Much Ado about Nil: Reflection from Moving Mirrors and the Interferometry Experiments

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The emitter and receiver Doppler effects are re-examined from the point of view of boundary condition on a moving boundary. Formulas are derived for the frequencies of the waves excited on receiver's and emitter's surfaces by the waves traveling thorough the medium. It is shown that if the emitting source and the reflection mirror are moving with the same speed in the same direction relative to a medium at rest, there is no observable Doppler effect. Hence, the nil effect of Michelson and Morley experiment (MME) is the only possible outcome and cannot be construed as an indication about the existence or nonexistence of an absolute continuum. The theory of a new experiment that can give conclusive information is outlined and the possible experimental set-up is sketched.

5 Introduction

Since the groundlaying work of Fizeau, interferometry has been one of the most often used methods to investigate the properties of light. The idea of interferometry was also applied to detecting the presence of an absolute medium in the Michelson and Morley experiment (MME) [1]. The expected effect was of second order $O(v^2/c^2)$ with respect to the ratio between the Earth speed v and speed of light c and it is generally accepted now that Michelson-Morley experiment yielded a nil result, in the sense that the fringes that were observed corresponded to a much smaller (assumed to be negligible) speed than actual Earth's speed. Around the end of Nineteen Century, the nil result of MME prompted Fitz-Gerald and Lorentz to surmise that the lengths are contracted in the direction of motion by the Lorentz factor $\sqrt{1 - v^2/c^2}$ that cancels exactly the expected effect. Since then the Lorentz contraction has been many times verified and can be considered now as an established fact. The Lorentz contraction does not need MME anymore in order to survive as the main vehicle of the modern physics of processes at high speeds.

On another note, the nil effect of MME was eventually interpreted as an indication that there exists no absolute (resting) medium where the light propagates. The problem with this conclusion is that nobody *actually* proposed a theory for MME in which a continuous medium was considered with the correct boundary conditions. Rather, the emission theory of light was used whose predictions contradicted the experimental evidence. In the present paper we show that if a medium at rest is assumed and if this medium is not entrained by the moving bodies, the exact effect from MME is nil, i. e., the expected second-order effect was an artifact from the fact that the emission theory of light (essentially corpuscular in its nature) was applied to model the propagation of light in a continuous medium.

The best way to judge about the existence of the absolute medium is to stage first-order experiments (one way experiments). Along these lines are organized many experimental works, most notably [2, 3] where the sought effect was the anisotropy of speed of light. In our opinion, it is not quite clear how one can discriminate between an anisotropic speed of light on one hand and a first-order Doppler effect, on the other. Yet, we believe that the solution of the conundrum about the existence or nonexistence of an absolute continuum will be solved by a first-order experiment. To this end we also propose an interference experiment that should be able to measure the first-order effect. The most important thing is that first-order effect has actually been observed (see [2, 3], among others). This being said, one should be aware that the "second-order" re-interpretations of the slightly nontrivial results of [4] are also a valid avenue of research in the quest for detecting the absolute medium (or as the modern euphemism goes "the preferred frame"). In this connection, an important contribution seems to be [5]. Another source of higher-order effects can also be the local dependence of speed of light on the strength of the gravitational field. This kind of dependence is very important in any experiment conducted on Earth and in order to figure out the more subtle effects, one should use a theory in which the fundamental tensor of space affects the propagation of light. In the framework of the present approach it will result into a wave equation for the light which has non-constant coefficients, the latter depending on the curvature tensor. It goes beyond the scope of the present short note to delve into this more complicated case.

The aim of the present paper is to be understood in a very limited fashion: we show that the main effect of MME must be zero when it is considered in a purely Euclidean space without gravitational effects on the propagation of light. We pose correctly the problem of propagation and reflection of waves in a resting medium when both the source and the mirror are moving with respect to the medium. We show that the strict result from the interference is nil which invalidates most of the conclusions drawn from the perceived nil effect of MME.

6 Conditions on moving boundaries

Here we follow [6] (see also [7] for application to MME) where emitter's Doppler effect was explained with boundary conditions (b. c.) on a moving boundary. Consider the $(1+1)\,D$ linear wave equation

$$\phi_{tt} = c^2 \phi_{xx}\,, \tag{1}$$

whose solution is the harmonic wave.

$$\phi\,(x,t) = e^{i\hat{k}x \pm i\hat{\omega}t}, \quad \text{where} \quad \hat{k} = \frac{\hat{\omega}}{c}\,, \tag{2}$$

where c is the characteristic speed and "$\pm$" signs refer to the left- and right-going waves, respectively.

Consider now a boundary (a point in 1D) moving with velocity u, at which a wave with temporal frequency ω is created. This means that the wave propagating inside the medium satisfies the following boundary condition

$$\begin{aligned} \phi\,(ut,t) = e^{i(\omega_1 t - k_1 x)} = e^{i\omega_1 (t - x/c)} = \\ = e^{i\omega_1 t(1-u/c)} = e^{i\omega t}, \end{aligned} \tag{3}$$

where it is tacitly assumed that the right going wave is of interest. The above b. c. gives that

$$\omega_1 \left(1 - \frac{u}{c}\right) = \omega, \quad \rightarrow \quad \omega_1 = \frac{\omega}{1 - u/c}\,. \tag{4}$$

The last formula is the well known emitter's Doppler effect which shows how the frequency of the propagating wave is related to the frequency of the moving emitter

If the receiver is at rest, it will measure a frequency ω_1. The situation is completely different if the receiver is also moving, say with velocity v in the positive x-direction (to the right). Then due to the b. c. $\phi\,(vt,t) = e^{i\omega_1 t - i\frac{\omega_1}{c}vt} = e^{i\omega_2 t}$, the traveling wave of frequency ω_1 and wave number $k_1 = \frac{\omega_1}{c}$ will generate an oscillation of frequency ω_2 at the moving boundary point $x = vt$:

$$\omega_2 = \omega_1 \left(1 - \frac{v}{c}\right) = \omega\,\frac{1 - v/c}{1 - u/c}\,, \tag{5}$$

i. e., the measuring instruments in the moving frame of the receiver will detect a standing wave of frequency ω_2. We observe here that if the receiver is moving exactly with the speed of the emitter, then the frequency measured in receiver's frame will be exactly equal to emitter's frequency. In other words, a receiver that is moving with the same speed as the emitter does not observe a Doppler effect and cannot discover the motion.

This conclusion appears in an implicit form in the standard texts, e. g. [8, 9, p.164], where it is claimed that a Doppler effect is observed only for relative motion of the emitter and the receiver. Unfortunately, this correct observation did not lead to posing the question about the relevance of MME despite of the conspicuous lack of *relative motion* between the emitter and the receiver (mirror) in MME. The explanation in [8] was that "[F]or electromagnetic waves there evidently exists *no preferred frame*". We believe that the rigorous statement is that absolute rest (the "preferred frame") *cannot be detected* from measurements of Doppler effect between a source and a receiver which are moving together with identical speed through the absolute continuum.

After a consensus has been reached between the present work and the literature that the luminiferous continuum cannot be detected from an experiment in which a single source and a receiver are moving together as a non-deformable system, then the interesting question which remains is whether the absolute continuum can be detected when the emitter and the mirror are in relative motion, i. e. when they move with different speeds relative to the resting frame. To this end, consider now the situation when the receiver is a mirror which sends back a left going wave $e^{i\omega_3 t + ik_3 x}$ generated by the oscillations with frequency ω_2 at the point $x = vt$ namely, $e^{i\omega_3 t + ik_3 vt} = e^{i\omega_2 t}$. Then

$$\omega_3\,(1 + v/c) = \omega_2, \quad \Rightarrow \quad \omega_3 = \omega\,\frac{1 - v/c}{(1+v/c)(1-u/c)}\,. \tag{6}$$

Now, the wave of frequency ω_3 is traveling through the continuum to the left. The frequency, ω_4, of the wave excited on the *moving* surface of the emitter by this traveling wave has to satisfy the moving b. c. $e^{i\hat{\omega}_4} = e^{i(\hat{\omega}_3 t + \omega_3 \frac{u}{c} t)}$. Then

$$\omega_4 = \omega_3 \left(1 + \frac{u}{c}\right), \quad \Rightarrow \quad \omega_4 = \omega\,\frac{(1-v/c)(1+u/c)}{(1+v/c)(1-u/c)}\,. \tag{7}$$

The above result is illustrated in Fig. 1.

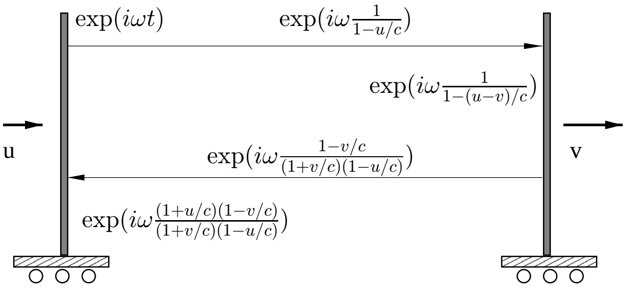

Fig. 1: Moving emitter and receiver

The case of waves propagating transversely to the emitter and receiver gives a trivial result in 1D, in the sense that the frequency and wave number of the propagating wave are not affected by the motion of the source or the receiver. The most general treatment for point source in 3D is given by the eikonal equation [6, p.225] for the inhomogeneous wave equation that is obtained in a frame moving with prescribed speed in certain direction.

An interesting limiting case is presented when $u, v \ll c$. Then the product uv/c^2 can be neglected in comparison with $(u-v)/c$ (provided that $u-v \simeq \mathrm{O}(u)$) and the above formula reduces to

$$\omega_4 = \frac{1+(u-v)/c}{1-(u-v)/c}\,,$$

which is the formula from [8, 9] for zero angle between the relative speed and the line of the emitter and observer. The discrepancies of order (uv/c^2) can be the cause of the so-called Pioneer anomaly [10]. It will be interesting to reexamine the raw data from Pioneer 10 eliminating the formula for relativistic Doppler effect and using in its place Eq. 7. Then what appears as an anomaly, can actually give the information about the absolute velocities of Earth and of the space ship. It is not necessary, of course, to go as far as Pioneer 10 and 11 went. The experiment can be done with an interferometer whose arms are the distances between two different satellites moving with different orbital speeds in the vicinity of Earth.

7 Michelson-Morley experiment (MME)

It was argued that because of the motion of the experimental equipment (the interferometer), the time taken by light to travel in the direction of motion will be different from the time needed to return, and these times together will differ from the time to travel in lateral direction. The argument that led to the prediction that the effect is of second order (see, [11, p.149], [1]) was typically corpuscular in its nature. The emission theory of light assumed that the "particles" of light were supposed to move in a resting continuum with velocity c. However when these particles were emitted by a moving surface in the direction of motion, they acquired speed $c+v$, whereas the particles emitted against the motion would move with speed $c-v$. The emission theory claimed that the total time for a ray to complete the full path in longitudinal direction is

$$t_1 = \frac{l}{c+v} + \frac{l}{c-v} = \frac{2l}{c(1-v^2/c^2)}\,, \tag{8}$$

where l is the length of the longitudinal and transverse arms of the interferometer. The arguments about the nature of reflections in the transverse arm of the interferometer are similarly based on the emission theory. In the transverse direction the length of the path traveled by one light corpuscle is calculated using the Pythagorean theorem and the total time needed for the light particle to complete the return trip to the lateral mirror is given by (see [1])

$$t_2 = \frac{2l}{c}\sqrt{1+\frac{v^2}{c^2}}\,. \tag{9}$$

Then the difference in the times needed to traverse the longitudinal and the transverse arms is

$$t_1 - t_2 \approx \frac{2l}{c}\left[1 + 2\frac{v^2}{c^2} - 1 - \frac{v^2}{c^2} + \mathrm{O}\left(\frac{v^4}{c^4}\right)\right] \approx l\,\frac{v^2}{c^2}\,. \tag{10}$$

Under the standard analogies of corpuscular approach, at this point the arguments usually go back to the wave theory of light assuming that the change in travel time of light particles somehow materializes as change of the emitted or received frequency.

Although the scientific community gradually elevated MME to the status of one of the *experimenta crucis* for the theory of relativity, the above argument was never critically revisited after the postulate of the constancy of speed of light was accepted. The only work known to the present author is [12] where the emission theory and wave theory of Doppler effect are compared and shown to coincide within the first order in v/c but no conclusions about the actual applicability of the above corpuscular-based formula are made.

The problem with applying a corpuscular approach to a wave phenomenon in a medium is that a propagation speed $c+v$ is impossible since all propagation speeds are limited by the characteristic speed of the medium. Yet, the above derivations were repeated in [11, 13] and now feature prominently in many of the most authoritative modern text-books, such as [9, 14]. So we are faced with a very peculiar situation: The formula used to explain the results of one of the most important for relativity theory experiments contradict the second postulate of the same theory.

The fallacy of the argumentation is as follows:

(i) The existence of a continuous medium in which the light propagates is stipulated (luminiferous continuum);

(ii) An irrelevant to continuum description theoretical formula is derived using the corpuscular concept of light (emission theory of light);

(iii) An experiment is designed for which it is believed that it can allow the measurement of the variable involved in the irrelevant theoretical formula;

(iv) Measurements obtained from the experiment do not show the expected effect;

(v) Conclusion is drawn that the contradiction is due to the fact that the original assumption of the presence of a continuum at rest is wrong;

(vi) The concept of existing of a material luminiferous continuum (i) is abandoned altogether.

This kind of fallacy is called *ignoratio elenchi* ("pure and simple irrelevance") and consists in using an argument that is supposed to prove one proposition but succeeds only in proving a different one. Clearly, there can be at least two causes for the nil result of the experiment. Before assuming that (i) is wrong, one has to examine (ii) from the point of view of the wave theory of light under the condition of constancy of speed of light. The only way to pass judgment on the presence or absence of an absolute continuum is to derive a formula for the interference effect that is based on the assumption that the space between the different parts of the equipment is filled with a continuous medium in which the propagation speed of linear waves is a given constant. In doing so, the reflection from the mirror has to be treated as an excitation of a wave on moving material surface. Then the frequency of the excited wave (which then travels back as the reflected wave) is subject to the motion of the mirror itself. In this short note we make an attempt to correctly pose the problem (using the adequate mathematical approach to solving the wave equation with b. c. on moving boundaries) and to show the consequences of this for the interpretation of interferometry experiments involving moving mirrors that are moving translatory with respect to the supposed absolute continuum. Only after the proper theoretical formula based on the idea that the continuum is at rest and that the equipment is moving relative to it, is derived and only after the predictions of this *relevant* formula are found to contradict the experimental evidence, one rule out the existence of an absolute continuum at rest in which the light waves are propagating as shear waves in a material medium.

It has been shown above that if the source of light and the mirror are moving together with the same velocity relative to the resting medium, then the Doppler effect is strictly equal to zero. This means that no Doppler effect can be detected from an experiment in which the emitter and the mirror are moving together through a quiescent continuum. This means that a nil effect from the celebrated experiment of Michelson and Morley should be interpreted as an evidence about the existence of a material continuum at rest and that this absolute continuum is not entrained by the moving bodies. The flawed arguments of the emission theory of light introduced an error of $O(v^2/c^2)$ in the formulas which was, in fact, the perceived effect in MME. At the same time, the correct solution (see the previous section) shows that the effect must be strictly nil provided that an absolute continuum fills the space between the different parts of the interferometer and that this continuum is not entrained.

8 A possible experimental set-up

If MME is irrelevant to detecting the absolute medium, then the question arises of is it possible at all to detect the latter by means of an interferometry experiment whose parts are moving together with the Earth. The answer (as already suggested in [7]) is in the positive if one can use two *independent* sources of light of virtually identical frequencies and avoid reflections. This means that one has to aim the beams against each other as shown in Fig. 2.

Assume now that two waves of identical frequencies are excited at two *different* points that are moving together in the same direction with the same velocity relative to the resting medium. The interference between the right-going wave from the left source and the left-going wave from the right source is given by

$$\begin{aligned} &e^{i\omega(t-x/c)/(1-u/c)} + e^{i\omega(t+x/c)/(1+u/c)} = \\ &= \left[\cos\left(\omega_1 t - k_1 x\right) + \cos\left(\omega_2 t + k_2 x\right)\right] + \\ &+ i\left[\sin\left(\omega_1 t - k_1 x\right) + \sin\left(\omega_2 t + k_2 x\right)\right] = \\ &= 2\cos\left(\tilde{\omega} t + \hat{k} x\right)\exp\left[i(\hat{\omega} t + \tilde{k} x)\right], \end{aligned} \tag{11}$$

where

$$\tilde{\omega} = \frac{\omega_1 + \omega_2}{2} = \omega\left(1 - \frac{u^2}{c^2}\right), \quad \hat{\omega} = \frac{\omega_2 - \omega_1}{2} = -\frac{u}{c}\,\tilde{\omega},$$

are the carrier and beat frequencies, and $\tilde{k} = \tilde{\omega}/c$, $\hat{k} = \hat{\omega}/c$. The wave excited at certain point, say $x = 0$, is

$$2\cos\left(\tilde{\omega} t\right)\exp\left(i\hat{\omega} t\right). \tag{12}$$

In Fig. 2 we show a possible experimental set-up which makes use of two independent sources of coherent light. Note that using two lasers, does not make our experiment similar to the set-up used in [15] because the latter involves mirrors and as it has been shown above, using mirrors dispels any possible effect.

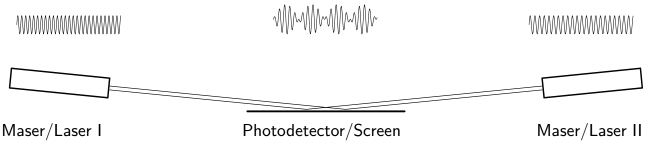

Fig. 2: Experimental set-up involving two lasers/masers

One of the ways to find the beat frequency is to use a photodetector in a point of the region of interference of the two waves. Note that the carrier frequency of the visible light is very high and cannot be detected in principle. The problem is that and even the beat frequency, Eq. 12, can be too high for the resolution of the available photodetectors. Apart from the fact that mirrors were used in [16], the high beat frequency could be another reason why it was not detected in those experiments. In fact they were after the beat frequency connected with the second-order effects and found practically no beat which is exactly what is to be expected in the light of the theory above presented. This is additional confirmation of the theory proposed here because we claim that no effect (neither first- nor second-order not higher-order) can exist if reflections are involved.

The other way to conduct the experiment is to measure the beat wave number $\hat{k}$ by taking a snapshot of the wave at certain moment of time. Then the spatial distribution of the wave amplitude is

$$2\cos\left(\hat{k} x\right)\,\exp\left(i\tilde{k} x\right), \tag{13}$$

which will produce an interference pattern in the resting continuum that can be observed on a screen (as shown alternatively in Fig. 2). Note that in this case the screen is "parallel" or "tangent" to the vibrating part of the absolute continuum, and what is observed, are the dark and light strips corresponding the the different values of the amplitude of the beat wave. Clearly, the effect will be best observed if the two lasers beams have identical polarization.

The requirements for the frequency stabilization of the sources of light stem from the magnitudes of the beat frequency. It is accepted nowadays that the speed of the so-called Local Standard of Rest (LSR) to which solar system belongs, is of order of $v \approx 300$ km/s relative to the center of the local cluster of galaxies [17]. The speed of LSR is an upper estimate of the speed with respect to the absolute medium. This maximum can be reached only if the center of cluster of galaxies is at rest relative to the medium. Thus, the upper limit for the dimensionless parameter $\varepsilon = v/c$ is 10^{-3}, which places very stringent requirements on the resolution in case that a photodetector is involved. For red-light lasers, the beat frequency is of order of 600 GHz which is well beyond the sensitivity of the available photodetectors. This means that one should opt for terahertz masers when the beat frequency ω_b will be smaller than 1–3 GHz.

In the alternative implementation of the experimental set-up a detecting screen is used to get the spatial distribution of the interference pattern. In such a case, one can use standard visible-light lasers. For instance, the red light has wavelength approximately in the range of $600\,\mathrm{m}^{-9}$, then the beat wave length is expected to be $\varepsilon^{-1} \approx 1000$ times longer. This means 0.6 mm which is technically feasible to observe on a screen. Conversely, using terahertz masers in this case could make the wave length of the beat wave of order of 20–50 cm.

Now, in order to have reliable results from the proposed interferometry experiment, one needs frequency stabilization a couple of orders of magnitude better than the sought effect. To be on the safe side, we mention that the lowest value for ε is 10^{-4} which corresponds to the orbital speed of Earth. Then the best stabilization of the frequency needed is 10^{-7}. This is well within the stabilization limits for the currently available low-power lasers. For example, Coherent, Inc. offers the series 899-21 that are Actively Stabilized, Scanning Single-Frequency Ring Lasers with stabilization 10^{-9}.

9 Conclusion

The theory of Michelson-Morley interference experiment is revisited from the point of view of the wave theory of light. The fallacy of using the accepted formula based on the emission theory of light is shown and new formulas are derived based on the correct posing of the boundary conditions at moving boundaries for a hyperbolic equation. It is shown that when the source of light and the reflector are moving with the same speed through a non-entrained absolute continuum, the reflected wave as received back at the emitter's place shows no Doppler shift, and hence no fringes can be expected. The situation is different if the emitter and the reflector are in relative motion with respect to each other. The meaning of the results of the present work is that the only correct conclusion from a nil effect from interferometry experiment involving reflection is not that absolute medium does not exist, but that an absolute continuum exist which is not entrained by the motion of the measuring instrument (the system of emitters and mirrors). Naturally, the nil effect of Michelson-Morley experiment should not be used as the sole verification of the absolute medium and to this end a new experimental set-up is proposed.

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