Exact Mapping of Quantum Waves between Unruh's and Afshar's Setup (Reply to W. Unruh)

Danko Dimchev Georgiev

Kanazawa University Graduate School of Natural Science and Technology,
Kakuma-machi, Kanazawa-shi, Ishikawa-ken 920-1192, Japan
E-mail: danko@p.kanazawa-u.ac.jp

In a recent letter, Unruh argued that I have misrepresented his position and I have "*put words into his mouth*" which distort Unruh's original analysis of Unruh's setup. Unfortunately such a complaint is ungrounded. I have presented a mathematical argument that Unruh's which way claim for the discussed setup is equivalent to the claim for a mixed density matrix of the experiment. This is a mathematical proof, and has nothing to do with misrepresentation. Unruh clearly accepts the existence of the interference pattern at paths 5 and 6, accepts that the setup is described by pure state density matrix, and at the same time insists on existing which way bijection, therefore his position is provably mathematically inconsistent.

1 Direct calculation of detector states

Unruh in [6, 7] clearly has accepted the existence of unmeasured destructive interference at path 5 (pure state density matrix) plus a direct which way claim stating that $|\psi_1\rangle$ and $|\psi_2\rangle$ are respectively eigenstates of the detectors $\mathfrak{D}_1$ and $\mathfrak{D}_2$, thus it is easy for one to show that Unruh's analysis is mathematically inconsistent [2]. Despite of the fact that the mathematical analysis in my previous paper is rigorous, it was based on retrospective discussion deciding which waves shall annihilate, and which shall remain to be squared according to Born's rule. The choice for such a purely mathematical discussion was done in order to provide insight *why* Unruh's confusion arises. In this comment I will present concise physical description of the evolution of the photon based on direct forward-in-time calculation of Unruh's setup described in detail in [2], and will spot several troublesome claims made by Unruh, which appear to be severe mathematical misunderstandings.

For a coherent setup the quantum state in Unruh's interferometer after exit of beamsplitter 2 (BS2) is $|\Psi(t_1)\rangle = -1|\psi_6\rangle$, where $|\psi_6\rangle$ denotes the wavefunction evolving along path 6.* After reflection at mirror 3 (M3) the state evolves into $|\Psi(t_2)\rangle = -\imath|\psi_6\rangle$, which meets BS3 and splits into coherent superposition of two parts each going to one of the detectors

$$|\Psi(t_3)\rangle = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\imath\right)|\psi_6\rangle = \frac{1}{\sqrt{2}}|\mathfrak{D}_1\rangle - \frac{1}{\sqrt{2}}\imath|\mathfrak{D}_2\rangle \quad (1)$$

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*Here explictly should be noted that $|\psi_6\rangle$ is not just eigenstate of the position operator describing location at path 6, it is a wavefunction describing the photon state including its energy (wavelength), position, momentum, etc., that evolves in time and which may be represented as a vector (ket) in Hilbert space. As we speak about arbitrary photon with arbitrary energy, etc., the definition of the vector $|\psi_6\rangle$ is left flexible with the comprehension that it must describe fully the characteristics of the real photon. Also $|\psi_6\rangle$ is an unit vector, and as easily can be seen it must be multiplied by -1 in order for one to get the real state of the qubit at path 6.

from which follows that $|\mathfrak{D}_1\rangle = |\mathfrak{D}_2\rangle = |\psi_6\rangle$. Since $|\psi_6\rangle = \frac{1}{2}(|\psi_1\rangle + |\psi_2\rangle)$ it is obvious that $|\psi_1\rangle$ and $|\psi_2\rangle$ are *not* eigenstates of the detectors $\mathfrak{D}_1$ and $\mathfrak{D}_2$. That is why there is no which way information in coherent version of Unruh's setup. To suggest that the BS3 can selectively only reflect or only transmit the components $|\psi_1\rangle$ and $|\psi_2\rangle$ in a fashion preserving the which way correspondence is *mathematically equivalent* to detect photons at path 6, and then determine just a single path 1 or 2 along which the photon has arrived. Since it is impossible for one to distinguish the $|\psi_1\rangle$ component from the $|\psi_2\rangle$ component of a photon detected at path 6 it is perfectly clear that the BS3 cannot distinguish these components either, so standard QM prediction is that BS3 will "see" photon coming at path 6 but BS3 will not make any difference for $|\psi_1\rangle$ or $|\psi_2\rangle$ component of the photon state. BS3 will reflect both $|\psi_1\rangle$ and $|\psi_2\rangle$ to both detectors. The evolution of the state $-\imath|\psi_6\rangle$ into a coherent superposition going to both detectors providing no which way information is straightforward and can be characterized as "back-of-an-envelope calculation".[†]

Now let us investigate why if one prevents the interference along path 5 by converting the setup into a mixed one, the which way information will be preserved and the states $|\psi_1\rangle$ and $|\psi_2\rangle$ *will be* eigenstates of the corresponding detectors. First, one must keep in mind how the quantum entanglements (correlations) work in QM — due to the fact the photon wavefunction is entangled with the state of external system it is possible if one investigates only the reduced density matrix of the photon to see mixed state with all off-diagonal elements being zeroes, hence no interference effects manifested. This is the essence of Zeh's decoherence theory which does not violate Schrödinger equation and one

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[†]This expression was used by Prof. Tabish Qureshi (Jamia Millia Islamia, New Delhi, India) to describe how in just a few lines one can disprove Afshar's analysis and the calculation can be performed on the back side of an envelope for letters.

ends up with states that are not true classical mixtures, but have the same mathematical description satisfying the XOR gate. Thus let us put vertical polarizator **V** on path 1 and horizontal polarizator **H** on path 2. The state after BS2 will have non-zero component at path 5

$$|\Psi(t_1)\rangle = \left[-\frac{1}{2}\,\imath|\psi_1\rangle|\mathrm{V}\rangle + \frac{1}{2}\,\imath|\psi_2\rangle|\mathrm{H}\rangle\right] + \\ + \left[-\frac{1}{2}|\psi_1\rangle|\mathrm{V}\rangle - \frac{1}{2}|\psi_2\rangle|\mathrm{H}\rangle\right]. \quad (2)$$

Now as both wavefunctions ψ_1 and ψ_2 are orthogonal and distinguishable because of spatial separation (no overlap) in the interferometer arms 1 and 2, and because they get entangled with orthogonal states of the two different polarizators **V** and **H**, in the future spatial overlapping of the wavefunctions ψ_1 and ψ_2 cannot convert them into non-orthogonal states. Due to entanglement with polarizators the photon state is such that as if for ψ_1 the wavefunction ψ_2 does not exist, hence ψ_1 cannot overlap with ψ_2, and the state will be $\psi_1 \mathrm{XOR} \psi_2$.* At the detectors due to destructive quantum interference the ψ_2 waves will self-annihilate at $\mathfrak{D}_1$ and ψ_1 waves will self-annihilate at $\mathfrak{D}_2$. Thus $|\psi_1\rangle|\mathrm{V}\rangle$ and $|\psi_2\rangle|\mathrm{H}\rangle$ *will be* eigenstates of the corresponding detectors $\mathfrak{D}_1$ and $\mathfrak{D}_2$ (see details in [2]). This which way information is only existent because of the existent which way label which is the mixed state of photon polarization due to entanglements with the polarizators. In Unruh's single path setups the mixture of the photon states is result of obstacles on one of the interferometer paths, and then taking fictitious statistical average i.e. photons from the two alternative setups run in two distinguishable time intervals t_1 vs t_2. So in the classical mixture of two single path trials investigated by Unruh the time intervals t_1 and t_2 have the equivalent function of $|\mathrm{V}\rangle$ and $|\mathrm{H}\rangle$ entanglements. In order to complete the analogy one may explicitly write entanglements with orthogonal kets $|t_1\rangle$ and $|t_2\rangle$ describing the *interferometer quantum state* with obstacles on one of the two paths 1 or 2. Thus actually in the classical mixture discussed by Unruh it is $|\psi_1\rangle|t_1\rangle$ and $|\psi_2\rangle|t_2\rangle$ that are the eigenstates of the detectors. Destroying the mixture leads to loss of the which way information at the detectors.

Where was the essential step in the mixed setup that allowed us to recover the which way information? It was exactly the nonzero value of path 5. If in a coherent setup one allows for a state $0|\psi_5\rangle$ it is obvious that the vector $|\psi_5\rangle$ cannot be recovered without division to *zero*. Recovering of the which way information requires components included in the vector $|\psi_5\rangle$, thus one will be mathematically inconsistent if keeps the which way claim, and also claims that the state at path 5 is $0|\psi_5\rangle$ i.e. from that moment $|\psi_5\rangle$ is erased. It is obvious that in any QM calculation one can write the real state as a sum of infinite number of such terms of arbitrary

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*If however one erases the polarization the spatial overlap of the two waves will manifest interference and will erase the which way information.

vector states multiplied by *zero* without changing anything e.g. $|\Psi\rangle = |\Psi\rangle + 0|\Lambda\rangle + \cdots + 0|\Theta\rangle$. However all these *zeroed* components do not have physical significance.

And last but not least, it is clear that puting obstacle on place where the quantum amplitudes are expected to be *zero* does not change the mathematical description of the setup. Formally one may think as if having *Renninger negative-result experiment* [4] with the special case of measuring at place where the probability is *zero*. This is the only QM measurement that does not collapse the wavefunction of the setup! Analogously one may put obstacles in the space outside of the Unruh's interferometer. As the photon wavefunction is *zero* outside the interferometer it is naive one to expect that the photon wavefunction inside the interferometer is collapsed by the obstacles located around the interferometer. So puting obstacle or not, at place where the quantum amplitudes are *zero*, does not change the mathematical description. As this is always true, Unruh's idea that having obstacle or not at the negative interference area at path 5 will change the final conclusions of the which way information is wrong. As we have defined the which way information as provable bijection, it is unserious for one to believe that from a difference that has no effect on photon's wavefunction and does not change the mathematical description, one may change a mathematical proof of existent bijection.

2 Which way information as provable bijection

Now we will show that the naive statement that which way information and quantum interference are incompatible with each other is generally *false*. First one must define the which way information as a *provable bijection* between at least two distinguishable wavefunctions and two observables. Alternatively no which way information will be *disprovable bijection* i.e. the bijection is provably *false*. Then one can only say that *if* the bijection is *true* then quantum *cross-interference* of the two wavefunctions did not occur, yet *self-interference* is always possible! This was explicitly formulated in [2] however in the text bellow we discuss the idea in depth with the proposed Georgiev's four-slit experiment.

Let us us have four equally spaced identical slits A, B, C, D, and let us detect the interference pattern of photon at the *far-field Fraunhofer limit*. In case of coherent setup one will have coherent wavefunction $\Phi \equiv \Psi_A + \Psi_B + \Psi_C + \Psi_D$ and will observe a single four-slit interference $P = |\Psi_A + \Psi_B + \Psi_C + \Psi_D|^2$. This is a no which way distribution as far as we know that the photon amplitudes have passed through all four slits at once in quantum superposition.

Now let us put **V** polarizators on slits A and B, and **H** polarizators on slits C and D. There will be no *cross-interference* between the wavefunctions $\Phi_1 \equiv \Psi_A + \Psi_B$ and $\Phi_2 \equiv \Psi_C + \Psi_D$ and the observed intensity distribution will be mixed one $\mathcal{P} = |\Psi_A + \Psi_B|^2 + |\Psi_C + \Psi_D|^2$. In this case one can establish provable bijection $\Phi_1 \to P_1 \equiv |\Psi_A + \Psi_B|^2$,
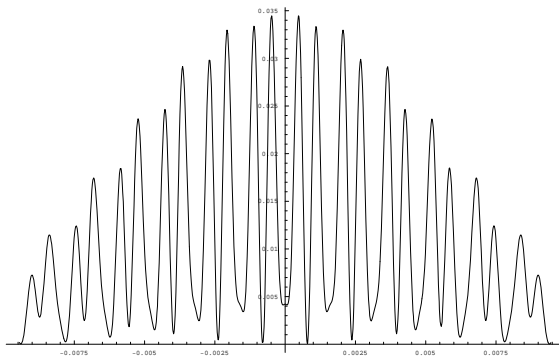
Fig. 1: The four slit interference pattern $P = |\Psi_A + \Psi_B + \Psi_C + \Psi_D|^2$ of non-polarized or identically polarized photons.
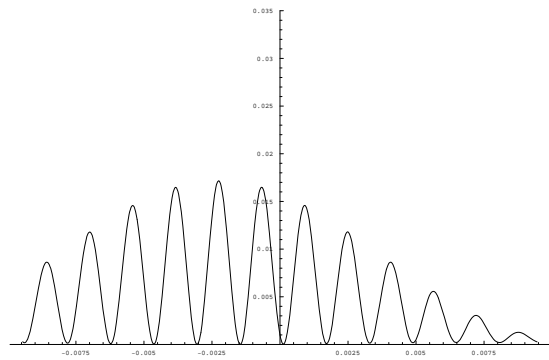
$\Phi_2 \rightarrow P_2 \equiv |\Psi_C + \Psi_D|^2$. Thus there is which way information $\Phi_1 \rightarrow P_1$, $\Phi_2 \rightarrow P_2$ only because there is no cross-interference between Φ_1 and Φ_2. The self-interferences of Φ_1 and Φ_2 are always there e.g. the cross-interference between Ψ_A and Ψ_B does not allows us to further prove existent bijection in which only slit A wavefunction, or only slit B wavefunction participates. In order to illustrate the discussion we have performed numerical plotting with *Wolfram's Mathematica 5.2* for photons with wavelength $\lambda = 850$ nm, slit width $s = 0.25$ mm, interslit distance $d = 2$ mm, at the Fraunhofer limit $z = 4.2$ m behind the four slits. Results are presented in Figures 1–3.

This section on the which way information as existent provable bijection was added for clarity. From the presented details it does *not* follow that Bohr's complementarity principle is wrong, we have just explicitly reformulated the principle providing strict definitions for *which way claims* as *bijections*, and have clarified the useful terms *self-interference* and *cross-interference*. If one investigates existent bijection then self-interference is always there, only certain cross-interferences are ruled out.

3 Quantum states as vectors

In this section we point out that QM can be approached in three ways. One way is to use wave equations with the prototype being the *Schrödinger equation*. One may write down a wave function $\Psi(x, t)$ that evolves both in space and time, where x is defined in $\mathbb{R}^3$. It is clear that the *history* of such mathematical function can be "traced" in time t, because the very defining of the wavefunction should be done by specifying its temporal evolution. Every wavefunction can be represented as a vector (ket) in Hilbert space. This is just second equivalent formulation, and changes nothing to the above definition. As the wavefunction evolves in time, it is clear that the vector representing the function will evolve in time too. It is the wavefunction that is referred to as *quantum state*, and it is the equivalent vector representing the wavefunction that is called *state vector*. Third way to represent the quantum state is with the use of *density*
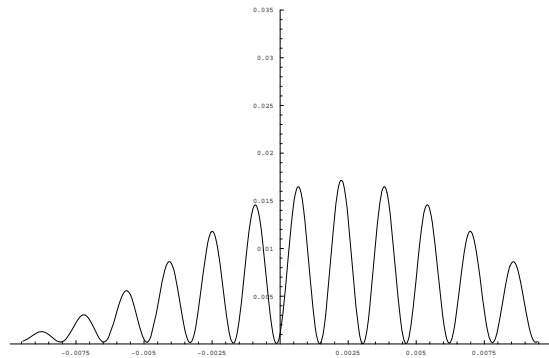

Fig. 2: Shifted to the left $P_1 = |\Psi_A + \Psi_B|^2$ double-slit interference pattern of vertically polarized photons.

Fig. 3: Shifted to the right $P_2 = |\Psi_C + \Psi_D|^2$ double-slit interference pattern of horizontally polarized photons.

matrices. In the previous work [2] we have used all three representations in order to provide more clear picture of Unruh's setup.

Namely, we have shown that the different wavefunctions if they manifest cross-interference are no more described by orthogonal vectors in Hilbert space. What is more the wavefunctions were "traced" in time in order for one to prove possible bijections. Surprisingly Unruh makes the following claim:

> "*Certainly amplitudes for the particle travelling along both path 1 and 5, say, exist, but amplitudes are just complex numbers. They are not states. And complex numbers can be added and subtracted no matter where they came from.*"

Such a misunderstanding of mathematical notation is not tolerable. As written in Eqs. 7–8 in [2] the usage of Dirac's ket notation is clear. All kets denote vectors (wavefunctions), hence all these are quantum states, and nowhere I have discussed only the quantum amplitude itself.

First, one should be aware that all kets are time dependent, as for example instead of writing $|\psi_1(t_1)\rangle$, $|\psi_1(t_2)\rangle$, $|\psi_1(t_3)\rangle$, $\cdots$ the notation was concisely written as $|\psi_1\rangle$ with the understanding that the state is a function of time. Even for two different points along the same interferometer arm, the spread of the laser beam (or the single photon wave-

packet) is different, yet this time dependence should be kept in mind without need for explicitly stating it. It is the time dependence of the state vectors that has been overlooked by Unruh. If one rejects the possibility to "trace" the history of the discussed wavefunctions in time, then he must accept the bizarre position that it is meaningless for one to speak about bijections and which way correspondences at first place.

Another target of Unruh's comment is the reality of the states $|\psi_{15}\rangle$, $|\psi_{16}\rangle$, $|\psi_{25}\rangle$, $|\psi_{26}\rangle$ in Eqs. 7–8 in [2].

> "[*Georgiev in*] *his equations 7 and 8 ascribes a state to the photon both passing along arm 1 or 2 and arm 5 or 6. In no conventional quantum formalism do such states exist.*"

Unfortunately this is wrong. Mathematically one can always represent a wavefunction as a sum of suitably defined functions. As it was clearly stated in [2] e.g. the state $|\psi_{15}\rangle$ is a wavefunction (vector, and not a scalar as erroneously argued by Unruh) which is branch of the wavefunction ψ_1 that evolves at arm 5. Therefore the mathematical definition is rigorous $\psi_1 = \alpha(t)(\psi_{15} + \psi_{16})$. One may analitically continue both functions ψ_{15} and ψ_{16} along path 1 as well, in this case the two functions are indistinguishable for times before BS2 with $\alpha = \frac{1}{2}$, while after BS2 the wavefunctions become distinguishable with $\alpha = \frac{1}{\sqrt{2}}$. The time dependence of $\alpha(t)$ is because the orthogonality of the two states is function of time. The usage of the same Greek letter with different numerical index as a *name* of a new function is standard mathematical practice in order to keep minimum the numer of various symbols used. The fact that the vector $|\psi_{15}\rangle$ is not orthogonal with the vector $|\psi_{25}\rangle$ in the coherent version of Unruh's setup is not a valid argument that it is not a valid quantum state. Mathematically it is well defined and whether it can be observed directly is irrelevant. Analogously, at path 6 the wavefunctions ψ_1 and ψ_2 are indistinguishable however mathematically they are still valid quantum states. Indistinguishability of states does not mean their non-existence as argued by Unruh. Indeed exactly because the two quantum functions $|\psi_{15}\rangle$ and $|\psi_{25}\rangle$ are defined in different way and have different time history, one may make them orthogonal by physical means. Simply putting obstacle at path 2, and then registering photon at path 5 one observes photons with intensity distribution $P_{15} = |\psi_{15}|^2$ which are solely contributed by ψ_{15}. And each photon only manifests "*passing along arm 1 and arm 5*". The other method to create mixed state where one can have bijective association of observables to each of the states $|\psi_{15}\rangle$, $|\psi_{16}\rangle$, $|\psi_{25}\rangle$, $|\psi_{26}\rangle$ is to put different polarizators **V** and **H** on paths 1 and 2, and then detect photons at paths 5 and 6. Due to polarizator entanglements there will be four observables and provable bijection $\psi_{15} \to P_{15}$, $\psi_{16} \to P_{16}$, $\psi_{25} \to P_{25}$, $\psi_{26} \to P_{26}$, where each probability distribution P is defined by the corresponding wavefunction squared and polarization of the photon dependent on the passage either through path 1

or path 2.

If Unruh's argument were true then it obviously can be applied to Unruh's own analysis, disproving the reality of the states $|\psi_1\rangle$ and $|\psi_2\rangle$ after BS2. As noted earlier, in the mixed state discussed by Unruh the state of the photon is either $|\psi_1\rangle|t_1\rangle$ or $|\psi_2\rangle|t_2\rangle$, where by $|t_1\rangle$ and $|t_2\rangle$ we denote two different distinguishable states of the Unruh's interferometer one with obstacle at path 2, and one with obstacle at path 1. It is exactly these entanglements with the external system being the interferometer itself and the obstacles that make the states $|\psi_1\rangle$ and $|\psi_2\rangle$ orthogonal at the detectors. If Unruh's logic were correct then removing the obstacles and making the two states not orthogonal at path 6 should be interpreted as non-existence for the two states. Fortunately, we have shown that Unruh's thesis is incorrect as is based on misunderstanding the difference between vector and scalar in the ket notation. All mentioned wavefunctions in [2] are well-defined mathematically and they are valid quantum states, irrespective of whether they are orthogonal with other states or not.

4 Classical language and complementarity

Unruh's confusion concerning the reality of quantum states, is grounded on some early *antirealist* misunderstandings of QM formalism. Still in some QM textbooks one might see expressions such as "*if the position of a qubit is precisely measured the momentum is largely unknown*", or "*if in the double slit setup a photon is detected at the Fraunhofer limit one will observe interference pattern but will not know which slit the photon has passed*". Such expressions are based on simple logical error — knowledge that "*the photon has not passed either only through slit 1, or only through slit 2*" is *not* mathematically equivalent to "*lack of knowledge which slit the photon has passed*".

Let us discuss a statistical mixture of two single slit experiments with shutter on one of the slits. What knowledge do we have? Certainly this is XOR knowledge, which means either one slit, or the other one, but not both! The truth-table was given in Table 1 in [2]. It is clear that exactly one of the statements "*passage through slit 1*" or "*passage through slit 2*" is *true*.

Now investigate the logical negation of the XOR gate. This essentially describes two possibilities. The first one is trivial with both slits *closed*. The photon does not pass through any slit, so no detection will occur at the Fraunhofer limit. A photon passed through slit 1 will be indistinguishable from photon passed through slit 2, but this is *vacuously true*. Simply no such photons exist! Much more interesting is however the coherent setup in which both slits are open. Logically one proves that the photon has passed through both slits at once. This is the essence of the quantum superposition and is described by AND logical gate. The statements "*passage through slit 1*" and "*passage through slit 2*" are

simultaneously *true*, and it is ruled out that only one of them is true but not the other. Therefore the *antirealist* position based on classical physical intuition, and/or classical language is erroneous when it comes to describe superposed state. The logical negation (NOT gate) of the XOR gate i.e. the XOR gate is *false*, is wrongly interpreted as "*lack of knowledge on the slit passage*" i.e. XOR gate possibly might be *true* or might be *false*. As this lack of knowledge is contradicting the QM formalism one runs directly into inconsistency with the theory.

Let us now see the implications for Unruh's objection e.g. against the ψ_{15} state. As in a coherent setup this state is superposed with the ψ_{25} state along path 5, Unruh argues that they are both nonexistent. This conclusion is *non sequitur*, because the quantum superposition is described by AND logical gate and this means that ψ_{15} and ψ_{25} are both true, hence existent states. Unruh relies on von Neumann formulation of QM, which is *antirealist* one, and rejects to accept the reality of quantum superposed states. This is untenable position because the antirealist vision interpreted as lack of precise knowledge of one of two non-commuting observables is mathematically inconsistent with the underlying mathematical formalism. It exactly the opposite — if one knows precisely the spatial region of the localization of qubit (having XOR knowledge ruling out other possible localizations) then mathematically it will follow that the momentum will be spread widely amongst numerous possible values (hence having AND knowledge). What is the reality of the AND state is outside the scope of the present article and depends on the interpretation - in MWI the superposed states reside in different Universes, in Penrose's OR model the quantum coherent state resides in a single Universe with superposed space-time curvatures, etc.

From the preceding discussion follows that expressions as "*which way information*" and "*no which way information*" are just *names* and have precise mathematical definitions as provable bijection b, and respectively disprovable bijection $\neg b$. Also we have logically proved that non-commuting observables are always existent and well-defined mathematically. However in contrast with classical intuition necessarily at least one of the two non-commuting observables should be described by AND gate, hence being quantum superposed.

5   Qureshi's waves mapped onto Georgiev's waves

One of the major differences between works of Georgiev [2] and Qureshi [4] is that in our previous paper we have introduced explicitly the idea of XOR and AND states in QM, and we have explicitly formulated the need of provable bijection. Otherwise Qureshi's argument is identical to the presented here forward-in-time calculation. Yet for the sake of clarity, we will provide one-to-one mapping of *Qureshi's waves for Afshar's setup* with *Georgiev's waves for Unruh's setup*. This one-to-one mapping is mathematically clear evi-

dence for existence of the quantum waves (states) described by Georgiev in [2] and leave no other alternative but one in which Unruh must confess his confusion in the complementarity debate.

As shown in [2] in retrospective discussion on wave annihilation, there will be eight waves that shall interfere. This is purely mathematical method, because mathematical truth is atemporal, and as explained before one either chooses self-interference of ψ_1 and self-interference of ψ_2 at detectors, or chooses destructive cross-interference between ψ_1 and ψ_2 at earlier times (path 5). Here we will show that the canceled sinh terms in Qureshi's calculation provide four more waves that go to both detectors and that one-to-one mapping exists with Georgiev's waves.

Let us denote all eight waves in Georgiev's description of Unruh's setup with ψ_{151}, ψ_{152}, ψ_{161}, ψ_{162}, ψ_{251}, ψ_{252}, ψ_{261}, ψ_{262}. As these are only *names*, the precise meaning for each one should be explicitly defined e.g. ψ_{151} is wavefunction whose history traced in time is passage along path 1, then passage along path 5, and ending at detector 1. Definitions for rest of the waves is analogous.

Now let us write again the Qureshi's equation for Afshar's setup

$$\Psi(y,t) = a\,C(t)\,e^{-\frac{y^2+y_0^2}{\Omega(t)}}\left[\cosh\frac{2yy_0}{\Omega(t)} + \sinh\frac{2yy_0}{\Omega(t)}\right] +$$
$$+ b\,C(t)\,e^{-\frac{y^2+y_0^2}{\Omega(t)}}\left[\cosh\frac{2yy_0}{\Omega(t)} - \sinh\frac{2yy_0}{\Omega(t)}\right]$$

where $C(t) = \frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon + 2\imath\hbar t/m\epsilon}}$, $\Omega(t) = \epsilon^2 + \frac{2\imath\hbar t}{m}$, a is the amplitude contribution from pinhole 1, b is the amplitude contribution from pinhole 2, ϵ is the width of the wavepackets, $2y_0$ is the slit separation. Qureshi's analysis continues directly with annihilation of four of the waves contributed by the sinh terms i.e. for Afshar's setup $a = b = \frac{1}{\sqrt{2}}$ so the sinh terms cancel out at the dark fringes. What is left at the bright fringes are the cosh terms, which can be expanded as a sum of exponential functions, namely $\cosh x = \frac{1}{2}(e^x + e^{-x})$, and after simplification we arrive at*:

$$\Psi(y,t) = \frac{1}{2}\,a\,C(t)\left[e^{-\frac{(y-y_0)^2}{\Omega(t)}} + e^{-\frac{(y+y_0)^2}{\Omega(t)}}\right] +$$
$$+ \frac{1}{2}\,b\,C(t)\left[e^{-\frac{(y-y_0)^2}{\Omega(t)}} + e^{-\frac{(y+y_0)^2}{\Omega(t)}}\right].$$

If a lens is used after the cross-interference has occurred to take the $e^{-\frac{(y-y_0)^2}{\Omega(t)}}$ part to detector 1, and the part $e^{-\frac{(y+y_0)^2}{\Omega(t)}}$ to detector 2, one easily sees that the amplitudes from each slit evolve into a superposition of two *identical parts* that go to both detectors. The waves that shall be responsible for which way information in *mixed setups* and make possible the bijection $a \to \mathfrak{D}_1$, $b \to \mathfrak{D}_2$ are hidden in the erased sinh

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*The following equation actually is the intended Eq. 10 in [2], where unfortunately *typesetting error* occurred.

terms. Taking into account that $\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$, one may recover the four zeroed sinh components in the form:

$$0 = \frac{1}{2}\,aC(t)\left[e^{-\frac{(y-y_0)^2}{\Omega(t)}} - e^{-\frac{(y+y_0)^2}{\Omega(t)}}\right] +$$
$$+ \frac{1}{2}\,bC(t)\left[-e^{-\frac{(y-y_0)^2}{\Omega(t)}} + e^{-\frac{(y+y_0)^2}{\Omega(t)}}\right].$$

If the eight interfering Qureshi's waves are denoted with Q, where Q_{1-4} arise from the cosh terms and Q_{5-8} arise from the sinh terms, then the one-to-one mapping with the eight Georgiev's waves is

$$Q_1 \equiv \frac{1}{2}\,aC(t)\,e^{-\frac{(y-y_0)^2}{\Omega(t)}} \rightarrow \psi_{161} \qquad (3)$$

$$Q_2 \equiv \frac{1}{2}\,aC(t)\,e^{-\frac{(y+y_0)^2}{\Omega(t)}} \rightarrow \psi_{162} \qquad (4)$$

$$Q_3 \equiv \frac{1}{2}\,bC(t)\,e^{-\frac{(y-y_0)^2}{\Omega(t)}} \rightarrow \psi_{261} \qquad (5)$$

$$Q_4 \equiv \frac{1}{2}\,bC(t)\,e^{-\frac{(y+y_0)^2}{\Omega(t)}} \rightarrow \psi_{262} \qquad (6)$$

$$Q_5 \equiv \frac{1}{2}\,aC(t)\,e^{-\frac{(y-y_0)^2}{\Omega(t)}} \rightarrow \psi_{151} \qquad (7)$$

$$Q_6 \equiv -\frac{1}{2}\,aC(t)\,e^{-\frac{(y+y_0)^2}{\Omega(t)}} \rightarrow \psi_{152} \qquad (8)$$

$$Q_7 \equiv -\frac{1}{2}\,bC(t)\,e^{-\frac{(y-y_0)^2}{\Omega(t)}} \rightarrow \psi_{251} \qquad (9)$$

$$Q_8 \equiv \frac{1}{2}\,bC(t)\,e^{-\frac{(y+y_0)^2}{\Omega(t)}} \rightarrow \psi_{252} \qquad (10)$$

To our knowledge this is the first exact one-to-one mapping between Unruh's setup and Afshar's setup, all previous discussions were much more general and based on analogy [2, 6]. Now one can explicitly verify that a and b terms in Qureshi's calculation have the same meaning as path 1 and path 2 in Unruh's setup; sinh and cosh terms have the meaning of the path 5 and path 6, and $e^{-\frac{(y-y_0)^2}{\Omega(t)}}$ and $e^{-\frac{(y+y_0)^2}{\Omega(t)}}$ terms have the meaning of detection at $\mathfrak{D}_1$ or $\mathfrak{D}_2$. The provided exact mapping between Qureshi's and Georgiev's work is clear evidence that Unruh's complaint for Georgiev's waves not being valid quantum states is invalid. None of the proposed by Georgiev states is being *zero*. Only couples of Georgiev's states can be collectively zeroed, but which members will enter in the zeroed couples depends on the density matrix of the setup. And this is just the complementarity in disguise.

6 Conclusions

In recent years there has been heated debate whether complementarity is more fundamental than the uncertainty principle [5, 8], which ended with conclusion that complementarity is enforced by quantum entanglements and not by uncertainty principle itself [1]. Indeed the analysis of the proposed here Georgiev's four-slit experiment, as well as the analysis of Unruh's and Afshar's setups, show that which way claims defined as provable bijections are just another mathematical expression of the underlying density matrix of the setup, and as discussed earlier diagonalized mixed density matrices in standard Quantum Mechanics are possible only if one considers quantum entanglements in the context of Zeh's decoherence theory [9].

Unruh's error is that he uses results from mixed state setup to infer which way correspondence in coherent setup, overlooking the fact that bijections must be mathematically proved. Therefore it is not necessary for one to measure the interference in order to destroy the which way claim, it is sufficient only to *know* the interference is existent in order to disprove the claimed bijection. Indeed in the presented calculations for Unruh's setup we have proved that Unruh's which way bijection is *false*. Hence Unruh's analysis is mathematically inconsistent.

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