most sure only that, because we observe motion of the line of apsides in binaries, the stars have no strong concentration of matter at the centre.

Blackett supposed a law according to which the ratio between the magnetic momentum P_H and angular momentum U is constant for all rotating space bodies. If this law is correct, we could have a possibility of determining the structures of stars in an independent way. We denote by k the ratio between the moments of the inertia of an arbitrary structured star rotating with the angular velocity ω and of the same star if it would be homogeneous throughout. Then

$$U = \frac{2}{5}\, k\omega M R^2, \qquad k = \frac{5}{3}\,\frac{I_{x_0}}{x^2 M_{x_0}}\,,$$

where I_{x_0} is the dimensionless moment of inertia. Using Blanchett's formula [24]

$$\frac{P_H}{U} = \beta\,\frac{G^{1/2}}{2c} \tag{3.27}$$

(β is a dimensionless multiplier, equal to about 1), and having the magnetic magnitude at the pole $H = 2P_H/R^3$, we can calculate k. For the Earth ($k = 0.88$), we obtain $\beta = 0.3$. Supposing $k = 0.16$ for stars, Blackett has found: $\beta = 1.14$ for the Sun and $\beta = 1.16$ for 78 Virginis (its magnetic field has been measured by Babcock).

If Blackett's law (3.27) is valid throughout the Universe and $\beta = 0.3$ for all space bodies, not just for the Earth, then $k = 0.60$ should be accepted for stars. Comparing $k = 0.60$ with Table 6, we come to the same conclusion that we have obtained by completely different methods: that stars have polytropic structures of class $n = 3/2$.

Table 6

n	k
0	1.00
1	0.65
3/2	0.52
2	0.40
2.5	0.28
3	0.20

For the convective model of a star (calculated in §2.8) we obtain k=0.26. This is much less than required. The same convective model with a variable absorption coefficient (Cowling's model) gives even less: $k = 0.19$.

The agreement of our value $n = 3/2$ with other data, obtained by very different methods, verifies Blackett's law. It is possible his formula (3.27) should be written without β, but with the denominator $2\pi c$.

3.5 Conclusions about the internal constitution of stars

The most certain conclusions about the structure of stars are derived from the theory of pulsation of Cepheids. We have concluded that Cepheids have structures close to the polytropic one of class $n = 3/2$, for which $\rho_c = 6\,\bar{\rho}$. This conclusion is verified by other data, whereas each of them could be doubtful when being considered in isolation. At the same time all the data, characterizing stars of different classes, lead to the same result. It is probable that stars are really close to being homogeneous, having a low concentration of matter at the centre like the bulky planets, Jupiter and Saturn. Such a distribution of matter, as we saw in the ultimate case of the convective model, cannot be explained by a strong concentration of an energy source at the centre, or by a special kind of absorption coefficient. The real reason is that the radiant pressure B is included in the mechanical equilibrium equation through the gaseous pressure in the exponent $1/4$. Therefore the structural characteristics M_{x_0} and X_0, determined by the function ρ_1, have small changes even in very different models. Hence, in order to obtain the observable low concentration of matter at the centre of stars, we can search for the reason only in the heat equilibrium equation. The polytropic model $n = 3/2$ differs from other polytropic models by a smaller value of x_0. In order to make x_0 smaller, the gaseous pressure should decrease more strongly in the upper layers of a star. Such a rapid decrease in the pressure is possible only if the surface layers are heavy. In other words, in the case of the strong increase of the molecular weight in the surface layers of a star. Such an explanation is in complete agreement with our conclusion about the high concentration of hydrogen in the internal regions of stars. If the average molecular weight changes from $\mu = 1/2$ at the centre to $\mu = 2$ at the surface of a star, such a change of the molecular weight can be sufficient.

What is the goal of introducing the variable μ? Let us assume that μ depends on the temperature as

$$\mu_1 = \frac{1}{T_1^s}\,, \tag{3.28}$$

where s is a positive determined exponent. Increase of the molecular weight at the surface should result in an increase of the absorption coefficient κ (transition from $\kappa = \kappa_{\mathrm{T}}$ to $\kappa = \kappa_{\mathrm{Ch}}$). At the same time, under energy sources concentrated at the centre, the quantity $\kappa_1 L_x / M_x$ can remain almost the same. If $\kappa_1 L_x / M_x = \mathrm{const} = 1$, equation (2.22a) leads to

$$p_1 = B_1 = T_1^4\,, \qquad \lambda = 1\,. \tag{3.29}$$

Instead of T_1 we introduce the characteristics

$$u_1 = \frac{T_1}{\mu_1} = T_1^{1+s} = \mu_1^{-\frac{1+s}{s}}\,, \tag{3.30}$$

which keeps the ideal gas equation in the regular form $p_1 = u_1 \rho_1$. According to (3.29), we have

$$p_1 = u_1^{\frac{4}{1+s}}. \tag{3.31}$$

where we should equate the exponent $4/(1+s)$ to $n+1$ according to formula (2.7a).

Thus we have

$$\rho_1 = u_1^n\,, \qquad n = \frac{3-s}{1+s}\,, \tag{3.32}$$

so the function u_1 is determined by Emden's equation of class n. Hence, in order to obtain the structure $n = 3/2$, there should be $s = 3/5$ — the very low increase of the molecular weight: for instance, under such s the molecular weight μ increases 4 times at the distance x_1 where

$$\mu_1 = \left(\frac{1}{4}\right)^{\frac{8}{3}} = 0.025\,, \qquad T_1 = \left(\frac{1}{4}\right)^{\frac{5}{3}} = 0.10\,. \tag{3.33}$$

At $x > x_1$ the molecular weight remains unchanged, the equilibrium of a star is determined by the regular system of the equilibrium equations. However at the numerical values (3.33) almost the whole mass of a star is accounted for (see Table 4, for instance), so we obtain small corrections to the polytropic structure $n = 3/2$. Naturally, tables of Emden's function taken under $n = 3/2$ show that $x_1 = 5.6$ and $M_{x_1} = 11.0$. Applying formula (2.27a), we obtain $x_0 = 7.0$ instead of $x_0 = 6$, as expected for such a polytropic structure. These calculations show that the observed structures of stars* verify our result about the high content of hydrogen in the internal regions of a star, obtained from the "mass-luminosity" relation. At the same time, it should be taken into account that the hydrogen content in the surface layers of stars is also substantial. Therefore on the average we have $\mu < 2$ inside a star, so the problem about homogeneity of the molecular weight of stars is not completely solved with the above.

*The fact that the molecular weight is variable does not change the formulas, determining the pulsation period of Cepheids. The variability of μ can include a goal only if the whole structure of star has been changed.

We saw that the dimensionless mass M_{x_0} is almost the same in completely different models of stars. For polytropic structures of the classes $n = 3/2$ and $n = 2$, convective models, and models described in Table 2, we obtained approximately the same numerical values of M_{x_0}. Therefore we can surely accept $M_{x_0} = 11$. What about x_0? According to observed structures of stars, we accept $x_0 = 6$. Hence $\bar{\rho}_c = 6.5\,\bar{\rho}$. In order to obtain $\kappa = \kappa_1$ from the observed "mass-luminosity" relation, we should have $\lambda L_{x_0} / M_{x_0}^3 = 1.0{\times}10^{-3}$. Thus we obtain $\lambda L_{x_0} = 1.5$. As a result, using these numerical values in formulae (2.28), (2.30), and (2.31), we have a way of calculating the physical conditions at the centre of any star. We now make this calculation for the Sun. Assuming $\mu_{c\odot} = 1/2$, we obtain

$$\begin{gathered} \rho_{c\odot} = 9.2\,, \qquad p_{c\odot} = 9.5{\times}10^{15}\ \text{dynes/cm}^2, \\ \gamma_{c\odot} = 0.4{\times}10^{-3}, \qquad B_{c\odot} = 3.8{\times}10^{12}\ \text{dynes/cm}^2, \\ T_{c\odot} = 6.3{\times}10^{6}\ \text{degrees}. \end{gathered} \tag{3.34}$$

Of the data the most soundly calculated is $\gamma_{c\odot}$, because it is dependent only on M_{x_0}. Thus a low temperature at the centre of the Sun, about 6 million degrees, is obtained because of low numerical values of $\mu_{c\odot}$ and x_0. Having such low temperatures, it is scarcely possible to explain the origin of stellar energy by thermonuclear reactions.

The results indicate possible ways to continue our research into the internal constitution of stars. They open a way for a physical interpretation of the Russell-Hertzsprung diagram, which is directly linked to the origin of stellar energy.

PART II

Chapter 1

The Russell-Hertzsprung Diagram and the Origin of Stellar Energy

1.1 An explanation of the Russell-Hertzsprung diagram by the theory of the internal constitution of stars

The Russell-Hertzsprung diagram connects the luminosity L of a star to its spectral class or, in other words, the reduced temperature T_{eff}. The theory of the internal constitution of stars uses the radius R of a star instead of the effective temperature T_{eff}. It follows from the Stephan-Boltzmann law

$$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4\,, \qquad \sigma = \frac{1}{4}\,\alpha c\,,$$

where c is the velocity of light, α is the radiant energy density constant. Thus, the Russell-Hertzsprung diagram is the same for the correlation $L(R)$ or $M(R)$, if we use the "mass-luminosity" relation. Due to the existence of numerous sequences in the Russell-Hertzsprung diagram (the main sequence, the sequences of giants, dwarfs, etc.) the correlations $L(R)$ and $M(R)$ are not sufficiently clear. In this paragraph we show that for most stars the correlations $L(R)$ and $M(R)$ are directly connected to the mechanism generating stellar energy. The essence of the correlation $L(R)$ becomes clear, as soon as we replace the observable characteristics of stars (the masses M, the luminosities L, and the radii R) with the parameters which determine the physical conditions inside stars. The method of such calculations and the precision of the obtained results were discussed in detail in Part I of this research.

First we calculate the average density of a star

$$\rho = \frac{3M}{4\pi R^3}\,. \tag{1.1}$$

Then, having the mechanical equilibrium of a star, we calculate the average pressure within. This internal pressure is in equilibrium with the weight of the column whose aperture is one square centimeter and whose length is the radius of the star. The pressure is $p = g\rho R$. Because of $g = GM/R^2$,

$$p = \frac{3G}{4\pi}\frac{M^2}{R^4}\,. \tag{1.2}$$

What can be said about the temperature of a star? It should be naturally calculated by the energy flow of excess radiation F_R

$$F_R = \frac{L}{4\pi R^2}\,, \tag{1.3}$$

because the flow is connected to the gradient of the temperatures. If we know what mechanism transfers energy inside a star, we can calculate the temperature T by formula (1.1) or (1.2)

$$T = f\,(L, M, R)\,. \tag{1.4}$$

For instance, if energy is dragged by radiations, according to §1.2, we have

$$F_R = -\frac{c}{\kappa\rho}\frac{dB}{dr}\,, \tag{1.5}$$

where κ is the absorption coefficient per unit mass, B is the radiant pressure

$$B = \frac{1}{3}\,\alpha T^4. \tag{1.6}$$

We often use the radiant pressure B instead of the temperature. By formula (1.3) we can write

$$B \simeq \frac{\kappa F_R}{c}\,\rho R\,,$$

which, by using (1.1) and (1.3), gives

$$B \simeq \frac{3LM}{(4\pi)^2 cR^4}\,\kappa\,. \tag{1.4a}$$

If we know how κ depends on B and ρ, formula (1.4a) leads to equation (1.4). So formulae (1.1), (1.2), and (1.4a) permit calculation of the average numerical values of the density, the pressure, and the temperature for any star. Exact numerical values of the physical parameters at a given point inside a star (at the centre, for instance) can be obtained, if we multiply the formulae by dimensionless "structural" coefficients. We studied the structural coefficients in detail in Part I of this research. We studied them by both mathematical methods (solving the system of the dimensionless differential equations and mechanical equilibrium and heat equilibrium of a star) and empirical methods (the analysis of observable properties of stars).

Values of ρ, p, and T, calculated by formulae (1.1), (1.2), and (1.4), should be connected by the equation of the phase state of matter. Hence, we obtain the first theoretical correlation

$$F_1\,(L, M, R) = 0\,, \tag{1.7}$$

which almost does not depend on the kind of energy generation in stars.

For instance, a star built on an ideal gas has

$$p = \frac{\Re T}{\mu}\,\rho\,.$$

Dividing (1.2) by (1.1), we obtain

$$T \simeq \frac{G}{\Re}\,\mu\,\frac{M}{R}\,, \qquad B \simeq \frac{\alpha G^4}{3\Re^4}\,\mu\,\frac{M^4}{R^4}\,, \tag{1.8}$$

$$\gamma = \frac{B}{p} \simeq M^2\mu^4. \tag{1.9}$$

Comparing (1.8) to formula (1.4a), obtained for the energy transfer by radiation, we obtain the correlation (1.7) in clear form

$$L \simeq M^3\,\frac{\mu^4}{\kappa}\,. \tag{1.7a}$$

Another instance — a star built on a degenerate gas

$$p \simeq \rho^{\frac{5}{3}}\,,$$

then formulae (1.1) and (1.2) lead to

$$RM^{1/3} = \text{const}\,, \tag{7.b}$$

so in this case we just obtain the correlation like (1.7), where there is no L.

Formula (1.7a), which is true for an ideal gas, can include R only through κ. Therefore this formula is actually the "mass-luminosity" relation, which is in good agreement with observational data $L \sim M^3$, if $\mu^4/\kappa = \text{const} = 0.08$. The calculations are valid under the low radiant pressure $\gamma < 1$. As we see from formula (1.9), inside extremely bulky stars the value of γ can be more than 1. In such cases formula (1.2) will determine the radiant pressure

$$B \simeq \frac{M^2}{R^4}\,,$$

not the gaseous one. Comparing to formula (1.4a), we have

$$L \simeq \frac{M}{\kappa}\,. \tag{1.7b}$$

Astronomical observations show that super-giants do not have the huge variations of M which are predicted by this formula. Therefore, in Part I, we came to the conclusion that $\gamma \leqslant 1$ for stars of regular masses $M \leqslant 100\,M_{\odot}$, so formula (1.9) gives for them: $\mu = 1/2$. Hence, $\kappa = 0.8$, which is approximately equal to Thomson's absorption coefficient. This is very interesting, for we have obtained that the radiant pressure places a barrier to the existence of extremely large masses for stars, although there is no such barrier in the theory based on the equilibrium equations of stars.

Until now, we hardly used the heat equilibrium equation, which requires that the energy produced inside a star should be equal to its radiation into space. According to the heat equilibrium equation, the average productivity of energy by one gram of stellar matter can be calculated by the formula

$$\varepsilon = \frac{L}{M}\,. \tag{1.10}$$

On the other hand, if the productivity of energy is determined by some other reactions, ε would be a function of ρ and T. This function would also be dependent on the kinetics of the supposed reaction. Thus formulae (1.10), (1.1), (1.4), and the equation of the reaction demand the existence of the second correlation

$$F_2\,(L, M, R) = 0\,, \tag{1.11}$$

which is fully determined by the mechanism that generates energy in the reaction. For an ideal gas, R disappears from the first correlation $F_1 = 0$ (1.7). For this reason formula (1.11) transforms into the relation $L(R)$ or $M(R)$, which become directly dependent on the kind of energy sources in stars. For a degenerate gas we obtain another picture: as we saw above, in this case $M(R)$ is independent of energy sources, and then M and L are connected by equation (1.11).

1.2 Transforming the Russell-Hertzsprung diagram to the physical characteristics specific to the central regions of stars

Our task is to find those processes which generate energy in stars. In order to solve this problem, we must know physical conditions inside stars. In other words, we should proceed from the observed characteristics L, M, R to physical parameters.

We denote by a bar all the quantities expressed in terms of their numerical values in the Sun. Assuming, according to our conclusion in Part I, that stars have the same structure, we can, by formulae (1.1), (1.2), and (1.10), strictly calculate the central characteristics of stars

$$\bar{p}_c = \frac{\bar{M}^2}{\bar{R}^4}\,, \qquad \bar{\rho}_c = \frac{\bar{M}}{\bar{R}^3}\,, \qquad \bar{\varepsilon}_c = \frac{\bar{L}}{\bar{M}}\,. \tag{1.12}$$

Even for very different structures of stars, it is impossible to obtain distorted results by the formulae. As we saw in the previous paragraph, we can calculate the temperature (or, which is equivalent, the radiant pressure) in two ways, either way being connected to suppositions. First, the radiant pressure can be obtained through the flow of energy, i. e. through ε by formula (1.4a). The exact formula of that relation, by equations (1.27) in §1.3 (Part I), is

$$B_c = \frac{\varepsilon_c \kappa_c}{4\pi G c\,\lambda}\, p_c\,, \tag{1.13}$$

where λ is the structural parameter of the main system of the dimensionless equations of equilibrium: its numerical value is about 1. Second, for an ideal gas, the radiant pressure can be calculated directly from formulae (1.12)

$$\frac{\bar{B}_c}{\bar{\mu}^4} = \left(\frac{\bar{p}_c}{\bar{\rho}_c}\right)^4 = \frac{\bar{M}^4}{\bar{R}^4}\,. \tag{1.14}$$

Formulae (1.13) and (1.14) must lead to the same result. This requirement leads to the "mass-luminosity" relation. Our conclusion that all stars (except for while dwarfs) are built on an ideal gas is so well grounded that it is fair to use formula (1.14) in order to calculate the temperature or the radiant pressure in stars. Naturally, Eddington [21] showed: under temperatures of about a few million degrees, because of the ionization of matter, the atoms of even heavy elements take up so little space (about one millionth of their normal sizes) that van der Waals' corrections are negligible if the density is even much more than 1. However, because of plasma, there could be substantial electrostatic interactions between particles, making the pressure negative, and the gas approaches properties of a super-ideal one. The approximate theory of such phenomena in strong electrolytes has been developed by Debye and Hückell. Eddington and Rosseland applied the theory to a gas inside stars. They came to the conclusion that the electric pressure cannot substantially change the internal constitution of stars. Giving no details of that theory, we can show directly that the electric pressure is negligible in stars built on hydrogen. We compare the kinetic energy of particles to the energy of Coulomb interaction

$$kT > \frac{z^2 e^2}{r}\,.$$

As soon as the formula becomes true in a gas, the gas becomes ideal. Cubing the equation we obtain

$$\frac{(kT)^3}{n} = \frac{(kT)^4}{p} > z^6 e^6,$$

where n is the number of particles in a unit volume. Because the radiant pressure is given by the formula

$$B = \frac{\pi^2}{45}\frac{(kT)^4}{(\hbar c)^3}\,, \tag{1.6a}$$

a gas becomes ideal as soon as the ratio between the radiant pressure and the gaseous pressure becomes

$$\gamma > \frac{\pi^2 z^6}{45}\left(\frac{e^2}{\hbar c}\right)^3.$$

Because of formula (1.9), this ratio is determined by the mass of a star. Because $\gamma = 1$ under $\bar{M} = 100$, we obtain $\gamma^{\frac{1}{2}} \approx \bar{M}/100$. So, for an ideal gas, we obtain the condition

$$100 M_\odot > M > \frac{100\pi}{\sqrt{45}}\, z^3 \left(\frac{e^2}{\hbar c}\right)^{3/2} M_\odot\,. \tag{1.15}$$

which is dependent only on the mass of a star.

For hydrogen or singly ionized elements, we have $z = 1$. Hence, for hydrogen contents of stars, the electric pressure can play a substantial rôle only in stars with masses less than 0.01–0.02 of the mass of the Sun.

It is amazing that of all possible states of matter in stars there are realized those states which are the most simple from the theoretical point of view.

Now, if we know $\bar{M}$ and $\bar{R}$ for a star, assuming the same molecular weight $\bar{\mu} = 1$ for all stars (by our previous conclusions), we can calculate its central characteristics $\bar{\rho}_c$ and $\bar{T}_c$ by formulae (1.12) and (1.14). The range, within which the calculated physical parameters are located, is so large ($10^{-8} < \bar{\rho}_c < 10^6$, $10^{-2} < \bar{T}_c < 10^2$, $10^{-3} < \bar{\varepsilon}_c < 10^4$), that we use logarithmic scales. We use the abscissa for $\log \bar{\rho}_c$, while the ordinate is used for $\log \bar{B}_c$ (or equivalently, $4 \log \bar{T}_c$). If an energy generation law like $\varepsilon_c = f(\rho_c, T_c)$ exists in Nature, the points $\log \bar{\varepsilon}_c$ plotted along the z-coordinate axis will build a surface. On the other hand, the equilibrium condition requires formula (1.13), so the equilibrium states of stars should be possible only at the transection of the above surfaces*. Hence, stars should be located in the plane ($\log \bar{\rho}_c$, $\log \bar{B}_c$) along a line which is actually the relation $M(R)$ transformed to the physical characteristics inside stars. There in the diagram, we draw the numerical values of $\log \bar{\varepsilon}_c$ in order to picture the whole volume.

1.3 The arc of nuclear reactions

The equation for the generation of energy by thermonuclear reactions is

$$\varepsilon = A\,\rho\tau^2\varepsilon^{-\tau}\,, \qquad \tau = \frac{a}{T_{\mathrm{m}}^{1/3}}\,, \tag{1.16}$$

where T_{m} is temperature expressed in millions of degrees. For instance, for the proton-proton reaction, the constants a and A take the values

$$a = 33.8, \qquad A = 4{\times}10^3. \tag{1.17}$$

In order to find the arc of the relation between ρ_c and B_c, on which stars should be located if nuclear reactions are the sources of their energy, we eliminate ε_c from formula (1.16) by formula (1.13)

$$\lambda 4\pi G c B_c = A\kappa_c\, p_c\, \rho_c \tau_c^2 e^{-\tau_c}. \tag{1.18}$$

As the exponent indicates (see formula (1.16)), ε is very sensitive to temperature. Therefore, inside such stars, a core of free convection should exist, as was shown in detail in Part I, §2.8. We showed there that λ cannot be calculated separately for stars within which there is a convective core: the equilibrium equations determine only λL_{x_0}, where L_{x_0} is the dimensionless luminosity

$$L_{x_0} = \int_0^{x_0} \varepsilon_1\, \rho_1 x^2 dx\,. \tag{1.19}$$

In this formula x_0 is the dimensionless radius (see Part I). The subscript 1 on ε and ρ means that the quantities are taken in terms of their numerical values at the centre of a star. In the case under consideration (stars inside which thermonuclear reactions occur).

$$L_{x_0} = \int_0^{x_0} \rho_1^2 \tau_1^2\, e^{-(\tau_1 - \tau_c)} x^2 dx\,.$$

Because this integral includes the convective core (where $\rho_1 = T_1^{3/2}$),

$$L_{x_0}(\tau_c) = \int_0^{x_0} T_1^{1/3} x^2 e^{-\tau_c\left(T_1^{-1/3}-1\right)} dx\,. \tag{1.20}$$

The integral $L_{x_0}(\tau_c)$ can be easily taken by numerical methods, if we use Emden's solution $T_1(x)$ for stars of the polytropic structure 3/2. The calculations show that numerical values of the integral taken under very different τ_c are very little different from 1. For instance,

$$L_{x_0}(33.8) = 0.67, \qquad L_{x_0}(7.3) = 1.15.$$

For the proton-proton reactions formula (1.17), the first value of L_{x_0} is 1 million degrees at the centre of a star, the second value is one hundred million degrees. Assuming $L_{x_0} \approx 1$ (according to our conclusions in Part I), Table 3 gives $\lambda \approx 3$ in stars where the absorption coefficient is constant.

In Part I of this research we found the average molecular weight 1/2 for all stars. We also found that all stars have structures very close to the polytropic structure of the class 3/2. Under these conditions, the central temperature of the Sun should be $6{\times}10^6$ degrees. Therefore Bethe's carbon-nitrogen

*The energy generation surface, drawn from the energy generation law $\varepsilon_c = f(\rho_c, T_c)$, and the energy drainage surface, drawn from formula (1.13).

cycle is improbable as the source of stellar energy. As an example, we consider proton-proton reactions. Because of the numerical values obtained for the constants a and A (1.17), formula (1.18) gives

$$\log \rho_c = 0.217\,\tau_c - 5.5 \log \tau_c + 5.26 - \frac{1}{2} \log \frac{\kappa_c}{\mu} \,. \quad (1.21)$$

Taking κ_c/μ constant in this formula, we see that ρ_c has the very slanting minimum (independent of the temperature) at $\tau_c = 11$ that is $T_c = 30{\times}10^6$ degrees. In a hydrogen star where the absorption coefficient is Thomson, the last term of (1.21) is zero and the minimal value of ρ_c is 100. Hence, stars undergoing proton-proton reactions internally should be located along the line $\rho_c \approx 100$ in the diagram for (ρ_c, B_c). It appears that stars of the main sequence satisfy the requirement (in a rude approximation). Therefore, it also appears that the energy produced by thermonuclear reactions could explain the luminosity of most of stars. But this is only an illusion. This illusion disappears completely as soon as we construct the diagram for ($\log \bar{\rho}_c$, $\log \bar{B}_c$) using the data of observational astronomy.

1.4 Distribution of stars on the physical conditions diagram

Currently we know all three parameters (the mass, the bolometric absolute stellar magnitude, and the spectral class) for approximately two hundred stars. In our research we should use only independent measurements of the quantities. For this reason, we cannot use the stellar magnitudes obtained by the spectroscopic parallax method, because the basis of this method is the "mass-luminosity" relation.

For stars of the main sequence we used the observational data collection published in 1948 by Lohmann [26], who generalized data by Parenago and Kuiper. For eclipse variable stars we used data collections mainly by Martynov [27], Gaposchkin [28], and others. Finally, we took particularly interesting data about super-giants from collections by Parenago [29], Kuiper [7], and Struve [30]. Some important data about the masses of sub-dwarfs were given to the writer by Prof. Parenago in person, and I'm very grateful to him for his help, and critical discussion of the whole research. Consequently, we used the complete data of about 150 stars.

The stellar magnitudes were obtained by the above mentioned astronomers by the trigonometric parallaxes method and the empirically obtained bolometric corrections (Petit, Nickolson, Kuiper). In order to go from the spectral class to the effective temperature, we used Kuiper's temperature scale. Then we calculated the radius of a star by the formula

$$5 \log \bar{R} = 4.62 - m_b - 10 \log \bar{T}_{\text{eff}} \,, \quad (1.22)$$

where m_b is the bolometric stellar magnitude of the star. Then, by formulae (1.12) and (1.14), we calculated $\log \bar{\rho}_c$, $\log \bar{B}_c$, $\log \bar{\varepsilon}_c$. We calculated the characteristics for every star on our list. The results are given in Fig. 2*. There the abscissa takes the logarithm of the matter density, $\log \bar{\rho}_c$, while the ordinate takes the logarithm of the radiant energy density, $\log \bar{B}_c$, where both values are taken at the centre of a star†. Each star is plotted as a point in the numerical value of $\log \bar{\varepsilon}_c$ — the energy productivity per second from one gramme of matter at the centre of a star with respect to the energy productivity per second at the centre of the Sun. In order to make exploration of the diagram easier, we have drawn the net values of the fixed masses and radii. Bold lines at the left side and the right side are the boundaries of that area where the ideal gas law is true (stars land in exactly this area). The left bold line is the boundary of the ultimately large radiant pressure ($\gamma = 1$). The bold line in the lower part of the diagram is the boundary of the ultimately large electric pressure, drawn for hydrogen by formula (1.15). This line leads to the right side bold lines, which are the boundaries of the degeneration of gas calculated for hydrogen (the first line) and heavy elements (the second line).

We built the right boundary lines in the following way. We denote by n_e the number of free electrons inside one cubic centimetre, and μ_e the molecular weight per electron. Then

$$\rho = \mu_e m_{\text{H}} n_e \,,$$

so Sommerfeld's condition of degeneration

$$\frac{n_e \hbar^3}{2} \frac{1}{\left(2\pi m_e k T\right)^{3/2}} > 1 \quad (1.23)$$

can be re-written as

$$\rho > 10^{-8} \mu_e T^{3/2}. \quad (1.24)$$

For the variables p and ρ, we obtain the degeneration boundary equation‡

$$\begin{aligned} p &= k\,\mu_e^{5/3} \rho^{5/3}, \\ \bar{p} &= k\,\frac{\rho_\odot^{5/3}}{p_\odot}\,\bar{\rho}^{\,5/3} \mu_e^{5/3} \,, \end{aligned} \quad (1.25)$$

which coincides with the Fermi gas state equation $p = K\rho^{5/3} = K_{\text{H}}\mu_e^{5/3}\rho^{5/3}$ (formula 1.9 in Part I), if

$$K \approx K_{\text{H}} = 9.89{\times}10^{12}.$$

*Of course not all the stars are shown in the diagram, because that would produce a very dense concentration of points. At the same time, the plotted points show real concentrations of stars in its different parts. — Editor's remark.

†The bar means that both values are expressed in multiples of the corresponding values at the centre of the Sun. — Editor's remark.

‡The degeneration boundary equation is represented here in two forms: expressed in absolute values of p and in multiples of the pressure in the Sun. — Editor's remark.

At the centre of the Sun, as obtained in Part I of this research (see formula 3.34),

$$\rho_{c\odot} = 9.2\,, \qquad p_{c\odot} = 9.5\times10^{15},$$
$$\gamma_{c\odot} = 0.4\times10^{-3}, \qquad B_{c\odot} = 3.8\times10^{12}, \qquad (1.26)$$
$$T_{c\odot} = 6.3\times10^{6},$$

then we obtain

$$\bar{p} = 4\times10^{-2}\,\bar{\rho}^{\,5/3}\mu_e^{5/3}.$$

The right side boundaries drawn in the diagram are constructed for $\mu_e = 1$ and $\mu_e = 2$. At the same time these are lines along which stars built on a degenerate gas (the lines of Chandrasekhar's "mass-radius" relation) should be located. In this case the ordinate axis has the meaning $\log\,(\bar{p}/\bar{\rho})^4$ that becomes the logarithm of the radiant energy density $\log \bar{B}$ for ideal gases only. In this sense we have drawn white dwarfs and Jupiter on the diagram. Under low pressure, near the boundary of strong electric interactions, the degeneration lines bend. Then the lines become constant density lines, because of the lowering of the ionization level and the appearance of normal atoms. The lines were constructed according to Kothari's "pressure-ionization" theory [31]. Here we see a wonderful consequence of Kothari's theory: the maximum radius which can be attained by a cold body is about the radius of Jupiter.

Finally, this diagram contains the arc along which should be located stars whose energy is generated by proton-proton reactions. The arc is built by formula (1.21), where we used the central characteristics of the Sun (1.26) obtained in Part I.

The values $\log \bar{\varepsilon}_c$ plotted for every star builds the system of isoergs — the lines of the same productivity of energy. The lines were drawn through the interval of ten changes of $\bar{\varepsilon}_c$. If a "mass-luminosity" relation for stars does not contain their radii, $\bar{\varepsilon}_c$ should be a function of only the masses of stars. Hence, the isoergs should be parallel to the constant mass lines. In general, we can suppose the "mass-luminosity" relation as the function

$$L \sim M^{\alpha}, \qquad (1.27)$$

then the interval between the neighbouring isoergs should decrease with increasing α according to the picture drawn in the upper left part of the diagram. We see that the real picture does not correspond to formula (1.27) absolutely. Only for giants, and the central region of the main sequence (at the centre of the diagram) do the isoergs trace a path approximately parallel to the constant mass lines at the interval $\alpha = 3.8$. In all other regions of the diagram the isoergs $\bar{\varepsilon}_c$ are wonderfully curved, especially in the regions of super-giants (the lower left part of the diagram) and hot sub-dwarfs (the upper right part). As we will soon see, the curvilinearity can be explained. In the central concentration of stars we see two opposite tendencies of the isoergs to be curved. We have a large dataset here, so the isoergs were drawn very accurately. The twists are in exact agreement with the breaks, discovered by Lohmann [26], in the "mass-luminosity" relation for stars of the main sequence. It is wonderful that this tendency, intensifying at the bottom, gives the anomalously large luminosities for sub-giants (the satellites of Algol) — the circumstance, considered by Struve [30]. For instance, the luminosity of the satellite of XZ Sagittarii, according to Struve, is ten thousand times more than that calculated by the regular "mass-luminosity" relation. There we obtain also the anomalously large luminosity, discovered by Parenago [29], for sub-dwarfs of small masses. The increase of the opposite tendency at the top verifies the low luminosity of extremely hot stars, an increase which leads to Trumpler stars. It is very doubtful that masses of Trumpler stars measured through their Einstein red shift are valid. For this reason, the diagram contains only Trumpler stars of "intermediate" masses. Looking at the region of sub-giants and sub-dwarfs (of large masses and of small ones) we see that ε is almost constant there, and independent of the masses of the stars. Only by considering altogether the stars located in the diagram we can arrive at the result obtained in Part I of this research: $L \sim M^3$.

So, the first conclusion that can be drawn from our consideration of the diagram is: deviations from the "mass-luminosity" relation are real, they cannot be related to systematic errors in the observational data. The possibility of drawing the exact lines of constant $\bar{\varepsilon}_c$ itself is wonderful: it shows that ε is a simple function of ρ and B. Hence, the luminosity L is a simple function of M and R. Some doubts can arise from the region located below and a little left of the central region of the diagram, where the isoergs do not coincide with L for sub-dwarfs of spectral class F–G and L of normal dwarfs of class M. It is most probable that the inconsistency is only a visual effect, derived from errors in experimental measurements of the masses and radii of the sub-dwarfs.

As a whole our diagram shows the plane image of the surface $\varepsilon\,(\rho, B)$. We obtained much more than expected: we should obtain only one section of the surface, but we obtained the whole surface, beautifully seen in the central region of the diagram. Actually, we see no tendency for stars to be distributed along a sequence $\varepsilon = \text{const}$. Thus, of the two equations determining ε, there remains only one: *the energy productivity in stars is determined by the energy drainage (radiation) only*. This conclusion is very important. Thus the mechanism that generates energy in stars is not of any kid of reactions, but is like the generation of energy in the process of its drainage. The crude example is the energy production when a star, radiating energy into space, is cooling down: the star compresses, so the energy of its gravitational field becomes free, cooling the star (the well-known Helmholtz-Kelvin mechanism). Naturally, in a cooling down (compress-

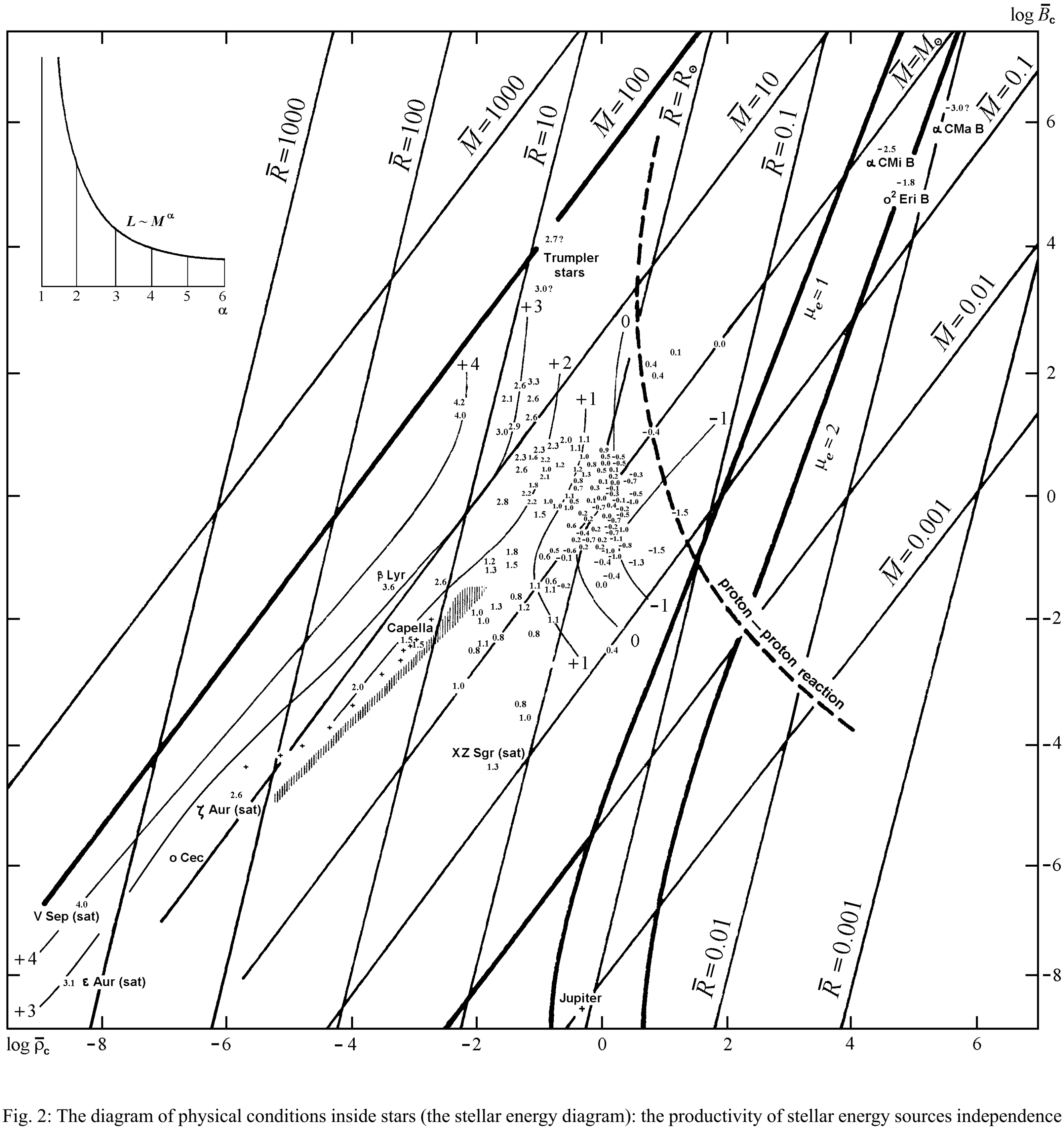


Fig. 2: The diagram of physical conditions inside stars (the stellar energy diagram): the productivity of stellar energy sources independence of the physical conditions in the central regions of stars. The abscissa is the logarithm of the density of matter, the ordinate is the logarithm of the radiant energy density (both are taken at the centre of stars in multiples of the corresponding values at the centre of the Sun). The small diagram at the upper left depicts the intervals between the neighbouring isoergs.

ing) star the quantity of energy generated is determined by the speed of this process. At the same time the speed is regulated by the heat drainage. Of course, the Helmholtz-Kelvin mechanism is only a crude example, because of the inapplicable short period of the cooling (a few million years). At the same time the mechanism that really generates energy in stars should also be self-regulating by the radiation. In contrast to reaction, such a mechanism should be called a *machine*.

It should be noted that despite many classes of stars in the diagram, the filling of the diagram has some limitations.

First there is the main direction along which stars are concentrated under a huge range of physical conditions — from the sequence of giants, then the central concentration in our diagram (the so-called main sequence of the Hertzsprung-Russel diagram), to sub-dwarfs of class A and white dwarfs. In order to amplify the importance of this direction, we indicated the main location of normal giants by a hatched strip. The main direction wonderfully traces an angle of exactly 45°. Hence, all stars are concentrated along the line, determined by the equation*

$$B \sim \rho\mu^4. \tag{1.28}$$

Because stars built on a degenerate gas satisfy this direction, a more accurate formula is

$$p \sim \rho^{5/5} \tag{1.28a}$$

Second, there is in the main direction (1.28) a special point — the centre of the main sequence†, around which stars are distributed at greater distances, and in especially large numbers.

Thus, there must exist two fundamental constants which determine the generation of energy in stars:

1. The coefficient of proportionality of equation (1.28);
2. One of the coordinates of the "main point", because its second coordinate is determined by the eq. (1.28).

The above mentioned symmetry of the surface $\varepsilon(\rho, B)$ is connected to the same two constants.

Concluding the general description of the diagram, we note: this diagram can also give a practical profit in calculations of the mass of a star by its luminosity and the spectral class. Naturally, having the radius calculated, we follow the line $R = \text{const}$ to that point where $\log\bar{\varepsilon} + \log\bar{M}$ gives the observed value of $\log\bar{L}$.

1.5 Inconsistency of the explanation of stellar energy by Bethe's thermonuclear reactions

It is seemingly possible that the existence of the uncovered main direction along which stars are concentrated in our diagram support a stellar energy mechanism like reactions. In the real situation the equation of the main direction (1.28) contradicts the kinetics of any reaction. Naturally, equation (1.28) can be derived from the condition of energy drainage (1.13) only if

$$\varepsilon \sim \frac{1}{T}, \quad \text{under} \quad \rho \sim T^4, \tag{1.29}$$

i. e. only if the energy productivity increases with decrease in temperature and hence the density. The directions of all the isoergs in the diagram, and also the numerical values $\varepsilon = 10^3$–10^4 in giants and super-giants under the low temperatures inside them (about a hundred thousands degrees) cannot be explained by nuclear reactions. It is evident therefore, that the possibility for nuclear reactions is just limited by the main sequence of the Russell-Hertzsprung diagram (the central concentration of stars in our diagram).

The proton-proton reaction arc is outside the main sequence of stars. If we move the arc to the left, into the region of the main sequence stars, we should change the constant A in the reaction equation (1.16) or change the physical characteristics at the centre of the Sun (1.26) as we found in Part I. Equation (1.18) shows that the shift of the proton-proton reaction arc along the density axis is proportional to the square of the change of the reaction constant A. Hence, in order to build the proton-proton reaction arc through the main concentration of stars we should take at least $A = 10^5$–10^6 instead of the well-known value $A = 4{\times}10^3$. This seems very improbable, for then we should ignore the central characteristics of the Sun that we have obtained, and hence all conclusions in Part I of this research which are in fine agreement with observational data. Only in a such case could we arrive at a temperature of about 20 million degrees at the centre of the Sun; enough for proton-proton reactions and also Bethe's carbon-nitrogen cycle.

All theoretical studies to date on the internal constitution of stars follow this approach. The sole reason adduced as proof of the high concentration of matter in stars, is the slow motion of the lines of apsides in compact binaries. However the collection published by Luyten, Struve, and Morgan [32] shows no relation between the velocity of such motion and the ratio of the star radius to the orbit semi-axis. At the same time, such a relation would be necessary if the motion of the lines of apsides in a binary system is connected to the deformations of the stars. Therefore we completely agree with the conclusion of those astronomers, that no theory correctly explains the observed motions of apsides. Even if we accept that the arc of nuclear reactions could intersect the central concentration of stars in our diagram (the stars of the main sequence in the Russell-Hertzsprung diagram), we should explain why the stars are distributed not along this arc, but fill some region around it. One could explain this circumstance by a "dispersion" of the parameters included in the main equations. For instance, one relates this dispersion

*See formula (1.14). — Editor's remark.

†The main sequence in the sense of the Russell-Hertzsprung diagram, is here the central concentration of stars. — Editor's remark.

to possible differences in the chemical composition of stars, their structure etc. Here we consider the probability of such explanations.

The idea that stars can have different chemical compositions had been introduced into the theory in 1932 by Strömgren [16], before Bethe's hypothesis about nuclear sources for stellar energy. He used only the heat drainage condition (1.13), which leads to the "mass-luminosity" relation (1.7a) for ideal gases. In chapter 2 of Part I we showed in detail that the theoretical relation (1.7a) is in good agreement (to within the accuracy of Strömgren's data) with the observed correlation for hydrogen stars (where we have Thomson's absorption coefficient, which is independent of physical conditions). Introducing some a priori suppositions (see §2.7, Part I), Eddington, Strömgren and other researchers followed another path; they attempted to explain non-transparency of stellar matter by high content of heavy elements, which build the so-called Russell mix. At the same time the absorption theory gives such a correlation $\kappa(\rho, B)$ for this mix which, being substituted into formula (1.7a), leads to incompatibility with observational data. Strömgren showed that such a "difficulty" can be removed if we suppose different percentages of heavy elements in stars, which substantially changes the resulting absorption coefficient κ. Light element percentages X can be considered as the hydrogen percentage. Comparing the theoretical formula to the observable "mass-luminosity" relation gives the function $X(\rho, B)$ or $X(M, R)$. Looking at the Strömgren surface from the physical viewpoint we can interpret it as follows. As we know, the heat drainage equation imposes a condition on the energy generation in stars. This is condition (1.13), according to which κ and μ depend on the chemical composition of a star. Let us suppose that the chemical composition is determined by one parameter X. Then

$$\varepsilon = f_1(\rho, B, X). \tag{I}$$

For processes like a reaction, the energy productivity ε is dependent on the same variables by the equation of this reaction

$$\varepsilon = f_2(\rho, B, X). \tag{II}$$

So we obtain the condition $f_1 = f_2$, which will be true only if a specific relation $X(\rho, B)$ is true in the star. The parameter X undergoes changes within the narrow range $0 \leqslant X \leqslant 1$, so stars should fill a region in the plane (ρ, B). Some details of the Russell-Hertzsprung diagram can be obtained as a result of an additional condition, imposed on $X(\rho, B)$: Strömgren showed that arcs of $X = \text{const}$ can be aligned with the distribution of stars in the Russell-Hertzsprung diagram. Kuiper's research [33] is especially interesting in this relation. He discovered that stars collected in open clusters are located along one of Strömgren's arcs $X = \text{const}$ and that the numerical values of X are different for different clusters. Looking at this result, showing that stellar clusters are different according to their hydrogen percentage, one can perceive an evolutionary meaning — the proof of the nuclear transformations of elements in stars.

Strömgren's research prepared the ground for checking the whole nuclear hypothesis of stellar energy: substituting the obtained correlation $X(\rho, B)$ into the reaction equation (II), we must come to the well-known relation (I). The nuclear reaction equation (1.16), where X is included through A, had not passed that examination. Therefore they introduced the second parameter Y into the theory — the percentage of helium. As a result, every function f_1 and f_2 can be separately equated to the function $\varepsilon(\rho, B)$ known from observations. Making the calculations for many stars, it is possible to obtain two surfaces: $X(\rho, B)$ and $Y(\rho, B)$. However, both surfaces are not a consequence of the equilibrium conditions of stars. It remains unknown as to why such surfaces exist, i. e. why the observed ε is a simple function of ρ and B? It is very difficult to explain this result by evolutionary transformations of X and Y, if the transformation of elements procedes in only one direction. Of course, taking a very small part of the plane (ρ, B), the evolution of elements can explain changes of X and Y. For instance, calculations made by Masevich [34] gave a monotone decrease of hydrogen for numerous stars located between the spectral classes B and G. To the contrary, from the class G to the class M, the hydrogen percentage increases again (see the work of Lohmann work [26] cited above). As a result we should be forced to think that stars evolve in two different ways. In such a case the result that the chemical composition of stars is completely determined by the physical conditions inside them can only be real if there is a balanced transformation of elements. Then the mechanism that generates energy in stars becomes the Helmholtz-Kelvin mechanism, not reactions. Nuclear transformations of elements only become an auxiliary circumstance which changes the thermal capacity of the gas. At the same time, the balanced transformation of elements is excluded from consideration, because it is possible only if the temperature becomes tens of billions of degrees, which is absolutely absent in stars.

All the above considerations show that the surfaces $X(\rho, B)$ and $Y(\rho, B)$ obtained by the aforementioned researchers are only a result of the trimming of formulae (I) and (II) to the observed relation $\varepsilon(\rho, B)$. Following this approach, we cannot arrive at a solution to the stellar energy problem and the problem of the evolution of stars. This conclusion is related not only to nuclear reactions; it also shows the impossibility of any sources of energy whose productivity is not regulated by the heat drainage condition. Naturally, the coincidence of the surfaces (I) and (II) manifests their identity. In a real situation the second condition is not present*.

*For reactions, the energy productivity increases with the increase of the density. In the heat drainage condition we see the opposite: equation (1.13). Therefore the surfaces (I) and (II), located over the plane (ρ, B), should be oppositely inclined — their transection should be very sharp.

So we get back to our conclusion of the previous paragraph: there are special physical conditions, the main direction (1.28) and the main point in the plane (ρ, B), about which stars generate exactly as much energy as they radiate into space. In other words, stars are *machines* which generate radiant energy. The heat drainage is the power regulation mechanism in the machines.

1.6 The "mass-luminosity" relation in connection with the Russell-Hertzsprung diagram

The luminosity of stars built on an ideal gas, radiant transfer of energy and low radiant pressure, is determined by formula (1.7a). This formula is given in its exact form by (2.38) in Part I. We re-write formula (2.38) as

$$\bar{\varepsilon} = \frac{\bar{L}}{\bar{M}} = 1.04\times10^4 \, \frac{\mu^4}{\kappa_c} \left(\frac{\lambda L_{x_0}}{M_{x_0}^3}\right) \bar{M}^2, \tag{1.30}$$

where M_{x_0} is the dimensionless mass of a star, κ_c is the absorption coefficient at its centre. It has already been shown that the structural multiplier of this formula has approximately the same numerical value

$$\frac{\lambda L_{x_0}}{M_{x_0}^3} \simeq 2\times10^{-3} \tag{1.31}$$

for all physically reasonable models of stars. The true "mass-luminosity" relation is shown in Fig. 2 by the system of isoergs $\bar{\varepsilon} = \bar{L}/\bar{M} = \text{const}$. If we do not take the radius of a star into account, we obtain the correlation shown in Fig. 1, Part I. There L is approximately proportional to the cube of M, although we saw a dispersion of points near this direction $L \sim M^3$. As we mentioned before, in Part I, the comparison of this result to formula (1.30) indicates that: (1) the radiant pressure plays no substantial rôle in stars, (2) stars are built on hydrogen.

Now we know that the dispersion of points near the average direction $L \sim M^3$ is not stochastic. So we could compare the exact correlation to the formula (1.30), and also check our previous conclusions.

Our first conclusion about the negligible rôle of the radiant pressure is confirmed absolutely, because of the mechanical equilibrium of giants. Naturally, comparing formula (1.7b) to (1.7a), we see that the greater the rôle of the radiant pressure, the less ε is dependent on M, so the interval between the neighbouring isotherms should increase for large masses. Such a tendency is completely absent for bulky stars (see the stellar energy diagram, Fig. 2). This result, in combination with formula (1.9) (its exact form is formula 2.47, Part I), leads to the conclusion that giants are built mainly on hydrogen (the molecular weight ½). Thus we calculate the absorption coefficient for giants. We see in the diagram that red giants of masses $\approx 20 M_\odot$ have $\log\bar{\varepsilon} = 3$. By formulae (1.30) and (1.31), we obtain

$$\frac{\kappa_c}{\mu^4} = 8\,. \tag{1.32}$$

If $\mu = 1/2$, we obtain $\kappa_c = 0.5$. This result implies that the non-transparency of giants is derived from Thomson's dispersion of light in free electrons ($\kappa_{\mathrm{T}} = 0.40$), as it should be in a pure hydrogen star.

The main peculiarity of the "mass-luminosity" relation is the systematic curvilinearity of the isoergs in the plane (ρ, B). Let us show that this curvilinearity cannot be explained by the changes of the coefficient in formula (1.30). First we consider the multiplier containing the molecular weight and the absorption coefficient.

The curvilinearity of the isoergs shows that for the same mass the diagram contains anomalous low luminosity stars at the top and anomalous bright stars at the bottom. Hence, the left part of (1.32) should increase under higher temperatures, and should decrease with lower temperatures. Looking from the viewpoint of today's physics, such changes of the absorption coefficient are impossible. Moreover, for the ultimate inclinations of the isoergs, we obtain absolutely impossible numerical values of the coefficient (1.32). For instance, in the case of super-giants, the lower temperature stars, this coefficient is 100 times less than that in giants. Even if we imagine a star built on heavy elements, we obtain that κ is about 1. In hot super-giants (the direction of Trumpler stars) the coefficient (1.32) becomes 200. Because of high temperatures in such stars, the absorption coefficient cannot be so large.

In order to explain the curvilinearity by the structural multiplier (1.31), we should propose that it be anomalously large in stars of high luminosity (sub-giants) and anomalously small in stars like Trumpler stars. We note that the dimensionless mass M_{x_0} included in (1.31) cannot be substantially changed, as shown in Part I. So the structural multiplier (1.31) can be changed by only λL_{x_0}. Employing the main system of the dimensionless equations of equilibrium of stars, we easily obtain the equation

$$\frac{dB_1}{dp_1} = \frac{\lambda L_x}{M_x}\,, \tag{1.33}$$

which is equation (2.22) of Part I, where B_1 and p_1 are the radiant pressure and the gaseous pressure expressed in multiples of their values at the centre of a star. Here the absorption coefficient κ is assumed constant from the centre to the surface, i. e. $\kappa_1 = 1$. Applying this equation to the surface layers of a star, we deduce that the structural coefficient is

$$\frac{\lambda L_{x_0}}{M_{x_0}} = \frac{B_1}{p_1}\,. \tag{1.34}$$

We denote the numerical values of the functions at the boundary between the surface layer and the "internal" layers of a star by the subscript 0. We consider two ultimate cases of the temperature gradients within the "internal" layers:

1. The "internal" zone of a star is isothermal:

$$\frac{\lambda L_{x_0}}{M_{x_0}} = \frac{1}{p_{1_0}}\,, \tag{1.34a}$$

2. The "internal" zone of a star is convective ($B_1 = p_1^{8/5}$):

$$\frac{\lambda L_{x_0}}{M_{x_0}} = p_{1_0}\,. \tag{1.34b}$$

In the first theoretical case, spreading the isothermal zone to almost the surface of a star, we can make the structural coefficient as large as we please. This case is attributed to sub-giants and anomalous bright stars in general. The second theoretical case can explain stars of anomalously low luminosity. Following this way, i. e. spreading the convective zone inside stars, Tuominen [35] attempted to explain the low luminosity of Trumpler stars.

The isothermy can appear if energy is generated mainly in the upper layers of a star. The spreading of the convective zone outside the Schwarzschild boundary can occur if energy is generated in moved masses of stellar gas, i. e. under forced convection. A real explanation by physics should connect the above peculiarities of the energy generation to the physical conditions inside stars or their general characteristics L, M, R. Before attempting to study the theoretical possibility of such relations, it is necessary to determine them first from observational data. Dividing $\bar{\varepsilon}$ by $\bar{M}^2$ for every star, we obtain the relation of the structural coefficient of formula (1.30) for ρ and B. But, at the same time, the determination of this relation in this way is somewhat unclear. There are no clear sequences or laws, so we do not show it here. Generally speaking, a reason should be simpler than its consequences. Therefore, it is most probable that the structural coefficient is not the reason. It is most probable that the reason for the incompatibility of the observed "mass-luminosity" relation with formula (1.30) is that equation (1.30) itself is built incorrectly. This implies that the main equations of equilibrium of stars are also built incorrectly. This conclusion is in accordance with our conclusion in the previous paragraph: energy is generated in stars like in machines — their workings are incompatible with the standard principles of today's mechanics and thermodynamics.

1.7 Calculation of the main constants of the stellar energy state

The theoretical "mass-luminosity" relation (1.30) is obtained as a result of comparing the radiant energy B calculated by the excess energy flow (formula 1.4a or 1.13) to the same B calculated by the phase state equation of matter (through p and ρ by formula 1.14). Therefore the incompatibility of the theoretical correlation (1.30) to observational data can be considered as the incompatibility of both the values of B. So we denote by B^* the radiant pressure calculated by the ideal gas equation. For the radiant transport of energy in a star, formulae (1.4a) and (1.13) lead to

$$\frac{\bar{B}^*}{\bar{\kappa}} = \bar{\varepsilon}\bar{p}\,. \tag{1.35}$$

By this formula we can calculate $\bar{B}^*/\bar{\kappa}$ for every star of the stellar energy diagram (Fig. 2). As a result we can find the correlation of the quantity $\bar{B}^*/\bar{\kappa}$ to $\bar{p}$ and $\bar{\rho}$. Fig. 3 shows the stellar energy diagram transformed in this fashion. Here the abscissa is $\log\bar{\rho}$, while the ordinate is $\log\bar{p}$. In order to make the diagram readable, we have not plotted all stars. We have plotted only the Sun and a few giants. At the same time we drawn the lines of constant $\bar{B}^*/\bar{\kappa}$ through ten intervals. The lines show the surface $\log\bar{B}^*/\bar{\kappa}\,(\log\bar{\rho}, \log\bar{p})$. For the constant absorption coefficient κ, the lines show the system of isotherms. If $B^* = B$, there should be a system of parallel straight lines, inclined at 45° to the $\log\bar{p}$ axis and following through the interval 0.25. As we see, the real picture is different in principle. There is in it a wonderful symmetry of the surface $\log\bar{B}^*/\bar{\kappa}$. Here the origin of the coordinates coincides with the central point of symmetry of the isoergs. At the same time it is the main point mentioned in relation in the stellar energy diagram. The coordinates of the point with respect to the Sun are

$$\begin{aligned} &\log\bar{\rho}_0 = -0.58\,, \quad \log\bar{p}_0 = -0.53\,, \\ &\log\bar{B}_0 = +0.22\,, \quad \log\bar{B}_0^* = +0.50\,. \end{aligned} \tag{1.36}$$

Using the data, we deduce that the main point is attributed to a star of the Russell-Hertzsprung main sequence, which has spectral class A4. Rotating the whole diagram around the main point by 180°, we obtain almost the same diagram, only the logarithms of the isotherms change their signs. Hence, if

$$\frac{\frac{B^*}{\kappa}}{\frac{B_0^*}{\kappa}} = f\left(\frac{p}{p_0}, \frac{\rho}{\rho_0}\right),$$

we have

$$f\left(\frac{p}{p_0}, \frac{\rho}{\rho_0}\right) f\left(\frac{p_0}{p}, \frac{\rho_0}{\rho}\right) = 1\,. \tag{1.37}$$

The relation (1.37) is valid in the central region of the diagram. An exception is white dwarfs, in which B^*/κ is 100 times less than that required by formula (1.37), i. e. 100 times less than that required for the correspondence to giants after the 180° rotation of the diagram. It is probable that this circumstance is connected to the fact that white dwarfs are located close to the boundary of degenerate gas.

Besides the isotherms, we have drawn the main direction along which stars are distributed. Now the equation of the direction (1.28) can be written in the more precise form

$$\log\frac{\bar{B}}{\bar{\rho}} = +0.80\,. \tag{1.38}$$

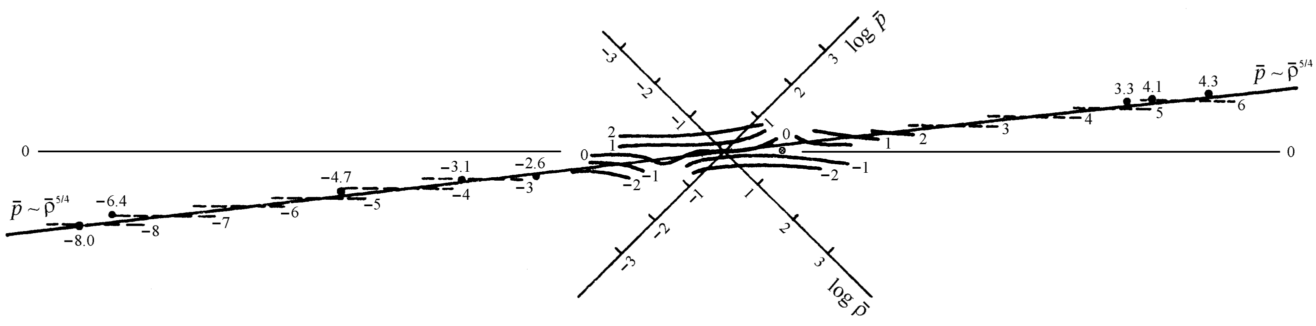


Fig. 3: Isotherms of stellar matter. The coordinate axes are the logarithms of the matter density and the gaseous pressure. Dashed lines show isotherms of an ideal gas.

Because of the very large range of the physical states in the diagram, the main direction is drawn very precisely (to within 5%). It should be noted that, despite their peculiarities, white dwarfs satisfy the main direction like all regular stars.

A theory of the internal constitution of stars, which could explain observational data (the relation 1.37, for instance), should be built on equations containing the coordinates of the main point. This circumstance is very interesting: it shows that there is an absolute system of "physical coordinates", where physical quantities of absolutely different dimensions can be combined. Such combinations can lead to a completely unexpected source of stellar energy. Therefore it is very important to calculate the absolute numerical values of the constants (1.36). Assuming in (1.36) a mostly hydrogen content for stars $\mu = 1/2$, and using the above calculated physical characteristics at the centre of the Sun (1.26), we obtain

$$\rho_0 = 2.4, \quad p_0 = 2.8{\times}10^{15}, \quad B_0 = 6.3{\times}10^{12}. \tag{1.39}$$

We calculate B_0^* by formula (1.13). Introducing the average productivity of energy ε

$$B_c^* = \frac{\varepsilon \kappa_c p_c}{4\pi G c} \frac{M_{x_0}}{\lambda L_{x_0}}, \tag{1.40}$$

assuming κ_c equal to Thomson's absorption coefficient, $\varepsilon_\odot = 1.9$, $M_{x_0} = 11$, and the structural multiplier according to (1.31). We then obtain for the Sun, $B_{c\odot}^* = 1.1{\times}10^{12}$ instead of $B_{c\odot} = 3.8{\times}10^{12}$. Hence,

$$B_0^* = 4.1{\times}10^{12} \approx B_0\,. \tag{1.41}$$

We introduce the average number of electrons in one cubic centimetre n_e instead of the density of matter: $\rho = 1.66 \times 10^{-24} n_e$. Then the equation of the main direction becomes

$$\frac{3B}{n_e} = 1.4{\times}10^{-11} = 8.7 \text{ eV}, \tag{1.42}$$

which is close to the hydrogen ionization potential, i. e. $\chi_0 = 13.5$ eV. Thus the average radiant energy per particle in stars (calculated by the ideal gas formula) is constant and is about the ionization energy of the hydrogen atom. Fig. 3 shows that, besides the main direction, the axis $\rho = \rho_0$ is also important. Its equation can be formulated through the average distance between particles in a star

$$r = 0.55\,(n_e)^{-1/3}$$

as follows

$$r = 0.51{\times}10^{-8} = r_{\text{H}} = \frac{e^2}{2\chi_0}, \tag{1.43}$$

where r_{H} is the radius of the hydrogen atom, e is the charge of the electron. As a result we obtain the very simple correlation between the constants of the lines (1.42) and (1.43), which bears a substantial physical meaning.

In the previous paragraph we showed that the peculiarities of the "mass-luminosity" relation* cannot be explained by changes of the absorption coefficient κ. Therefore the lines $B^*/\kappa = $ const should bear the properties of the isotherms. The isotherms drawn in Fig. 3 are like the isotherms of the van der Waals gas. The meaning of this analogy is that there is a boundary near which the isotherms become distorted, at which the regular laws of thermodynamics are violated. The asymptotes of the boundary line (the boundary between two different phases in the theory of van der Waals) are axes (1.42) and (1.43). The distortion of the isotherms increases with approach to the axis $\rho = \rho_0$ or $r = r_{\text{H}}$. That region is filled by stars of the Russell-Hertzsprung main sequence. The wonderful difference from van der Waals' formula is the fact that there are two systems of the distortions, equation (1.37), which become smoothed with the distance from the axis $\rho = \rho_0$ (for both small densities and large densities).

Stars can radiate energy for a long time only under conditions close to the boundaries (1.42) and (1.43). This most probably happens because the mechanism generating energy in stars works only if the standard laws of classical physics are broken.

The results are completely unexpected from the viewpoint of contemporary theoretical physics. The results show

*The dispersion of showing-stars points around the theoretically calculated direction "mass-luminosity". — Editor's remark.

that in stars the classical laws of mechanics and thermodynamics are broken much earlier than predicted by Einstein's theory of relativity, and it occurs under entirely different circumstances. The main direction constants (1.42) and (1.43) show that the source of stellar energy is not Einstein's conversion of mass and energy (his mass-energy equivalence principle), but by a completely different combination of physical quantities.

Here we limit ourselves only to conclusions which follow from the observational data. A generalization of the results and subsequent theoretical consequences will be dealt with in the third part of this research. In the next chapter we only consider some specific details of the Russell-Hertzsprung diagram, not previously discussed.

Chapter 2

Properties of Some Sequences in the Russell-Hertzsprung Diagram

2.1 The sequence of giants

The stellar energy diagram (see Fig. 2) shows that the "mass-luminosity" relation has the most simple form for stars of the Russell-Hertzsprung main sequence

$$L \sim M^{\alpha}, \qquad \alpha = 3.8. \tag{2.1}$$

Cepheids, denoted by crosses in the diagram, also satisfy the relation (2.1). Using the pulsation equation $P\sqrt{\rho} = c_1$ we obtained (see formula 3.25 of Part I)

$$\left(0.30 - \frac{1}{5\alpha}\right)\left(m_b - 4.62\right) + \log P + 3\log \bar{T}_{\text{eff}} = \log c_1 \,, \tag{2.2}$$

where $\bar{T}_{\text{eff}}$ is the reduced temperature of a star, expressed in multiples of the reduced temperature of the Sun, m_b is the absolute stellar magnitude, P is the pulsation period (days). We plot stars in a diagram where the abscissa is $m_b - 4.62$, while the ordinate is $\log P + \log \bar{T}_{\text{eff}}$. As a result we should obtain a straight line, which gives both the constant c_1 (see §3.3 of Part I) and the angular coefficient $0.30 - 1/5\alpha$. Fig. 4 shows this diagram, built using the collected data of Becker [36], who directly calculated $\bar{T}_{\text{eff}}$ and m_b by the radiant velocities arc (independently of the distances). As a result the average straight line satisfying all the stars has the angular coefficient 0.25 and $c_1 = 0.075$. Hence, $\alpha = 4$, which is in fine accordance with the expected result ($\alpha = 3.8$). Such a coincidence makes Melnikov's conclusion unreasonable: that Cepheids have the same masses ($\alpha = \infty$), as shown by the dashed line in Fig. 4.

In §1.5 of Part I we showed that the "mass-luminosity" relation for giants is explained by the fact that the structural coefficient $\lambda L_{x_0}/M_{x_0}^3$ has the same value $\simeq 2{\times}10^{-3}$ (1.31) for all stars. In order to obviate difficulties which appear if

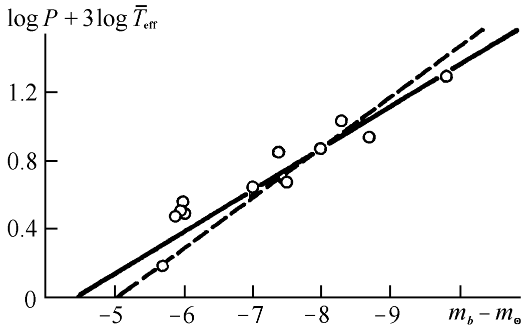


Fig. 4: Finding the exponent index α in the $L \sim M^{\alpha}$ relation for Cepheids.

one attempts to explain the luminosity of giants by nuclear reactions, one attributes to them an exotic internal constitution (the large shell which covers a normal star). Therefore, the simple structure of giants we have obtained gives an additional argument for the inconsistency of the nuclear sources of stellar energy. At the same time, because of their simple structure, giants and super-giants are quite wonderful. For instance, for a giant like the satellite of ε Aurigae we obtain its central density at 10^{-4} of the density of air, and the pressure at about 1 atmosphere. Therefore, it is quite possible that in moving forward along the main direction we can encounter nebulae satisfying the condition (1.42). Such nebulae can generate their own energy, just like stars.

Because of the physical conditions in giants, obtained above, the huge amounts of energy radiating from them cannot be explained by nuclear reactions. Even if this were true, their life-span would be very short. For reactions, the upper limit of the life-span of a star (the full transformation of its mass into radiant energy) can be obtained as the ratio of $\bar{\varepsilon}$ to c^2. So, by formula (1.40), we obtain

$$t = \frac{t_0}{4\gamma_c}\left(\frac{M_{x_0}}{\lambda L_{x_0}}\right), \tag{2.3}$$

where

$$t_0 = \frac{\kappa_{\text{T}}\, c}{\pi G} = 6{\times}10^{16}\,\text{sec} = 2{\times}10^{9}\,\text{years} \tag{2.4}$$

and $\gamma_c = B_c/p_c$ is the ratio of the radiant pressure to the gaseous one. As obtained, the structural multiplier here is about 4. Therefore

$$t = \frac{t_0}{\gamma_c}\,. \tag{2.5}$$

In giants $\gamma_c \approx 1$, so we obtain that t is almost the same as t_0. At the same time, as we know, the percentage of energy which could be set free in nuclear reactions is no more than 0.008. Hence, the maximum life-span of a giant is about $1.6{\times}10^{7}$ years, which is absolutely inapplicable. This gives additional support for our conclusion that the mechanism of stellar energy is not like reactions.

It is very interesting that the constant (2.4) has a numerical value similar to the time constant in Hubble's relation (the red shift of nebulae). It is probable that the exact form of the Hubble equation should be

$$\nu = \nu_0 \, e^{-t/t_0}, \tag{2.6}$$

where ν is the observed frequency of a line in a nebula spectrum when it is located at t light years from us, ν_0 is its normal frequency. According to the General Theory of Relativity the theoretical correlation between the constant t_0 and the average density $\bar{\rho}$ of matter in the visible part of the Universe

$$t_0 \simeq \frac{1}{\sqrt{\pi G \bar{\rho}}}\,, \tag{2.7}$$

which, independently of its theoretical origin, is also the very interesting empirical correlation. Because of (2.4) and (2.7), we re-write equation (2.6) as follows

$$\nu = \nu_0 \, e^{-\kappa_{\mathrm{T}} \bar{\rho} x}, \tag{2.8}$$

where $x = ct$ is the path of a photon. Formula (2.8) is like the formula of absorption, and so may give additional support to the explanation of the nebula red shift by unusual processes which occur in photons during their journey towards us. It is possible that in this formula $\bar{\rho}$ is the average density of the intergalactic gas.

2.2 The main sequence

The contemporary data of observational astronomy has sufficiently filled the Russell-Hertzsprug diagram, i. e. the "luminosity — spectral class" plane. As a result we see that there are no strong arcs $L(\bar{T}_{\mathrm{eff}})$ and $L(R)$, but regions filled by stars. In the previous chapter we showed that such a dispersion of points implies that the energy productivity in stars is regulated exclusively by the energy drainage (the radiation). So the mechanism generating stellar energy is not like any reactions. It is possible that only the main sequence of the Russell-Hertzsprung diagram can be considered a line along which stars are located. According to Parenago [38], this direction is

$$m_b = m_\odot - 1.62\,x, \qquad x = 10 \log \bar{T}_{\mathrm{eff}}\,. \tag{2.9}$$

An analogous relation had been found by Kuiper [8] as the $M(R)$ relation

$$\log \bar{R} = 0.7 \log \bar{M}\,. \tag{2.10}$$

Using formulae (1.12) and (1.14), we could transform formula (2.10) to a correlation $B(\rho)$. At the same time, looking at the stellar energy diagram (Fig. 2), we see that the stars of the Russell-Hertzsprung "main sequence" have no $B(\rho)$ correlation, but fill instead a ring at the centre of the diagram. This incompatibility should be considered in detail.

In the stellar energy diagram, the Russell-Hertzsprung main sequence is the ring of radius c filled by stars. The boundary equation of this region is

$$\log^2 \bar{B} + \log^2 \bar{\rho} = c^2. \tag{2.11}$$

We transform this equation to the variables $\bar{M}$ and $\bar{R}$ by formulae (1.12) and (1.14). We obtain

$$17 \log^2 \bar{M} - 38 \log \bar{M} \log \bar{R} + 25 \log^2 \bar{R} = c^2. \tag{2.12}$$

As we have found, for stars located in this central region (the Russell-Hertzsprung main sequence), the exponent of the "mass-luminosity" relation is about 4. Therefore, using formulae

$$\log \bar{M} = -0.1\,m_b\,, \qquad 5 \log \bar{R} = -m_b - x\,,$$

we transform (2.12) to the form

$$m_b^2 + 2{\times}1.51\,m_b x + 2.44 x^2 = c_1^2\,. \tag{2.13}$$

The left side of this equation is almost a perfect square, hence we have the equation of a very eccentric ellipse, with an angular coefficient close to 1.51. The exact solution can be found by transforming (2.13) to the main axes using the secular equation. As a result we obtain

$$\frac{a}{b} = 8.9, \qquad \alpha = -1.58\,, \tag{2.14}$$

where a and b are the main axis and the secondary axis of the ellipse respectively, α is the angle of inclination of its main axis to the abscissa's axis. Because of the large eccentricity, there is in the Russell-Hertzsprung diagram the illusion that stars are concentrated along the line a, the main axis of the ellipse. The calculated angular coefficient $\alpha = -1.58$ (2.14) is in close agreement with the empirically determined $\alpha = -1.62$ (2.9).

Thus the Russell-Hertzsprung main sequence has no physical meaning: it is the result of the scale stretching used in observational astrophysics. In contrast, the reality of the scale used in our stellar energy diagram (Fig. 2) is confirmed by the homogeneous distribution of the isoergs.

As obtained in Part I of this research, from the viewpoint of the internal constitution of stars, stars located at the opposite ends of the main sequence (the spectral classes O and M) differ from each other no more than stars of the same spectral class, but of different luminosity. Therefore the "evolution of a star along the main sequence" is a senseless term.

The results show that the term "sequence" was applied very unfortunately to groups of stars in the Russell-Hertzsprung diagram. It is quite reasonable to change this terminology, using the term "region" instead of "sequence": the region of giants, the main region, etc.

2.3 White dwarfs

There is very little observational data related to white dwarfs. Only for the satellite of Sirius and for o^2 Eridani do we know values of all three quantities L, M, and R. For Sirius' satellite we obtain

$$\begin{aligned} &\bar{M} = 0.95, \quad \bar{R} = 0.030, \quad \varepsilon = 1.1\times10^{-2}, \\ &\rho = 10^4, \quad \rho_c = 3\times10^5, \quad p_c = 1\times10^{22}. \end{aligned} \tag{2.15}$$

For an ideal gas and an average molecular weight $\mu = 1/2$, we obtain $T_c = 2\times10^8$ degrees. The calculations show that white dwarfs generate energy hundreds of times smaller than regular stars. Looking at the isoergs in Fig. 2 and the isotherms in Fig. 3, we see that the deviation of white dwarfs from the "mass-luminosity" relation is of a special kind; not the same as that for regular stars. At the same time white dwarfs satisfy the main direction in the stellar energy diagram: they lie in the line following giants. Therefore it would be natural to start our brief research into the internal constitution of white dwarfs by proceeding from the general supposition that they are hot stars whose gas is at the boundary of degeneration

$$\rho = A\,T^{3/2}, \qquad A = 10^{-8}\,\mu_e\,. \tag{2.16}$$

We now show that, because of high density of matter in white dwarfs, the radiant transport of energy F_R is less than the transport of energy by the electron conductivity F_T

$$F_R = -\frac{1}{3}\,\bar{v}_e\,\bar{\lambda}\,\bar{c}_v\,n_e\,\frac{dT}{dr}\,,$$

where $\bar{\lambda}$ is the mean free path of electrons moved at the average velocity $\bar{v}_e$, $\bar{c}_v$ is the average heat capacity per particle. Also

$$\lambda = \frac{1}{n_i\,\sigma}\,, \quad n_i = \frac{n_e}{z}\,, \quad \sigma = \pi r^2, \quad c_v = \frac{3}{2}\,k\,, \tag{2.18}$$

where n_i is the number of ions deviating the electrons, σ is the ion section determined by the 90° deviation condition

$$m_e v_e^2 = \frac{ze^2}{r}\,, \tag{2.19}$$

i. e. the condition to move along a hyperbola.

Substituting (2.19) and (2.18) into formula (2.17) and eliminating $\bar{v}$ by the formula

$$\bar{v}^5 = \frac{12}{\sqrt{\pi}}\left(\frac{2\,kT}{m_e}\right)^{5/2},$$

we obtain

$$F_T = -\frac{24}{ze^4}\left(\frac{2\,k^7T^5}{\pi^3 m_e}\right)^{1/2}\frac{dT}{dr}\,. \tag{2.20}$$

The radiant flow can be written as

$$F_R = -\frac{4}{3}\frac{c\alpha T^3}{\kappa\rho}\frac{dT}{dr}\,, \tag{2.21}$$

hence

$$\frac{F_R}{F_T} = \frac{zT^{1/2}}{\kappa\rho}\left(\frac{\alpha ce^4\pi^{3/2}m_e^{1/2}}{k^{7/2}18\sqrt{2}}\right) = \frac{2.6\,zT^{1/2}}{\kappa\rho}\,. \tag{2.22}$$

Using (2.15) it is easily seen that even if $\kappa \simeq 1$, $F_R < F_T$ in the internal regions of white dwarfs. We can apply the formulae obtained to the case of the conductive transport of energy, if we eliminate κ with the effective absorption coefficient κ^*

$$\kappa^* = \frac{2.6\,zT^{1/2}}{\rho}\,. \tag{2.23}$$

Thus, if white dwarfs are built on an ideal gas whose state is about the degeneration boundary, their luminosity should be more than that calculated by the "mass-luminosity" formula (the heat equilibrium condition).

We consider the regular explanation for white dwarfs, according to which they are stars built on a fully degenerate gas. For the full degeneration, we use Chandrasekhar's "mass-radius" formula (see formula 2.32, Part I). With $\bar{M} = 1$ we obtain

$$\bar{R} = 0.042 \quad (\mu_e = 1)\,, \qquad \bar{R} = 0.013 \quad (\mu_e = 2)\,.$$

The observable radius (2.15) cannot be twice as small, so we should take Sirius' satellite as being composed of at least 50% hydrogen. From here we come to a serious difficulty: because of the high density of white dwarfs, even for a few million degrees internally, they should produce much more energy than they can radiate.We now show that such temperatures are necessary for white dwarfs.

Applying the main equations of equilibrium to the surface layer of a star, we obtain

$$\frac{B}{p} = \frac{L\kappa}{4\pi GcM} = \frac{\varepsilon\kappa}{4\pi Gc}\,, \tag{2.24}$$

where κ is the absorption coefficient in the surface layer. At the boundary of degeneration we can transform the left side by (2.16)

$$\rho_0 = \frac{3\varepsilon\kappa}{4\pi Gc}\frac{A^2\Re}{\mu\alpha}$$

so that

$$\begin{aligned} \rho_0 &= 125\,\varepsilon\kappa\left(\frac{\mu_e^2}{\mu}\right), \\ T_0^{3/2} &= 1.25\times10^{10}\,\varepsilon\kappa\left(\frac{\mu_e}{\mu}\right). \end{aligned} \tag{2.25}$$

We see from formula (2.22) that even in the surface layer the quantity F_T can be greater than F_R. Substituting κ^* (2.23) into (2.25), we obtain

$$T_0 = 2.5\times10^7\,\varepsilon^{2/5}\left(\frac{z}{\mu}\right)^{2/5}. \tag{2.26}$$

For $\varepsilon = 10^{-2}$, $\mu = 1$, and $z = 1$, we obtain

$$T_0 = 4{\times}10^6, \quad \rho_0 = 80, \quad \kappa_0^* = 65,$$

thus for such conditions, $\kappa > \kappa^*$.

We know that in the surface layer the temperature is linked to the depth h as follows

$$T = \frac{g\mu}{4\Re}\, h\,. \tag{2.27}$$

In the surface of Sirius' satellite we have $g = 3{\times}10^7$. Hence $h_0 = 3{\times}10^7$. Therefore the surface layer is about 2% of the radius of the white dwarf, so we can take the radius at the observed radius of the white dwarf.

It should be isothermal in the degenerated core, because the absorption coefficient rapidly decreases with increasing density. For a degenerate gas we can transform formula (2.23) in a simple way, if we suppose the heat capacity proportional to the temperature. Then, in the formula for F_R (2.20), the temperature remains in the first power, while $T_0^{3/2}$ should be eliminated with the density by (2.16). As a result we obtain $F_R \sim \rho T$ and also

$$\kappa_1^* \simeq 2.6{\times}10^{-8}\left(\frac{T}{\rho}\right)^2 z\mu_e\,. \tag{2.28}$$

Even for $4{\times}10^6$ degrees throughout a white dwarf, the average productivity of energy calculated by the proton-proton reaction formula (1.16) is $\varepsilon = 10^2$ erg/sec, which is much more than that observed. In order to remove the contradiction, we must propose a very low percentage of hydrogen, which contradicts the calculation above,* which gives hydrogen as at least 50% of its contents. So the large observed value of the radius of Sirius' satellite remains unexplained.

So we should return to our initial point of view, according to which white dwarfs are hot stars at the boundary of degeneration, but built on heavy elements. The low luminosity of such stars is probably derived from the presence of endothermic phenomena inside them. That is, besides energy generating processes, there are also processes where ε is negative. This consideration shows again that the luminosity of stars is unexplained within the framework of today's thermodynamics.

*By Chandrasekhar's formula for a fully degenerate gas. — Editor's remark.

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