Forces of Space Non-Holonomity as the Necessary Condition for Motion of Space Bodies

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The motion of a satellite in the gravitational field of the Earth is studied. The condition of weightlessness in terms of physical observable quantities is formulated. It is shown that the motion of all planets in the Solar system satisfy this condition. The exact solution of non-null geodesic lines describing the motion of a satellite in a state of weightlessness is obtained. It is shown that two kinds of rotational forces (forces of non-holonomity) exist: the inner force is linked to a gravitational potential, the outer force changes geometric properties of a space. The latter force causes both anisotropy of the velocity of light and additional displacement of mass-bearing bodies.

1 Introduction

We continue studies commenced in [1], where, using General Relativity, the space metric along the Earth's trajectory in the Galaxy was constructed. This metric was constructed in two steps: (i) the metric along the Earth's transit in the gravitational field of the Sun; (ii) using the Lorenz transformation to change to the reference frame moving along the z-axis coinciding with the direction in which the Earth moves in the Galaxy. The behaviour of a light ray in a reference body's space described by the obtained metric was studied in [1]. It follows from exact solutions of the isotropic geodesic lines equations for the obtained metric, that an anisotropy of the velocity of light exists in the z-direction. This anisotropy is due to the motion of the Earth in the Galaxy. The Earth's motion in the Galaxy causes additional spreading of the light ray in this direction: harmonic oscillations with a 24-hour period and amplitude $\frac{\mathrm{v}}{2}$, where v is the velocity of concomitant motion of the Earth with the Solar system in the Galaxy.

The metric describing a satellite's motion around the Earth as it moves concomitantly with the Earth in the gravitational field of the Sun is applied in this paper. The motion of a satellite by means of non-isotropic (non-null) geodesic lines equations is described. The motion of a satellite in a state of weightlessness is realised. The strong mathematical definition of this state in terms of physically observed (chronometrically invariant) quantities of A. L. Zelmanov [2, 3] is formulated. It is shown that the condition of weightlessness means that gravitational-inertial forces are absent in the region in which a satellite moves. The condition of weightlessness is a *condition of a equilibrium* between the gravitational (Newtonian) force F_N attracting a satellite towards the Earth's centre and the force F_ω directing it from the Earth. We called it the *inner force of non-holonomity*. We describe this force as a vector product of two quantities: (1) a pseudo-vector of the angular velocity of the Earth's daily rotation ω; (2) a vector of the linear velocity V of orbital motion of a satellite. The result of vectorial multiplication of these quantities is a pseudo-vector, directed always in the direction opposite to the force of gravitational attraction. If the forces of attraction and rejection are not equal one to other, a satellite: (1) falls to Earth if $F_\omega < F_N$; (2) escapes Earth if $F_\omega > F_N$. It is shown that the condition of weightlessness applies to all planets of the Solar system. Moreover, it is in accordance with Kepler's third law: *the cube of the mean distance of a planet from the Sun is proportional to the square of the period of rotation of the planet around the Sun*.

We obtain the exact solution of the non-isotropic (non-null) geodesic lines equations. It follows from them that the relativistic mass of a satellite in a state of weightlessness is constant; space velocities and space displacements in the r- and z-directions include additions caused by the Earth's daily rotation; the motion in the z-direction coinciding with the Earth's motion in the Solar system includes the effect which is described by harmonic oscillations having a period of 24 hours and an amplitude of 13 cm.

The question as to why the z-direction is preferred, is studied. It is shown that motion along the z-axis is also a rotational motion, with the angular velocity Ω, around the gravitational centre of a greater body. This body attracts the studied body and the gravitational centre around which the studied body rotates with the angular velocity ω. In order that this situation can be realized it is necessary that both these motions satisfy the condition of weightlessness.

It is shown that two kinds of forces exist, linked to a rotational motion. Because rotation of a space means that this space is non-holonomic [2, 3], we called these forces the inner and the outer force of non-holonomity, respectively. They have a different physical nature. From the physical viewpoint the inner force F_ω counteracts the Newtonian force F_N, the outer force F_Ω causes the motion in the z-direction. This action is an interaction of two rotations with the angular velocities ω and Ω, respectively. From the mathematical viewpoint these forces are different, because they are included in different terms of the space-time metric.

2 The weightlessness condition in terms of physical observable quantities

We consider, using the methods of General Relativity, the space of a body which: (1) rotates on its own axis, passing through its centre of gravity; (2) moves as a whole around the centre of gravity of a greater body. For example, the Earth rotates on its axis and simultaneously rotates around the Sun. The period of one rotation of the Earth on its axis is *one astronomical day*, or 86,400 sec. The linear velocity v_{rot} of this rotation depends on geographic latitude ϕ: $\mathrm{v}_{rot} = 500\cos\phi$ m/sec ($\mathrm{v}_{rot} = 0$ at the Earth's poles). The Earth rotates around the Sun with the velocity $\mathrm{v} = 30$ km/sec. The period of this rotation is *one astronomical year*, or 365.25 of astronomical days. The Earth's radius is 6,370 km, the distance between the Earth and the Sun is 150×10^6 km, and therefore we can consider the orbital motion of the Earth approximately as a forward motion.

We will consider every parallel of the Earth as a cylinder oriented in interplanetary space along the Earth's axis, passing through its poles. Every point of the Earth: (1) rotates around the axis with a velocity depending on its geographic latitude; (2) moves together with the Earth in the Sun's space with the velocity 30 km/sec. It is necessary to note that the points of the Earth space, which are on the Earth axis, move forward only. It is evident that, not only for the Earth's poles but also for all points along this direction, the linear velocity of rotation is zero. The combined motion of every point of the Earth's space (except axial points) is a very elongated spiral [1].

This metric is applicable to the general case of one body rotating around another body, moving concomitantly with the latter in the gravitational field of a greater body. For example, the Earth rotates around the Sun with the velocity 30 km/sec and simultaneously moves together with the Sun in the galactic space with the velocity 222 km/sec. The combined motion of the Earth motion in the Galaxy is described by a very elongated spiral. This case is studied in detail in [1]. The combined motion of every point of the Earth's surface in the Galaxy is more complicated trajectory.

The metric describing the space of a body which rotates around another body (or around its own centre of gravity) and moves together with the latter in the gravitational field of a greater body is [1]:

$$\begin{aligned} ds^2 &= \left(1-\frac{2GM}{c^2r}\right)c^2dt^2+\frac{2\omega r^2}{c}cdtd\varphi- \\ &-\left(1+\frac{2GM}{c^2r}\right)dr^2-r^2d\varphi^2+\frac{2\omega \mathrm{v} r^2}{c^2}d\varphi dz-dz^2, \end{aligned} \tag{1}$$

where $G = 6.67\times10^{-8}$ cm³/g×sec² is Newton's gravitational constant, ω is the angular velocity of the rotation around the axis, v is the orbital velocity of the body, r, φ and z are cylindrical coordinates. We direct the z-axis along a direction of a forward motion. This metric describes the motion of all points of the rotating body, besides axial points.

We apply Zelmanov's theory of physically observed quantities (chronometrically invariants) [2, 3] in order to describe this gravitational field. The three-dimensional observed space of the space-time (1) has a metric h_{ik} $(i = 1, 2, 3)$. Its components are

$$\begin{aligned} &h_{11}=1+\frac{2GM}{c^2r}\,, \quad h_{22}=r^2\left(1+\frac{\omega^2r^2}{c^2}\right), \\ &h_{23}=-\frac{\omega r^2\mathrm{v}}{c^2}\,, \qquad h_{33}=1; \\ &h^{11}=1-\frac{2GM}{c^2r}\,, \quad h^{22}=\frac{1}{r^2}\left(1-\frac{\omega^2r^2}{c^2}\right), \\ &h^{23}=\frac{\omega\mathrm{v}}{c^2}\,, \qquad h^{33}=1\,. \end{aligned} \tag{2}$$

Physically observed (chronometrically invariant) characteristics of this space are

$$F^1=\left(\omega^2r-\frac{GM}{r^2}\right)\left(1+\frac{\omega^2r^2}{c^2}\right), \tag{3}$$

$$A^{12}=-\frac{\omega}{r}\left(1-\frac{2GM}{c^2r}+\frac{\omega^2r^2}{2c^2}\right), \quad A^{31}=\frac{\omega^2\mathrm{v}r}{c^2}\,, \tag{4}$$

where F^i is a vector of a gravitational-inertial force, A^{ik} is a tensor of an angular velocity of a rotation (a tensor of a non-holonomity). The third characteristic is a tensor of velocities of a deformation $D_{ik} = 0$.

Geometric space characteristics of (2) are chronometrically invariant Christoffel symbols Δ^k_{ij} of the second kind:

$$\Delta^k_{ij}=h^{km}\Delta_{ij,m}=\frac{1}{2}h^{km}\left(\frac{{}^*\partial h_{im}}{\partial x^j}+\frac{{}^*\partial h_{jm}}{\partial x^i}-\frac{{}^*\partial h_{ij}}{\partial x^m}\right), \tag{5}$$

where $\Delta_{ij,m}$ are Christoffel symbols of the first kind, while $\frac{{}^*\partial}{\partial x^i}=\frac{\partial}{\partial x^i}-\frac{1}{c^2}v_i\frac{{}^*\partial}{\partial t}$ is chronometric differentiation with respect to spatial coordinates, and $\frac{{}^*\partial}{\partial t}$ is chronometric differentiation with respect to time. Because the gravitational field described by the metric (1) is stationary, we have $\frac{{}^*\partial}{\partial x^i}=\frac{\partial}{\partial x^i}$

The non-zero components of Δ^k_{ij} for (1) are

$$\begin{aligned} &\Delta^1_{11}=\frac{GM}{c^2r}\,, \quad \Delta^1_{22}=-r\left(1-\frac{2GM}{c^2r}+\frac{2\omega^2r^2}{c^2}\right), \\ &\Delta^1_{23}=\frac{\omega\mathrm{v}r}{c^2}\,, \quad \Delta^2_{12}=\frac{1}{r}\left(1+\frac{\omega^2r^2}{c^2}\right), \quad \Delta^2_{13}=-\frac{\omega\mathrm{v}}{c^2r}\,. \end{aligned} \tag{6}$$

Let's consider the particular case of this motion when the gravitational-inertial force is absent:

$$F^i=0\,. \tag{7}$$

We rewrite it for the metric (1) in the form

$$\frac{GM}{r}=\omega^2r^2=V^2, \tag{8}$$

where V is the linear velocity of a rotational motion.

Substituting into (8) the Earth's mass $M_\oplus = 6\times10^{27}$ g and the Earth's radius $R_\oplus = 6.37\times10^{8}$ cm we obtain the value of a velocity of a rotation $\mathrm{V} = 7.9$ km/sec. This value is the *first space velocity*, which we denote by V_I. If we accelerate a body located on the Earth in this way, so that its velocity acquires the value 7.9 km/sec, it will move freely in the gravitational field of the Earth as an Earth satellite. This means that condition (7) is the *weightlessness condition* in General Relativity, formulated in terms of physically observed quantities.

Substituting into (8) the mass of the Sun $M_\odot = 2\times10^{33}$ g and the distance between the Earth and the Sun $r = 15\times10^{12}$ cm we obtain $\mathrm{v} = 30$ km/sec — the orbital velocity of the Earth in the gravitational field of the Sun. This means that *the Earth rotates around the Sun in the state of weightlessness*.

Analogous calculations show [4] that *the orbital motion of the Moon around the Earth and orbital motions of all planets of the Solar system satisfy the weightlessness condition*. We conclude that the weightlessness condition is the condition by which the force of Newton's attraction F_N equals the force F_ω connected with a rotational motion. It is evident that this force must be directed opposite to that of the Newtonian force. It is possible to consider this force as a vector product of two quantities: (1) a pseudo-scalar of an angular velocity of rotation ω directed along the Earth's axis; (2) a vector $V = \omega \times r$ in a direction tangential to the satellite's orbit. Thus we have

$$F_\omega = \omega \times V = \omega \times \left[\omega \times r\right]. \tag{9}$$

This force is directed opposite to the Newtonian force in a right coordinate frame. Its value is $\omega \mathrm{V} \sin\alpha$, α the angle between these vectors; it equals $\omega^2 r$ if these quantities are orthogonal to one another.

We call this force the *inner force of non-holonomity*, because it acts on a body moving in the inner gravitational field of another body. This force is included in the g_{00}-component of the fundamental metric tensor $g_{\alpha\beta}$.

It is necessary to explain why we consider ω a pseudovector. In general, Zelmanov defines a pseudo-vector of an angular velocity $\Omega^i = \frac{1}{2}\,\varepsilon^{imn} A_{mn}$, where ε^{imn} is a completely antisymmetric chronometrically invariant unit tensor. For it we have $\varepsilon^{123} = \frac{1}{\sqrt{h}}$, where h is the determinant of a three-dimensional fundamental metric tensor h_{ik}.

Taking (8) into account, we calculate for the metric (1):

$$h = r^2\left(1 + \frac{3\omega^2 r^2}{c^2}\right), \tag{10}$$

$$A_{12} = -\omega r\left(1 + \frac{3\omega^2 r^2}{2c^2}\right), \tag{11}$$

with the other components of A_{ik} all zero. Consequently only the component $\Omega^3 = -\omega$ is not zero for this metric.

It is easy to calculate for all planets that orbital motion satisfies Kepler's third law.

3 The motion of a satellite in the gravitational field of the Earth

We consider the motion of a satellite in the gravitational field of the Earth rotating around the its own axis and moving in the gravitational field of the Sun (rotating around its centre). This is a motion of a free body, so it is consequently described by the geodesic equations

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0, \tag{12}$$

where $\frac{dx^\alpha}{ds}$ is a vector of a four-dimensional velocity, $\Gamma^\alpha_{\mu\nu}$ are four-dimensional Christoffel symbols. In terms of observed quantities, these equations have the form

$$\left.\begin{aligned} &\frac{dm}{d\tau} - \frac{m}{c^2}\,F_iV^i + \frac{m}{c^2}\,D_{ik}V^iV^k = 0, \\ &\frac{d(mV^i)}{d\tau} + 2m\left(D^i_k + A^{\cdot i}_{k\cdot}\right)V^k - mF^i + m\Delta^i_{nk}V^nV^k = 0, \end{aligned}\right\} \tag{13}$$

where τ is proper (observed) time, $V^i = \frac{dx^i}{d\tau}$ is a three-dimensional observed velocity, m is the relativistic mass of a satellite. It is evident that its gravitational field is negligible.

Substituting into these equations the calculated values of A^{ik} and Δ^k_{ij} for the metric (1) and taking into account the condition of weightlessness (8), we obtain a system of equations

$$\frac{dm}{d\tau} = 0\,, \tag{14}$$

$$\begin{aligned} &\frac{d}{d\tau}\left(m\frac{dr}{d\tau}\right) + 2m\omega r\left(1 - \frac{\omega^2r^2}{2c^2}\right)\frac{d\varphi}{d\tau} + \frac{m\omega^2 r}{c^2}\left(\frac{dr}{d\tau}\right)^2 - \\ &\qquad - mr\left(\frac{d\varphi}{d\tau}\right)^2 + \frac{2m\omega \mathrm{v} r}{c^2}\frac{d\varphi}{d\tau}\frac{dz}{d\tau} = 0\,, \end{aligned} \tag{15}$$

$$\begin{aligned} &\frac{d}{d\tau}\left(m\frac{d\varphi}{d\tau}\right) - \frac{2m\omega}{r}\left(1 + \frac{\omega^2r^2}{2c^2}\right)\frac{dr}{d\tau} + \\ &\quad + \frac{2m}{r}\left(1 + \frac{\omega^2r^2}{c^2}\right)\frac{dr}{d\tau}\frac{d\varphi}{d\tau} - \frac{2m\omega\mathrm{v}}{c^2 r}\frac{dr}{d\tau}\frac{dz}{d\tau} = 0\,, \end{aligned} \tag{16}$$

$$\frac{d}{d\tau}\left(m\frac{dz}{d\tau}\right) - \frac{2m\omega^2 \mathrm{v} r}{c^2}\frac{dr}{d\tau} = 0. \tag{17}$$

We obtain from equation (14) that the relativistic mass of a space body is, by a condition of weightlessness, constant: $m = \mathrm{const}$. Using this condition we calculate the first integral of equation (17)

$$\dot z = \dot z_0 + \frac{\omega^2 \mathrm{v}\left(r^2 - r_0^2\right)}{c^2}, \tag{18}$$

where $\dot z$ denotes differentiation with respect to τ, $\dot z_0$ and r_0 are initial values.

Taking into account that $m = \text{const}$ and also $\frac{GM}{c^2 r} = \frac{\omega^2 r^2}{c^2}$ (the condition of weightlessness) we rewrite (15) and (16) as

$$\ddot{r} + 2\omega r\left(1 - \frac{\omega^2 r^2}{2c^2}\right)\dot{\varphi} + \frac{\omega^2 r}{c^2}\dot{r}^2 - r\dot{\varphi}^2 + \frac{2\omega \mathrm{v} r}{c^2}\dot{\varphi}\dot{z} = 0, \qquad (19)$$

$$\ddot{\varphi} - \frac{2\omega}{r}\left(1 + \frac{\omega^2 r^2}{c^2}\right)\dot{r} + \frac{2}{r}\left(1 + \frac{\omega^2 r^2}{c^2}\right)\dot{r}\dot{\varphi} - \frac{2\omega \mathrm{v}}{c^2 r}\dot{r}\dot{z} = 0. \qquad (20)$$

The linear velocity of the Earth's rotation around its axis, $\omega r \cos\phi$, has the maximum value, at the equator, $\omega r = 500$ m/sec, and consequently the maximum value of $\frac{\omega^2 r^2}{c^2} = 6\times10^{-11}$. Substituting (18) into (19–20) and neglecting the terms $\frac{\omega^2 r^2}{c^2}$ and $\frac{\omega^2 r \dot{r}^2}{c^2}$, we obtain

$$\ddot{r} + 2\omega r\dot{\varphi} - r\dot{\varphi}^2 + \frac{2\omega \mathrm{v} r}{c^2}\,\dot{\varphi}\dot{z}_0 = 0\,, \qquad (21)$$

$$\ddot{\varphi} - 2\omega\,\frac{\dot{r}}{r} + 2\dot{\varphi}\,\frac{\dot{r}}{r} - \frac{2\omega \mathrm{v}\dot{z}_0}{c^2}\,\frac{\dot{r}}{r} = 0\,. \qquad (22)$$

We rewrite (24) in the form

$$\ddot{\varphi} + 2\left(\dot{\varphi} - \widetilde{\omega}\right)\frac{\dot{r}}{r} = 0\,, \qquad (23)$$

where $\widetilde{\omega} = \omega\left(1 + \frac{\mathrm{v}\dot{z}_0}{c^2}\right)$. The quantity $\widetilde{\omega}$ is the angular velocity of an Earth point daily rotation containing a correction $\frac{\mathrm{v}\dot{z}_0}{c^2}$ which is due to the orbital motion of the Earth around the Sun. It is necessary that we do not neglect the term $\frac{\mathrm{v}\dot{z}_0}{c^2}$, because its order is 2.7×10^{-9}: we propose $\mathrm{v} = 30$ km/sec (the orbital velocity of the Earth) and $\dot{z}_0 = 8$ km/sec (the initial value of the satellite velocity).

The variable in equation (23) can be separated, and therefore it is easily integrated. The first integral is

$$\dot{\varphi} = \widetilde{\omega} + \frac{\left(\dot{\varphi}_0 - \widetilde{\omega}\right) r_0^2}{r^2}\,, \qquad (24)$$

where $\dot{\varphi}$ and r_0 are initial values.

Substituting (24) into (21) we obtain, after transformations, the second order differential equation relative to r

$$\ddot{r} + \widetilde{\omega}^2 r - \frac{\left(\dot{\varphi}_0 - \widetilde{\omega}\right) r_0^4}{r^3} = 0\,. \qquad (25)$$

We introduce the new variable $p = \dot{r}$. Then $\ddot{r} = p\,\frac{dp}{dr}$ and (25) becomes

$$p\,dp = \frac{\left(\dot{\varphi}_0 - \widetilde{\omega}\right)^2 r_0^4}{r^3}\,dr - \widetilde{\omega}^2 r dr = 0\,, \qquad (26)$$

the variables of which are also separable. It is easily integrated to

$$\dot{r}^2 = \left(\frac{dr}{d\tau}\right)^2 = -\omega^2 r^2 - \frac{\left(\dot{\varphi}_0 - \widetilde{\omega}\right)^2 r_0^4}{r^2} + K, \qquad (27)$$

where the constant of integration K is

$$K = \dot{r}_0^2 + r_0^2\left[2\widetilde{\omega}^2 + \dot{\varphi}_0\left(\dot{\varphi}_0 - 2\widetilde{\omega}\right)\right]. \qquad (28)$$

We obtain

$$\dot{r} = \frac{dr}{d\tau} = \pm\sqrt{K - \widetilde{\omega}^2 r^2 - \frac{\left(\dot{\varphi}_0 - \widetilde{\omega}\right)^2 r_0^4}{r^2}}\,. \qquad (29)$$

This too is an equation with separable variables. Considering the positive sign we obtain, after elementary transformations,

$$d\tau = \frac{r dr}{\sqrt{-\widetilde{\omega}^2 r^4 + K r^2 - \left(\dot{\varphi}_0 - \widetilde{\omega}\right)^2 r_0^4}}\,. \qquad (30)$$

Introducing the new variable $y = r^2$ we have

$$d\tau = \frac{1}{2}\frac{dy}{\sqrt{-\widetilde{\omega}^2 y^2 + K y - \left(\dot{\varphi}_0 - \widetilde{\omega}\right)^2 r_0^4}}. \qquad (31)$$

Integrating (31) and returning to the old variable r we obtain the expression for τ

$$\tau = -\frac{1}{2\widetilde{\omega}}\arcsin\frac{K - 2\widetilde{\omega}^2 r^2}{\sqrt{K^2 - 4\widetilde{\omega}^2\left(\dot{\varphi}_0 - \widetilde{\omega}\right) r_0^4}} + B, \qquad (32)$$

where B is a constant of integration. Calculating $B = 0$ for the initial value of $\tau_0 = 0$ we rewrite (32) as

$$\sin 2\widetilde{\omega}\tau = \frac{2\widetilde{\omega}^2\left(r^2 - r_0^2\right)}{\sqrt{K^2 - 4\widetilde{\omega}^2\left(\dot{\varphi}_0 - \omega\right)^2 r_0^4}}\,, \qquad (33)$$

where r_0 is the initial value of r. It is easy to express r^2 as

$$r^2 = r_0^2 + \frac{\sqrt{K^2 - 4\widetilde{\omega}^2\left(\dot{\varphi}_0 - \widetilde{\omega}\right)}}{2\widetilde{\omega}^2}\sin 2\widetilde{\omega}\tau. \qquad (34)$$

Expressing $K^2 - 4\widetilde{\omega}^2 B^2$ through initial values we obtain

$$r = \sqrt{r_0^2 + \frac{Q}{2\widetilde{\omega}^2}\sin 2\widetilde{\omega}\tau}\,, \qquad Q = \text{const}, \qquad (35)$$

where $Q = \sqrt{\left(\dot{r}_0^2 + r_0^2\dot{\varphi}_0^2\right)\left[\dot{r}_0^2 + r_0^2(\dot{\varphi}_0 + 2\widetilde{\omega})^2\right]}$.

Substituting (35) into (18) and integrating the resulting expression we have

$$z = \dot{z}_0\tau + \frac{\mathrm{v}Q}{2\widetilde{\omega}c^2}\left(1 - \cos 2\widetilde{\omega}\tau\right) + z_0\,, \qquad (36)$$

where z_0 and $\dot{z}_0$ are initial values.

Substituting (34) into (24) and integrating we obtain for φ the expression

$$\varphi = \widetilde{\omega}\tau + \frac{2\widetilde{\omega}^2 r_0^2\left(\dot{\varphi}_0 - \widetilde{\omega}\right)}{\sqrt{Q^2 - 4\widetilde{\omega}^4 r_0^4}} \times \\ \times \ln\left|\frac{2\widetilde{\omega}^2 r_0^2\tan\widetilde{\omega}\tau + Q - \sqrt{Q^2 - 4\widetilde{\omega}^4 r_0^4}}{2\widetilde{\omega}^2 r_0^2\tan\widetilde{\omega}\tau + Q + \sqrt{Q^2 - 4\widetilde{\omega}^4 r_0^4}}\right| + P\,, \qquad (37)$$

where the integration constant equals $P = \varphi_0 - \frac{2\tilde{\omega}^2 r_0^2(\dot{\varphi}_0 - \tilde{\omega})}{Q^2 - 4\tilde{\omega}^4 r_0^4} \times \ln\left|\frac{Q - \sqrt{Q^2 - 4\tilde{\omega}^4 r_0^4}}{Q + \sqrt{Q^2 - 4\tilde{\omega}^4 r_0^4}}\right|$.

We see from (35–37) that trajectories of a freely falling satellite in the Earth's gravitational field conclude corrections for the daily rotation of the Earth. Besides that, the motion in the z-direction coinciding with a forward motion of the Earth includes the velocity of the orbital motion of the Earth around the Sun. Let's estimate the correction in the z-direction caused the orbital motion of the Earth with velocity 30 km/sec. In order to estimate the value Q, we propose that the satellite moved vertically at the initial moment. This means that only the radial component of the initial velocity is not zero: $\dot{r}_0 \neq 0$. Let it be equal to the first space velocity: $\dot{r}_0 \simeq V_I = 8$ km/sec. In this case $Q = V_I^2 \simeq 64\,\text{km}^2/\text{sec}^2$. Taking into account the angular velocity of the daily rotation $\omega = 8\times10^{-5}\,\text{sec}^{-1}$ we obtain the correction $\frac{\mathrm{v}Q}{2\tilde{\omega}c^2} = 13$ cm. This means that a satellite not only moves forward with a velocity $\dot{z}_0$ in the z-direction, it also undergoes harmonic oscillations with the amplitude 13 cm during the 24-hour period.

It is necessary to take into account these corrections in relation to some experiments with satellites. For example, experiments, the aim of which is to discover gravitational waves: two geostationary satellites are considered as two free particles. Measuring changes of the distance between them by means of laser interferometer, scientists propose to discover gravitational waves emitted by different space sources. It is evident, it is necessary to take into account changes of this distance caused by motion of satellites in the gravitational field of the Sun.

Let's study in detail why the z-direction is preferred. The displacement in the z-direction includes the velocity $\mathrm{v} = 30$ km/sec of the Earth's motion in the gravitational field of the Sun. We consider this motion as a "forward" motion. On the other hand, this motion is as well rotation, because the Earth rotates around the Sun. Therefore we can consider v as a vector product of two quantities

$$\mathrm{v} = \Omega \times R\,, \tag{38}$$

where $\Omega = 2\times10^{-7}\,\text{sec}^{-1}$ is the angular velocity of the Earth's orbital rotation, $R = 150\times10^6$ km is the distance between the Earth and the Sun.

Now we define the *outer force of non-holonomity* F_Ω as a force of a kind different toF_ω. This definition corresponds to the case where one body rotates around another as the latter rotates around a greater body. We define this force also as a force of non-holonomity, because Zelmanov proved that a rotation of a three-dimensional space means that this space is non-holonomic. The metric of the corresponding space-time in this case necessarily includes the mixed (space-time) terms g_{0i}, because it is impossible to transform coordinates in such a way that all $g_{0i} = 0$.

We define the outer force of non-holonomity as

$$F_\Omega = \omega \times \left[\Omega \times R\right], \tag{39}$$

where ω and Ω are angular velocities of two different rotations: ω is the angular velocity of rotation of a space body around a centre of attraction; Ω is the angular velocity of rotation of the concomitant rotation of a space body and its centre of attraction around a greater space body. The interaction of both rotations produces a real force, acting on masses the fields of which are in the region of this force.

We see that this force is included in metric (1) as an off-diagonal term $\frac{\omega \mathrm{v} r^2}{c^2}$. It is also contained in the chronometrically invariant Christoffel symbols (6). Solving the null geodesic lines equations for this metric, we obtained in [1] that an anisotropy of the velocity of light exists in the z-direction. The z-axis in (1) coincides with the direction of the concomitant motion of the Earth with the Solar system. This motion realises the velocity 222 km/sec. The anisotropy correction appearing in this direction as

$$\Delta\dot{z} = \frac{\mathrm{v}}{2}\,\sin 2\tilde{\omega}\tau\,, \tag{40}$$

where $\dot{z} = \frac{dz}{d\tau}$, $\tilde{\omega}$ is the angular velocity of the Earth's orbital motion. thus the value of $\dot{z}$ is realised during one astronomic year harmonic oscillation, with the amplitude 111 km/sec.

4 Conclusion. Further perspectives

We studied in this paper the motion of a satellite in the Earth's gravitational field. This motion is realised by the condition of weightlessness, defined as a state of equilibrium between two forces: the Newtonian force of attraction F_N and the force of repulsion F_ω, caused by a rotational motion. The existence of a rotation means the existence of a field of non-holonomity, and, consequently, the existence of forces of non-holonomity. The inner force of non-holonomity F_ω is a pseudo-tensor, always directed opposite to the direction to the centre of attraction. This is a real force countering the Newtonian force F_N. The equality of these two forces means that a satellite moves around the Earth in a state of weightlessness.

A satellite moves freely, and consequently moves along non-isotropic geodesic lines. We obtain from these equations that the relativistic mass of a satellite is constant. Displacements of a satellite in the r- and φ-directions include components caused by the daily rotation of the Earth. Solving the non-null geodesic lines equations describing its motion, we obtained from formula (36)

$$\Delta\dot{z} = \frac{\mathrm{v}Q}{2c^2} \simeq \frac{\mathrm{v}V_I^2}{2c^2} = 10^{-8}\ \text{km}, \tag{41}$$

where $\mathrm{v} = 30$ km/sec, $V_I = 8$ km/sec is the first space velocity. This correction is very small, but it has the same origin as the anisotropy of the velocity of light. Calculating the displacement of a satellite in the z-direction, we obtain the correction as a harmonic oscillation with the amplitude 13 cm and of 24-hour period.

The expression (36) is the exact solution of the equation (17). It is easy to see that the second term of (17) includes the quantity $m\omega\mathrm{v} = m\omega\Omega R$, where R is the distance between the Earth and the Sun. We can rewrite it as the angular momentum L of the outer force of non-holonomity

$$L = mF_{\Omega}R = m\omega\Omega R\,. \tag{42}$$

We conclude that:

1. If a body rotating around a centre of attraction also rotates with the latter around a greater origin of attraction, these fields of rotation interact.

This interaction exists only by the condition that both bodies rotate. The interaction of two fields of non-holonomity (the inner and the outer) causes an anisotropy in the velocity of light in the direction of the motion in the gravitational field of a greater body [1]. This interaction causes the displacement in this equation of a mass-bearing body (a satellite) obtained in the present paper. Both effects have the same nature: the angular moment of the outer force of non-holonomity deviates null and non-null trajectories of light-like particles and mass-bearing bodies.

We conclude that the inner and the outer forces of non-holonomity have a different nature, and therefore produce different effects on the motion of space bodies (and light).

2. The inner force of non-holonomity counters the Newtonian force of attraction. It is included in a three-dimensional potential of a gravitational field

$$\mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\right) \simeq \frac{GM}{r} + \frac{\omega^2 r^2}{2}\,. \tag{43}$$

This field of non-holonomity is linked to the weightlessness condition: the motion of a space body satisfies the weightlessness condition if $\frac{\partial \mathrm{w}}{\partial r} = 0$. This result follows from the definition of a gravitational-inertial force vector

$$F_i = \frac{1}{1 - \frac{\mathrm{w}}{c^2}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right). \tag{44}$$

We see that if a rotation is stationary (i. e. $\frac{\partial v_i}{\partial t} = 0$) the condition of weightlessness has the form $\frac{\partial \mathrm{w}}{\partial x^i} = 0$. It is evident that if a rotation is non-stationary, the condition of the weightlessness takes the form

$$\frac{\partial \mathrm{w}}{\partial x^i} = \frac{\partial v_i}{\partial t}\,. \tag{45}$$

It is interesting to note that a stationary rotation of a three-dimensional space is linked with motions of the space-time. It is shown in [4] that a stationary rotation of a three-dimensional space is a motion of the space-time itself due to the fact that a Lie derivative for this metric is zero.

3. The outer force of non-holonomity acts on the geometry of the space of transit of a body which rotates around another body and moves with the latter in the gravitational field of a greater body. It imparts energy to a moving (rotating) system of bodies, the gravitational fields of which are part of the gravitational field. We obtain the following chain: the gravitational field of the Earth (and all other planets) is a part of the Sun's gravitational field; the gravitational field of the Sun is a part of the galactic gravitational field, etc. All these space bodies are linked by gravitational forces and forces of non-holonomity. Its equilibrium is a necessary condition of existence for the Universe.

A study of spaces with non-stationary rotation is the theme of further papers. A necessary consideration of this problem involves the microwave radiation in the observed Universe. We have shown in the second part of [1] that the space-time satisfying to metric (1) can be permeated only by matter with stationary characteristics: a density, a stream of energy, a stress tensor, etc. Proposing that the Universe is filled by an ideal fluid (gas) and electromagnetic radiation, we showed that the electromagnetic field can only be stationary. If we consider this electromagnetic field as an electromagnetic wave, we conclude that these waves can only be standing waves. But observations show that in our Universe a microwave electromagnetic radiation exists. We therefore must initially choice a non-stationary metric. Such a metric can allow non-stationary electromagnetic radiation. It is possible that microwave radiation is linked with non-stationary fields of non-holonomity. But this is a theme for further studies.

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