Preferred Spatial Directions in the Universe: a General Relativity Approach

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Herein is constructed, using General Relativity, the space metric along the Earth's trajectory in the Galaxy, where the Earth traces outs a complicated spiral in its orbital motion around the Sun and its concomitant motion with the solar system around the centre of the Galaxy. It is deduced herein that this space is inhomogeneous and anisotropic. The observable properties of the space, characterizing its gravitation, rotation, deformation, and curvature, are obtained. The theory predicts that the observable velocity of light is anisotropic, due to the anisotropy and inhomogeneity of space caused by the presence of gravitation and the space rotation, despite the world-invariance of the velocity of light remaining unchanged. It is calculated that two pairs of synchronised clocks should record a different speed of light for light beams travelling towards the Sun and orthogonal to this direction, of about $4\times10^{-4}\,c$ (i. e. 120 km/sec, 0.04% of the measured velocity of light c). This effect should have oscillations with a 12-hour period (due to the daily rotation of the Earth) and 6 month period (due to the motion of the Earth around the Sun). The best equipment for detecting the effect is that being used by R. T. Cahill (Flinders University, Australia) in his current experiments measuring the velocity of light in an RF coaxial-cable equipped with a pair of high precision synchronized Rb atomic clocks.

The geniality of geometry, its applicability to our real world, can be verified by observation or experiment, not logical deduction.

N. A. Kozyrev

1 Introduction

We construct herein, by General Relativity, a mathematical model for a space body moving around another body (the centre of attraction), both moving in an observer's reference space. The Earth rotates around the Sun, and orbits in common with it around the centre of the Galaxy; the Sun rotates around the centre of the Galaxy and orbits in common with the Galaxy around the centre of the Local Group of galaxies; etc. As a result there are preferred directions determined by orbital motions, so the real Universe is *anisotropic* (inequivalence of directions). Because there are billions of centres of gravitational attraction, the Universe is also *inhomogeneous* (inequivalence of points). Hence, for the real Universe, we cannot ignore the anisotropy of space and gravitation.

On the other hand, most cosmologists use the concept of a homogeneous isotropic Universe wherein all points and directions are equivalent. Such a model can be built only by an observer who, observing matter in the Universe from afar, doesn't see such details as stars and galaxies. Such conceptions lead to a vicious circle — most cosmologists are sure that our Universe is a homogeneous isotropic ball expanding from an initial point-like state (singularity); they ignore the anisotropy of space and gravitation in such models.

Relativistic models of a homogeneous isotropic universe (which include the Friedmann solutions) are only a few partial solutions to Einstein's equations. Besides, as shown during the last decade, many popular cosmological metrics (including the Friedmann solutions) are inadmissible, because the difference between the radial coordinate and the proper radius isn't taken into account there (see [1, 2] and References therein).

And so forth, we shall show that the homogeneous isotropic metric spaces contain no rotation and gravitation, and that they can only undergo deformation: no stars, galaxies or other space bodies exist in such a universe*. Why do the scientists use such solutions? The answer is clearly evident: such solutions are simple, and thereby easier to study.

We shall consider another problem statement, the case of an inhomogeneous anisotropic universe as first set up in 1944 by A. Zelmanov [4, 5]. Such a consideration is applicable to any local part of the Universe. We show in this paper that along such a preferred direction, caused by the orbital motion of a space body, an *anisotropy of the observable velocity of light* can be deduced, despite the world-invariance of the velocity of light remaining unchanged†. Using this result as a basis, we will show in a subsequent paper (now in preparation) that not only is the anisotropy of the velocity of light expected along a satellite's trajectory, but even its motion is permitted only in a non-empty space filled by a distribution of matter and a λ-field (both derived from the right side of Einstein's equations). This conclusion leads to

*This situation is similar to the standard solution of the gravitational wave problem, which considers them as space deformation waves in a space free of rotation and gravitation [3].

†The observable velocity of light is different to the world-invariant velocity of light if considered by means of the mathematical apparatus of physically observed quantities in General Relativity — so-called chronometric invariants [4, 5].

the possibility of a new source of energy working in a rotating (non-holonomic) space, and has a direct link to the conclusion that stars produce energy due to the background space non-holonomity (as recently derived by means of General Relativity in [6, 7]).

2 Observed characteristics of space in the Earth's motion in the Galaxy

How do the Earth and the planets move in space? The Earth rotates around its own axis at 465 m/sec at the equator, with an approximately 24-hour period, and moves at 30 km/sec around the Sun with a 365.25-day period (astronomical year). The Sun, in common with the planets, moves at 250 km/sec around the centre of the Galaxy with an ~ 200 million year period. And so the Earth's orbit traces a cylinder, the axis of which is the galactic trajectory of the Sun. As a result, the local space of the Earth draws a very stretched spiral, spanned over the "galactic" cylinder of the Earth's orbit. Each planet traces a similar spiral in the Galaxy.

We aim to build a metric for the space along the Earth's transit in the Galaxy. We do this in two steps. First, the metric along the Earth's transit in the gravitational field of the Sun. Second, using the Lorentz transformation to change to the reference frame moving (with respect to the first frame) along the axis coinciding with the direction in which the Earth moves in the Galaxy.

We use a reference frame which rotates and moves forwards in a weak gravitational field. We therefore use cylindrical coordinates. Then the metric along the Earth's transit in the gravitational field of the Sun has the form*

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 - \frac{2\omega r^2}{c}\, c dt\, d\varphi - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2, \tag{1}$$

where ω is the angular velocity of the Earth's rotation around the Sun: $\omega = \frac{v_{orb}}{r} = 2\times10^{-7}\ \text{sec}^{-1}$.

We now change to a reference frame that rotates in a weak gravitational field and moves uniformly with a velocity v (associated with the motion of the Sun in the Galaxy) along the z-axis. We apply the Lorentz transformations

$$\tilde{z} = \frac{z + \mathrm{v}t}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}, \qquad \tilde{t} = \frac{t + \frac{\mathrm{v}z}{c^2}}{\sqrt{1 - \frac{\mathrm{v}}{c^2}}}, \tag{2}$$

where $\tilde{z}$ and $\tilde{t}$ are corresponding coordinates in the new reference frame. We differentiate $\tilde{z}$ and $\tilde{t}$, then substitute the resulting $d\tilde{z}^2$, $d\tilde{t}^2$ and $d\tilde{t}$ into (2). For $\mathrm{v} = 250$ km/sec we have $\mathrm{v}^2/c^2 = 7\times10^{-7}$, hence $\frac{1}{\sqrt{1-\mathrm{v}^2/c^2}} \approx 1 + \mathrm{v}^2/2c^2$. We ignore terms in powers higher than $\frac{1}{c^2}$. As a result we obtain the metric along the Earth's trajectory in the Galaxy (dropping the tilde from the formulae)

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 - \frac{2\omega r^2}{c}\, c dt\, d\varphi - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - \frac{2\omega \mathrm{v} r^2}{c^2}\, d\varphi\, dz - dz^2. \tag{3}$$

This metric differs from (1), because of a spatial term $2\omega r^2 \mathrm{v}/c^2$ depending upon the linear velocity v.

In order to obtain really observable effects expected in the metric (3), we use the mathematical method of physical observed quantities [4, 5], which considers a fixed spatial section connected to a real reference frame of an observer. For such an observer the fundamental metrical tensor† has the three-dimensional invariant form

$$h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i v_k\,, \qquad i,k = 1,2,3, \tag{4}$$

dependent upon the linear velocity of the space rotation $v_i = -\frac{c g_{0i}}{\sqrt{g_{00}}}$. In (3) the metric tensor has the components

$$h_{11} = 1 + \frac{2GM}{c^2 r}\,, \quad h_{22} = r^2 \left(1 + \frac{\omega^2 r^2}{c^2}\right), \\ h_{23} = \frac{\omega r^2 \mathrm{v}}{c^2}\,, \qquad h_{33} = 1\,, \tag{5}$$

while its contravariant components are

$$h^{11} = 1 - \frac{2GM}{c^2 r}\,, \quad h^{22} = \frac{1 - \frac{\omega^2 r^2}{c^2}}{r^2}\,, \\ h^{23} = -\frac{\omega \mathrm{v}}{c^2}\,, \qquad h^{33} = 1\,. \tag{6}$$

According to the theory [4, 5], any reference space has principal observable (chronometrically invariant) characteristics: the chr.inv.-vector of gravitational inertial force

$$F_i = \frac{1}{1 - \frac{\mathrm{w}}{c^2}} \left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right); \tag{7}$$

the chr.inv.-tensor of the angular velocity of the space rotation

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right); \tag{8}$$

and the chr.inv.-tensor of the rates of the space deformation

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \tag{9}$$

*See any textbook on relativity. Note that the gravitational field is included in the components of the fundamental metric tensor $g_{\alpha\beta}$ as $\frac{GM}{c^2 r}$. The mass of the Sun is $M_\odot = 2\times10^{33}$ g, the mass of the Earth is $M_\oplus = 6\times10^{27}$ g; the distance between the Sun and the Earth is 15×10^{11} cm, the Earth's radius is 6.37×10^8 cm. We obtain $\frac{GM_\odot}{c^2 r} = 10^{-8}$, $\frac{GM_\oplus}{c^2 r} = 10^{-10}$. So, in this consideration we mean the daily rotation of the Earth and its gravitational field neglected (quasi-Newtonian approximation).

†The spatial indices $1,2,3$ are denoted by Roman letters, while the space-time indices $0,1,2,3$ are denoted by Greek letters.

where $\mathrm{w} = c^2\left(1-\sqrt{g_{00}}\right)$, while $\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ is the so-called chronometrically invariant time derivative.

Calculating these for the metric space (3), we obtain

$$F^1 = \left(\omega^2 r - \frac{GM}{r^2}\right)\left(1+\frac{\omega^2 r^2}{c^2}\right); \tag{10}$$

$$A^{12} = \frac{\omega}{r}\left(1-\frac{2GM}{c^2 r}+\frac{\omega^2 r^2}{2c^2}\right), \quad A^{31} = \frac{\omega^2 \mathrm{v} r}{c^2}. \tag{11}$$

All components of D_{ik} equal zero. Hence the reference body gravitates, rotates, and moves forward at a constant velocity. Appropriate characteristics of the metrics (1) and (3) coincide, aside for A^{31}: $A^{31}=0$ in (3).

The observable time interval $d\tau$ contains v_i [4, 5]:

$$d\tau = \left(1-\frac{\mathrm{w}}{c^2}\right)dt - \frac{1}{c^2}\,v_i\,dx^i. \tag{12}$$

Within an area wherein $A_{ik}=0$ (holonomic space) the time coordinate $x^0 = ct$ can be transformed so that all $v_i = 0$. In other words, the time interval between two events at different points does not depend on the path of integration: time is *integrable*, so a global synchronization of clocks is possible. In such a space the spatial section $x^0 = \text{const}$ is everywhere orthogonal to time lines $x^i = \text{const}$. If $A_{ik} \neq 0$ (non-holonomic space), it is impossible for all v_i to be zero: the spatial section is not orthogonal to the time lines, and the time interval between two events at different points depends on the path of integration (time is *non-integrable*).

Zelmanov also introduced the chr.inv.-pseudovector of the angular velocity of the space rotation [4]

$$\Omega_i = \frac{1}{2}\,\varepsilon_{ijk}\,A^{jk}, \tag{13}$$

where $\varepsilon_{ijk} = \frac{e_{ijk}}{\sqrt{h}}$ is the three-dimensional discriminant tensor, e_{ijk} is the completely antisymmetric three-dimensional tensor, $h = \det\|h_{ik}\|$. Hence, $\Omega_1 = A^{23}$, $\Omega_2 = A^{31}$, $\Omega_3 = A^{12}$.

In our statement we have two bodies, both rotating and gravitating. The first body is at rest with respect to the observer, whilst the second body moves with a linear velocity. As seen from (11), for the rest body only $\Omega_3 \neq 0$. For the moving body we also obtain $\Omega_2 \neq 0$ and $\Omega_3 \neq 0$.* In other words, any linear motion of an observer with respect to his reference body provides an additional degree of freedom to rotations of his reference space.

Besides the aforementioned observable "physical" characteristics F_i, A_{ik}, and D_{ik}, every reference space also has an observable geometric characteristic [4]: the chr.inv.-tensor of the three-dimensional space curvature

$$C_{lkij} = H_{lkij} - \frac{1}{c^2}\big(2A_{ki}D_{jl} + A_{ij}D_{kl} + A_{jk}D_{il} + A_{kl}D_{ij} + A_{li}D_{kj}\big), \tag{14}$$

*This is because any linear motion leads to an additional term in the observable metric tensor h_{ik}: see formulae (5) and (6).

which possesses all the properties of the Riemann-Christoffel curvature tensor $R_{\alpha\beta\gamma\delta}$ in the spatial section. Here $H_{lkij} = h_{jm}H_{lki\cdot}^{\cdots m}$, where $H_{lki\cdot}^{\cdots m}$ is the chr.inv-tensor similar to Schouten's tensor [8]:

$$H_{lki\cdot}^{\cdots j} = \frac{^*\partial\Delta_{il}^{j}}{\partial x^k} - \frac{^*\partial\Delta_{kl}^{j}}{\partial x^i} + \Delta_{il}^{m}\Delta_{km}^{j} - \Delta_{kl}^{m}\Delta_{im}^{j}. \tag{15}$$

If all A_{ik} or D_{ik} are zero in a space, $C_{iklj} = H_{iklj}$. Zelmanov also introduced $H_{ik} = h^{mn}H_{imkn}$, $H = h^{ik}H_{ik}$, $C_{ik} = h^{mn}C_{imkn}$ and $C = h^{ik}C_{ik}$.

The chr.inv.-Christoffel symbols of the first and second kinds, by Zelmanov, are

$$\Delta_{ij}^{k} = h^{km}\Delta_{ij,m} = \frac{1}{2}\left(\frac{^*\partial h_{im}}{\partial x^j} + \frac{^*\partial h_{jm}}{\partial x^i} - \frac{^*\partial h_{ij}}{\partial x^m}\right), \tag{16}$$

where $\frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} - \frac{1}{c^2}\frac{^*\partial}{\partial t}$ is the so-called chr.inv.-spatial derivative.

Calculating the components of Δ_{ij}^{k} for the metric (3), we obtain

$$\begin{gathered}
\Delta_{22}^{1} = -r\left(1-\frac{2GM}{c^2 r}+\frac{2\omega^2 r^2}{c^2}\right),\\
\Delta_{11}^{1} = \frac{GM}{c^2 r^2}, \qquad \Delta_{23}^{1} = -\frac{\omega \mathrm{v} r}{c^2},\\
\Delta_{12}^{2} = \frac{1}{r}\left(1+\frac{\omega^2 r^2}{c^2}\right), \quad \Delta_{13}^{2} = \frac{\omega \mathrm{v}}{c^2 r},
\end{gathered} \tag{17}$$

while non-zero components of C_{iklj}, C_{ik} and C are

$$\begin{gathered}
C_{1212} = -\frac{GM}{c^2 r} + \frac{3\omega^2 r^2}{c^2},\\
C_{11} = -\frac{GM}{c^2 r^3} + \frac{3\omega^2}{c^2}, \quad C_{22} = -\frac{GM}{c^2 r} + \frac{3\omega^2 r^2}{c^2},\\
C = 2\left(-\frac{GM}{c^2 r^3} + \frac{3\omega^2}{c^2}\right).
\end{gathered} \tag{18}$$

We have thus calculated by the theory of observable quantities, that:

> The observable space along the Earth's trajectory in the Galaxy is non-holonomic, inhomogeneous, and curved due to the space rotation and/or Newtonian attraction. This should be true for any other planet (or its satellite) as well, or any other body considered within the framework this analysis.

3 Deviation of light in the field of the Galactic rotation

We study how a light ray behaves in a reference body space described by the metric (3). Light moves along isotropic geodesic lines. Such geodesics are trajectories of the parallel transfer of the four-dimensional isotropic wave vector

$$K^\alpha = \frac{\Omega}{c}\frac{dx^\alpha}{d\sigma}, \qquad g_{\alpha\beta}\,K^\alpha K^\beta = 0, \tag{19}$$

where Ω is the proper frequency of the radiation, $d\sigma = h_{ik}\,dx^i dx^k$ is the three-dimensional observable interval*. The equations of geodesic lines in chr.inv.-form are [4, 5]

$$\left.\begin{aligned} &\frac{d\Omega}{d\tau} - \frac{\Omega}{c^2}F_i c^i + \frac{\Omega}{c^2}D_{ik}c^i c^k = 0\,,\\ &\frac{d(\Omega c^i)}{d\tau} + 2\omega(D^i_k + A^{\cdot i}_{k\cdot})c^k - \Omega F^i + \Omega\Delta^i_{kn}c^k c^n = 0\,, \end{aligned}\right. \tag{20}$$

where $c^i = \frac{dx^i}{d\tau}$ is the observable chr.inv.-velocity of light (its square is invariant $c_i c^i = h_{ik}c^i c^k = c^2$).

Substituting the chr.inv.-characteristics of the reference space (3) into equations (20), we obtain

$$\frac{1}{\Omega}\frac{d\Omega}{d\tau} - \frac{1}{c^2}\left(\omega^2 r - \frac{GM}{r^2}\right)\frac{dr}{d\tau} = 0\,, \tag{21}$$

$$\begin{aligned} &\frac{d}{d\tau}\left(\Omega\frac{dr}{d\tau}\right) - 2\Omega\omega r\left(1 - \frac{2GM}{c^2 r} + \frac{3\omega^2 r^2}{2c^2}\right)\frac{d\varphi}{d\tau} -\\ &- \Omega\left(\omega^2 r - \frac{GM}{r^2}\right)\left(1 + \frac{\omega^2 r^2}{c^2}\right) - \frac{2\Omega\omega \mathrm{v} r}{c^2}\frac{d\varphi}{d\tau}\frac{dz}{d\tau} -\\ &- \Omega r\left(1 - \frac{2GM}{c^2 r} + \frac{2\omega^2 r^2}{c^2}\right)\left(\frac{d\varphi}{d\tau}\right)^2 = 0\,, \end{aligned} \tag{22}$$

$$\begin{aligned} &\frac{d}{d\tau}\left(\Omega\frac{d\varphi}{d\tau}\right) + \frac{2\Omega\omega}{r}\left(1 + \frac{GM}{2c^2 r} + \frac{\omega^2 r^2}{2c^2}\right)\frac{dr}{d\tau} +\\ &+ \frac{2\omega}{r}\left(1 + \frac{\omega^2 r^2}{c^2}\right)\frac{dr}{d\tau}\frac{d\varphi}{d\tau} + \frac{2\Omega\omega \mathrm{v}}{c^2 r}\frac{dr}{d\tau}\frac{dz}{d\tau} = 0\,, \end{aligned} \tag{23}$$

$$\frac{d}{d\tau}\left(\Omega\frac{dz}{d\tau}\right) - \frac{2\Omega\omega^2 \mathrm{v} r}{c^2}\frac{dr}{d\tau} = 0\,. \tag{24}$$

Integrating (21) we obtain the observable proper frequency of the light beam at the moment of observation

$$\Omega = \frac{\Omega_0}{\sqrt{1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}}} \approx \Omega_0\left(1 + \frac{GM}{c^2 r} + \frac{\omega^2 r^2}{2c^2}\right), \tag{25}$$

where Ω_0 is its "initial" proper frequency (in the absence of external affects). We integrate (22)–(24) with the use of (25).

Rewrite (24) as

$$\frac{d}{d\tau}\left(\Omega\frac{dz}{d\tau}\right) = \frac{\Omega\omega^2 \mathrm{v}}{c^2}\frac{d}{d\tau}\left(r^2\right), \tag{26}$$

integration of which gives

$$\Omega\frac{dz}{d\tau} = \frac{\Omega\omega^2 \mathrm{v} r^2}{c^2} + Q\,, \qquad Q = \text{const}, \tag{27}$$

where $\dot z_0 = \left(\frac{dz}{d\tau}\right)_0$ is the initial value of $\frac{dz}{d\tau}$, while the integration constant is $Q = \Omega_0\left(\dot z_0 - \frac{\omega^2 \mathrm{v} r_0^2}{c^2}\right)$.

*So the space-time interval $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$ in chr.inv.-form is $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$. Therefore, because $ds^2 = 0$ along isotropic trajectories by definition, there $d\sigma = c\,d\tau$.

Substituting (27) into (23) and (24) and using Ω from (25), we obtain the system of equations with respect to r and φ,

$$\left.\begin{aligned} &\frac{d}{d\tau}\left(\Omega\frac{d\varphi}{d\tau}\right) + \frac{2\Omega\omega}{r}\left(1 + \frac{GM}{2c^2 r} + \frac{\omega^2 r^2}{2c^2}\right)\frac{dr}{d\tau} +\\ &+ \frac{2\omega}{r}\left(1 + \frac{\omega^2 r^2}{c^2}\right)\frac{dr}{d\tau}\frac{d\varphi}{d\tau} + \frac{2\Omega_0\omega \mathrm{v}\dot z_0}{c^2 r}\frac{dr}{d\tau} = 0\,,\\ &\frac{d}{d\tau}\left(\Omega\frac{dr}{d\tau}\right) - 2\Omega\omega r\left(1 - \frac{2GM}{c^2 r} + \frac{3\omega^2 r^2}{2c^2}\right)\frac{d\varphi}{d\tau} -\\ &- \Omega\left(\omega^2 r - \frac{GM}{r^2}\right)\left(1 + \frac{\omega^2 r^2}{c^2}\right) - \frac{2\Omega_0\omega \mathrm{v}\dot z_0 r}{c^2}\frac{d\varphi}{d\tau} -\\ &- \Omega r\left(1 - \frac{2GM}{c^2 r} + \frac{2\omega^2 r^2}{c^2}\right)\left(\frac{d\varphi}{d\tau}\right)^2 = 0\,. \end{aligned}\right\} \tag{28}$$

We are looking for an approximate solution to this system. The last term has the dimensionless factor $\frac{\mathrm{v}\dot z_0}{c^2}$. For a light beam, $\dot z_0$ (the initial value of the light velocity along the z-axis) is c. Hence $\frac{\mathrm{v}\dot z_0}{c^2} = \frac{\mathrm{v}}{c}$. At 250 km/sec, attributed to the Earth moving in the Galaxy, $\frac{\mathrm{v}}{c} = 8.3\times10^{-4}$. The terms $\frac{GM}{c^2 r}$ and $\frac{\omega^2 r^2}{c^2}$, related to the orbital motion of the Earth, are in order of 10^{-8}. We therefore drop these terms from consideration, so equations (28) become

$$\begin{aligned} &\frac{d}{d\tau}\left(\Omega\frac{dr}{d\tau}\right) - 2\Omega\omega r\frac{d\varphi}{d\tau} - \Omega\left(\omega^2 r - \frac{GM_\odot}{r^2}\right) -\\ &- \Omega r\left(\frac{d\varphi}{d\tau}\right)^2 - \frac{2\Omega_0\omega \mathrm{v}\dot z_0 r}{c^2}\frac{d\varphi}{d\tau}\,, \end{aligned} \tag{29}$$

$$\frac{d}{d\tau}\left(\Omega\frac{d\varphi}{d\tau}\right) + \frac{2\Omega\omega}{r}\frac{dr}{d\tau} + \frac{2\omega}{r}\frac{dr}{d\tau}\frac{d\varphi}{d\tau} + \frac{2\Omega_0\omega \mathrm{v}\dot z_0}{c^2 r}\frac{dr}{d\tau} = 0. \tag{30}$$

We rewrite (30) as

$$\ddot\varphi + 2(\dot\varphi + \tilde\omega)\frac{\dot r}{r} = 0\,, \tag{31}$$

where $\tilde\omega = \omega\left(1 + \frac{\mathrm{v}\dot z_0}{c^2}\right)$, $\dot\varphi = \frac{d\varphi}{d\tau}$, $\ddot\varphi = \frac{d^2\varphi}{d\tau^2}$. This is an equation with separable variables, so its first integral is

$$\dot\varphi = \frac{B}{r^2} - \tilde\omega, \qquad B = \text{const} = (\dot\varphi_0 + \tilde\omega)r_0^2\,, \tag{32}$$

where $\dot\varphi_0$ and r_0 are the initial values of $\dot\varphi$ and r.

We rewrite (29) as

$$\ddot r - 2\tilde\omega r\dot\varphi + \frac{GM}{r^2} - \omega^2 r - r\dot\varphi^2 = 0\,, \tag{33}$$

where $\dot r = \frac{dr}{d\tau}$, $\ddot r = \frac{d^2 r}{d\tau^2}$. In our consideration, $\frac{GM}{r^2} - \omega^2 r$ is zero, so the motion of the Earth around the Sun satisfies the weightlessness condition [9, 10]† — a balance between the

†Each planet, in its orbital motion, should satisfy the *weightlessness condition* $\mathrm{w} = v_i u^i$, where w is the potential of the field attracting the planet to a body around which this planet is orbiting, v_i is the linear velocity of the body's space rotation in this orbit, and $u^i = dx^i/dt$ is the coordinate velocity of the planet in its orbit. The orbital velocity is the same as the space rotation velocity. Hence the weightiness condition can be written as $GM/r = v^2 = v_i v^i$ [9, 10].

acting forces of gravity $\frac{GM}{r^2}$ and inertia $\omega^2 r$. Taking this into account, and substituting (32) into (33), we obtain

$$\ddot{r} + \tilde{\omega}^2 r - \frac{B^2}{r^3} = 0\,. \qquad (34)$$

We replace the variables as $\dot{r} = p$. So $\ddot{r} = p\frac{dp}{dr}$ and the equation (36) takes the form

$$p\,\frac{dp}{dr} = \frac{B^2}{r^3} - \tilde{\omega}^2 r^2, \qquad (35)$$

which can be easily integrated:

$$p^2 = \left(\frac{dr}{d\tau}\right)^2 = -\frac{B^2}{r^2} - \tilde{\omega}^2 r^2 + K\,, \qquad K = \text{const}, \quad (36)$$

where the integration constant is $K = \dot{r}_0^2 + (\dot{\varphi}_0 + \tilde{\omega})^2 r_0^2 + \tilde{\omega}^2 r_0^2$, so we obtain

$$\frac{dr}{d\tau} = \pm\sqrt{K - \tilde{\omega}^2 r^2 - \frac{B^2}{r^2}}\,. \qquad (37)$$

Looking for τ as a function of r, we integrate (37) taking the positive time flow into account (positive values of τ). We obtain

$$\tau = \int_{r_0}^{r} \frac{r\,dr}{\sqrt{-\tilde{\omega}^2 r^4 + K r^2 - B^2}}\,. \qquad (38)$$

Introducing a new variable $u = r^2$ we rewrite (38) as

$$\tau = \frac{1}{2}\int_{u_0}^{u} \frac{du}{\sqrt{-\tilde{\omega}^2 u^2 + K u - B^2}}\,, \qquad (39)$$

which integrates to

$$\tau = -\frac{1}{2\tilde{\omega}}\left[\arcsin\left(\frac{-2\tilde{\omega}^2 r^2 + K}{\sqrt{K^2 - 4\tilde{\omega}^2 B^2}}\right) - \arcsin\left(\frac{-2\tilde{\omega}^2 r_0^2}{\sqrt{K^2 - 4\tilde{\omega}^2 B^2}}\right)\right] \qquad (40)$$

where

$$K^2 - 4\tilde{\omega}^2 B^2 \equiv Q^2 = \left(\dot{r}_0^2 + r_0^2\dot{\varphi}_0^2\right)\left[\dot{r}_0^2 + 4\tilde{\omega}\left(\tilde{\omega} + \dot{\varphi}_0\right) r_0^2 + \dot{\varphi}_0^2 r_0^2\right], \qquad (41)$$

so we obtain r^2 and r

$$r^2 = \frac{Q}{2\tilde{\omega}^2}\sin 2\tilde{\omega}\tau + r_0^2\,, \quad r = \sqrt{\frac{Q}{2\tilde{\omega}^2}\sin 2\tilde{\omega}\tau + r_0^2}\,, \quad (42)$$

where r_0 is the initial displacement in the r-direction.

Substituting (42) into (32) we obtain φ,

$$\varphi = \int_0^{\tau}\left(\frac{B}{r^2} - \tilde{\omega}\right) d\tau = -\tilde{\omega}\tau + \frac{\tilde{\omega}B}{\sqrt{Q^2 - 4\omega^4 r_0^4}} \times \ln\left|\frac{\left(Q + \sqrt{Q^2 - 4\tilde{\omega}^4 r_0^4}\,\right)\tan\tilde{\omega}\tau + 2\tilde{\omega}^2 r_0^2}{\left(Q - \sqrt{Q^2 - 4\tilde{\omega}^4 r_0^4}\,\right)\tan\tilde{\omega}\tau + 2\tilde{\omega}^2 r_0^2}\right| + \varphi_0\,, \qquad (43)$$

where φ_0 is the initial displacement in the φ-direction.

Substituting Ω from (25) into (27), and eliminating the terms containing $\frac{GM}{c^2 r}$ and $\frac{\omega^2 r^2}{c^2}$, we obtain the observable velocity of the light beam in the z-direction

$$\dot{z} = \frac{\omega^2 \mathrm{v} r^2}{c^2} + \dot{z}_0 - \frac{\omega^2 \mathrm{v} r_0^2}{c^2}\,, \qquad (44)$$

the integration of which gives its observable displacement

$$z = \dot{z}_0\tau + \frac{\omega^2 Q\mathrm{v}}{4\tilde{\omega}^3 c^2}\left(1 - \cos 2\tilde{\omega}\tau\right) + z_0\,, \qquad (45)$$

which, taking into account that $\tilde{\omega} = \omega\left(1 + \frac{\mathrm{v}\dot{z}_0}{c^2}\right)$, is

$$z = \dot{z}_0\tau + \frac{\mathrm{v}Q}{4\tilde{\omega}c^2}\left(1 - \cos 2\tilde{\omega}\tau\right)\left(1 - \frac{\mathrm{v}\dot{z}_0}{c^2}\right)^2 + z_0\,. \quad (46)$$

We have obtained solutions for $\dot{r}$, $\dot{\varphi}$, $\dot{z}$ and r, φ, z. We see the galactic velocity of the Earth in only $\dot{z}$ and z.

Let's find corrections to the displacement of the light $\dot{z}$ and its displacement z caused by the motion of the Earth in the rotating and gravitating space of the Galaxy.

As follows from formula (41), Q doesn't include the initial velocity and displacement of the light beam in the z-direction. Besides, $Q = 0$ if $\dot{r}_0 = 0$ and $r_0 = 0$. In a real situation $\dot{r}_0 \neq 0$, because the light beam is emitted from the Earth so r_0 is the distance between the Sun and the Earth. Hence, in our consideration, $Q \neq 0$ always. If $\dot{\varphi}_0 = 0$, the light beam is directed strictly towards the Sun.

We calculate the correction to the light velocity in the r-direction $\Delta\dot{z}_0$ (we mean $\dot{\varphi}_0 = 0, \dot{z}_0 = 0$). Eliminating the term $1 - \frac{\mathrm{v}\dot{z}_0}{c^2}$ we obtain

$$\Delta\dot{z} = \frac{Q\mathrm{v}}{2c^2}\sin 2\tilde{\omega}\tau\,, \qquad Q = \dot{r}_0\sqrt{\dot{r}_0^2 + 4\tilde{\omega}^2 r_0^2}\,. \qquad (47)$$

We see that the correction $\Delta\dot{z}_0$ is a periodical function, the frequency of which is twice the angular velocity of the Earth's rotation around the Sun; $2\tilde{\omega} = 4\times 10^{-7}\,\text{sec}^{-1}$. Because the initial value of the light velocity is $\dot{r}_0 = c$, and also $4\tilde{\omega}^2 r_0^2 \ll c^2$, we obtain the amplitude of the harmonic oscillation

$$\frac{Q\mathrm{v}}{2c^2} = \frac{\dot{r}_0^2}{2c^2}\sqrt{1 + \frac{4\tilde{\omega}^2 r_0^2}{c^2}} \approx \frac{\mathrm{v}}{2}\,, \qquad (48)$$

then the correction to the light velocity in the r-direction $\Delta\dot{z}_0$ is,

$$\Delta\dot{z} = \frac{\mathrm{v}}{2}\sin 2\tilde{\omega}\tau = 4\times 10^{-4}\left(\sin 2\tilde{\omega}\tau\right) c\,. \qquad (49)$$

From this resulting "key formula" we have obtained we conclude that:

> The component of the observable vector of the light velocity directed towards the Sun (the r-direction) gains an addition (correction) in the z-direction, because the Earth moves in common with the Sun in the

Galaxy. The obtained correction manifests as a harmonic oscillation added to the world-invariant of the light velocity c. The expected amplitude of the oscillation is $4\times10^{-4}c$, i. e. 120 km/sec; the period $T=\frac{1}{2\bar{\omega}}$ is half the astronomical year. So the theory predicts an anisotropy of the observable velocity of light due to the inhomogeneity and anisotropy of space, caused by its rotation and the presence of gravitation.

In our statement the anisotropy of the velocity of light manifests in the z-direction. We therefore, in this statement, call the z-direction the *preferred direction*.

We can verify the anisotropy of the velocity of light by experiment. By the theory of observable quantities [4, 5], the invariant c is the length

$$c=\sqrt{h_{ik}\,c^i c^k}=\sqrt{\frac{h_{ik}\,dx^i dx^k}{d\tau}}=\frac{d\sigma}{d\tau} \tag{50}$$

of the chr.inv.-vector $c^i=\frac{dx^i}{d\tau}$ of the observable light velocity. Let a light beam be directed towards the Sun, i. e. in the r-direction. According to our theory, the Earth's motion in the Galaxy deviates the beam away from the r-axis so that we should observe an additional z-component to the light velocity invariant. Let's set up two pairs of detectors (synchronised clocks) along the r-direction and z-direction in order to measure time intervals during which the light beams travel in these directions. Because the distances $\Delta\sigma$ between the clocks are fixed, and c is constant, the measured time in the z-direction is expected to have a dilation with respect to that measured in the r-direction: by formula (49) the light velocity measured in both directions is expected to be differ by ~ 120 km/sec at the maximum of the effect.

The most suitable equipment for such an experiment is that used by R. T. Cahill (Flinders University, Australia) in his current experiments on the measurement of the velocity of light in an RF coaxial-cable equipped with a pair of high precision synchronized Rb atomic clocks [11]. This effect probably had a good chance of being detected in similar experiments by D. G. Torr and P. Colen (Utah State University, USA) in the 1980's [12] and, especially, by Roland De Witte (Belgacom Laboratory of Standards, Belgium) in the 1990's [13]. However even De Witte's equipment had a measurement precision a thousand times lower than that currently used by Cahill.

Because the Earth rotates around its own axis we should observe a weak daily variation of this effect. In order to register the complete variation of this value, we should measure it at least during half the astronomical year (one period of its variation).

4 Inhomogeneity and anisotropy of space along the Earth's transit in the Galaxy

We just applied the metric (3) to the Earth's motion in the Galaxy. Following this approach, we can also employ this metric to other preferred directions in the Universe, connected to the motion of another space body, for instance — the motion of our Galaxy in the Local Group of galaxies.

Astronomical observations show that the Sun moves in common with our Galaxy in the Local Group of galaxies at the velocity 700 km/sec.* The metric (3) can take into account this aspect of the Earth's motion as well. In such a case we should expect two weak maximums in the time dilation measured in the above described experimental system during the 24-hour period, when the z-direction coincides with the direction of the apex of the Sun. The amplitude of the variation of the observable light velocity should be 2.8 times the variation caused by the Earth's motion in the Galaxy.

Swedish astronomers in the 1950's discovered that the Local Group of galaxies is a part of an compact "cloud" called the Supercluster of galaxies, consisting of galaxies, small groups of galaxies, and two clouds of galaxies. The Supercluster has a diameter of $\sim$98 million light years, while our Galaxy is located at 62 million light years from the centre. The Supercluster rotates with a period of $\sim$100 billion years in the central area and $\sim$200 billion years at the periphery. As supposed by the Swedes, our Galaxy, located at $\sim$2/3 of the Supercluster's radius, from its centre, rotates around the centre at a velocity of $\sim$700 km/sec. (See Chapter VII, § 6 in [14] for the details.)

In any case, in any large scale our metric (3) gives the same result, because any of the spaces is non-holonomic (rotates) around its own centre of gravity. All the spaces are included, one into the other, and cause bizarre spirals in their motions. The greater the number of the space structures taken onto account by our metric (3), the more complicated is the spiral traced out by the Earth observer in the space — the spiral is plaited into other space spirals (the fractal structure of the Universe [15]).

This analysis of our theoretical results, obtained by General Relativity, and the well-known data of observational astronomy leads us to the obvious conclusion:

> The main factors forming the observable structure of the space of the Universe are gravitational fields of bulky bodies and their rotations, not the space deformations as previously thought.

Many scientists consider homogeneous isotropic models as models of the real Universe. A homogeneous isotropic space-time is described by Friedmann's metric

$$ds^2=c^2dt^2-R^2\frac{dx^2+dy^2+dz^2}{\left[1+\frac{k}{4}\left(x^2+y^2+z^2\right)\right]^2}\,, \tag{51}$$

where $R=R(t)$; $k=0,\pm1$. For such a space, the main observable characteristics are $F^i=0$, $A_{ik}=0$, $D_{ik}\neq0$. In other words, such a space can undergo deformation (expansion,

*The direction of this motion is pointed out in the sky as the apex of the Sun. Interestingly, the Sun has a slow drift of 20 km/sec in the same direction as the apex, but within the Galaxy with respect to its plane.

compression, or oscillation), but it is free of rotation and contains no gravitating bodies (fields). So the metric (51) is the necessary and sufficient condition for homogeneity and isotropy. This is a model constructed by an imaginary observer who is located so far away from matter in the real Universe that he sees no such details as stars and galaxies.

In contrast to them, we consider a cosmological model constructed by an Earth observer, who is carried away by all motions of our planet. Zelmanov, the pioneer of inhomogeneous anisotropic relativistic models, pointed out the mathematical conditions of a space's homogeneity and isotropy, expressed with the terms of physically observable characteristics of the space [4]. The conditions of isotropy are

$$F_i = 0\,, \quad A_{ik} = 0\,, \quad \Pi_{ik} = 0\,, \quad \Sigma_{ik} = 0\,, \qquad (52)$$

where $\Pi_{ik} = D_{ik} - \frac{1}{3}Dh_{ik}$ and $\Sigma_{ik} = C_{ik} - \frac{1}{3}Ch_{ik}$ are the factors of anisotropy of the space deformation and the three-dimensional (observable) curvature. In a space of the metric (3) we have $D_{ik} = 0$, hence there $\Pi_{ik} = 0$. However F_i and A_{ik} are not zero in such a space (see formulae 10 and 11). Besides these there are the non-zero quantities,

$$\begin{aligned} \Sigma_{11} &= -\frac{1}{3}\frac{GM}{c^2r^3} + \frac{\omega^2}{c^2}\,; \\ \Sigma_{22} &= -\frac{1}{3}\frac{GM}{c^2r^2} + \frac{\omega^2 r}{c^2}\,; \\ \Sigma_{33} &= \frac{2}{3}\frac{GM}{c^2r^3} - \frac{2\omega^2}{c^2}\,. \end{aligned} \qquad (53)$$

We see that a space of the metric (3) is anisotropic due to its rotation and gravitation.

The conditions of homogeneity, by Zelmanov [4], are

$$\nabla_j F_i = 0\,, \quad \nabla_j A_{ik} = 0\,, \quad \nabla_j D_{ik} = 0\,, \quad \nabla_j C_{ik} = 0\,. \qquad (54)$$

Calculating the conditions for the metric (3), we obtain

$$\begin{aligned} &\nabla_1 C_{11} = \frac{3GM}{c^2r^4}\,, \qquad \nabla_1 C_{22} = \frac{3GM}{c^2r^2}\,, \\ &\nabla_1 F_1 = \omega^2\left(1 + \frac{3\omega^2r^2}{c^2}\right) + \frac{2GM}{r^3}\left(1 + \frac{3GM}{c^2r}\right), \\ &\nabla_1 A_{12} = -\omega\left(\frac{2}{r^2} + \frac{\omega^2}{c^2} + \frac{3GM}{c^2r^3}\right). \end{aligned} \qquad (55)$$

This means, a space of the metric (3) is inhomogeneous due to its rotation and gravitation.

The results we have obtained manifest thus:

> The real space of our Universe, where space bodies move, is inhomogeneous and anisotropic. Moreover, the space inhomogeneity and anisotropy determine the bizarre structure of the Universe which we observe: the preferred directions along which the space bodies move, and the hierarchial distribution of the motions.

5 Conclusions

By means of General Relativity we have shown that the space metric (3) along the Earth's trajectory in the Galaxy, where the Earth follows a complicated spiral traced out by its orbital motion around the Sun and its concomitant motion with the whole solar system around the centre of the Galaxy. We have shown that this metric space is: (a) globally non-holonomic due to its rotation and the presence of gravitation, as manifested by the non-holonomic chr.inv.-tensor A_{ik} (11) calculated in the metric space*; (b) inhomogeneous, because the chr.inv.-Christoffel symbols Δ^k_{ij} indicating inhomogeneity of space, being calculated in the metric space as shown by (17), contain gravitation and space rotation; (c) curved due to gravitation and space rotation, represented in the formulae for the three-dimensional chr.inv.-curvature C_{iklj} calculated in the metric space as shown by (18).

Consequently, in real space there exist "preferred" spatial directions along which space bodies undergo their orbital motions.

We have deduced that the observable velocity of light should be anisotropic in space due to the anisotropy and inhomogeneity of space, caused by the aforementioned factors of gravitation and space rotation, despite the world-invariance of the velocity of light. It has been calculated that two pairs of synchronised clocks should record different values for the speed of light in light beams directed towards the Sun and orthogonal to this direction, at about $4\times10^{-4}c$ (0.04% of the measured velocity of light c, i.e. $\sim$120 km/sec). This effects should undergo oscillations with a 12-hour period (due to the daily rotation of the Earth) and with a 6-month period (due to the motion of the Earth around the Sun). Equipment most suitable for detecting the effect is that used by R. T. Cahill (Flinders University, Australia) in his current experiment on the measurement of the velocity of light in a one-way RF coaxial-cable equipped with a pair of high precision synchronized Rb atomic clocks.

The predicted anisotropy of the observable velocity of light has been deduced as a direct consequence of the geometrical structure of four-dimensional space-time. Therefore, if the predicted anisotropy is detected by experiment, it will be one more fact in support of Einstein's General Theory of Relativity.

The anisotropy of the observable velocity of light as a consequence of General Relativity was first pointed out by D. Rabounski in the editorial preface to [13], his papers [6, 7], and many private communications with the author, which commenced in Autumn, 2005. He has stated that the anisotropy results from the non-holonomity (rotation) of the

*Gravitation is represented by the mass of the Sun M, while the space rotation is represented by two factors: the angular velocity ω of the solar space rotation in the Earth's orbit (equal to the angular velocity of the Earth's rotation around the Sun), and also the linear velocity v of the rotation of the Sun in common with the whole solar system around the centre in the Galaxy.

local space of a real observer and/or the non-holonomity of the background space of the whole Universe. Moreover, the non-holonomic field of the space background can produce energy, if perturbed by a local rotation or oscillation (as this was theoretically found for stars [6, 7]).

Detailed calculations provided in the present paper show not only that the non-holonomity (rotation) of space is the source of the anisotropy of the observable velocity of light, but also gravitational fields.

This paper will be followed by a series of papers wherein we study the interaction between the fields of the space non-holonomity, and also consider these fields as new sources of energy. This means that we consider open systems. Naturally, given the case of an inhomogeneous anisotropic universe, it is impossible to study it as a closed system since such systems don't physically exist owing to the presence of space non-holonomity and gravitation*. In a subsequent paper we will consider the non-holonomic fields in a space of the metric (3) with the use of Einstein's equations. It is well known that the equations can be applied to a wide variety distributions of matter, even inside atomic nuclei. We can therefore, with the use of the Einstein equations, study the non-holonomic fields and their interactions in any scaled part of the Universe — from atomic nuclei to clusters of galaxies — the problem statement remains the same in all the considerations.

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*According to the Copernican standpoint, the solar system should be a closed system, because that perspective doesn't take into account the fact that the solar system moves around the centre of the Galaxy, which carries it into other, more complicated motions.