Dynamical Fractal 3-Space and the Generalised Schrödinger Equation: Equivalence Principle and Vorticity Effects

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The new dynamical "quantum foam" theory of 3-space is described at the classical level by a velocity field. This has been repeatedly detected and for which the dynamical equations are now established. These equations predict 3-space "gravitational wave" effects, and these have been observed, and the 1991 DeWitte data is analysed to reveal the fractal structure of these "gravitational waves". This velocity field describes the differential motion of 3-space, and the various equations of physics must be generalised to incorporate this 3-space dynamics. Here a new generalised Schrödinger equation is given and analysed. It is shown that from this equation the equivalence principle may be derived as a quantum effect, and that as well this generalised Schrödinger equation determines the effects of vorticity of the 3-space flow, or "frame-dragging", on matter, and which is being studied by the Gravity Probe B (GP-B) satellite gyroscope experiment.

1 Introduction

Extensive experimental evidence [1, 2, 3] has shown that a complex dynamical 3-space underlies reality. The evidence involves the repeated detection of the motion of the Earth relative to that 3-space using Michelson interferometers operating in gas mode [3], particularly the experiment by Miller [4] in 1925/26 at Mt.Wilson, and the coaxial cable RF travel time measurements by Torr and Kolen in Utah, and the DeWitte experiment in 1991 in Brussels [3]. All such 7 experiments are consistent with respect to speed and direction. It has been shown that effects caused by motion relative to this 3-space can mimic the formalism of spacetime, but that it is the 3-space that is "real", simply because it is directly observable [1].

The 3-space is in differential motion, that is one part has a velocity relative to other parts, and so involves a velocity field $\mathbf{v}(\mathbf{r}, t)$ description. To be specific this velocity field must be described relative to a frame of observers, but the formalism is such that the dynamical equations for this velocity field must transform covariantly under a change of observer. As shown herein the experimental data from the DeWitte experiment shows that $\mathbf{v}(\mathbf{r}, t)$ has a fractal structure. This arises because, in the absence of matter, the dynamical equations for $\mathbf{v}(\mathbf{r}, t)$ have no scale. This implies that the differential motion of 3-space manifests at all scales. This fractal differential motion of 3-space is missing from all the fundamental equations of physics, and so these equations require a generalisation. Here we report on the necessary generalisation of the Schrödinger equation, and which results in some remarkable results: (i) the equivalence principle emerges, as well as (ii) the effects of vorticity of this velocity field. These two effects are thus seen to be quantum-theoretic effects, i. e. consequences of the wave nature of matter. The equivalence principle, as originally formulated by Galileo and then Newton, asserts that the gravitational acceleration of an object is independent of its composition and speed. However we shall see that via the vorticity effect, the velocity of the object does affect the acceleration by causing rotations.

It has been shown [1, 5] that the phenomenon of gravity is a consequence of the time-dependence and inhomogeneities of $\mathbf{v}(\mathbf{r}, t)$. So the dynamical equations for $\mathbf{v}(\mathbf{r}, t)$ give rise to a new theory of gravity, when combined with the generalised Schrödinger equation, and the generalised Maxwell and Dirac equations. The equations for $\mathbf{v}(\mathbf{r}, t)$ involve the Newtonian gravitational constant G and a dimensionless constant that determines the strength of a new spatial self-interaction effect, which is missing from both Newtonian Gravity and General Relativity. Experimental data has revealed [1, 5] the remarkable discovery that this constant is the fine structure constant $\alpha \approx 1/137$. This dynamics then explains numerous gravitational anomalies, such as the bore hole g anomaly, the so-called "dark matter" anomaly in the rotation speeds of spiral galaxies, and that the effective mass of the necessary black holes at the centre of spherical matter systems, such as globular clusters and spherical galaxies, is $\alpha/2$ times the total mass of these systems. This prediction has been confirmed by astronomical observations [6].

The occurrence of α suggests that space is itself a quantum system undergoing on-going classicalisation. Just such a proposal has arisen in *Process Physics* [1] which is an information-theoretic modelling of reality. There quantum space and matter arise in terms of the Quantum Homotopic Field Theory (QHFT) which, in turn, may be related to the standard model of matter. In the QHFT space at this quantum level is best described as a "quantum foam". So we interpret the observed fractal 3-space as a classical approximation to this "quantum foam".

While here we investigate the properties of the generalised Schrödinger equation, analogous generalisations of the Maxwell and Dirac equations, and in turn the corresponding generalisations to the quantum field theories for such systems, may also be made. In the case of the Maxwell equations we obtain the light bending effects, including in particular gravitational lensing, caused by the 3-space differential and time-dependent flow.

2 The physics of 3-space

Because of the dominance of the spacetime ontology, which has been the foundation of physics over the last century, the existence of a 3-space as an observable phenomenon has been overlooked, despite extensive experimental detection over that period, and earlier. This spacetime ontology is distinct from the role of spacetime as a mathematical formalism implicitly incorporating some real dynamical effects, though this distinction is rarely made. Consequently the existence of 3-space has been denied, and so there has never been a dynamical theory for 3-space. In recent years this situation has dramatically changed. We briefly summarise the key aspects to the dynamics of 3-space.

Relative to some observer 3-space is described by a velocity field $\mathbf{v}(\mathbf{r},t)$. It is important to note that the coordinate $\mathbf{r}$ is not itself 3-space, rather it is merely a label for an element of 3-space that has velocity $\mathbf{v}$, relative to some observer. This will become more evident when we consider the necessary generalisation of the Schrödinger equation. Also it is important to appreciate that this "moving" 3-space is not itself embedded in a "space"; the 3-space is all there is, although as noted above its deeper structure is that of a "quantum foam".

In the case of zero vorticity $\nabla\times\mathbf{v}=0$ the 3-space dynamics is given by, in the non-relativistic limit,

$$\nabla\cdot\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right)+\frac{\alpha}{8}\left((\mathrm{tr}D)^2-\mathrm{tr}(D^2)\right)= = -4\pi G\rho\,, \tag{1}$$

where ρ is the matter density, and where

$$D_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right). \tag{2}$$

The acceleration of an element of space is given by the Euler form

$$\mathbf{g}(\mathbf{r},t)\equiv\lim_{\Delta t\to 0}\frac{\mathbf{v}\left(\mathbf{r}+\mathbf{v}(\mathbf{r},t)\Delta t,t+\Delta t\right)-\mathbf{v}(\mathbf{r},t)}{\Delta t}= =\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\,. \tag{3}$$

These forms are mandated by Galilean covariance under change of observer*. This non-relativistic modelling of the dynamics for the velocity field gives a direct account of the various phenomena noted above. A generalisation to include vorticity and relativistic effects of the motion of matter through this 3-space is given in [1]. From (1) and (2) we obtain that

$$\nabla\cdot\mathbf{g}=-4\pi G\rho-4\pi G\rho_{DM}\,, \tag{4}$$

where

$$\rho_{DM}(\mathbf{r})=\frac{\alpha}{32\pi G}\left((\mathrm{tr}D)^2-\mathrm{tr}(D^2)\right). \tag{5}$$

In this form we see that if $\alpha\to 0$, then the acceleration of the 3-space elements is given by Newton's Law of Gravitation, in differential form. But for a non-zero α we see that the 3-space acceleration has an additional effect, the ρ_{DM} term, which is an effective "matter density" that mimics the new self-interaction dynamics. This has been shown to be the origin of the so-called "dark matter" effect in spiral galaxies. It is important to note that (4) does not determine $\mathbf{g}$ directly; rather the velocity dynamics in (1) must be solved, and then with $\mathbf{g}$ subsequently determined from (3). Eqn. (4) merely indicates that the resultant non-Newtonian aspects to $\mathbf{g}$ could be mistaken as being the result of a new form of matter, whose density is given by ρ_{DM}. Of course the saga of "dark matter" shows that this actually happened, and that there has been a misguided and fruitless search for such "matter".

The numerous experimental confirmations of (1) imply that Newtonian gravity is not universal at all. Rather a key aspect to gravity was missed by Newton because it so happens that the 3-space self-interaction dynamics does not necessarily explicitly manifest outside of spherical matter systems, such as the Sun. To see this it is only necessary to see that the velocity field

$$\mathbf{v}(\mathbf{r})=-\sqrt{\frac{2GM'}{r}}\,\hat{\mathbf{r}}\,, \tag{6}$$

is a solution to (1) external to a spherical mass M, where $M'=(1+\frac{\alpha}{2})M+\ldots$ Then (6) gives, using (3), the resultant external "inverse square law" acceleration

$$\mathbf{g}(\mathbf{r})=-\frac{GM'}{r^2}\,\hat{\mathbf{r}}\,. \tag{7}$$

Hence in this special case the 3-space dynamics predicts an inverse square law form for $\mathbf{g}$, as confirmed in the non-relativistic regime by Kepler's laws for planetary motion, with only a modified value for the effective mass M'. So for this reason we see how easy it was for Newton to have overlooked a velocity formalism for gravity, and so missed the self-interaction dynamics in (1). Inside a spherical matter

*However this does not exclude so-called relativistic effects, such as the length contraction of moving rods or the time dilations of moving clocks.

system Newtonian gravity and the new gravity theory differ, and it was this difference that explained the bore hole g anomaly data [5], namely that g does not decrease down a bore hole as rapidly as Newtonian gravity predicts. It was this anomaly that lead to the discovery that α was in fact the fine structure constant, up to experimental errors. As well the 3-space dynamics in (1) has "gravitational wave" solutions [7]. Then there are regions where the velocity differs slightly from the enveloping region. In the absence of matter these waves will be in general fractal because there is no dimensioned constant, and so no natural scale. These waves were seen by Miller, Torr and Kolen, and by DeWitte [1, 7] as shown in Fig. 2.

However an assumption made in previous analyses was that the acceleration of the 3-space itself, in (3), was also the acceleration of matter located in that 3-space. The key result herein is to derive this result by using the generalised Schrödinger equation. In doing so we discover the additional effect that vorticity in the velocity field causes quantum states to be rotated, as discussed in sect. 7.

3 Newtonian gravity and the Schrödinger equation

Let us consider what might be regarded as the conventional "Newtonian" approach to including gravity in the Schrödinger equation [8]. There gravity is described by the Newtonian potential energy field $\Phi(\mathbf{r},t)$, such that $\mathbf{g} = -\nabla\Phi$, and we have for a "free-falling" quantum system, with mass m,

$$i\hbar\,\frac{\partial\psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + m\Phi(\mathbf{r},t)\,\psi(\mathbf{r},t) \equiv \equiv H(t)\psi\,, \tag{8}$$

where the hamiltonian is in general now time dependent, because the masses producing the gravitational acceleration may be moving. Then the classical-limit trajectory is obtained via the usual Ehrenfest method [9]: we first compute the time rate of change of the so-called position "expectation value"

$$\frac{d\langle\mathbf{r}\rangle}{dt} \equiv \frac{d}{dt}(\psi,\mathbf{r}\psi) = \frac{i}{\hbar}(H\psi,\mathbf{r}\psi) - \frac{i}{\hbar}(\psi,\mathbf{r}H\psi) = = \frac{i}{\hbar}(\psi,[H,\mathbf{r}]\psi)\,, \tag{9}$$

which is valid for a normalised state ψ. The norm is time invariant when H is hermitian ($H^\dagger = H$) even if H itself is time dependent,

$$\frac{d}{dt}(\psi,\psi) = \frac{i}{\hbar}(H\psi,\psi) - \frac{i}{\hbar}(\psi,H\psi) = = \frac{i}{\hbar}(\psi,H^\dagger\psi) - \frac{i}{\hbar}(\psi,H\psi) = 0\,. \tag{10}$$

Next we compute the matter "acceleration" from (9)

$$\frac{d^2\langle\mathbf{r}\rangle}{dt^2} = \frac{i}{\hbar}\frac{d}{dt}(\psi,[H,\mathbf{r}]\psi) = \left(\frac{i}{\hbar}\right)^2\big(\psi,[H,[H,\mathbf{r}]]\psi\big) + \frac{i}{\hbar}\left(\psi,\left[\frac{\partial H(t)}{\partial t},\mathbf{r}\right]\psi\right) = = -(\psi,\nabla\Phi\,\psi) = \big(\psi,\mathbf{g}(\mathbf{r},t)\psi\big) = \langle\mathbf{g}(\mathbf{r},t)\rangle, \tag{11}$$

where for the commutator

$$\left[\frac{\partial H(t)}{\partial t},\mathbf{r}\right] = \left[m\,\frac{\partial\Phi(\mathbf{r},t)}{\partial t},\mathbf{r}\right] = 0\,. \tag{12}$$

In the classical limit ψ has the form of a wavepacket where the spatial extent of ψ is much smaller than the spatial region over which $\mathbf{g}(\mathbf{r},t)$ varies appreciably. Then we have the approximation $\langle\mathbf{g}(\mathbf{r},t)\rangle \approx \mathbf{g}(\langle\mathbf{r}\rangle,t)$, and finally we arrive at the Newtonian 2nd-law equation of motion for the wavepacket,

$$\frac{d^2\langle\mathbf{r}\rangle}{dt^2} \approx \mathbf{g}(\langle\mathbf{r}\rangle,t)\,. \tag{13}$$

In this classical limit we obtain the equivalence principle, namely that the acceleration is independent of the mass m and of the velocity of that mass. But of course that followed by construction, as the equivalence principle is built into (8) by having m as the coefficient of Φ. In Newtonian gravity there is no explanation for the origin of Φ or $\mathbf{g}$. In the new theory gravity is explained in terms of a velocity field, which in turn has a deeper explanation within *Process Physics*.

4 Dynamical 3-space and the generalised Schrödinger equation

The key insight is that conventional physics has neglected the interaction of various systems with the dynamical 3-space. Here we generalise the Schrödinger equation to take account of this new physics. Now gravity is a dynamical effect arising from the time-dependence and spatial inhomogeneities of the 3-space velocity field $\mathbf{v}(\mathbf{r},t)$, and for a "free-falling" quantum system with mass m the Schrödinger equation now has the generalised form

$$i\hbar\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{1}{2}\nabla\cdot\mathbf{v}\right)\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t)\,, \tag{14}$$

which we write as

$$i\hbar\,\frac{\partial\psi(\mathbf{r},t)}{\partial t} = H(t)\,\psi(\mathbf{r},t)\,, \tag{15}$$

where now

$$H(t) = -i\hbar\left(\mathbf{v}\cdot\nabla + \frac{1}{2}\nabla\cdot\mathbf{v}\right) - \frac{\hbar^2}{2m}\nabla^2\,. \tag{16}$$

This form for H specifies how the quantum system must couple to the velocity field, and it uniquely follows from two considerations: (i) the generalised Schrödinger equation must remain form invariant under a change of observer, i. e. with $t \to t$, and $\mathbf{r} \to \mathbf{r} + \mathbf{v}t$, where $\mathbf{v}$ is the relative velocity of the two observers. Then we compute that $\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{1}{2}\nabla\cdot\mathbf{v} \to$

$\rightarrow \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{1}{2}\nabla\cdot\mathbf{v}$, i. e. that it is an invariant operator, and (ii) requiring that $H(t)$ be hermitian, so that the wavefunction norm is an invariant of the time evolution. This implies that the $\frac{1}{2}\nabla\cdot\mathbf{v}$ term must be included, as $\mathbf{v}\cdot\nabla$ by itself is not hermitian for an inhomogeneous $\mathbf{v}(\mathbf{r},t)$. Then the consequences for the motion of wavepackets is uniquely determined; they are fixed by these two quantum-theoretic requirements.

Then again the classical-limit trajectory is obtained via the position "expectation value", first with

$$\mathbf{v}_O \equiv \frac{d\langle\mathbf{r}\rangle}{dt} = \frac{d}{dt}(\psi, \mathbf{r}\psi) = \frac{i}{\hbar}(\psi, [H,\mathbf{r}]\psi) = \\ = \left(\psi, \left(\mathbf{v}(\mathbf{r},t) - \frac{i\hbar}{m}\nabla\right)\psi\right) = \langle\mathbf{v}(\mathbf{r},t)\rangle - \frac{i\hbar}{m}\langle\nabla\rangle, \tag{17}$$

on evaluating the commutator using $H(t)$ in (16), and which is again valid for a normalised state ψ.

Then for the "acceleration" we obtain from (17) that*

$$\begin{aligned} \frac{d^2\langle\mathbf{r}\rangle}{dt^2} &= \frac{d}{dt}\left(\psi, \left(\mathbf{v} - \frac{i\hbar}{m}\nabla\right)\psi\right) = \\ &= \left(\psi, \left(\frac{\partial\mathbf{v}(\mathbf{r},t)}{\partial t} + \frac{i}{\hbar}\left[H, \left(\mathbf{v} - \frac{i\hbar}{m}\nabla\right)\right]\right)\psi\right) = \\ &= \left(\psi, \frac{\partial\mathbf{v}(\mathbf{r},t)}{\partial t}\psi\right) + \\ &+ \left(\psi, \left(\mathbf{v}\cdot\nabla + \frac{1}{2}\nabla\cdot\mathbf{v} - \frac{i\hbar}{2m}\nabla^2\right)\left(\mathbf{v} - \frac{i\hbar}{m}\nabla\right)\psi\right) - \\ &- \left(\psi, \left(\mathbf{v} - \frac{i\hbar}{m}\nabla\right)\left(\mathbf{v}\cdot\nabla + \frac{1}{2}\nabla\cdot\mathbf{v} - \frac{i\hbar}{2m}\nabla^2\right)\right)\psi\right) = \\ &= \left(\psi, \left(\frac{\partial\mathbf{v}(\mathbf{r},t)}{\partial t} + ((\mathbf{v}\cdot\nabla)\mathbf{v}) - \frac{i\hbar}{m}(\nabla\times\mathbf{v})\times\nabla\right)\psi\right) + \\ &+ \left(\psi, \frac{i\hbar}{2m}\left(\nabla\times(\nabla\times\mathbf{v})\right)\psi\right) \approx \\ &\approx \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + (\nabla\times\mathbf{v})\times\left(\frac{d\langle\mathbf{r}\rangle}{dt} - \mathbf{v}\right) + \\ &+ \frac{i\hbar}{2m}\left(\nabla\times(\nabla\times\mathbf{v})\right) = \\ &= \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + (\nabla\times\mathbf{v})\times\left(\frac{d\langle\mathbf{r}\rangle}{dt} - \mathbf{v}\right) = \\ &= \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} + (\nabla\times\mathbf{v})\times\mathbf{v}_R\,, \end{aligned} \tag{18}$$

where in arriving at the 3rd last line we have invoked the small-wavepacket approximation, and used (17) to identify

$$\mathbf{v}_R \equiv -\frac{i\hbar}{m}\langle\nabla\rangle = \mathbf{v}_O - \mathbf{v}, \tag{19}$$

where $\mathbf{v}_O$ is the velocity of the wavepacket or object "O" relative to the observer, so then $\mathbf{v}_R$ is the velocity of the wavepacket relative to the local 3-space. Then all velocity field terms are now evaluated at the location of the wavepacket. Note that the operator

$$-\frac{i\hbar}{m}(\nabla\times\mathbf{v})\times\nabla + \frac{i\hbar}{2m}\left(\nabla\times(\nabla\times\mathbf{v})\right) \tag{20}$$

is hermitian, but that separately neither of these two operators is hermitian. Then in general the scalar product in (18) is real. But then in arriving at the last line in (18) by means of the small-wavepacket approximation, we must then self-consistently use that $\nabla\times(\nabla\times\mathbf{v}) = 0$, otherwise the acceleration acquires a spurious imaginary part. This is consistent with (27) outside of any matter which contributes to the generation of the velocity field, for there $\rho = 0$. These observations point to a deep connection between quantum theory and the velocity field dynamics, as already argued in [1].

We see that the test "particle" acquires the acceleration of the velocity field, as in (3), and as well an additional vorticity induced acceleration which is the analogue of the Helmholtz acceleration in fluid mechanics. Hence we find that the equivalence principle arises from the unique generalised Schrödinger equation and with the additional vorticity effect. This vorticity effect depends on the absolute velocity $\mathbf{v}_R$ of the object relative to the local space, and so requires a change in the Galilean or Newtonian form of the equivalence principle.

The vorticity acceleration effect is the origin of the Lense-Thirring so-called "frame-dragging" effect† [10] discussed in sect. 7. While the generation of the vorticity is a relativistic effect, as in (27), the response of the test particle to that vorticity is a non-relativistic effect, and follows from the generalised Schrödinger equation, and which is not present in the standard Schrödinger equation with coupling to the Newtonian gravitational potential, as in (8). Hence the generalised Schrödinger equation with the new coupling to the velocity field is more fundamental. The Helmholtz term in (18) is being explored by the Gravity Probe B gyroscope precession experiment, however the vorticity caused by the motion of the Earth is extremely small, as discussed in sect. 7.

An important insight emerges from the form of (15) and (16): here the generalised Schrödinger equation involves two fields $\mathbf{v}(\mathbf{r},t)$ and $\psi(\mathbf{r},t)$, where the coordinate $\mathbf{r}$ is merely a label to relate the two fields, and is not itself the 3-space. In particular while $\mathbf{r}$ may have the form of a Euclidean 3-geometry, the space itself has time-dependence and inhomogeneities, and as well in the more general case will exhibit vorticity $\omega = \nabla\times\mathbf{v}$. Only in the unphysical case does the description of the 3-space become identified with the coordinate system $\mathbf{r}$, and that is when the velocity field $\mathbf{v}(\mathbf{r},t)$ becomes uniform and time independent. Then by a suitable choice of observer we may put $\mathbf{v}(\mathbf{r},t) = 0$, and the generalised Schrödinger equation reduces to the usual "free"

*Care is needed to indicate the range of the various ∇'s. Extra parentheses (. . .) are used to limit the range when required.

†In the spacetime formalism it is mistakenly argued that it is "spacetime" that is "dragged".

Schrödinger equation. As we discuss later the experimental evidence is that $\mathbf{v}(\mathbf{r},t)$ is fractal and so cannot be removed by a change to a preferred observer. Hence the generalised Schrödinger equation in (15)–(16) is a major development for fundamental physics. Of course in general other non-3-space potential energy terms may be added to the RHS of (16). A prediction of this new quantum theory, which also extends to a generalised Dirac equation, is that the fractal structure to space implies that even at the scale of atoms etc there will be time-dependencies and inhomogeneities, and that these will affect transition rates of quantum systems. These effects are probably those known as the Shnoll effects [11].

5 Free-fall minimum proper-time trajectories

The acceleration in (18) also arises from the following argument, which is the analogue of the Fermat least-time formalism. Consider the elapsed time for a comoving clock travelling with the test particle. Then taking account of the Lamour time-dilation effect that time is given by

$$\tau[\mathbf{r}_0] = \int dt \left(1 - \frac{\mathbf{v}_R^2}{c^2}\right)^{1/2} \tag{21}$$

with $\mathbf{v}_R$ given by (19) in terms of $\mathbf{v}_O$ and $\mathbf{v}$. Then this time effect relates to the speed of the clock relative to the local 3-space, and that c is the speed of light relative to that local 3-space. We are using a relativistic treatment in (21) to demonstrate the generality of the results*. Under a deformation of the trajectory

$$\mathbf{r}_0(t) \to \mathbf{r}_0(t) + \delta\mathbf{r}_0(t), \quad \mathbf{v}_0(t) \to \mathbf{v}_0(t) + \frac{d\delta\mathbf{r}_0(t)}{dt}, \tag{22}$$

and then

$$\begin{aligned} &\mathbf{v}\left(\mathbf{r}_0(t) + \delta\mathbf{r}_0(t), t\right) = \\ &= \mathbf{v}\left(\mathbf{r}_0(t), t\right) + \left(\delta\mathbf{r}_0(t)\cdot\nabla\right)\mathbf{v}\left(\mathbf{r}_0(t), t\right) + \ldots \end{aligned} \tag{23}$$

Evaluating the change in proper travel time to lowest order

$$\begin{aligned} &\delta\tau = \tau[\mathbf{r}_0 + \delta\mathbf{r}_0] - \tau[\mathbf{r}_0] = \\ &= -\int dt\, \frac{1}{c^2}\, \mathbf{v}_R\cdot\delta\mathbf{v}_R \left(1 - \frac{\mathbf{v}_R^2}{c^2}\right)^{-1/2} + \cdots = \\ &= \int dt\, \frac{1}{c^2} \frac{\mathbf{v}_R\cdot(\delta\mathbf{r}_0\cdot\nabla)\mathbf{v} - \mathbf{v}_R\cdot\dfrac{d(\delta\mathbf{r}_0)}{dt}}{\sqrt{1 - \dfrac{\mathbf{v}_R^2}{c^2}}} = \\ &= \int dt\, \frac{1}{c^2} \left(\frac{\mathbf{v}_R\cdot(\delta\mathbf{r}_0\cdot\nabla)\mathbf{v}}{\sqrt{1 - \dfrac{\mathbf{v}_R^2}{c^2}}} + \delta\mathbf{r}_0\cdot\frac{d}{dt}\frac{\mathbf{v}_R}{\sqrt{1 - \dfrac{\mathbf{v}_R^2}{c^2}}} \right) = \\ &= \int dt\, \frac{1}{c^2}\delta\mathbf{r}_0\cdot \left(\frac{(\mathbf{v}_R\cdot\nabla)\mathbf{v} + \mathbf{v}_R\times(\nabla\times\mathbf{v})}{\sqrt{1 - \dfrac{\mathbf{v}_R^2}{c^2}}} + \frac{d}{dt}\frac{\mathbf{v}_R}{\sqrt{1 - \dfrac{\mathbf{v}_R^2}{c^2}}} \right). \end{aligned}$$

*A non-relativistic analysis may be alternatively pursued by first expanding (21) in powers of $1/c^2$.

Hence a trajectory $\mathbf{r}_0(t)$ determined by $\delta\tau = 0$ to $O\big(\delta\mathbf{r}_0(t)^2\big)$ satisfies

$$\frac{d}{dt}\frac{\mathbf{v}_R}{\sqrt{1 - \dfrac{\mathbf{v}_R^2}{c^2}}} = -\frac{(\mathbf{v}_R\nabla)\,\mathbf{v} + \mathbf{v}_R\times(\nabla\times\mathbf{v})}{\sqrt{1 - \dfrac{\mathbf{v}_R^2}{c^2}}}. \tag{24}$$

Substituting $\mathbf{v}_R(t) = \mathbf{v}_0(t) - \mathbf{v}\left(\mathbf{r}_0(t), t\right)$ and using

$$\frac{d\mathbf{v}\left(\mathbf{r}_0(t), t\right)}{dt} = \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}_0\cdot\nabla)\,\mathbf{v}\,, \tag{25}$$

we obtain

$$\begin{aligned} \frac{d\mathbf{v}_0}{dt} &= \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} + (\nabla\times\mathbf{v})\times\mathbf{v}_R - \\ &\quad - \frac{\mathbf{v}_R}{1 - \dfrac{\mathbf{v}_R^2}{c^2}}\,\frac{1}{2}\,\frac{d}{dt}\left(\frac{\mathbf{v}_R^2}{c^2}\right). \end{aligned} \tag{26}$$

Then in the low speed limit $v_R \ll c$ we may neglect the last term, and we obtain (18). Hence we see a close relationship between the geodesic equation, known first from General Relativity, and the 3-space generalisation of the Schrödinger equation, at least in the non-relativistic limit. So in the classical limit, i.e when the wavepacket approximation is valid, the wavepacket trajectory is specified by the least propertime geodesic.

The relativistic term in (26) is responsible for the precession of elliptical orbits and also for the event horizon effect. Hence the trajectory in (18) is a non-relativistic minimum travel-time trajectory, which is Fermat's Principle. The relativistic term in (26) will arise from a generalised Dirac equation which would then include the dynamics of 3-space.

6 Fractal 3-space and the DeWitte experimental data

In 1991 Roland DeWitte working within Belgacom, the Belgium telecommunications company, accidently made yet another detection of absolute motion, and one which was 1st-order in v/c. 5 MHz radio frequency (RF) signals were sent in both directions through two buried coaxial cables linking the two clusters of cesium atomic clocks.

Changes in propagation times were observed and eventually observations over 178 days were recorded. A sample of the data, plotted against sidereal time for just three days, is shown in Fig. 1. The DeWitte data was clear evidence of absolute motion with the Right Ascension for minimum/maximum propagation time agreeing almost exactly with

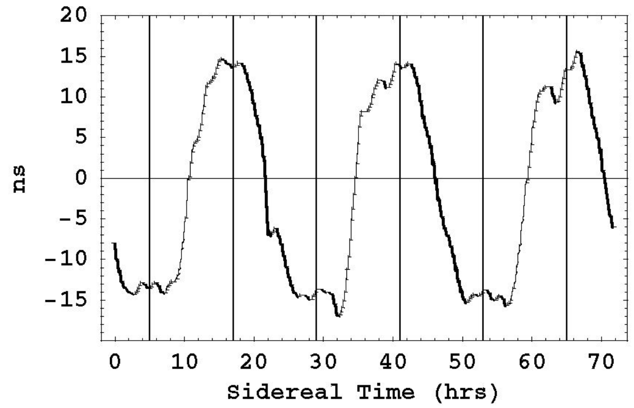

Fig. 1: Variations in twice the one-way travel time, in ns, for an RF signal to travel 1.5 km through a buried coaxial cable between Rue du Marais and Rue de la Paille, Brussels. An offset has been used such that the average is zero. The cable has a North-South orientation, and the data is ± difference of the travel times for NS and SN propagation. The sidereal time for maximum effect of ∼5hr (or ∼17hr) (indicated by vertical lines) agrees with the direction found by Miller[4]. Plot shows data over 3 sidereal days and is plotted against sidereal time. The main effect is caused by the rotation of the Earth. The superimposed fluctuations are evidence of turbulence i.e gravitational waves. Removing the Earth induced rotation effect we obtain the first experimental data of the fractal structure of space, and is shown in Fig. 2. DeWitte performed this experiment over 178 days, and demonstrated that the effect tracked sidereal time and not solar time[1].

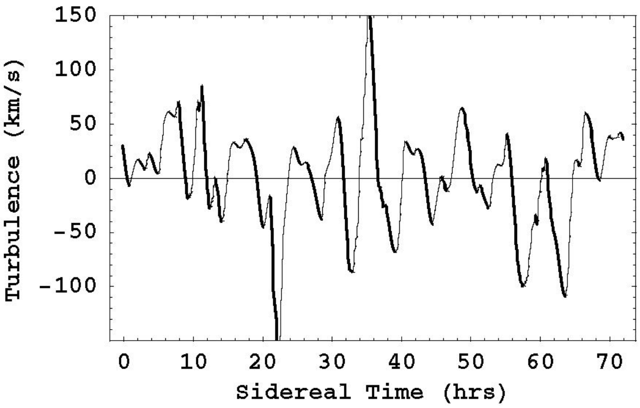

Fig. 2: Shows the velocity fluctuations, essentially "gravitational waves" observed by DeWitte in 1991 from the measurement of variations in the RF coaxial-cable travel times. This data is obtained from that in Fig. 1 after removal of the dominant effect caused by the rotation of the Earth. Ideally the velocity fluctuations are three-dimensional, but the DeWitte experiment had only one arm. This plot is suggestive of a fractal structure to the velocity field. This is confirmed by the power law analysis shown in Fig. 3.

Miller's direction* ($\alpha = 5.2^{\mathrm{hr}}$, $\delta = -67^\circ$)†, and with speed 420 ± 30 km/s. This local absolute motion is different from the CMB motion, in the direction ($\alpha = 11.20^{\mathrm{hr}}, \delta = -7.22^\circ$) with speed of 369 km/s, for that would have given the data a totally different sidereal time signature, namely the times for maximum/ minimum would have been shifted by 6hrs. The CMB velocity is motion relative to the distant early universe, whereas the velocity measured in the DeWitte and related experiments is the velocity relative to the local space. The declination of the velocity observed in this DeWitte experiment cannot be determined from the data as only three days of data are available. However assuming exactly the same declination as Miller the speed observed by DeWitte appears to be also in excellent agreement with the Miller speed. The dominant effect in Fig. 1 is caused by the rotation of the Earth, namely that the orientation of the coaxial cable with respect to the direction of the flow past the Earth changes as the Earth rotates. This effect may be approximately unfolded from the data, leaving the gravitational waves shown in Fig. 2. This is the first evidence that the velocity field describing 3-space has a complex structure, and is indeed fractal.

The fractal structure, i. e. that there is an intrinsic lack of scale, to these speed fluctuations is demonstrated by binning the absolute speeds $|v|$ and counting the number of speeds $p(|v|)$ within each bin. A least squares fit of the log–log plot to a straightline was then made. Plotting $\log[p(|v|)]$ vs $\log|v|$, as shown in Fig. 3 we see that the fit gives $p(v) \propto |v|^{-2.6}$. With the new experiment considerably more data will become available.

7 Observing 3-space vorticity

The vorticity effect in (18) can be studied experimentally in the Gravity Probe B (GP-B) gyroscope satellite experiment in which the precession of four on-board gyroscopes has been measured to unprecedented accuracy [12, 13]. In a generalisation of (1) [1] the vorticity $\nabla\times\mathbf{v}$ is generated by matter in motion through the 3-space, where here $\mathbf{v}_R$ is the absolute velocity of the matter relative to the local 3-space.

$$\nabla\times(\nabla\times\mathbf{v}) = \frac{8\pi G\rho}{c^2}\mathbf{v}_R\,. \tag{27}$$

We then obtain from (27) the vorticity (ignoring homogeneous vortex solutions)

$$\vec{\omega}(\mathbf{r},t) = \frac{2G}{c^2}\int d^3r'\,\frac{\rho(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|^3}\,\mathbf{v}_R(\mathbf{r}',t)\times(\mathbf{r}-\mathbf{r}')\,. \tag{28}$$

*This velocity arises after removing the effects of the Earth's orbital speed about the Sun, 30 km/s, and the gravitational in-flow past the Earth towards the Sun, 42 km/s, as in (6).

†The opposite direction is not easily excluded due to errors within the data, and so should also be considered as possible. A new experiment will be capable of more accurately determining the speed and direction, as well as the fractal structure of 3-space. The author is constructung a more compact version of the Torr-Kolen - DeWitte coaxial cable RF travel-time experiment. New experimental techniques have been developed to increase atomic-clock based timing accuracy and stability, so that shorter cables can be used, which will permit 3-arm devices.

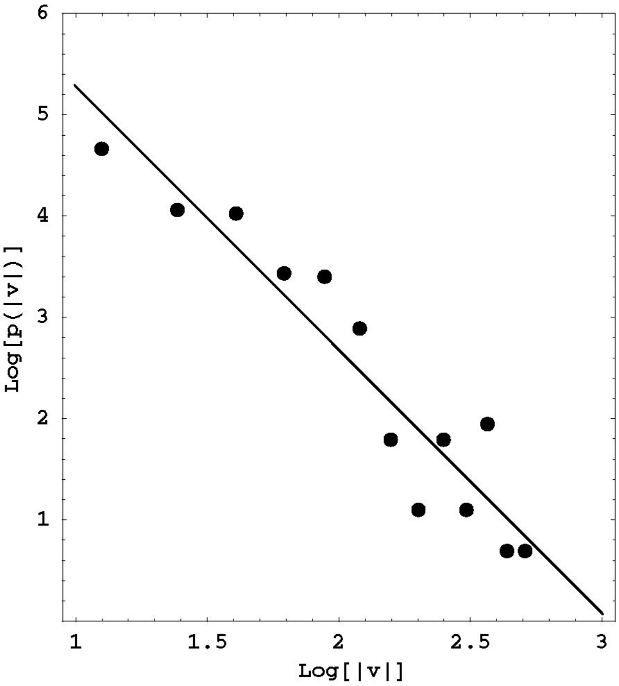

Fig. 3: Shows that the velocity fluctuations in Fig. 2 are scale free, as the probability distribution from binning the speeds has the form $p(v) \propto |v|^{-2.6}$. This plot shows $\log[p(v)]$ vs $\log|v|$. This shows that the velocity field has a fractal structure, and so requiring the generalisation of the Schrödinger equation, as discussed herein, and also the Maxwell and Dirac equations (to be discussed elsewhere).

For the smaller Earth-rotation induced vorticity effect $\mathbf{v}_R(\mathbf{r}) = \mathbf{w} \times \mathbf{r}$ in (28), where $\mathbf{w}$ is the angular velocity of the Earth, giving

$$\vec{\omega}(\mathbf{r})_{\mathrm{rot}} = 4\frac{G}{c^2}\frac{3(\mathbf{r}\cdot\mathbf{L})\,\mathbf{r} - r^2\mathbf{L}}{2r^5}, \tag{29}$$

where $\mathbf{L}$ is the angular momentum of the Earth, and $\mathbf{r}$ is the distance from the centre.

In general the vorticity term in (18) leads to a apparent "torque", according to a distant observer, acting on the angular momentum $\mathbf{S}$ of the gyroscope,

$$\vec{\tau} = \int d^3r\,\rho(\mathbf{r})\;\mathbf{r}\times\big(\vec{\omega}(\mathbf{r})\times\mathbf{v}_R(\mathbf{r})\big)\,, \tag{30}$$

where ρ is its density, and where now $\mathbf{v}_R$ is used here to describe the motion of the matter forming the gyroscope relative to the local 3-space. Then $d\mathbf{S} = \vec{\tau}dt$ is the change in $\mathbf{S}$ over the time interval dt. For a gyroscope $\mathbf{v}_R(\mathbf{r}) = \mathbf{s}\times\mathbf{r}$, where $\mathbf{s}$ is the angular velocity of the gyroscope. This gives

$$\vec{\tau} = \frac{1}{2}\,\vec{\omega}\times\mathbf{S} \tag{31}$$

and so $\vec{\omega}/2$ is the instantaneous angular velocity of precession of the gyroscope. The component of the vorticity in (29) has been determined from the laser-ranged satellites LAGEOS (NASA) and LAGEOS 2 (NASA-ASI) [14], and the data implies the indicated coefficient on the RHS of (27) to $\pm 10\%$. For GP-B the direction of $\mathbf{S}$ has been chosen so that this precession is cumulative and, on averaging over an orbit, corresponds to some 7.7×10^{-6} arcsec per orbit, or 0.042 arcsec per year. GP-B has been superbly engineered so that measurements to a precision of 0.0005 arcsec are possible.

However for the Earth-translation induced precession if we use $v_R = 430$ km/s (in the direction RA $= 5.2^{\mathrm{hr}}$, Dec $= -67^\circ$), (28) gives

$$\vec{\omega}(\mathbf{r})_{\mathrm{trans}} = \frac{2GM}{c^2}\frac{\mathbf{v}_R\times\mathbf{r}}{r^3}\,, \tag{32}$$

and then the total vorticity is $\vec{\omega} = \vec{\omega}_{\mathrm{rot}} + \vec{\omega}_{\mathrm{trans}}$. The maximum magnitude of the speed of this precession component is $\omega_{\mathrm{trans}}/2 = gv_C/c^2 = 8\times10^{-6}$ arcsec/s, where here g is the usual gravitational acceleration at the altitude of the satellite. This precession has a different signature: it is not cumulative, and is detectable by its variation over each single orbit, as its orbital average is zero, to first approximation.

Essentially then these spin precessions are caused by the rotation of the "wavepackets" describing the matter forming the gyroscopes, and caused in turn by the vorticity of 3-space. The above analysis shows that the rotation is exactly the same as the rotation of the 3-space itself, just as the acceleration of "matter" was exactly the same as the acceleration of the 3-space. We this obtain a much clearer insight into the nature of motion, and which was not possible in the spacetime formalism.

8 Conclusions

We have seen herein that the new theory of 3-space has resulted in a number of fundamental developments, namely that a complex "quantum foam" dynamical 3-space exists and has a fractal "flow" structure, as revealed most clearly by the extraordinary DeWitte coaxial-cable experiment. This fractal structure requires that the fundamental equations of physics be generalised to take account of, for the first time, the physics of this 3-space and, in particular, here the inclusion of that dynamics within the dynamics of quantum systems. We saw that the generalisation of the Schrödinger equation is unique, and that from an Ehrenfest wavepacket analysis we obtained the equivalence principle, with the acceleration of "matter" being shown to be identical to the acceleration of the 3-space; which while not unexpected, is derived here for the first time. This result shows that the equivalence principle is really a quantum-theoretic effect. As well we obtained by that same analysis that any vorticity in the 3-space velocity field will result in a corresponding rotation of wavepackets, and just such an effect is being studied in the GP-B gyroscope experiment. So for the first time we see that the original Schrödinger equation actually lacked a key dynamical ingredient. We saw that self-consistency within the small-wavepacket approximation imposed restrictions on the dynamical equations that determine the vorticity, giving yet another indication of the close connection between quantum theory and the phenomena of 3-space and gravity. As well because the 3-space is fractal the generalised Schrödinger equation now contains a genuine element of stochasticity.

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