The Spacetime Structure of Open Quantum Relativity

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In the framework of the Open Quantum Relativity, we discuss the geodesic and chronological structures related to the embedding procedure and dimensional reduction from 5D to 4D spacetime. The emergence of an extra-force term, the deduction of the masses of particles, two-time arrows and closed time-like solutions are considered leading to a straightforward generalization of causality principle.

1 Introduction

Open Quantum Relativity [1] is a theory based on a dynamical unification scheme [2] of fundamental interactions achieved by assuming a 5D space which allows that the conservation laws are always and absolutely valid as a natural necessity. What we usually describe as violations of conservation laws can be described by a process of embedding and dimensional reduction, which gives rise to an induced-matter theory in the 4D space-time by which the usual masses, spins and charges of particles, naturally spring out. At the same time, it is possible to build up a covariant symplectic structure directly related to general conservation laws [3, 4]. Finally, the theory leads to a dynamical explanation of several paradoxes of modern physics (*e.g.* entanglement of quantum states, quantum teleportation, gamma ray bursts origin, black hole singularities, cosmic primary antimatter absence and a self-consistent fit of all the recently observed cosmological parameters [2, 5, 7, 8, 9]). A fundamental rôle in this approach is the link between the geodesic structure and the field equations of the theory *before* and *after* the dimensional reduction process. The emergence of an Extra Force term in the reduction process and the possibility to recover the masses of particles, allow to reinterpret the Equivalence Principle as a dynamical consequence which naturally "selects" geodesics from metric structure and vice-versa the metric structure from the geodesics. It is worth noting that, following Schrödinger [10], in the Einstein General Relativity, geodesic structure is "imposed" by choosing a Levi-Civita connection [12] and this fact can be criticized considering a completely "affine" approach like in the Palatini formalism [13]. As we will show below, the dimensional reduction process gives rise to the generation of the masses of particles which emerge both from the field equations and the embedded geodesics. Due to this result, the coincidence of chronological and geodesic structure is derived from the embedding and a new dynamical formulation of the Equivalence Principle is the direct consequence of dimensional reduction. The dynamical structure is further rich since two time arrows and closed time-like paths naturally emerge. This fact leads to a reinterpretation of the standard notion of causality which can be, in this way, always recovered, even in the case in which it is questioned (like in entanglement phenomena and quantum teleportation [5, 6]), because it is generalized to a *forward* and a *backward causation*.

The layout of the paper is the following. In Sec.2, we sketch the 5D approach while in Sec.3 we discuss the rôle of conservation laws. Sec.4 is devoted to the discussion of geodesic structure and to the emergence of the Extra Force term. The field equations, the masses of the particles and time-like solutions are discussed in Sec.5. Conclusions are drawn in Sec.6.

2 The 5D-field equations

Open Quantum Relativity can be framed in a 5D space-time manifold and the 4D reduction procedure induces a scalar-tensor theory of gravity where conservation laws (*i.e.* Bianchi identities) play a fundamental rôle into dynamics. The 5D-manifold which we are taking into account is a Riemannian space provided with a 5D-metric of the form

$$dS^2 = g_{AB}\, dx^A dx^B\,, \tag{1}$$

where the Latin indexes are $A, B = 0, 1, 2, 3, 4$. We do not need yet to specify the 5D signature, because, in 4D, it is dynamically fixed by the reduction procedure as we shall see below. The curvature invariants, the field equations and the conservation laws in the 5D-space can be defined as follows. In general, we ask for a space which is a singularity free, smooth manifold, where conservation laws are always valid [7]. The 5D-Riemann tensor is

$$R^D_{ABC} = \partial_B \Gamma^D_{AC} - \partial_C \Gamma^D_{AB} + \Gamma^D_{EB}\Gamma^E_{AC} - \Gamma^D_{EC}\Gamma^E_{AB} \tag{2}$$

and the Ricci tensor and scalar are derived from the contractions

$$R_{AB} = R^C_{ACB}\,, \qquad {}^{(5)}R = R^A_A\,. \tag{3}$$

The field equations can be obtained from the 5D-action

$$ {}^{(5)}\mathcal{A} = -\frac{1}{16\pi\,{}^{(5)}G}\int d^5x\sqrt{-g^{(5)}}\left[{}^{(5)}R\right], \qquad (4) $$

where ${}^{(5)}G$ is the 5D-gravitational coupling and $g^{(5)}$ is the determinant of the 5D-metric [2]. The 5D-field equations are

$$ G_{AB} = R_{AB} - \frac{1}{2}\,g_{AB}\,{}^{(5)}R = 0\,, \qquad (5) $$

so that at least the Ricci-flat space is always a solution. Let us define now a 5D-stress-energy tensor for a scalar field Φ:

$$ T_{AB} = \nabla_A\Phi\,\nabla_B\Phi - \frac{1}{2}\,g_{AB}\,\nabla_C\Phi\,\nabla^C\Phi\,, \qquad (6) $$

where only the kinetic terms are present. As standard, such a tensor can be derived from a variational principle

$$ T^{AB} = \frac{2}{\sqrt{-g^{(5)}}}\frac{\delta\left(\sqrt{-g^{(5)}}\,\mathcal{L}_\Phi\right)}{\delta g_{AB}}\,, \qquad (7) $$

where $\mathcal{L}_\Phi$ is a Lagrangian density related to the scalar field Φ. Because of the definition of 5D space itself, based on the conservation laws [7], it is important to stress now that no self-interaction potential $U(\Phi)$ has to be taken into account so that T_{AB} is a completely symmetric object and Φ is, by definition, a cyclic variable. In this situation the Noether theorem always holds for T_{AB}. With these considerations in mind, the field equations can assume the form

$$ R_{AB} = \chi\left(T_{AB} - \frac{1}{2}\,g_{AB}\,T\right), \qquad (8) $$

where T is the trace of T_{AB} and $\chi = 8\pi\,{}^{(5)}G$.

3 The rôle of conservation laws

Eqs. (8) are useful to put in evidence the rôle of the scalar field Φ, if we are not simply assuming Ricci-flat 5D-spaces. Due to the symmetry of the stress-energy tensor T_{AB} and the Einstein field equations G_{AB}, the contracted Bianchi identities

$$ \nabla_A\,T^A_B = 0\,, \qquad \nabla_A\,G^A_B = 0\,, \qquad (9) $$

must always hold. Developing the stress-energy tensor, we obtain

$$ \nabla_A\,T^A_B = \Phi_B\,{}^{(5)}\Box\Phi\,, \qquad (10) $$

where ${}^{(5)}\Box$ is the 5D d'Alembert operator defined as $\nabla_A\Phi^A \equiv \equiv g^{AB}\Phi_{,A;B} \equiv {}^{(5)}\Box\Phi$. The general result is that the conservation of the stress-energy tensor T_{AB} (*i.e.* the contracted Bianchi identities) implies the Klein-Gordon equation which assigns the dynamics of Φ, that is

$$ \nabla_A\,T^A_B = 0 \qquad \Longleftrightarrow \qquad {}^{(5)}\Box\Phi = 0\,. \qquad (11) $$

Let us note again the absence of self-interactions due to the absence of potential terms. The relations (11) give a physical meaning to the fifth dimension. Splitting the 5D-problem in a $(4+1)$-description, it is possible to generate the mass of particles in 4D. Such a result can be deduced both from Eq. (11) and from the analysis of the geodesic structure, as we are going to show.

4 The 5D-geodesics and the Extra Force

The geodesic structure of the theory can be derived considering the action

$$ \mathcal{A} = \int dS\left(g_{AB}\frac{dx^A}{dS}\frac{dx^B}{dS}\right)^{1/2}, \qquad (12) $$

whose Euler-Lagrange equations are the geodesic equations

$$ \frac{d^2x^A}{dS^2} + \Gamma^A_{BC}\frac{dx^B}{dS}\frac{dx^C}{dS} = 0\,. \qquad (13) $$

Γ^A_{BC} are the 5D-Christoffel symbols. Eq. (13) can be split in the $(4+1)$ form

$$ 2g_{\alpha\mu}\left(\frac{dx^\alpha}{ds}\right)\left(\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\beta\gamma}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds}\right) + {} + \frac{\partial g_{\alpha\beta}}{dx^4}\frac{dx^4}{ds}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0\,, \qquad (14) $$

where the Greek indexes are $\mu,\nu = 0,1,2,3$ and $ds^2 = = g_{\alpha\beta}dx^\alpha dx^\beta$. Clearly, in the 4D reduction (*i.e.* in the usual spacetime) we ordinarily experience only the standard geodesics of General Relativity, *i.e.* the 4D component of Eq. (14)

$$ \frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\beta\gamma}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds} = 0\,, \qquad (15) $$

so that, under these conditions, the last part of the representation given by Eq. (14) is not detectable in 4D. In other words, for standard laws of physics, the metric $g_{\alpha\beta}$ does not depend on x^4 in the embedded 4D manifold. On the other hand, the last component of Eq. (14) can be read as an "Extra Force" which gives the motion of a 4D frame with respect to the fifth coordinate x^4. This fact shows that the fifth dimension has a *real physical meaning* and any embedding procedure scaling up in 5D-manifold (or reducing to 4D spacetime) has a dynamical description. The Extra Force

$$ \mathcal{F} = \frac{\partial g_{\alpha\beta}}{dx^4}\frac{dx^4}{ds}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}\,, \qquad (16) $$

is related to the mass of moving particles in 4D and to the motion of the whole 4D frame. This means that the emergence of this term in Eq. (14), leaving the 5D-geodesic equation verified, gives a new interpretation to the Equivalence Principle in 4D as a dynamical consequence. Looking at Eqs. (14) and (15), we see that in the ordinary 4D spacetime no term, in Eq. (15), is directly related to the masses which are, on the contrary, existing in Eq. (14). In other words, it is the quantity $\mathcal{F}$, which gives the masses to the particles, and this means that the Equivalence Principle can be formulated on a dynamical base by an embedding process. Furthermore the massive particles are different but massless in 5D while, for the physical meaning of the fifth coordinate, they assume mass in 4D thanks to Eq. (16).

Let us now take into account a 5D-null path given by

$$dS^2 = g_{AB}\, dx^A dx^B = 0\,. \tag{17}$$

Splitting Eq. (17) into the 4D part and the fifth component, gives

$$dS^2 = ds^2 + g_{44}\left(dx^4\right)^2 = 0\,. \tag{18}$$

An inspection of Eq. (18) tells that a null path in 5D can result, in 4D, in a time-like path, a space-like path, or a null path depending on the sign and the value of g_{44}. Let us consider now the 5D-vector $u^A = dx^A/dS$. It can be split as a vector in the ordinary 3D-space v, a vector along the ordinary time axis w and a vector along the fifth dimension z. In particular, for 5D null paths, we can have the velocity $v^2 = w^2 + z^2$ and this should lead, in 4D, to super-luminal speed, explicitly overcoming the Lorentz transformations. The problem is solved if we consider the 5D-motion as *a-luminal*, because all particles and fields have the same speed (being massless) and the distinction among super-luminal, luminal and sub-luminal motion (the standard causal motion for massive particles) emerges only *after* the dynamical reduction from 5D-space to 4D spacetime. In this way, the fifth dimension is the entity which, by assigning the masses, is able to generate the different dynamics which we perceive in 4D. Consequently, it is the process of mass generation which sets the particles in the 4D light-cone. Specifically, let us rewrite the expression (16) as

$$\mathcal{F} = \frac{\partial g_{\mu\nu}}{\partial x^4}\frac{dx^4}{ds}\, u^\mu u^\nu. \tag{19}$$

As we said, seen in 4D, this is an Extra Force generated by the motion of the 4D frame with respect to the extra coordinate x^4. This fact shows that all the different particles are massless in 5D and acquire their rest masses m_0 in the dynamical reduction from the 5D to 4D. In fact, considering Eqs. (14) and (18), it is straightforward to derive

$$\mathcal{F} = u^\mu u^\nu\, \frac{\partial g_{\mu\nu}}{\partial x^4}\frac{dx^4}{ds} = \frac{1}{m_0}\frac{dm_0}{ds} = \frac{d\ln(m_0)}{ds}\,, \tag{20}$$

where m_0 has the rôle of a rest mass in 4D, being, from General Relativity,

$$\frac{dx^\mu}{ds} - \frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial x^\mu}\, u^\alpha u^\beta = 0 \tag{21}$$

and

$$p^\mu = m_0 u^\mu, \qquad p_\mu p^\mu = m_0^2\,, \tag{22}$$

which are, respectively, the definition of linear momentum and the mass-shell condition. Then, it is

$$d\ln(m_0) = \frac{\partial g_{\mu\nu}}{\partial x^4}\, u^\mu u^\nu dx^4 \tag{23}$$

that is

$$m_0 = \exp\int\left(\frac{\partial g_{\mu\nu}}{\partial x^4}\, u^\mu u^\nu dx^4\right) = \exp\int\left(\mathcal{F}dx^4\right). \tag{24}$$

In principle, the term $\int\left(\frac{\partial g_{\mu\nu}}{\partial x^4}\, u^\mu u^\nu dx^4\right)$ never gives a zero mass. However, this term can be less than zero and, with large absolute values, it can asymptotically produce a m_0 very close to zero. In conclusion the Extra Force induced by the reduction from the 5D to the 4D is equal to the derivative of the natural logarithm of the rest mass of a particle with respect to the $(3+1)$ line element and the expression

$$\int\left(\frac{\partial g_{\mu\nu}}{\partial x^4} u^\mu u^\nu dx^4\right) = \int\left(\mathcal{F}dx^4\right) \tag{25}$$

can be read as the total "work" capable of generating masses in the reduction process from 5D to 4D.

5 The field structure and the chronological structure

The results of previous section assume a straightforward physical meaning considering the fifth component of the metric as a scalar field. In this way, the pure "geometric" interpretation of the Extra Force can be framed in a "material" picture. In order to achieve this goal, let us consider the Campbell theorem [15] which states that it is always possible to consider a 4D Riemannian manifold, defined by the line element $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, embedded in a 5D one with $dS^2 = g_{AB}\, dx^A dx^B$. We have $g_{AB} = g_{AB}\left(x^\alpha, x^4\right)$ with x^4 the extra coordinate. The metric g_{AB} is covariant under the group of 5D coordinate transformations $x^A \to \overline{x}^A(x^B)$, but not under the restricted group of 4D transformations $x^\alpha \to \overline{x}^\alpha(x^\beta)$. This means, from a physical point of view, that the choice of the 5D coordinate can be read as the *gauge* which specifies the 4D physics. On the other hand, the signature and the value of the fifth coordinate is related to the dynamics generated by the physical quantities which we observe in 4D (mass, spin, charge). Let us start considering the variational principle

$$\delta\int d^{(5)}x\sqrt{-g^{(5)}}\left[{}^{(5)}R + \lambda(g_{44} - \epsilon\Phi^2)\right] = 0\,, \tag{26}$$

derived from (4) where λ is a Lagrange multiplier, Φ a generic scalar field and $\epsilon = \pm 1$. This procedure allows to derive the physical gauge for the 5D metric. The above 5D metric can be immediately rewritten as

$$\begin{aligned} dS^2 = g_{AB}\, dx^A dx^B = g_{\alpha\beta} dx^\alpha dx^\beta + g_{44}\left(dx^4\right)^2 = \\ = g_{\alpha\beta}\, dx^\alpha dx^\beta + \epsilon\, \Phi^2\left(dx^4\right)^2, \end{aligned} \tag{27}$$

where the signature $\epsilon = -1$ can be interpreted as "particle like" solutions while $\epsilon = +1$ gives rise to wave-like solutions. The physical meaning of these distinct classes of solutions, as we will see below, is crucial. Assuming a standard signature $(+ - - -)$ for the 4D component of the metric, the 5D metric can be written as the matrix

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & \epsilon\,\Phi^2 \end{pmatrix}, \tag{28}$$

and the 5D Ricci curvature tensor is

$$ {}^{(5)}R_{\alpha\beta} = R_{\alpha\beta} - \frac{\Phi_{,\alpha;\beta}}{\Phi} + \frac{\epsilon}{2\Phi^2}\left(\frac{\Phi_{,4}\, g_{\alpha\beta,4}}{\Phi} - \right. \\ \left. - g_{\alpha\beta,44} + g^{\lambda\mu} g_{\alpha\lambda,4}\, g_{\beta\mu,4} - \frac{g^{\mu\nu} g_{\mu\nu,4}\, g_{\alpha\beta,4}}{2}\right) \tag{29}$$

where $R_{\alpha\beta}$ is the 4D Ricci tensor. After the projection from 5D to 4D, $g_{\alpha\beta}$, derived from g_{AB}, no longer explicitly depends on x^4, and then the 5D Ricci scalar assumes the remarkable expression:

$$ {}^{(5)}R = R - \frac{1}{\Phi}\,\Box\Phi\,, \tag{30}$$

where the $\Box$ is now the 4D d'Alembert operator. The action in Eq. (26) can be recast in a 4D Brans-Dicke form

$$\mathcal{A} = -\frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,\left[\Phi R + \mathcal{L}_\Phi\right], \tag{31}$$

where the Newton constant is given by

$$G_N = \frac{{}^{(5)}G}{2\pi l} \tag{32}$$

where l is a characteristic length in 5D. Defining a generic function of a 4D scalar field φ as

$$-\frac{\Phi}{16\pi G_N} = F(\varphi) \tag{33}$$

we get a 4D general action in which gravity is nonminimally coupled to a scalar field [2, 16, 17]:

$$\mathcal{A} = \int_{\mathcal{M}} d^4x\, \times \\ \times \sqrt{-g}\left[F(\varphi)R + \frac{1}{2}\, g^{\mu\nu}\varphi_{;\mu}\,\varphi_{;\nu} - V(\varphi) + \mathcal{L}_m\right] \tag{34}$$

$F(\varphi)$ and $V(\varphi)$ are a generic coupling and a self interacting potential respectively. The field equations can be derived by varying with respect to the 4D metric $g_{\mu\nu}$

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}R = \tilde{T}_{\mu\nu}\,, \tag{35}$$

where

$$\tilde{T}_{\mu\nu} = \frac{1}{F(\varphi)}\left\{-\frac{1}{2}\,\varphi_{;\mu}\varphi_{;\nu} + \frac{1}{4}\, g_{\mu\nu}\varphi_{;\alpha}\varphi^{;\alpha} - \right. \\ \left. - \frac{1}{2}\, g_{\mu\nu}V(\varphi) - g_{\mu\nu}\Box F(\varphi) + F(\varphi)_{;\mu\nu}\right\} \tag{36}$$

is the effective stress–energy tensor containing the nonminimal coupling contributions, the kinetic terms and the potential of the scalar field φ. By varying with respect to φ, we get the 4D Klein-Gordon equation

$$\Box\varphi - RF'(\varphi) + V'(\varphi) = 0\,, \tag{37}$$

where primes indicate derivatives with respect to φ.

Eq. (37) is the contracted Bianchi identity demonstrating the recovering of conservation laws also in 4D [2]. This feature means that the effective stress-energy tensor at right hand side of (35) is a zero-divergence tensor and this fact is fully compatible with Einstein theory of gravity also starting from a 5D space. Specifically, the reduction procedure from 5D to 4D preserves all the features of standard General Relativity. In order to achieve the physical identification of the fifth dimension, let us recast the generalized Klein-Gordon equation (37) as

$$\left(\Box + m^2_{\text{eff}}\right)\varphi = 0\,, \tag{38}$$

where

$$m^2_{\text{eff}} = \left[V'(\varphi) - RF'(\varphi)\right]\varphi^{-1} \tag{39}$$

is the effective mass, *i.e.* a function of φ, where self-gravity contributions $RF'(\varphi)$ and scalar field self interactions $V'(\varphi)$ are taken into account [18]. This means that a natural way to generate the masses of particles can be achieved starting from a 5D picture and the concept of *mass* can be recovered as a geometric derivation according to the Extra Force of previous section. In other words, the chronological structure and the geodesic structure of the reduction process from 5D to 4D naturally coincide since the the masses generated in both cases are equivalent. From an epistemological point of view, this new result clearly demonstrates why geodesic structure and chronological structure can be assumed to coincide in General Relativity using the Levi-Civita connection in both the Palatini and the metric approaches [13]. Explicitly the 5D d'Alembert operator can be split, considering the 5D metric in the form (27) for particle-like solutions:

$$ {}^{(5)}\Box = \Box - {\partial_4}^2\,. \tag{40}$$

This means that we are considering $\epsilon = -1$. We have then

$$ {}^{(5)}\Box\Phi = \left[\Box - {\partial_4}^2\right]\Phi = 0\,. \tag{41}$$

Separating the variables and splitting the scalar field Φ into two functions

$$\Phi = \varphi(t,\vec{x})\,\chi(x^4)\,, \tag{42}$$

the field φ depends on the ordinary space-time coordinates, while χ is a function of the fifth coordinate x^4. Inserting (42) into Eq. (41), we get

$$\frac{\Box\varphi}{\varphi} = \frac{1}{\chi}\left[\frac{d^2\chi}{dx_4^2}\right] = -k_n^2 \tag{43}$$

where k_n is a constant. From Eq. (43), we obtain the two field equations

$$\left(\Box + k_n^2\right)\varphi = 0\,, \tag{44}$$

and

$$\frac{d^2\chi}{dx_4^2} + k_n^2\,\chi = 0\,. \tag{45}$$

Eq. (45) describes a harmonic oscillator whose general solution is

$$\chi(x_4) = c_1 e^{-ik_n x^4} + c_2 e^{ik_n x^4}\,. \tag{46}$$

The constant k_n has the physical dimension of the inverse of a length and, assigning boundary conditions, we can derive the eigenvalue relation

$$k_n = \frac{2\pi}{l}\,n\,, \tag{47}$$

where n is an integer and l a length which we have previously defined in Eq. (32) related to the gravitational coupling. As a result, in standard units, we can recover the physical lengths through the Compton lengths

$$\lambda_n = \frac{\hbar}{2\pi m_n c} = \frac{1}{k_n} \tag{48}$$

which always assign the masses to the particles depending on the number n. It is worth stressing that, in this case, we have achieved a dynamical approach because the eigenvalues of Eq. (45) are the masses of particles which are generated by the process of reduction from 5D to 4D. The solution (46) is the superposition of two mass eigenstates. The 4D evolution is given by Eq. (38) or, equivalently, (44). Besides, the solutions in the coordinate x^4 give the associated Compton lengths from which the effective physical masses can be derived. Specifically, different values of n fix the families of particles, while, for any given value n, different values of parameters $c_{1,2}$ select the different particles within a family. With these considerations in mind, the effective mass can be obtained integrating the modulus of the scalar field Φ along the x^4 coordinate. It is

$$m_{\text{eff}} \equiv \int |\Phi| dx^4 = \int |\Phi\,(dx^4/ds)|\,ds \tag{49}$$

where ds is the 4D affine parameter used in the derivation of geodesic equation. This result means that the rest mass of a particle is derived by integrating the Extra Force along x^4 (see Eq. 24) while the effective mass is obtained by integrating the field Φ along x^4. In the first case, the mass of the particle is obtained starting from the geodesic structure of the theory, in the second case, it comes out from the field structure. In other words, the coincidence of geodesic structure and chronological structure (the causal structure), supposed as a principle in General Relativity, is due to the fact that masses are generated in the reduction process.

At this point, from the condition (42), the field 5D Φ results to be

$$\Phi(x^\alpha, x^4) = \sum_{n=-\infty}^{+\infty}\left[\varphi_n(x^\alpha)e^{-ik_n x^4} + \varphi_n^*(x^\alpha)e^{ik_n x^4}\right], \tag{50}$$

where φ and φ^* are the 4D solutions combined with the fifth-component solutions $e^{\pm ik_n x^4}$. In general, every particle mass can be selected by solutions of type (46). The number $k_n x^4$, *i.e.* the ratio between the two lengths x^4/λ_n, fixes the interaction scale. Geometrically, such a scale is related to the curvature radius of the embedded 4D spacetime where particles can be identified and, in principle, detected. In this sense, Open Quantum Relativity is an *induced-matter* theory, where the extra dimension cannot be simply classified as "compactified" since it yields all the 4D dynamics giving origin to the masses. Moreover, Eq. (50) is not a simple "tower of mass states" but a spectrum capable of explaining the hierarchy problem [7]. On the other hand, gravitational interaction can be framed in this approach considering as its fundamental scale the Planck length

$$\lambda_P = l = \left(\frac{\hbar G_N}{c^3}\right)^{1/2}, \tag{51}$$

instead of the above Compton length. It fixes the vacuum state of the system ant the masses of all particles can be considered negligible if compared with the Planck scales. Finally, as we have seen, the reduction mechanism can select also $\epsilon = 1$ in the metric (27). In this case, the 5D-Klein Gordon equation (11), and the 5D field equations (5) have wave-like solutions of the form

$$\begin{aligned} dS^2 = dt^2 - \Omega\,(t, x_1)\left(dx^1\right)^2 - \Omega\,(t, x_2)\left(dx^2\right)^2 - \\ - \Omega\,(t, x_3)\left(dx^3\right)^2 + \left(dx^4\right)^2, \end{aligned} \tag{52}$$

where

$$\Omega\,(t, x_j) = \exp i(\omega t + k_j x^j)\,, \quad j = 1, 2, 3\,. \tag{53}$$

In this solution, the necessity of the existence of two times arrows naturally emerges and, as a direct consequence, due to the structure of the functions $\Omega\,(t, x_j)$, closed time-like paths (*i.e.* circular paths) are allowed. The existence of closed time-like paths means that Anti-De Sitter [14] and Gödel [11] solutions are naturally allowed possibilities in the dynamics.

6 Discussion and conclusions

In this paper, we have discussed the reduction process which allows to recover the 4D spacetime and dynamics starting from the 5D manifold of Open Quantum Relativity. Such a theory needs, to be formulated, a *General Conservation Principle*. This principle states that conservation laws are always and absolutely valid also when, to maintain such a validity, phenomena as topology changes and entanglement can emerge in 4D. In this way, we have a theory without singularities (like conventional black holes) and unphysical spacetime regions are naturally avoided [8, 6]. The dimensional reduction can be considered from the geodesic structure and the field equations points of view. In the first case, starting from a 5D metric, it is possible to generate an Extra Force term in 4D which is related to the rest masses of particles and then to the Equivalence Principle. In fact, masses can be dynamically generated by the fifth component of the 5D space and the relation between inertial mass and gravitational mass is not an assumed principle, as in standard physics [10], but the result of the dynamical process of embedding. It is worth noting that an "amount of work" is necessary to give the mass to a particle. An effective mass is recovered also by splitting the field equations in a $(4+1)$ formalism. The fifth component of the metric can be interpreted as a scalar field and the embedding as the process by which the mass of particles emerges. The fact that particles acquire the mass from the embedding of geodesics and from the embedding of field equations is the reason why the chronological and geodesic structures of the 4D spacetime are the same: they can be both achieved from the same 5D metric structure which is also the solution of the 5D field equations. By taking into account such a result in 4D, the result itself naturally leads to understand why the metric approach of General Relativity, based on Levi-Civita connections, succeed in the description of spacetime dynamics even without resorting to a more general scheme as the Palatini-affine approach where connection and metric are, in principle, considered distinct. The reduction process leads also to a wide class of time solutions including two-time arrows and closed time-like paths. As a consequence, we can recover the concept of causality questioned by the EPR effect [6] thanks to the necessary introduction of backward and forward causation [1]. As a final remark, we can say that Open Quantum Relativity is an approach which allows to face Quantum Mechanics and Relativity under the same dynamical standard (a covariant symplectic structure [3]): this occurrence leads to frame several paradoxes of modern physics under the same dynamical scheme by only an assumption of the absolute validity of conservation laws and the generalization of the causal structure of spacetime.

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