Developing de Broglie Wave

J. X. Zheng-Johansson* and Per-Ivar Johansson†

*Institute of Fundamental Physics Research, 611 93 Nyköping, Sweden
E-mail: jxzj@iofpr.org

†Uppsala University, Studsvik, 611 82 Nyköping, Sweden
E-mail: Per-Ivar.Johansson@studsvik.uu.se

The electromagnetic component waves, comprising together with their generating oscillatory massless charge a material particle, will be Doppler shifted when the charge hence particle is in motion, with a velocity v, as a mere mechanical consequence of the source motion. We illustrate here that two such component waves generated in opposite directions and propagating at speed c between walls in a one-dimensional box, superpose into a traveling beat wave of wavelength $\Lambda_d = \frac{v}{c}\,\Lambda$ and phase velocity $c^2/v + v$ which resembles directly L. de Broglie's hypothetic phase wave. This phase wave in terms of transmitting the particle mass at the speed v and angular frequency $\Omega_d = 2\pi v/\Lambda_d$, with Λ_d and Ω_d obeying the de Broglie relations, represents a de Broglie wave. The standing-wave function of the de Broglie (phase) wave and its variables for particle dynamics in small geometries are equivalent to the eigen-state solutions to Schrödinger equation of an identical system.

1 Introduction

As it stood at the turn of the 20th century, M. Planck's quantum theory suggested that energy (ε) is associated with a certain periodic process of frequency (ν), $\varepsilon = h\nu$; and A. Einstein's mass-energy relation suggested the total energy of a particle (ε) is connected to its mass (m), $\varepsilon = mc^2$. Planck and Einstein together implied that mass was associated with a periodic process $mc^2 = h\nu$, and accordingly a larger ν with a moving mass. Incited by such a connection but also a clash with this from Einstein's relativity theory which suggested a moving mass is associated with a slowing-down clock and thus a smaller ν, L. de Broglie put forward in 1923 [1] a hypothesis that a matter particle (moving at velocity v) consists of an internal periodic process describable as a packet of phase waves of frequencies dispersed about ν, having a phase velocity $W = \frac{\nu}{k} = c^2/v$, with c the speed of light, and a group velocity of the phase-wave packet equal to v. Despite the hypothetic phase wave appeared supernatural and is today not held a standard physics notion, the de Broglie wave has proven in modern physics to depict accurately the matter particles, and the de Broglie relations proven their fundamental relations.

So inevitably the puzzles with the de Broglie wave persist, involving the hypothetic phase waves or not, and are unanswered prior to our recent unification work [2]: What is waving with a de Broglie wave and more generally Schrödinger's wave function? If de Broglie's phase wave is indeed a reality, what is then transmitted at a speed (W) being $\frac{c}{v}$ times the speed of light c? How is the de Broglie (phase) wave related to the particle's charge, which if accelerated generates according to Maxwell's theory electromagnetic (EM) waves of speed c, and how is it in turn related to the EM waves, which are commonplace emitted or absorbed by a particle which changes its internal state? In [2] we showed that a physical model able to yield all of the essential properties of a de Broglie particle, in terms of solutions in a unified framework of the three basic mechanics, is provided by a single harmonic oscillating, massless charge $+e$ or $-e$ (termed a *vaculeon*) and the resulting electromagnetic waves. The solutions for a basic material particle generally in motion, with the charge quantity (accompanied with a spin) and energy of the charge as the sole inputs, predict accurately the inertial mass, total wave function, total energy equal to the mass times c^2, total momentum, kinetic energy and linear momentum of the particle, and that the particle is a de Broglie wave, it obeys Newton's laws of motion, de Broglie relations, Schrödinger equation in small geometries, Newton's law of gravitation, and Galilean-Lorentz-Einstein transformation at high velocities. In this paper we give a self-contained illustration of the process by which the electromagnetic component waves of such a particle in motion superpose into a de Broglie (phase) wave.

2 Particle; component waves; dynamic variables

A free massless vaculeon charge (q) endowed with a kinetic energy $\mathcal{E}_q$ at its creation, being not dissipatable except in a pair annihilation, will tend to move about in the vacuum, and yet at larger displacement restored, fully if $\mathcal{E}_q$ below a threshold, toward equilibrium by the potential field of the surrounding dielectric vacuum being here polarized under the charge's own field [2]. As a result the charge oscillates in the vacuum, at a frequency Ω_q; once in addition unidirectionally driven, it will also be traveling at a velocity v here in a one-dimensional box of length L along X-axis firstly in $+X$-direction. Let axis X' be attached to the moving charge, $X' = X - vT$; let v be low so that $(v/c)^2 \to 0$, with c the velocity of light; accordingly $T' = T$. The charge will according to Maxwell's theory generate electromagnetic waves to both $+X$ and $-X$-directions, being by the standard solution a plane wave, given in dimensionless displacements (of the medium or fields in it):

$$\varphi^\dagger(X', T) = C_1 \sin\left[K^\dagger X' - \Omega^\dagger T + \alpha_0\right], \quad \text{(a)}$$
$$\varphi^\ddagger(X', T) = -C_1 \sin\left[K^\ddagger X' + \Omega^\ddagger T - \alpha_0\right], \quad \text{(b)} \qquad (1)$$

where $\begin{bmatrix} K^\dagger \\ K^\ddagger \end{bmatrix} = \lim_{(v/c)^2 \to 0} \begin{bmatrix} k^\dagger \\ k^\ddagger \end{bmatrix} = K \pm K_d$, $\begin{bmatrix} k^\dagger \\ k^\ddagger \end{bmatrix} = \frac{K}{1 \mp v/c}$ being wavevectors Doppler-shifted due to the source motion from their zero-v value, K; $\Lambda = 2\pi/K$, and $\Omega = cK$; $\Omega = \Omega_q$ for the classical electromagnetic radiation. On defining $k_d = \sqrt{(k^\dagger - K)(K - k^\ddagger)} = \left(\frac{v}{c}\right) k$, with $k = \gamma K$, $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$, we have at classic-velocity limit:

$$K_d = \lim_{(v/c)^2 \to 0} k_d = \left(\frac{v}{c}\right) K; \qquad (2)$$

$\begin{bmatrix} \Omega^\dagger \\ \Omega^\ddagger \end{bmatrix} = \begin{bmatrix} K^\dagger c \\ K^\ddagger c \end{bmatrix} = \Omega \pm K_d c$, and α_0 is the initial phase. Assuming $\mathcal{E}_q$ is large and radiated in $J\,(\gg 1)$ wave periods if without re-fuel, the wavetrain of φ^j of a length $L_\varphi = JL$ will wind about the box L in $J \gg 1$ loops.

The electromagnetic wave φ^j of an angular frequency $\omega^j = k^j c$, $j = \dagger$ or $\ddagger$, has according to M. Planck a wave energy $\varepsilon^j = \hbar\omega^j$, with $2\pi\hbar$ the Planck constant. The waves are here the components of a particle; the geometric mean of their wave energies, $\sqrt{\varepsilon^\dagger \varepsilon^\ddagger} = \hbar\sqrt{\omega^\dagger \omega^\ddagger} = \gamma\hbar\Omega$ gives thereby the total energy of the particle. $\varepsilon_v = \gamma\hbar\Omega - \hbar\Omega = \frac{1}{2}\hbar\Omega_d\big[1 + \frac{3}{4}\left(\frac{v}{c}\right)^2 + \ldots\big]$ gives further the particle's kinetic energy and in a similar fashion its linear momentum p_v (see [2]), and

$$\mathcal{E}_v = \lim_{(v/c)^2 \to 0} \varepsilon_v = \tfrac{1}{2}\hbar\left(\tfrac{v}{c}\right)^2 \Omega, \qquad (3)$$
$$P_v = \lim_{(v/c)^2 \to 0} p_v = \sqrt{2m_0 \mathcal{E}_v} = \hbar\left(\tfrac{v}{c}\right) K. \qquad (4)$$

The above continues to indeed imply as L. de Broglie noted that a moving mass has a larger $\gamma\Omega/2\pi$ $(=\nu)$, and thus a clash with the time-dilation of Einstein'smoving clock. This conflict however vanishes when the underlying physics becomes clear-cut [2, 2006c].

3 Propagating total wave of particle

A tagged wave front of say $\varphi^\dagger(X', T)$ generated by the vaculeon charge, of $v > 0$, to its right at location X' at time T, will after a round-trip of distance $2L$ in time $\delta T = 2L/c$ return from left and propagate again to the right to X' at time $T^* = T + \delta T$. Here it gains a total extra phase $\alpha' = K2L + 2\pi$ due to $2L$ (with $\frac{(K^\dagger + K^\ddagger)}{2} = K$) and the twice reflections at the massive walls, and becomes

$$\varphi_r^\dagger(X', T^*) = C_1 \sin\left[K^\dagger X' - \Omega^\dagger T + \alpha_0 + \alpha'\right]. \qquad (1a)'$$

$\varphi_r^\dagger$ meets $\varphi^\dagger(X', T^*)$ just generated to the right, an identical wave except for an α', and superposes with it to a maximum if assuming $K2L = N2\pi$, $N = 0, 1, \ldots$, returning the same $\varphi^\dagger$ (assuming normalized). Meanwhile, $\varphi_r^\dagger(X', T^*)$ meets $\varphi^\ddagger(X'_1, T^*)$ just generated to the left (Fig. 1a) and superposes with it as $\widetilde{\psi} = \varphi_r^\dagger + \varphi^\ddagger$. Using the trigonometric identity (TI), denoting $\widetilde{\psi}(X', T) = \widetilde{\psi}(X', T^*)$, this is $\widetilde{\psi}(X', T) = 2C_1 \cos(KX' - K_d cT)\sin(K_d X' - \Omega T + \alpha_0)$. With $X' = X - vT$, we have on the X-axis:

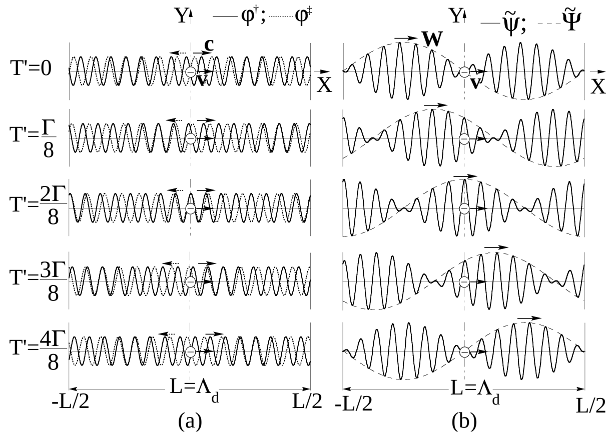

Fig. 1: (a) The time development of electromagnetic waves with wave speed c and wavelength Λ, $\varphi^\dagger$ generated to the right of (1a)$'$ and $\varphi^\ddagger$ to the left of (1b) by a charge ($\ominus$) traveling at velocity v in $+X$ direction in a one-dimensional box of side L. (b) $\varphi^\dagger$ and $\varphi^\ddagger$ superpose to a beat, or de Broglie phase wave $\widetilde{\psi}$ of (5) traveling at phase velocity $W \simeq \frac{c^2}{v}$, of wavelength Λ_d. For the plot: $\Lambda = 0.067\Lambda_d$, and $\alpha_0 = -\frac{\pi}{2}$; $T' = T - \frac{\Gamma}{4}$; $v = (\frac{\Lambda}{\Lambda_d})c \ll c$.

$$\widetilde{\psi}(X, T) = \widetilde{\Phi}(X, T)\,\widetilde{\Psi}(X, T), \qquad (5)$$
$$\widetilde{\Phi}(X, T) = 2C_1 \cos(KX - 2K_d cT), \qquad (6)$$
$$\widetilde{\Psi}(X, T) = \sin\left[K_d X - (\Omega + \Omega_d)T + \alpha_0\right], \qquad (7)$$

where $Kv = K_d c$, and

$$\Omega_d = K_d v = \left(\frac{v}{c}\right)^2 \Omega. \qquad (8)$$

$\widetilde{\psi}$ expressed by (5) is a *traveling beat wave*, as plotted versus X in Fig. 1b for consecutive time points during $\Gamma/2$, or Fig. 2a during $\Gamma_d/2$. $\widetilde{\psi}$ is due to all the component waves of the particle while its charge is moving in one direction, and thus represents the (propagating) total wave of the particle, to be identified as a *de Broglie phase wave* below.

$\widetilde{\psi}$ has one product component $\widetilde{\Phi}$ oscillating rapidly on the X-axis with the wavelength $\Lambda = 2\pi/K$, and propagating at the speed of light c at which the total wave energy is transported. The other, $\widetilde{\Psi}$, envelops about $\widetilde{\Phi}$, modulating it into a slow varying beat $\widetilde{\psi}$ which has a wavevector, wavelength and angular frequency given by:

$$K_b = K_d, \quad \Lambda_b = \frac{2\pi}{K_b} = \frac{2\pi}{K_d} = \Lambda_d, \quad \Omega_b = \Omega + \Omega_d; \qquad (9)$$

$$\text{where} \quad \Lambda_d = \left(\frac{c}{v}\right) \Lambda. \qquad (10)$$

As follows (9), the beat $\widetilde{\psi}$ travels at the *phase velocity*

$$W = \frac{\Omega_{\mathrm{b}}}{K_{\mathrm{b}}} = \frac{\Omega}{K_d} + v = \left(\frac{c}{v}\right) c + v\,. \tag{11}$$

4 De Broglie wave

Transmitted along with its beat wave, of a wavelength Λ_d, with $K_d = 2\pi/\Lambda_d$, is the mass of the particle at the velocity v. The beat wave conjoined with its transportation of the particle's mass represents thereby a periodic process of the particle, of a wavelength and wavevector equal to Λ_d and K_d of the beat wave. K_d and v define for the particle dynamics an angular frequency, $K_d v = \Omega_d$, as expressed by (8). Combining (10) and (4), and (8) and (3) respectively yield just the *de Broglie relations*:

$$P_v = \hbar K_d\,; \tag{12}$$

$$\mathcal{E}_v = \frac{1}{2}\,\hbar\Omega_d\,. \tag{13}$$

Accordingly K_d, Λ_d, and Ω_d represent the de Broglie wave-vector, wavelength and angular frequency. The beat wave $\widetilde{\psi}$ of a phase velocity W resembles thereby the *de Broglie phase wave* and it in the context of transmitting the particle mass represents the *de Broglie wave* of the particle.

5 Virtual source. Reflected total particle wave

At an earlier time $T_1 = T - \Delta T$, at a distance L advancing its present location X, with $\Delta T = L/v$, the actual charge was traveling to the left, let axis $X''(=X + vT)$ be fixed to it. This past-time charge, said being virtual, generated at location X'' at time T_1 similarly one component wave $\varphi^{\dagger\,\mathrm{vir}}(X'', T_1^*)$ to the right, which after traversing $2L$ returned from left to X'' at time $T_1^* = T_1 + \delta T$ as $\varphi^{\dagger\,\mathrm{vir}}_r(X'', T_1^*) = C_1 \sin(K^\dagger_{-v}X'' - \Omega^\dagger_{-v}T_1^* + \alpha_0 + \alpha')$, where $K^\dagger_{-v} = K - K_d$, $K^\ddagger_{-v} = K + K_d$, and $\Omega^j_{-v} = K^j_{-v}c$ are the Doppler shifted wavevectors and angular frequencies; $\alpha' = (2N+1)\,\pi$ as earlier. Here at X'' and T_1^*, $\varphi^{\dagger\,\mathrm{vir}}_r$ meets the wave the virtual charge just generated to the left, $\varphi^{\ddagger\,\mathrm{vir}}(X'', T_1^*) = -C_1 \times \sin(K^\ddagger_{-v}X'' + \Omega^\dagger_{-v}T_1^* - \alpha_0)$, and superpose with it to $\widetilde{\psi}^{\mathrm{vir}}(X, T_1^*) = \varphi^{\dagger\,\mathrm{vir}}_r + \varphi^{\ddagger\,\mathrm{vir}} = 2C_1 \cos(KX'' + K_d c T_1) \times \sin\left[-K_d X'' - 2\Omega T_1 - \alpha_0\right]$.

With $J \gg 1$ and being nondamping, $\widetilde{\psi}^{\mathrm{vir}}$ will be looping continuously, up to the present time T. Its present form $\widetilde{\psi}^{\mathrm{vir}}(X'', T)$ is then as if just produced by the virtual charge at time T but at a location of a distance L advancing the actual charge; it accordingly has a phase advance $\beta = \frac{(K^\dagger - K^\dagger_{-v})}{2} L = K_d L$ relative to $\widetilde{\psi}$ (the phase advance in time yields no never feature). Including this β, using TI and with some algebra, $\widetilde{\psi}^{\mathrm{vir}}(X'', T)$ writes on axis X as

$$\widetilde{\psi}^{\mathrm{vir}}(X,T) = \widetilde{\Phi}^{\mathrm{vir}}(X,T)\,\widetilde{\Psi}^{\mathrm{vir}}(X,T)\,, \tag{14}$$

$$\widetilde{\Phi}^{\mathrm{vir}}(X,T) = 2C_1 \cos\left[(KX + 2K_d cT\right]\,, \tag{15}$$

$$\widetilde{\Psi}^{\mathrm{vir}}(X,T) = -\sin\left[K_d X + (\Omega + \Omega_d)T + \alpha_0 + \beta\right]. \tag{16}$$

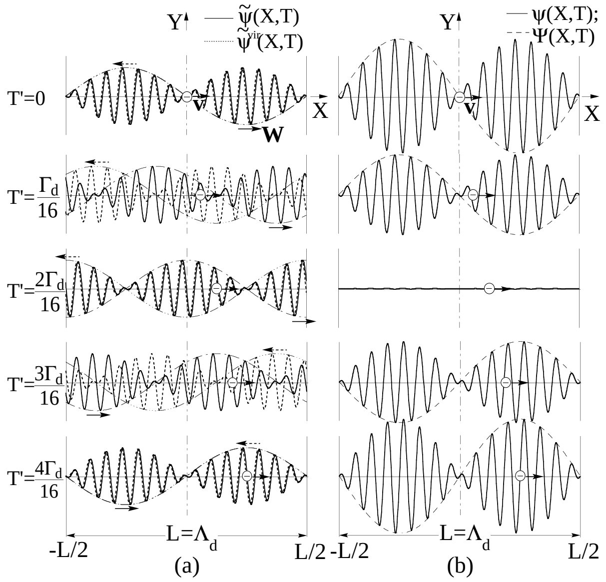

Fig. 2: (a) The beat waves $\widetilde{\psi}$ traveling at a phase velocity W to the right as in Fig. 1b and $\widetilde{\psi}^{\mathrm{vir}}$ at $-W$ to the left, of a wavelength Λ_d, due to the right- and left- traveling actual and virtual sources respectively. (b) $\widetilde{\psi}$ and $\widetilde{\psi}^{\mathrm{vir}}$ superpose to a standing beat or de Broglie phase wave ψ of wavelength Λ_d, angular frequency $\sim \Omega$. Along with the ψ process, the particle's center of mass ($\ominus$) is transported at the velocity v, of a period $\frac{2\pi}{\Omega_d} = \Lambda_d/v$.

$\widetilde{\psi}^{\mathrm{vir}}$ of the virtual or reflected charge is seen to be similarly a traveling beat or de Broglie phase wave to the left of a phase velocity $-W$ and wave parameters K_{b}, Λ_{b} and Ω_{b} as of (9).

6 Standing total wave and de Broglie wave

Now if $K_d L (= \beta) = n\pi$, i. e.

$$K_{dn} = \frac{n\pi}{L}\,, \quad n = 1, 2, \ldots, \tag{17}$$

and accordingly $\Lambda_{dn} = \frac{2L}{n}$, then $\widetilde{\psi}^{\mathrm{vir}}$ and $\widetilde{\psi}$ superposed onto themselves from different loops are each a maximum. Also at (X, T), $\widetilde{\psi}^{\mathrm{vir}}$ and $\widetilde{\psi}$ meet and superpose, as $\psi = \widetilde{\psi} + \widetilde{\psi}^{\mathrm{vir}} = \widetilde{\Phi}\widetilde{\Psi} + \widetilde{\Phi}^{\mathrm{vir}}\widetilde{\Psi}^{\mathrm{vir}}$. On the scale of Λ_d, or K_d, the time variations in $\widetilde{\Phi}$ and $\widetilde{\Phi}^{\mathrm{vir}}$ are higher-order ones; thus for $K \gg K_d$, we have to a good approximation $\widetilde{\Phi}(X,T) \simeq \widetilde{\Phi}^{\mathrm{vir}}(X,T) \simeq 2C_1 \cos(KX) = F(X)$ and $\psi(X,T) = F(X)\left[\widetilde{\Psi} + \widetilde{\Psi}^{\mathrm{vir}}\right] = C_4 \cos(KX) \sin\left[(\Omega + \Omega_d)T\right] \cos(K_d X + \alpha_0)$; $C_4 = 4C_1$. As a mechanical requirement at the massive walls,

$$\psi(0,T) = \psi(L,T) = 0\,. \tag{18}$$

Condition (18) requires $\alpha_0 = -\frac{\pi}{2}$; ψ is thus now

$$\psi(X,T) = \Phi(X,T)\,\Psi_X(X)\,; \tag{19}$$

$$\Psi_X(X) = \sin(K_d X)\,, \tag{20}$$

$$\Phi(X,T) = C_4 \cos(KX) \sin\left[(\Omega + \Omega_d)T\right]. \tag{21}$$

ψ of (18) is a standing beat or standing de Broglie phase wave; it includes all of the component waves due to both the actual and virtual charges and hence represents the (standing) total wave of the particle.

7 Eigen-state wave function and variables

Equation (13) showed the particle's kinetic energy is transmitted at the angular frequency $\frac{1}{2}\Omega_d$, half the value Ω_d for transporting the particle mass, and is a source motion effect of order $(\frac{v}{c})^2$. This is distinct from, actually exclusive of, the source motion effect, of order v, responsible for the earlier beat wave formation. We here include the order $(\frac{v}{c})^2$ effect only simply as a multiplication factor to ψ, and thus have $\psi' = \psi\,(X,T)\,e^{-i\frac{1}{2}\hbar\Omega_d T}$ which describes the particle's kinetic energy transmission. Furthermore, in typical applications $K \gg K_d$, $\Omega \gg \Omega_d$; thus on the scale of $(K_d,\, \Omega_d)$, we can to a good approximation ignore the rapid oscillation in Φ of (21), and have

$$\Phi(X,T) \simeq C_4 \equiv \text{constant} \qquad (21)'$$

and $\psi(X,T) = C\Psi_X(X)$. The time-dependent wave function, in energy terms, is thus $\Psi(X,T) = \psi'(X,T) = \psi e^{-i\frac{\Omega_d}{2}T} = C\Psi_X(X)e^{-i\frac{\Omega_d}{2}T}$, or

$$\Psi(X,T) = C\sin(K_d X)\,e^{-i\frac{1}{2}\Omega_d T}\,, \qquad (22)$$

where $C = \frac{1}{\int_0^L \psi^2 dX} = \frac{\sqrt{2/L}}{C_4}$ is a normalization constant. With (17) for K_{dn} in (12)–(13), for a fixed L we have the permitted dynamic variables

$$P_{vn} = \frac{n\hbar\pi}{L}\,, \qquad (23) \qquad\qquad \mathcal{E}_{vn} = \frac{n^2\hbar^2\pi^2}{2ML^2}\,, \qquad (24)$$

where $n = 1, 2, \ldots$ These dynamic variables are seen to be quantized, pronouncingly for L not much greater than Λ_d, as the direct result of the standing wave solutions. As shown for the three lowest energy levels in Fig. 3a, the permitted $\Psi(X)$, $\equiv \Psi(X, T_0)$ with T_0 a fixed time point, describing the envelopes (dotted lines) of $\psi(X) \equiv \psi(X, T_0)$ which rapid oscillations have no physical consequence to the particle dynamics, are in complete agreement with the corresponding solution of Schrödinger equation for an identical system, $\Psi_S(X)$, indicated by the same dotted lines.

The total wave of a particle, hence its total energy, mass, size, all extend in (real) space throughout the wave path. A portion of the particle, hence the probability of finding the particle, at a given position X in real space is proportional to the wave energy stored in the infinitesimal volume at X, $\mathcal{E}(X) = B\big(\psi(X)\big)^2$, with B a conversion constant [2], $\psi(X)$ as shown in Fig. 3b.

With (23) in $\Delta P_v = P_{v.n+1} - P_{v.n}$ we have $\Delta P_v\, 2L = h$, which reproduces Heisenberg's uncertainty relation. It follows from the solution that the uncertainty in finding a particle in real space results from the particle is an extensive wave over L, and in momentum space from the standing wave solution where waves interfering destructively are cancelled and inaccessible to an external observer.

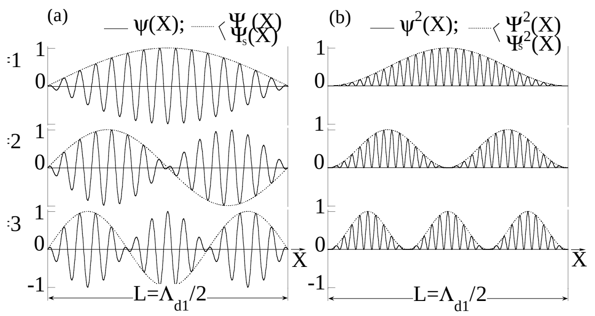

Fig. 3: (a) The total wave of particle $\psi(X)$ of (19) with rapid oscillation, and the de Broglie wave $\Psi(X)$ as the envelop, for three lowest energy levels $n = 1, 2, 3$; Ψ coincides with Schrödinger eigen-state functions Ψ_S. (b) The corresponding probabilities.

8 Concluding remarks

We have seen that the total wave superposed from the electromagnetic component waves generated by a traveling oscillatory vaculeon charge, which together make up our particle, has actually the requisite properties of a de Broglie wave. It exhibits in spatial coordinate the periodicity of the de Broglie wave, by the wavelength Λ_d, facilitated by a beat or de Broglie phase wave traveling at a phase velocity $\sim c^2/v$, with the beat in the total wave resulting naturally from the source-motion resultant Doppler differentiation of the electromagnetic component waves. Λ_d conjoined with the particle's center-of-mass motion leads to a periodicity of the de Broglie wave on time axis, the angular frequency Ω_d. The Λ_d and Ω_d obey the de Broglie relations. The particle's standing wave solutions in confined space agree completely with solutions for Schrödinger equation for an identical system.

References

1. De Broglie L., *Comptes rendus*, 1923, v. 177, 507–510; PhD Thesis, Univ. of Paris, 1924; *Phil. Mag.* 1924, v. 47, 446.
2. Zheng-Johansson J. X. and Johansson P.-I., Unification of classical, quantum and relativistic mechanics and of the four forces (Foreword by R. Lundin). Nova Science, N.Y., 2006a, 2nd printing (later 2006); *Quantum Theory and Symmetries IV*, ed. by V. K. Dobrev, Heron Press, Sofia, 2006b (one of two papers with R. Lundin); arxiv: phyiscs/0411134(v4); phyiscs/0411245; physics/0501037(v3); Inference of basic laws of classical, quantum and relativistic mechanics from first-principles classical-mechanics solutions (updated title), Nova Science, N.Y., 2006c.