Frequency Resolved Detection over a Large Frequency Range of the Fluctuations in an Array of Quantum Dots

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Quantum noise effects in an array of quantum dots coupled to superconducting leads are studied. The effect of broadband fluctuations on the inelastic rate in such tunable system has been taken into account. The quantum shot noise spectrum is expressed in terms of the time dependent fluctuations of the current around its average value. Numerical calculation has been performed over a wide range of frequencies of the induced photons. Our results show an asymmetry between absorption and emission processes. This research is very important for optoelectronic nanodevices.

1 Introduction

Semiconductor nanostructures based on two dimensional electron gas (2 DEG) could form the basis of future nanodevices for sensing, information processing and quantum computation. Coherent electron transport through mesoscopic system in the presence of a time-varying potential has been a subject of increasing interest in the past recent years [1]. Applying the microwave field with frequency, ω, to an electron with energy, E, the electron wavefunction possesses sideband components with energies $E + n\hbar\omega$ ($n = 0, \pm 1, \pm 2, \dots$). The coherence of sideband components characterizes transport properties of electrons such as photon-assisted tunneling (PAT) [2–6]. Shot noise measurements provide a powerful tool to study electron transport in mesoscopic systems [7]. Shot noise can be enhanced in devices with superconducting leads by virtue of the Andreev-reflection process taking place at the interface between a semiconductor and superconductor [8–10]. A remarkable feature of the current noise in the presence of timedependent potentials is its dependence on the phase of the transmission amplitudes [11]. Moreover, for high driving frequencies, the driving can be treated within a self-consistent perturbation theory [12]. In the present paper, a shot noise spectrum of a mesoscopic device is derived and analyzed over a wide range of frequencies of the induced microwave field.

2 Model of calculations

The present studied mesoscopic device is formed of an array of semiconductor quantum dots coupled weakly to two superconducting leads via tunnel barriers. Electrical shot noise is the time-dependent fluctuation of the current around its average value, due to the discreteness of the charge carriers. The nonsymmetrized shot noise spectrum is given by [13]:

$$P(\omega) = 2 \int_{-\infty}^{\infty} dt\, e^{i\omega t} \left\langle \Delta\hat{I}(t)\, \Delta\hat{I}(0) \right\rangle , \tag{1}$$

where $\Delta\hat{I}(t)$ is the time-dependent fluctuations of the current around its average value [14]. The average current operator is given by [15]:

$$\left\langle \hat{I}(t) \right\rangle = \frac{e}{h} \sum_{\alpha,\beta} \int_0^{\infty} d\varepsilon \int_0^{\infty} d\varepsilon' I_{\alpha,\beta}(\varepsilon,\varepsilon') \times \\ \times\, \hat{a}^{+}_{\alpha}(\varepsilon)\, \hat{a}_{\beta}(\varepsilon')\, e^{i(\varepsilon-\varepsilon')t/\hbar}, \tag{2}$$

where $\hat{a}^{+}_{\alpha}(\varepsilon)$ and $\hat{a}_{\alpha}(\varepsilon)$ are the creation and annihilation operators of the scattering states $\psi_\alpha(\varepsilon)$ respectively. $I_{\alpha\beta}(\varepsilon,\varepsilon')$ is the matrix element of the current operator between states $\psi_\alpha(\varepsilon)$ and $\psi_\beta(\varepsilon')$. The indices α and β (Eq. 2) denote mode number (m) as well as whether it concerns electron $\alpha = (m, e)$ or hole $\alpha = (m, h)$ propagation, due to Andreev reflection processes at semiconductor-superconductor interface [16]. The scattering states $\psi_\alpha(\varepsilon)$ and $\psi_\beta(\varepsilon')$ are determined by solving the Bogoliubov-deGennes equation (BdG) [17, 18] and are given by:

$$\Psi_{\alpha j}(x,\varepsilon) = \left[A_j \exp(ik_j x) \binom{1}{0} + B_j \exp(-ik_j x) \binom{0}{1} \right] \times \\ \times \sum_{n=0}^{\infty} J_n\left(\frac{eV_0}{\hbar\omega}\right) \exp\left[-i\left(\varepsilon + n\hbar\omega\right)t/\hbar\right] \tag{3}$$

where ω is the frequency of the induced microwave field, J_n is the n-th order Bessel function of first kind and V_0 is the amplitude of the ac-voltage. Eq. (3) represents the eigenfunction inside the quantum dot in the j-th region and the corresponding eigenfunction inside the superconducting leads is given by:

$$\Psi_{\alpha}(x,\varepsilon) = \left[C \exp(ik'x) \binom{u}{v} + D \exp(-ik'x) \binom{v}{u} \right] \times \\ \times \sum_{n=-\infty}^{\infty} J_n\left(\frac{eV_0}{\hbar\omega}\right) \exp\left[-i\left(\varepsilon + n\hbar\omega\right)t/\hbar\right]. \tag{4}$$

The wave vectors k_j and k' are the wave vectors inside j-th quantum dot and inside the superconducting leads and they are given by:

$$k_j = \frac{\left(2m^*(V_{\text{eff}} \pm \varepsilon + n\hbar\omega)\right)^{0.5}}{\hbar} \tag{5}$$

and

$$k' = \frac{\left(2m^*(E_F - V_b \pm \sqrt{(\varepsilon + n\hbar\omega)^2 - \Delta^2}\right)^{0.5}}{\hbar} \tag{6}$$

where V_{eff} is expressed as:

$$V_{\text{eff}} = V_b + \frac{U_c N^2}{2} + E_F + e\eta V_g \tag{7}$$

in which V_b is the Schottky barrier height, U_c is the charging energy of the quantum dot, E_F is the Fermi-energy, Δ is the energy gap of superconductor, V_g is the gate voltage and η is the lever arm. The eigenfunctions u and v (Eq. 4) of the corresponding electron/hole due to Andreev reflection process at the semiconductor-superconductor interface are given by:

$$u = \sqrt{\frac{1}{2}\left(1 + \frac{\left((\varepsilon + n\hbar\omega)^2 - \Delta^2\right)^{0.5}}{\varepsilon + n\hbar\omega}\right)} \tag{8}$$

and

$$v = \sqrt{\frac{1}{2}\left(1 - \frac{\left((\varepsilon + n\hbar\omega)^2 - \Delta^2\right)^{2}}{\varepsilon + n\hbar\omega}\right)}\,. \tag{9}$$

Now, in order to evaluate the shot noise spectrum, this can be achieved by substituting the current operator Eq. 2 into Eq. 1 and determining the expectation value [19] and after simple algebraic steps, we get a formula for the shot noise spectrum $P(\omega)$ [20] as:

$$\begin{aligned} P(\omega) = \frac{2eP_0}{h} \sum_{\alpha,\beta} \int_0^\infty d\varepsilon \, |\Gamma(\varepsilon)| \, f_{\alpha FD}(\varepsilon) \times \\ \times \left[1 - f_{\beta FD}(\varepsilon + n\hbar\omega)\right], \end{aligned} \tag{10}$$

where P_0 is the Poissonian shot noise spectrum and $f_{\beta FD}(\varepsilon + n\nabla\omega)$ are the Fermi distribution functions.

The tunneling rate, $\gamma(\varepsilon)$ through the barrier must be modified due to the influence of the induced microwave field as [21]:

$$\tilde{\gamma}(\varepsilon) = \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{eV_0}{\hbar\omega}\right)\gamma(\varepsilon + n\hbar\omega)\,. \tag{11}$$

The tunneling rate, $\gamma(\varepsilon)$ is related to the tunneling probability, $\Gamma(\varepsilon)$ [21] as:

$$\begin{aligned} \gamma(\varepsilon) = \frac{2\pi}{\hbar} \int_{E_F}^{E_F + 2\Delta\hbar\omega} d\varepsilon \, \Gamma(\varepsilon)\, \rho(\varepsilon)\, f_{FD}(\varepsilon) \times \\ \times \left(1 - f_{FD}(\varepsilon - \Delta F)\right) \end{aligned} \tag{12}$$

in which ΔF is the difference in final and initial free energy after and before the influence of microwave field. The tunneling probability, $\Gamma(\varepsilon)$, Eq. 10 has been determined by the authors [22, 23] using the transfer matrix method and it is expressed as:

$$\Gamma(\varepsilon + n\hbar\omega) = \frac{1}{\left(1 + C_1^2 C_2^2\right)} \tag{13}$$

where C_1 and C_2 are expressed as:

$$C_1 = \frac{V_{\text{eff}} \sinh(kb)}{2\sqrt{L_1}} \tag{14}$$

and

$$C_2 = 2\cosh(kb)\cos(k'a) - C_3\,. \tag{15}$$

We have used the following notations:

$$\begin{aligned} L_1 &= (\varepsilon + n\hbar\omega)\,(V_{\text{eff}} - \varepsilon - n\hbar\omega)\,, \\ C_3 &= \left(\frac{L_2}{\sqrt{L_1}}\right)\sin(k'a)\exp(2kb)\,, \\ L_2 &= 2(\varepsilon + n\hbar\omega) - V_{\text{eff}}\,. \end{aligned} \tag{16}$$

The parameters a and b represent the quantum dot size and the width of the barrier.

Now substituting Eq. 13 into Eq. 10 an expression for the frequency dependent shot noise spectrum and it depends on the geometrical dimension of the device under study.

3 Results and discussion

The shot noise spectrum, $P(\omega)$, Eq. 10 has been computed over a wide range of frequencies of the induced microwave field and at different temperatures. We considered a double quantum dots which they are a fully controllable two-level system. These quantum dots are AlGaAs-GaAs heterostructure and the leads are Nb superconductor. The calculations were performed for the cases: absorption of quanta from the environment (Fig. 1) and emission case (Fig. 2). The Schottky barrier height, V_b, was calculated by using a Monte-Carlo technique [24] and found to be equal to 0.47 eV. This value is in good agreement with those found by the authors [25]. As shown in Fig. 1 and Fig. 2, the normalized shot noise spectrum exhibits resonances at certain frequencies for both absorption and emission processes. The present results show that the Coulomb oscillations are modified by frequency of the induced microwave field over a wide range. Also, the Andreev reflection processes at the semiconductor-superconductor interface plays very important role for the appearance of these resonances. Our results show that the interplay between electronic transport and excitation by microwave is a particular interest. As high frequency perturbations are expected to yield a new nonequilibrium situation resulted from additional phase variations in energy states [26, 27, 28].

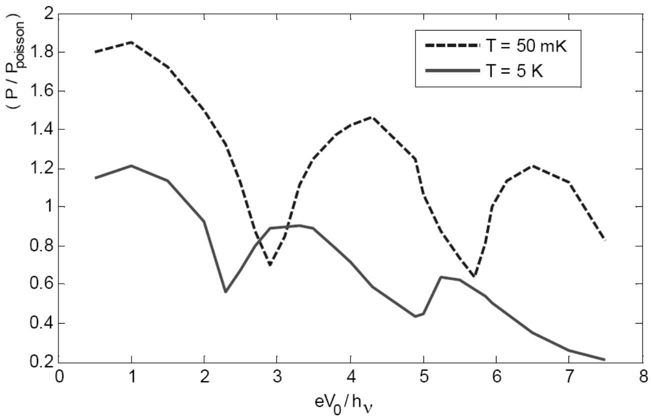

Fig. 1: The dependence of the normalized shot noise spectrum (P/P_{poisson}) on the normalized strength of the driving field (absorption case).

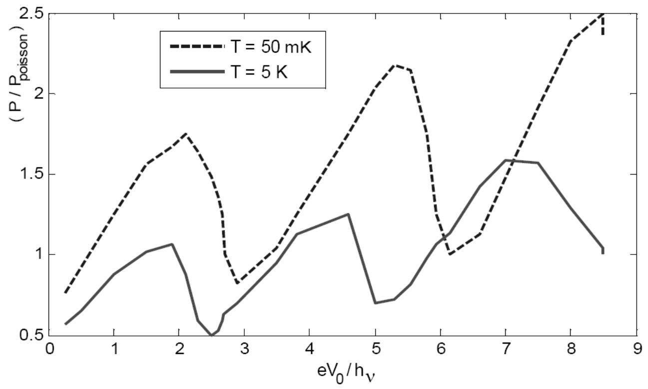

Fig. 2: The dependence of the normalized shot noise spectrum (P/P_{poisson}) on the normalized strength of the driving field (emission case).

4 Conclusions

In present paper, an expression for the shot noise spectrum has been deduced. The present studied mesoscopic device is modeled as double quantum dots coupled weakly to superconducting leads. The tunneling through the device is induced by microwave field of wide range of frequencies. The effect of both Andreev reflection processes and the Coulomb blockade had been taken into consideration. The resonances show the interplay between the forementioned effects and the photon induced microwave field. Our results show a concordant with those in the literature.

References

1. Toyoda E., Takayanagi H. and Nakano H. *J. Phys. Soc. Jpn*, 2000, v. 69, 1801.
2. Dayem A. H. and Martin R. J. *Phys. Rev. Lett.*, 1962, v. 8, 246.
3. Tien P. K. and Gordon J. P. *Phys. Rev.*, 1963, v. 129, 647.
4. Kouwenhoven L. P., Jauhar S., Orenstein J., McEuen P. L., Nagamune Y., Motohisa J., and Sakaki H. *Phys. Rev. Lett.*, 1994, v. 73, 3443.
5. Wiel V., Fujisawa W. G., Osterkamp T. H., and Kouwenhoven L. P. *Physica B*, 1999, v. 272, 31.
6. Keay B. J., Zeuner S., and Allen S. J. *Phys. Rev. Lett.*, 1995, v. 75, 4102.
7. Blanter Ya. M. and Buttiker M. *Phys. Rep.*, 2000, v. 336, 1.
8. Nagaev K. E. and Buttiker M. *Phys. Rev.*, 2001, v. B63, 081301(R).
9. Hoffmann C., Lefloch F., and Sanquer M. *Eur. Phys. J.*, 2002, v. B29, 629.
10. Phillips A. H. *Second Spring School on Current Activities of Materials Science*, Assiut University, Assiut, Egypt, 2000.
11. Camalet S., Lehmann J., Kohler S., and Hanggi P. *Phys. Rev. Lett.*, 2003, v. 90, 210602.
12. Brandes J. *Phys. Rev.*, 1997, v. B56, 1213.
13. DeJong M. J. M. and Beenakker C. W. Shot noise in mesoscopic systems. arXiv: cond-mat/9611140.
14. Aguado R. and Kouwenhoven L. P. *Phys. Rev. Lett.*, 2000, v. 84, 1986.
15. DeJong M. J. M. and Beenakker C. W. J. In: *Proceedings of Advanced Study Institute on Mesoscopic Electron Transport*, edited by Sohn L. L., Kouwenhoven L. P. and Schon G., Kluwer, Dordrecht, 1997.
16. Andreev A. F. *Sov. Phys. JETP*, 1964, v. 19, 1228.
17. Furusaki A., Takayanagi H., and Tsukada M. T. *Phys. Rev.*, 1992, v. B45, 10563.
18. Aly A. H., Phillips A. H., and Kamel R. *Egypt. J. Phys.*, 1999, v. 30, 32.
19. Beenakker C. W. J. and Buttiker M. *Phys. Rev.*, 1992, v. B46, R1889.
20. Blanter Ya. M. and Sukhorukov E. V. *Phys. Rev. Lett.*, 2000, v. 84, 1280.
21. Platero G. and Aguado R. Photon-assisted transport in semiconductor nanostructures. arXiv: cond-mat/0311001.
22. Mina A. N. *Egypt. J. Phys.*, 2001, v. 32, 253.
23. Atallah A. S. and Phillips A. H. *Proceedings of The International Conference on Materials Science and Technology*, Faculty of Science, Cairo University, Beni-Suef, Egypt, April 2–4, 2001.
24. Aly A. H. and Phillips A. H. *Phys. Stat. Sol. (B)*, 2002, v. 232, 238.
25. Becker Th., Muck M., and Heiden Ch. *Physica B*, 1995, v. 204, 14286.
26. Fujisawa T. and Hirayama Y. *Appl. Phys. Lett.*, 2000, v. 77, 543.
27. Camalet S., Kohler S., and Hanggi P. arXiv: cond-mat/0402182.
28. Phillips A. H., Mina A. N., Sobhy M. S., and Fouad E. A. Accepted for publ. in *J. Comput. & Theor. Nanoscience*, 2006.