Quantum Spin Transport in Mesoscopic Interferometer

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Spin-dependent conductance of ballistic mesoscopic interferometer is investigated. The quantum interferometer is in the form of ring, in which a quantum dot is embedded in one arm. This quantum dot is connected to one lead via tunnel barrier. Both Aharonov-Casher and Aharonov-Bohm effects are studied. Our results confirm the interplay of spin-orbit coupling and quantum interference effects in such confined quantum systems. This investigation is valuable for spintronics application, for example, quantum information processing.

1 Introduction

The flexibility offered by semiconductor spintronics [1] is anticipated to lead to novel devices and may eventually become used for quantum information processing. Another advantage offered by spin systems in semiconductors is their long coherence times [2, 3]. In recent years, much attention has been devoted towards the interplay of the spin-orbit interaction and quantum interference effects in confined semiconductor heterostructures [4, 5, 6]. Such interplay can be exploited as a mean to control and manipulate the spin degree of freedom at mesoscopic scale useful for phase-coherent spintronics applications.

Since the original proposal of the spin field effect transistor (SFET) [7] by Datta and Das, many proposals have appeared based on intrinsic spin splitting properties of semiconductors associated with the Rashba spin-orbit interaction [8, 9, 10].

In the present paper, a quantum interference effect in coherent Aharonov-Casher ring is investigated. In such devices quantum effects are affecting transport properties.

2 The model

The mesoscopic device proposed in the present paper is in the form of quantum dot embedded in one arm of the Aharonov-Casher interferometer. This interferometer is connected to two conducting leads. The form of the confining potential in such spintronics device is modulated by an external gate electrode, allowing for direct control of the electron spin-orbit interaction. The main feature of the electron transport through such device is that the difference in the Aharonov-Casher phase of the electrons traveling clockwise and counterclockwise directions produces spin-sensitive interference effects [11, 12]. The quantum transport of the electrons occurs in the presence of Rashba spin-orbit coupling [13] and the influence of an external magnetic field. With the present proposed mesoscopic device, we can predict that the spin polarized current through such device is controlled via gate voltage.

The Hamiltonian, $\hat{H}$, describing the quantum transport through the present studied device could be written in the form as [14]

$$\hat{H} = \frac{P^2}{2m^*} + V(r) + \hat{H}_{soc}, \qquad (1)$$

where $\hat{H}_{soc}$ is the Hamiltonian due to the spin-orbit coupling and is expressed as

$$\hat{H}_{soc} = \frac{\hbar^2}{2m^* a^2} \left(-i \frac{\partial}{\partial \varphi} + \frac{\omega_{soc} m^* a^2}{\hbar} \sigma_r \right), \qquad (2)$$

where $\omega_{soc} = \frac{\alpha}{\hbar a}$ and it is called the frequency associated with the spin-orbit coupling, α is the strength of the spin-orbit coupling, a is the radius of the Aharonov-Casher ring and σ_r is the radial part of the Pauli matrices which expressed in the components of Pauli matrices σ_x, σ_y as

$$\sigma_r = \sigma_x \cos \varphi + \sigma_y \sin \varphi,$$
$$\sigma_\varphi = \sigma_y \cos \varphi - \sigma_x \sin \varphi. \qquad (3)$$

The parameter φ, Eq. (3) represents the phase difference of electrons passing through the upper and the lower arms of the ring. In Eq. (1), $V(r)$ is the effective potential for transmission of electrons through the quantum dot which depends, mainly, on the tunnel barrier between the quantum dot and the lead. Applying external magnetic field, B, normal to the plane of the device, then the Aharonov-Bohm phase picked up by an electron encircling this magnetic flux is given by

$$\Phi_{AB} = \frac{\pi e B a^2}{\hbar}. \qquad (4)$$

Then the Hamiltonian, $\hat{H}_{soc}$, due to the spin-orbit coupling Eq. (2) will take the form

$$H'_{soc} = \frac{\hbar^2}{2m^* a^2} \left(-i \frac{\partial}{\partial \varphi} - \frac{\Phi_{AB}}{2\pi} - \frac{\omega_{soc} m^* a^2}{\hbar} \sigma_r \right). \qquad (5)$$

Now in order to study the transport properties of the present quantum system, we have to solve Schrödinger equation and finding the eigenfunctions for this system as follows

$$\hat{H}\,\Psi = E\,\Psi\,. \tag{6}$$

The solution of Eq. (6) consists of four eigenfunctions [14], where $\Psi_L(x)$ is the eigenfunction for transmission through the left lead, $\Psi_R(x)$-for the right lead, $\Psi_{up}(\theta)$-for the upper arm of the ring and $\Psi_{low}(\theta)$-for the lower arm of the ring. Their forms will be as

$$\Psi_L(x) = \sum_\sigma \left[A\,e^{ikx} + B\,e^{-ikx} \right] \chi^\sigma(\pi)\,,$$
$$x \in [-\infty, 0]\,, \tag{7a}$$

$$\Psi_R(x) = \sum_\sigma \left[C\,e^{ikx'} + D\,e^{-ikx'} \right] \chi^\sigma(0)\,,$$
$$x \in [0, \infty]\,, \tag{7b}$$

$$\Psi_{up}(\varphi) = \sum_{\sigma,\mu} F_\mu\,e^{in'^\sigma_\mu \varphi}\,\chi^\sigma(\varphi)\,,$$
$$\varphi \in [0, \pi]\,, \tag{7c}$$

$$\Psi_{low}(\varphi) = \sum_{\sigma,\mu} G_\mu\,e^{in^\sigma_\mu \varphi}\,\chi^\sigma(\varphi)\,,$$
$$\varphi \in [\pi, 2\pi]\,. \tag{7d}$$

The mutually orthogonal spinors $\psi_n^\sigma(\varphi)$ are expressed in terms of the eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ of the Pauli matrix σ_z as

$$\chi_n^{(1)}(\varphi) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix}\,, \tag{7}$$

$$\chi_n^{(2)}(\varphi) = \begin{pmatrix} \sin\frac{\theta}{2} \\ -e^{i\varphi}\cos\frac{\theta}{2} \end{pmatrix}\,, \tag{8}$$

where the angle θ [15] is given by

$$\theta = 2\tan^{-1}\frac{\Omega - \sqrt{\Omega^2 + \omega_{soc}^2}}{\omega_{soc}} \tag{9}$$

in which Ω is given by

$$\Omega = \frac{\hbar}{2\,m^*\,a^2}\,. \tag{10}$$

The parameters n'^σ_μ and n^σ_μ are expressed respectively as

$$n'^\sigma_\mu = \mu\,k'a - \varphi + \frac{\Phi_{AB}}{2\pi} + \frac{\Phi_{AC}^\sigma}{2\pi}\,, \tag{11}$$

$$n^\sigma_\mu = \mu\,ka - \varphi + \frac{\Phi_{AB}}{2\pi} + \frac{\Phi_{AC}^\sigma}{2\pi}\,, \tag{12}$$

where $\mu = \pm 1$ corresponding to the spin up and spin down of transmitted electrons, Φ_{AB} is given by Eq. (4). The term Φ_{AC} represents the Aharonov-Casher phase and is given by

$$\Phi_{AC}^{(\mu)} = -\pi \left[1 + \frac{(-1)^\mu (\omega_{soc}^2 + \Omega^2)^{1/2}}{\Omega} \right]\,. \tag{13}$$

The wave numbers k', k are given respectively by

$$k' = \sqrt{\frac{2\,m^*\,E}{\hbar^2}}\,, \tag{14}$$

$$k = \sqrt{\frac{2\,m^*}{\hbar^2}\left(V_d + e\,V_g + \frac{N^2 e^2}{2C} + E_F - E \right)}\,, \tag{15}$$

where V_d is the barrier height, V_g is the gate voltage, N is the number of electrons entering the quantum dot, C is the total capacitance of the quantum dot, m^* is the effective mass of electrons with energy, E, and charge, e, and E_F is the Fermi energy.

The conductance, G, for the present investigated device will be calculated using landauer formula [16] as

$$G = \frac{2e^2 \sin\varphi}{h} \sum_{\mu=1,2} \int dE \left(-\frac{\partial f_{FD}}{\partial E} \right) |\Gamma_\mu(E)|^2\,, \tag{16}$$

where f_{FD} is the Fermi-Dirac distribution is function and $|\Gamma_\mu(E)|^2$ is tunneling probability. This tunneling probability could be obtained by applying the Griffith boundary conditions [15, 17, 18], which states that the eigenfunctions (Eqs. 7a, 7b, 7c, 7d) are continuous and that the current density is conserved at each intersection. Then the expression for $\Gamma_\mu(E)$ is given by

$$\Gamma_\mu(E) =$$
$$= \frac{8i\,\cos\frac{\Phi_{AB} + \Phi_{AC}^{(\mu)}}{2}\,\sin(\pi ka)}{4\cos(2\pi k'a) + 4\cos\left(\Phi_{AB} + \Phi_{AC}^{(\mu)} \right) + 4i\sin(2\pi k'a)}\,. \tag{17}$$

3 Results and discussion

In order to investigate the quantum spin transport characteristics through the present device, we solve Eqs. (17, 18) numerically. We use the heterostructures as InGaAs/InAlAs.

We calculate the conductance, G, at different both magnetic field and the ω_{soc} which depends on the Rashba spin-orbit coupling strength. The main features of our obtained results are:

1. Figs. 1 and Fig. 2 show the dependence of the conductance on the magnetic field, B, for small and large values of B at different ω_{soc}.

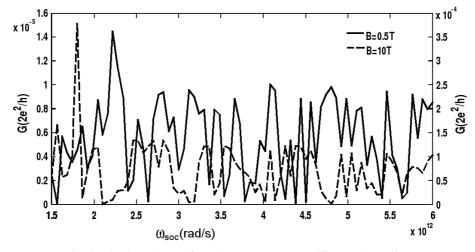2. Fig. 3 shows the dependence of the conductance on the parameter ω_{soc} at different values of B.

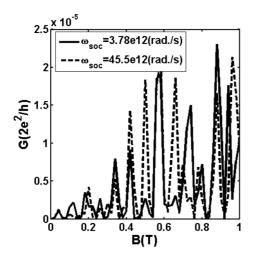Fig. 3: The dependence of conductance on ω_{soc} at different values of B.

Fig. 1: The dependence of conductance on B at different ω_{soc} (small B).
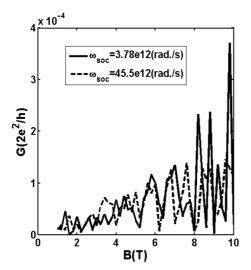

Fig. 2: The dependence of conductance on B at different ω_{soc} (large B).

From the figures we observe a quasi-periodic oscillations in the conductance (Fig. 1), and takes the form of satellite peaks. While for large values of B, the oscillations behave completely different from those in case of small values of B. The oscillatory behavior of $G\left(\omega_{soc}\right)$ shows a wide peaks and in some ranges of ω_{soc}, there is a splitting in the peaks.

The obtained results could be explained as follows: The oscillatory behavior of the conductance with B and ω_{soc} could be due to spin-sensitive quantum-interference effects caused by the difference in the Aharonov-Casher phase accumulated by the opposite spin states. Also the quantum interference effects in the present device could be due to Aharonov-Bohm effect. Our results are found concordant with those in the literatures [4, 5, 15, 19].

4 Conclusions

In the present paper an expression for the conductance has been deduced for the investigated mesoscopic device. The spin transport in such coherent device is investigated taking into consideration both Aharonov-Casher and Aharonov-Bohm effects in the quantum dot connected to conducting lead via a tunnel barrier. The present results are valuable for employing such devices in phase coherent spintronics applications.

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