On Isotropic Coordinates and Einstein's Gravitational Field

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It is proved herein that the metric in the so-called "isotropic coordinates" for Einstein's gravitational field is a particular case of an infinite class of equivalent metrics. Furthermore, the usual interpretation of the coordinates is erroneous, because in the usual form given in the literature, the alleged coordinate length $\sqrt{dx^2+dy^2+dz^2}$ is not a coordinate length. This arises from the fact that the geometrical relations between the components of the metric tensor are invariant and therefore bear the same relations in the isotropic system as those of the metric in standard Schwarzschild coordinates.

1 Introduction

Petrov [1] developed an algebraic classification of Einstein's field equations. Einstein's field equations can be written as,

$$R_{\alpha\beta} - \frac{1}{2}R\,g_{\alpha\beta} = \kappa\,T_{\alpha\beta} - \lambda\,g_{\alpha\beta}\,,$$

where κ is a constant, and λ the so-called cosmological constant. If $T_{\alpha\beta} \propto g_{\alpha\beta}$, the associated space is called an Einstein space. Thus, Einstein spaces include those described by partially degenerate metrics of this form. Consequently, such metrics become non-Einstein only when

$$g = \det\|g_{\alpha\beta}\| = 0\,.$$

A simple source is a spherically symmetric mass (a mass island), without charge or angular momentum. A simple source giving rise to a static gravitational field in vacuum, where space is isotropic and homogeneous, constitutes a Schwarzschild space. The associated field equations external to the simple source are

$$R_{\alpha\beta} - \frac{1}{2}R\,g_{\alpha\beta} = 0\,,$$

or, more simply,

$$R_{\alpha\beta} = 0\,.$$

Thus, a Schwarzschild space is an Einstein space. There are four types of Einstein spaces. The Schwarzschild space is a type 1 Einstein space. It gives rise to a spherically symmetric gravitational field.

The simple source interacts with a "test" particle, which has no charge, no angular momentum, and effectively no mass, or so little mass that its own gravitational field can be neglected entirely. A similar concept is utilised in electrodynamics in the notion of a "test" charge.

The only solutions known for Einstein's field equations involve a single gravitating source interacting with a test particle. There are no known solutions for two or more interacting comparable masses. In fact, it is not even known if Einstein's field equations admit of solutions for multi-body configurations, as no existence theorem has even been adduced. It follows that there is no theoretical sense to concepts such as black hole binaries, or colliding or merging black holes, notwithstanding the all too common practice of assuming them well-posed theoretical problems allegedly substantiated by observations.

The metric for Einstein's gravitational field in the usual isotropic coordinates is, in relativistic units ($c = G = 1$),

$$ds^2 = \frac{\left(1-\frac{m}{2r}\right)^2}{\left(1+\frac{m}{2r}\right)^2}\,dt^2 - \left(1+\frac{m}{2r}\right)^4\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)\right] = \tag{1a}$$

$$= \frac{\left(1-\frac{m}{2r}\right)^2}{\left(1+\frac{m}{2r}\right)^2}\,dt^2 - \left(1+\frac{m}{2r}\right)^4\left(dx^2+dy^2+dz^2\right), \tag{1b}$$

having set $r = \sqrt{x^2+y^2+z^2}$. This metric describes a Schwarzschild space.

By virtue of the factor $\left(dx^2+dy^2+dz^2\right)$ it is usual that $0 \leqslant r < \infty$ is taken. However, this standard range on r is due entirely to assumption, based upon the misconception that because $0 \leqslant r < \infty$ is defined on the usual Minkowski metric, this must also hold for (1a) and (1b). Nothing could be further from the truth, as I shall now prove.

2 Proof

Consider the standard Minkowski metric,

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \equiv$$

$$\equiv dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{2}$$

$$0 \leqslant r < \infty\,.$$

The spatial components of this metric describe a sphere of radius $r \geqslant 0$, centred at $r = 0$. The quantity r is an Efcleethean* distance since Minkowski space is pseudo-Efcleethean.

Now (2) is easily generalised [2] to

$$ds^2 = dt^2 - dr^2 - (r - r_0)^2(d\theta^2 + \sin^2\theta\, d\varphi^2) = \quad (3a)$$

$$= dt^2 - \frac{(r - r_0)^2}{|r - r_0|^2}\, dr^2 - |r - r_0|^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \quad (3b)$$

$$= dt^2 - d|r - r_0|^2 - |r - r_0|^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \quad (3c)$$

$$0 \leqslant |r - r_0| < \infty\,.$$

The spatial components of equations (3) describe a sphere of radius $R_c(r) = |r - r_0|$, centred at a point located anywhere on the 2-sphere r_0. Only if $r_0 = 0$ does (3) describe a sphere centred at the origin of the coordinate system. With respect to the underlying coordinate system of (3), $R_c(r)$ is the radial distance between the 2-spheres $r = r_0$ and $r \neq r_0$.

The usual practice is to supposedly generalise (2) as

$$ds^2 = A(r)dt^2 - B(r)\big(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2\big) \quad (4)$$

to finally obtain (1a) in the standard way, with the assumption that $0 \leqslant r < \infty$ on (2) must hold also on (4), and hence on equations (1). However, this assumption has never been proved by the theoreticians. The assumption is demonstrably false. Furthermore, this procedure does not produce a generalised solution in terms of the parameter r, but instead a particular solution.

Since (3) is a generalisation of (2), I use it to generalise (4) to

$$ds^2 = e^{\nu}dt^2 - e^{\mu}\big(dh^2 + h^2 d\theta^2 + h^2\sin^2\theta\, d\varphi^2\big) \quad (5)$$

$$h = h(r) = h(|r - r_0|), \quad \nu = \nu(h(r)), \quad \mu = \mu(h(r))\,.$$

Note that (5) can be written in the mixed form

$$ds^2 = e^{\nu}dt^2 - e^{\mu}\left[\left(\frac{dh}{dr}\right)^2 dr^2 + h^2 d\theta^2 + h^2\sin^2\theta d\varphi^2\right], \quad (6)$$

from which the particular form (4) usually used is recovered if $h(|r - r_0|) = r$. However, no particular form for $h(|r - r_0|)$ should be pre-empted. Doing so, in the routine fashion of the majority of the relativists, produces only a particular solution in terms of the Minkowski r, with all the erroneous assumptions associated therewith.

Now (5) must satisfy the energy-momentum tensor equations for the simple, static, vacuum field:

$$0 = e^{-\mu}\left(\frac{\mu'^2}{4} + \frac{\mu'\nu'}{2} + \frac{\mu' + \nu'}{h}\right)$$

$$0 = e^{-\mu}\left(\frac{\mu''}{2} + \frac{\nu''}{2} + \frac{\nu'^2}{r} + \frac{\mu' + \nu'}{2h}\right)$$

$$0 = e^{-\mu}\left(\mu'' + \frac{\mu'^2}{4} + \frac{2\mu'}{h}\right),$$

where the prime indicates d/dh. This gives, in the usual way,

$$ds^2 = \frac{\left(1 - \frac{m}{2h}\right)^2}{\left(1 + \frac{m}{2h}\right)^2}\, dt^2 - \left(1 + \frac{m}{2h}\right)^4 \left[dh^2 + h^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right], \quad (7)$$

from which the admissible form for $h(|r - r_0|)$ and the value of the constant r_0 must be rigorously ascertained from the intrinsic geometrical properties of the metric itself.

Now the intrinsic geometry of the metric (2) is the same on all the metrics given herein in terms of the spherical coordinates of Minkowski space, namely, the radius of curvature R_c in the space described by the metric is always the square root of the coefficient of the angular terms of the metric and the proper radius R_p is always the integral of the square root of the component containing the differential element of the radius of curvature. Thus, on (2),

$$R_c(r) \equiv r, \quad R_p(r) \equiv \int_0^r dr = r \equiv R_c(r)\,,$$

and on (3),

$$R_c(r) \equiv |r - r_0|\,,$$

$$R_p(r) \equiv \int_0^{|r - r_0|} dr = |r - r_0| \equiv R_c(r)\,,$$

whereby it is clear that $R_c(r)$ and $R_p(r)$ are identical, owing to the fact that the spatial coordinates of (2) and (3) are Efcleethean.

Now consider the general metric of the form

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\theta^2 + \sin^2\theta\, d\varphi^2) \quad (8)$$

$$A, B, C > 0.$$

In this case,

$$R_c(r) = \sqrt{C(r)}, \quad R_p(r) = \int \sqrt{B(r)}\, dr\,.$$

I remark that although (8) is mathematically valid, it is misleading. In the cases of (2) and (3), the respective metrics are given in terms of the radius of curvature and its differential element. This is not the case in (8) where the first and second components are in terms of the parameter r of the radius of curvature, not the radius of curvature itself. I therefore write (8) in terms of only the radius of curvature on (8), thus

$$ds^2 = A^*\big(\sqrt{C(r)}\big)dt^2 - B^*\big(\sqrt{C(r)}\big)d\sqrt{C(r)}^{\,2} - C(r)(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \quad (9a)$$

$$A^*, B^*, C > 0\,.$$

*Due to Efcleethees, incorrectly Euclid, so the geometry is rightly Efcleethean.

Note that (9a) can be written as,

$$ds^2 = A^*\left(\sqrt{C(r)}\right)dt^2 - B^*\left(\sqrt{C(r)}\right)\left(\frac{d\sqrt{C(r)}}{dr}\right)^2 dr^2 - \\ - C(r)\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (9b)$$

$$A^*, B^*, C > 0\,,$$

and by setting

$$B^*\left(\sqrt{C(r)}\right)\left(\frac{d\sqrt{C(r)}}{dr}\right)^2 = B(r)\,,$$

equation (8) is recovered, proving that (8) and equations (9) are mathematically equivalent, and amplifying the fact that (8) is a mixed-term metric. Note also that if $C(r)$ is set equal to r^2, the alleged general form used by most relativists is obtained. However, the form of $C(r)$ should not be pre-empted, for by doing so only a particular parametric solution is obtained, and with the form chosen by most relativists, the properties of r in Minkowski space are assumed (incorrectly) to carry over into the metric for the gravitational field.

It is also clear from (8) and equations (9) that $|r - r_0|$ is the Efcleethean distance between the centre of mass of the field source and a test particle, in Minkowski space, and which is mapped into $R_c(r)$ and $R_p(r)$ of the gravitational field by means of functions determined by the structure of the gravitational metric itself, namely the functions given by

$$R_c(r) = \sqrt{C(r)}\,,$$

$$R_p(r) = \int \sqrt{B^*\left(\sqrt{C(r)}\right)}\, d\sqrt{C(r)} = \int \sqrt{B(r)}\, dr\,.$$

In the case of the usual metric the fact that $|r - r_0|$ is the Efcleethean distance between the field source and a test particle in Minkowski space is suppressed by the choice of the particular function $\sqrt{C(r)} = r^2$, so that it is not immediately apparent that when r goes down to $\alpha = 2m$ on that metric, the parametric distance between field source and test particle has gone down to zero. Generally, as the parametric distance goes down to zero, the proper radius in the gravitational field goes down to zero, irrespective of the location of the field source in parameter space. Thus, the field source is always located at $R_p = 0$ as far as the metric for the gravitational field is concerned.

It has been proved elsewhere [3, 4] that in the case of the simple "point-mass" (a fictitious object), metrics of the form (8) or (9) are characterised by the following scalar invariants,

$$R_p(r_0) \equiv 0\,, \quad R_c(r_0) \equiv 2m\,, \quad g_{00}(r_0) \equiv 0\,, \qquad (10)$$

so that the actual value of r_0 is completely irrelevant.

Now (7) can be written as

$$ds^2 = \frac{\left(1 - \frac{m}{2h}\right)^2}{\left(1 + \frac{m}{2h}\right)^2} dt^2 - \\ - \left(1 + \frac{m}{2h}\right)^4 dh^2 - h^2\left(1 + \frac{m}{2h}\right)^4 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (11)$$

$$h = h(r) = h(|r - r_0|)\,.$$

Since the geometrical relations between the components of the metric tensor are invariant it follows that on (11),

$$R_c(r) = h(r)\left(1 + \frac{m}{2h(r)}\right)^2, \qquad (12a)$$

$$R_p(r) = \int \left(1 + \frac{m}{2h(r)}\right)^2 dh(r) = \\ = h(r) + m \ln h(r) - \frac{m^2}{2}\frac{1}{h(r)} + K\,,$$

where $K = \text{constant}$,

$$R_p(r) = h(r) + m \ln \frac{h(r)}{K_1} - \frac{m^2}{2}\frac{1}{h(r)} + K_2 \qquad (12b)$$

where K_1 and K_2 are constants.

It is required that $R_p(r_0) \equiv 0$, so

$$0 = h(r_0) + m \ln \frac{h(r_0)}{K_1} - \frac{m^2}{2}\frac{1}{h(r_0)} + K_2\,,$$

which is satisfied only if

$$h(r_0) = K_1 = K_2 = \frac{m}{2}\,. \qquad (13)$$

Therefore,

$$R_p(r) = h(r) + m \ln\left(\frac{2h(r)}{m}\right) - \frac{m^2}{2}\frac{1}{h(r)} + \frac{m}{2}\,. \qquad (14)$$

According to (12a), and using (13),

$$R_c(r_0) = \frac{m}{2}\left(1 + \frac{m}{2\frac{m}{2}}\right)^2 = 2m\,,$$

satisfying (10) as required.

Now from (11),

$$g_{00}(r) = \frac{\left(1 - \frac{m}{2h(r)}\right)^2}{\left(1 + \frac{m}{2h(r)}\right)^2}\,,$$

and using (13),

$$g_{00}(r_0) = \frac{\left(1 - \frac{2m}{2m}\right)^2}{\left(1 + \frac{2m}{2m}\right)^2} = 0\,,$$

satisfying (10) as required.

It remains now to ascertain the general admissible form of $h(r) = h(|r - r_0|)$.

By (6),

$$\frac{dh}{dr} \neq 0 \quad \forall \quad r \neq r_0\,.$$

It is also required that (11) become Minkowski in the infinitely far field, so

$$\lim_{|r-r_0|\to\infty} \frac{h^2(r)\left(1+\frac{m}{2h(r)}\right)^4}{|r-r_0|^2} \to 1\,,$$

must be satisfied.

When there is no matter present ($m = 0$), $h(r)$ must reduce the metric to Minkowski space.

Finally, $h(r)$ must be able to be arbitrarily reduced to r by a suitable choice of arbitrary constants so that the usual metric (1a) in isotropic coordinates can be recovered at will.

The only form for $h(r)$ that satisfies all the requirements is

$$h(r) = \left[|r-r_0|^n + \left(\frac{m}{2}\right)^n\right]^{\frac{1}{n}}, \qquad n \in \Re^+, \quad r_0 \in \Re, \quad r \neq r_0\,, \tag{15}$$

where n and r_0 are entirely arbitrary constants. The condition $r \neq r_0$ is necessary since the "point-mass" is not a physical object.

Setting $n = 1$, $r_0 = \frac{m}{2}$, and $r > r_0$ in (15) gives the usual metric (1a) in isotropic coordinates. Note that in this case $r_0 = \frac{m}{2}$ is the location of the fictitious "point-mass" in parameter space (i. e. in Minkowski space) and thus as the distance between the test particle and the source, located at $r_0 = \frac{m}{2}$, goes to zero in parameter space, the proper radius in the gravitational field goes to zero, the radius of curvature goes to $2m$, and g_{00} goes to zero. Thus, the usual claim that the term $dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$ (or $dx^2 + dy^2 + dz^2$) describes a coordinate length is false. Note that in choosing this case, the resulting metric suppresses the true nature of the relationship between the r-parameter and the gravitational field because, as clearly seen by (15), $r_0 = \frac{m}{2}$ drops out. Note also that (15) generalises the mapping so that distances on the real line are mapped into the gravitational field.

Consequently, there is no black hole predicted by the usual metrics (1) in isotropic coordinates. The black hole concept has no validity in General Relativity (and none in Newton's theory either since the Michell-Laplace dark body is not a black hole [5, 6]).

The singularity at $R_p(r_0) \equiv 0$ is insurmountable because

$$\lim_{|r-r_0|\to 0} \frac{2\pi R_c(r)}{R_p(r)} \to \infty\,,$$

according to the admissible forms of $R_p(r)$, $R_c(r)$, and $h(r)$.

Note also that only in the infinitely far field are $R_c(r)$ and $R_p(r)$ identical; where the field becomes Efcleethean (i. e. Minkowski),

$$\lim_{|r-r_0|\to\infty} \frac{2\pi R_c(r)}{R_p(r)} \to 2\pi\,.$$

It has been proved elsewhere [3, 2] that there are no curvature singularities in Einstein's gravitational field. In particular the Riemann tensor scalar curvature invariant (the Kretschmann scalar) $f = R_{\alpha\beta\sigma\rho}R^{\alpha\beta\sigma\rho}$ is finite everywhere, and and in the case of the fictitious point-mass takes the invariant value

$$f(r_0) \equiv \frac{12}{(2m)^4}\,,$$

completely independent of the value of r_0.

Since the intrinsic geometry of the metric is invariant, (11) with (15) must also satisfy this invariant condition. A tedious calculation gives the Kretschmann scalar for (11) at

$$f(r) = \frac{48\,m^2}{h^6\left(1+\frac{m}{2h}\right)^{12}}\,,$$

which by (15) is

$$f(r) = \frac{48\,m^2}{\left[|r-r_0|^n + \left(\frac{m}{2}\right)^n\right]^{\frac{6}{n}}\left(1 + \frac{m}{2\left[|r-r_0|^n + \left(\frac{m}{2}\right)^n\right]^{\frac{1}{n}}}\right)^{12}}\,.$$

Then

$$f(r_0) \equiv \frac{12}{(2m)^4}\,,$$

completely independent of the value of r_0, as required by the very structure of the metric.

The structure of the metric is also responsible for the Ricci flatness of Einstein's static, vacuum gravitational field (satisfying $R_{\alpha\beta} = 0$). Consequently, all the metrics herein are Ricci flat (i. e. $R = 0$). Indeed, all the given metrics can be transformed into

$$ds^2 = \left(1-\frac{2m}{R_c}\right)dt^2 - \left(1-\frac{2m}{R_c}\right)^{-1}dR_c^2 - R_c^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{16}$$

$$R_c = R_c(r) = \sqrt{C(r)}\,, \quad 2m < R_c(r) < \infty\,,$$

which is Ricci flat for any analytic function $R_c(r)$, which is easily verified by using the variables

$$x^0 = t\,, \quad x^1 = R_c(r)\,, \quad x^2 = \theta\,, \quad x^3 = \varphi\,,$$

in the calculation of the Ricci curvature from (16), using,

$$R = g^{\mu\nu}\left\{\frac{\partial^2}{\partial x^\mu \partial x^\nu}\left(\ln\sqrt{|g|}\right) - \frac{1}{\sqrt{|g|}}\frac{\partial}{\partial x^\rho}\left(\sqrt{|g|}\,\Gamma^\rho_{\mu\nu}\right) + \Gamma^\rho_{\mu\sigma}\Gamma^\sigma_{\rho\nu}\right\}.$$

Setting

$$\frac{\chi}{2\pi} = R_c(r) = h(r)\left(1+\frac{m}{2h(r)}\right)^2,$$

transforms the metric (7) into,

$$ds^2 = \left(1 - \frac{2\pi\alpha}{\chi}\right) dt^2 - \left(1 - \frac{2\pi\alpha}{\chi}\right)^{-1} \frac{d\chi^2}{4\pi^2} - \frac{\chi^2}{4\pi^2}\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad (17)$$

$$2\pi\alpha < \chi < \infty\,, \quad \alpha = 2m\,,$$

which is the metric for Einstein's gravitational field in terms of the only theoretically measurable distance in the field – the circumference χ of a great circle [2]. This is a truly coordinate independent expression. There is no need of the r-parameter at all.

Furthermore, equation (17) is clear as to what quantities are radii in the gravitational field, *viz.*

$$R_c(\chi) = \frac{\chi}{2\pi}\,,$$

$$R_p(\chi) = \int_{2\pi\alpha}^{\chi} \sqrt{\frac{\frac{\chi}{2\pi}}{\left(\frac{\chi}{2\pi} - \alpha\right)}}\,\frac{d\chi}{2\pi} = \sqrt{\frac{\chi}{2\pi}\left(\frac{\chi}{2\pi} - \alpha\right)} + \alpha \ln\left|\frac{\sqrt{\frac{\chi}{2\pi}} + \sqrt{\frac{\chi}{2\pi} - \alpha}}{\sqrt{\alpha}}\right|\,.$$

3 Epilogue

The foregoing is based, as has all my work to date, upon the usual manifold with boundary, $[0, +\infty[\times S^2$. By using the very premises of most relativists, including their $[0,+\infty[\times S^2$, I have demonstrated herein that black holes (see also [4, 7]), and elsewhere as a logical consequence [8], that big bangs are not consistent with General Relativity. Indeed, cosmological solutions for isotropic, homogeneous, type 1 Einstein spaces do not exist. Consequently, there is currently no valid relativistic cosmology at all. The Standard Cosmological Model, the Big Bang, is false.

Stavroulakis [9] has argued that $\big[0,+\infty\big[\times S^2$ is inadmissible because it destroys the topological structure of $\mathbb{R}^3$. He has maintained that the correct topological space for Einstein's gravitational field should be $\mathbb{R} \times \mathbb{R}^3$. He has also shown that black holes are not predicted by General Relativity in $\mathbb{R} \times \mathbb{R}^3$.

However, the issue of whether or not $\big[0,+\infty\big[\times S^2$ is admissible is not relevant to the arguments herein, given the objectives of the analysis.

Although χ is measurable in principle, it is apparently beyond measurement in practice. This severely limits the utility of Einstein's theory.

The historical analysis of Einstein's gravitational field proceeded in ignorance of the fact that only the circumference χ of a great circle is significant. It has also failed to realise that there are two different immeasurable radii defined in Einstein's gravitational field, as an inescapable consequence of the intrinsic geometry on the metric, and that these radii are identical only in the infinitely far field where space becomes Efcleethean (i. e. Minkowski). Rejection summarily of the oddity of two distinct immeasurable radii is tantamount to complete rejection of General Relativity; an issue I have not been concerned with.

Minkowski's metric in terms of χ is,

$$ds^2 = dt^2 - \frac{d\chi^2}{4\pi^2} - \frac{\chi^2}{4\pi^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

$$0 \leqslant \chi < \infty\,.$$

It is generalised to

$$ds^2 = A\left(\frac{\chi}{2\pi}\right) dt^2 - B\left(\frac{\chi}{2\pi}\right)^{-1} \frac{d\chi^2}{4\pi^2} - \frac{\chi^2}{4\pi^2}\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad (18)$$

$$\chi_0 < \chi < \infty\,, \qquad A, B > 0\,,$$

which leads, in the usual way, to the line-element of (17), from which χ_0 and the radii associated with the gravitational field are determined via the intrinsic and invariant geometry of the metric.

Setting $R_c(r) = \sqrt{C(r)}$ in (16) gives,

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)^{-1} d\sqrt{C(r)}^{\,2} - C(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \quad (19)$$

where [2]

$$C(r) = \left(\left|r - r_0\right|^n + \alpha^n\right)^{\frac{2}{n}}, \quad (20)$$

$$n \in \Re^+, \quad r_0 \in \Re, \quad \alpha = 2m, \quad r \neq r_0\,,$$

and where n and r_0 are entirely arbitrary constants. Note that if $n = 1$, $r_0 = \alpha$, $r > r_0$, the usual line-element is obtained, but the usual claim that r can go down to zero is clearly false, since when $r = \alpha$, the parametric distance between field source and test particle is zero, which is reflected in the fact that the proper radius on (19) is then zero, $R_c = \alpha = 2m$, and $g_{00} = 0$, as required. The functions (20) are called Schwarzschild forms [4, 7], and they produce an infinite number of equivalent Schwarzschild metrics.

The term $\sqrt{dx^2 + dy^2 + dz^2}$ of the standard metric in "isotropic coordinates" is not a coordinate length as commonly claimed. This erroneous idea stems from the fact that the usual choice of $C(r) = r^2$ in the metric (19) suppresses the true nature of the mapping of parametric distances into the true radii of the gravitational field. This arises from the additional fact that the location of the field source at $r_0 = \alpha$

in parameter space drops out of the functional form $C(r)$ as given by (20), in this particular case. The subsequent usual transformation to the usual metric (1a) carries with it the erroneous assumptions about r, inherited from the misconceptions about r in (19) with the reduction of $C(r)$ to r^2, which, in the usual conception, violates (20), and hence the entire structure of the metric for the gravitational field. Obtaining (1a) from first principles using the expression (5) with $h(r) = r^2$ and the components of the energy-momentum tensor, already presupposes the form of $h(r)$ and generates the suppression of the true nature of r in similar fashion.

The black hole, as proved herein and elsewhere [4, 7], and the Big Bang, are due to a serious neglect of the intrinsic geometry of the gravitational metric, a failure heretofore to understand the structure of type 1 Einstein spaces, with the introduction instead, of extraneous and erroneous hypotheses by which the intrinsic geometry is violated.

Since Nature does not make point-masses, the point-mass referred to Einstein's gravitational field must be regarded as merely the mathematical artifice of a centre-of-mass of the source of the field. The fact that the gravitational metric for the point-mass disintegrates at the point-mass is a theoretical indication that the point-mass is not physical, so that the metric is undefined when $r = r_0$ in parameter space, which is at $R_p(r_0) \equiv 0$ on the metric for the gravitational field. The usual concept of gravitational collapse itself collapses.

To fully describe the gravitational field there must therefore be two metrics, one for the interior of an extended gravitating body and one for the exterior of that field source, with a transition between the two at the surface of the body. This has been achieved in the idealised case of a sphere of incompressible and homogeneous fluid in vacuum [10, 11]. No singularities then arise, and gravitational collapse to a "point-mass" is impossible.

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