On the Nature of the Microwave Background at the Lagrange 2 Point. Part II

Larissa Borissova and Dmitri Rabounski
E-mail: lborissova@yahoo.com; rabounski@yahoo.com

In this work the mathematical methods of General Relativity are used to answer the following questions: if a microwave background originates from the Earth, what would be its density and associated dipole measured at the altitude of a U2 aeroplane (25 km), the COBE satellite (900 km), and the 2nd Lagrange point (1.5 million km, the position of the WMAP and PLANCK satellites)? The first problem is solved via Einstein's equations for the electromagnetic field of the Earth. The second problem is solved using the geodesic equations for light-like particles (photons) which are mediators for electromagnetic radiation. We have determined that a microwave background that originates at the Earth (the Earth microwave background) decreases with altitude so that the density of the energy of such a background at the altitude of the COBE orbit (900 km) is 0.68 times less than that at the altitude of a U2 aeroplane. The density of the energy of the background at the L2 point is only $\sim 10^{-7}$ of the value detected by a U2 aeroplane or at the COBE orbit. The dipole anisotropy of the Earth microwave background, due to the rapid motion of the Earth relative to the source of another field which isn't connected to the Earth but is located in depths of the cosmos, doesn't depend on altitute from the surface of the Earth. Such a dipole will be the same irrespective of the position at which measurements are taken.

1 Problem statement: the space of the Earth and the Earth microwave background

Here we solve two theoretical problems related to the measurement of the microwave background:

(1) What is the density of the Earth microwave background which one will observe at the COBE orbit and at the L2 point?

(2) What is the anisotropy of the Earth microwave background, due to a drift of the whole space of the Earth, which one will observe in the COBE orbit and at the L2 point?

In a sense, the anisotropy we are treating is the sum of the dipole and all other multipoles.

According to General Relativity, the result of an observation depends on the velocity of the observer relative to the object he observes, and also on the properties of the local space (such as the space rotation, gravitation, deformation, curvature, etc.) where the observation is made. Therefore, we are looking for a theoretical solution of the aforementioned problems using the mathematical methods, which are specific to General Relativity.

We solve the first problem using Einstein's equations, manifest in the energy and momentum of a field of distributed matter (an electromagnetic field, for instance), depending on the distance from the field's source, and also on the properties of the space e.g. the space rotation, gravitation, etc.

We solve the second problem using the geodesic equations for light-like particles (photons, which are mediators for microwave radiation, and for any electromagnetic radiation in general). The geodesic equations give a possibility of finding a preferred direction (anisotropy) in such a field due to the presence of a linear drift of the whole reference space of the observer relative to the source of another field, which isn't connected to the observer's space, but moves with respect to it [1, 2]. In the present case, such a linear drift is due to the motion of the observer, in common with the microwave background's source, the Earth, relative to the source of another field such as the common field of a group of galaxies or that of the Universe as a whole (a weak microwave background). Then we compare our theoretical result from General Relativity to the experimental data for the microwave background, obtained in space near the Earth by the COBE satellite, located in a 900 km orbit, and also by the WMAP satellite, located at the L2 point, as far as 1.5 million km from the Earth.

In order to obtain a theoretical result expressed in quantities measurable in practice, we use the mathematical apparatus of chronometric invariants — the projections of four-dimensional quantities on the time line and spatial section of a real observer, which are the physical observable quatities in General Relativity [3, 4].

First, we introduce a space where all the mesurements are taken. Both locations, of the COBE satellite and the L2 point, are connected, by gravitation, to the gravitational field of the Earth, so both observers are connected to the space of the Earth, whose properties (e.g. rotation, gravitation, deformation, etc.) affect the observations. We therefore consider different locations of an observer in the space of the Earth.

We construct the metric for the Earth's space, which is the superposition of the metric of a non-holonomic (self-rotating) space and a gravitating space.

The space of the Earth rotates with a frequency of one revolution per day. By the theory of non-holonomic spaces

[5], a non-holonomic space (space-time) has inclinations between the times lines and the three-dimensional spatial section, cosines of which are represent by the three-dimensional linear velocity of the rotation. The metric of a non-holonomic space (space-time), which rotation is given by a linear velocity v at a given point, is described at this point by

$$ds^2 = c^2 dt^2 + \frac{2v}{c}\, cdt\,(dx+dy+dz) - dx^2 - dy^2 - dz^2. \quad (1)$$

For clarity of further calculation, we change to the cylindrical coordinates r, φ, z, where

$$x = r\cos\varphi\,, \quad y = r\sin\varphi\,, \quad z = z\,, \quad (2)$$

so the metric (1) takes the form

$$\begin{aligned} ds^2 &= c^2 dt^2 + \frac{2v}{c}\,(\cos\varphi + \sin\varphi)\, cdtdr + \\ &+ \frac{2vr}{c}\,(\cos\varphi - \sin\varphi)\, cdtd\varphi + \frac{2v}{c}\, cdtdz - \\ &- dr^2 - r^2 d\varphi^2 - dz^2. \end{aligned} \quad (3)$$

The metric of a space, where gravitation is due to a body of a mass M, in quasi-Newtonian approximation and in the cylindrical coordinates, is

$$\begin{aligned} ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - \\ - r^2 d\varphi^2 - dz^2, \end{aligned} \quad (4)$$

where G is the Newtonian gravitational constant. We consider a satellite which rotates in the metric (4) around the gravitating body. Both observers, located on board the COBE satellite (a 900 km orbit) and the WMAP satellite (the L2 point) respectively, are in a state of weightlessness, which is described by the weightlessness condition

$$\frac{GM}{r} = \omega^2 r^2, \quad (5)$$

where r is the radius of the satellite's orbit, while ω is the angular velocity of the rotation of the observer (in common with the satellite on which he is located) around the gravitating body. So the metric (4) is

$$\begin{aligned} ds^2 = \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 - \frac{2\omega r^2}{c}\, cdtd\varphi - \\ - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2, \end{aligned} \quad (6)$$

where $\frac{GM}{r} = \omega^2 r^2$. The weightless state is common to all planets and their satellites. So the Earth's space from the point of an observer located on board the COBE satellite and the WMAP satellite is in the weightless state.

We use the cylindrical coordinates, because such an observer is located on board of a satellite which orbits the Earth.

The metric of the Earth's space at the point of location of such an observer is a superposition of the metric with rotation (3) and the metric with a gravitational field (6), which is

$$\begin{aligned} ds^2 &= \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2}\right) c^2 dt^2 + \\ &+ \frac{2v\,(\cos\varphi + \sin\varphi)}{c}\, cdtdr + \\ &+ \frac{2r\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c}\, cdtd\varphi + \frac{2v}{c}\, cdtdz - \\ &- \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\varphi^2 - dz^2. \end{aligned} \quad (7)$$

Because the Earth, in common with its space, moves relative to the source of the weak microwave background, this drift should also be taken into account in the metric. This is accomplished by choosing this motion to be in the z-direction and then applying Lorentz' transformations to the z coordinate and time t

$$\tilde{t} = \frac{t + \frac{\mathrm{v}z}{c^2}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\,, \qquad \tilde{z} = \frac{z + \mathrm{v}t}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}\,, \quad (8)$$

so the resulting metric of the space of the Earth, where such a drift is taken into account, is

$$\begin{aligned} ds^2 &= \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}\right) c^2 dt^2 + \\ &+ \frac{2v\,(\cos\varphi + \sin\varphi)}{c}\, cdtdr + \\ &+ \frac{2r\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c}\, cdtd\varphi + \frac{2v}{c}\, cdt\,dz - \\ &- \left(1 + \frac{2GM}{c^2 r}\right) dr^2 + \frac{2v\mathrm{v}\,(\cos\varphi + \sin\varphi)}{c^2}\, drdz - r^2 d\varphi^2 + \\ &+ \frac{2r\mathrm{v}\,[v\,(\cos\varphi - \sin\varphi) - \omega r]}{c^2}\, d\varphi dz - \left(1 - \frac{2v\mathrm{v}}{c^2}\right) dz^2, \end{aligned} \quad (9)$$

where we mean $1 - \frac{\mathrm{v}^2}{c^2} \simeq 1$, because the Earth's velocity v relative to the source of the weak microwave background is small to the velocity of light c.

This is the *metric of the real physical space of the Earth*, where we process our observations.

Now we apply this metric to the reference frames of two observers, one of which is located on board the COBE satellite, in an orbit with an altitude of 900 km, while the second observer is located on board of WMAP satellite, at the L2 point, which is far as 1.5 million km from the Earth.

2 The density of the Earth microwave background at the COBE orbit and at the L2 point

Here we answer the question: what is the density of the Earth microwave background that one will observe at the COBE orbit and at the L2 point? Using the main observable characteristics of the space of the Earth, pervaded by an electromagnetic field (the microwave background, for instance), we derive Einstein's equations for the space. Einstein's equations describe the energy and momentum of distributed matter, in this case the microwave background. So we will know precisely, through Einstein's equations, the density of the energy of the Earth microwave background which will be observed at the COBE orbit and at the L2 point.

2.1 The Earth space. Its physical properties manifest in observations of the Earth microwave background

In this particular problem we are interested in the distribution of the Earth microwave background with altitude, giving the difference in the measurement of the background at the COBE orbit and at the L2 point. We therefore neglect terms like $\frac{v\mathrm{v}}{c^2}$, which take into account the drift of the whole space of the Earth. The quantity $\frac{2GM}{c^2 r}$ has its maximum numerical value $\sim 10^{-9}$ at the Earth's surface, and the value substatially decreases with altitude. We therefore neglect the last terms in $g_{11} = -\left(1+\frac{2GM}{c^2 r}\right)$, but we do not neglect the last terms in $g_{00} = 1-\frac{2GM}{c^2 r}-\frac{\omega^2 r^2}{c^2}$, because they will be multiplied by c^2 later. In such a case the Earth space metric takes the simplified form

$$ds^2 = \left(1-\frac{2GM}{c^2 r}-\frac{\omega^2 r^2}{c^2}\right)c^2 dt^2 + \frac{2v\left(\cos\varphi+\sin\varphi\right)}{c}\,cdtdr + \frac{2r\left[v\left(\cos\varphi-\sin\varphi\right)-\omega r\right]}{c}\,cdtd\varphi + \frac{2v}{c}\,cdtdz - dr^2 - r^2 d\varphi^2 - dz^2 . \qquad (10)$$

We will use this metric to determine the density of the energy of the Earth microwave background at the COBE orbit and at the L2 point. We are loooking for the main observable characteristics of the space. By the theory of physical observable quantities in General Relativity [3, 4], the observable properties of a space are determined within the fixed three-dimensional spatial section of an observer. Those are the quatities invariant within the spatial section (the so-called *chronometric invariants*): the gravitational potential w, the linear velocity of the space rotation v_i, the gravitational inertial force F_i, the angular velocity of the space rotation A_{ik}, the three-dimensional metric tensor h_{ik}, the space deformation D_{ik}, the three-dimensional Christoffel symbols Δ^i_{kn}, and the three-dimensional curvature C_{iklj}. These characteristics can be calculated through the components of the fundamental metric tensor $g_{\alpha\beta}$, which can be easily obtained from a formula for the space metric (see [3, 4] for the details).

The substantially non-zero components of the characteristics of the space of the Earth, calculated though the components $g_{\alpha\beta}$ of the metric (10), are

$$\mathrm{w} = \frac{GM}{r} + \frac{\omega^2 r^2}{2}\,, \qquad (11)$$

$$\left.\begin{aligned} v_1 &= -v\left(\cos\varphi+\sin\varphi\right)\\ v_2 &= -r\left[v\left(\cos\varphi-\sin\varphi\right)-\omega r\right]\\ v_3 &= -v \end{aligned}\right\} \qquad (12)$$

$$\left.\begin{aligned} F_1 &= \left(\cos\varphi+\sin\varphi\right)v_t + \omega^2 r - \frac{GM}{r^2}\\ F_2 &= r\left(\cos\varphi-\sin\varphi\right)v_t\,, \qquad F_3 = v_t \end{aligned}\right\} \qquad (13)$$

$$\left.\begin{aligned} A_{12} &= \omega r + \frac{1}{2}\Big[\left(\cos\varphi+\sin\varphi\right)v_\varphi - \\ &\qquad - r\left(\cos\varphi-\sin\varphi\right)v_r\Big]\\ A_{23} &= -\frac{v_\varphi}{2}\,, \qquad A_{13} = -\frac{v_r}{2} \end{aligned}\right\} \qquad (14)$$

$$\left.\begin{aligned} &h_{11}=h_{33}=1\,, \quad h_{22}=r^2, \quad h^{11}=h^{33}=1\\ &h^{22}=\frac{1}{r^2}\,, \qquad h=r^2, \qquad \frac{\partial\ln\sqrt{h}}{\partial r}=\frac{1}{r}\\ &\Delta^1_{22}=-r\,, \qquad \Delta^2_{12}=\frac{1}{r} \end{aligned}\right\} \qquad (15)$$

while all components of the tensor of the space deformation D_{ik} and the space curvature C_{iklj} are zero, in the framework of our assumptions. Here we assume the plane in cylindrical coordinates wherein the space of the Earth rotates: we assume that v doesn't depend from the z-coordinate. This assumption is due to the fact that the Earth, in common with its space, moves relative to a weak (cosmic) microwave background in the direction of its anisotropy. The quantities v_r, v_φ, and v_t denote the partial derivatives of v by the respective coordinates and time.

2.2 Einstein's equations in the Earth space. The density of the energy of distributed matter

Einstein's general covariant equations

$$R_{\alpha\beta} - \frac{1}{2}\,g_{\alpha\beta}R = -\kappa\,T_{\alpha\beta} + \lambda\,g_{\alpha\beta}\,, \qquad (16)$$

in a reference frame of the fixed spatial section of an observer, are represented by their projections onto the observer's time line and spatial section [3, 4]. We omit the λ-term, the space deformation D_{ik}, and the space curvature, C_{iklj}, because they are zero in the framework of our problem. In such a case the projected Einstein equations, according to Zelmanov [3, 4], are

$$\left.\begin{aligned} &\frac{\partial F^i}{\partial x^i} + \frac{\partial\ln\sqrt{h}}{\partial x^i}F^i - A_{ik}A^{ik} = -\frac{\kappa}{2}\left(\rho c^2 + U\right)\\ &\frac{\partial A^{ik}}{\partial x^k} + \frac{\partial\ln\sqrt{h}}{\partial x^k}A^{ik} = -\kappa J^i\\ &2A_{ij}A^{\cdot j}_{k\cdot} + \frac{1}{2}\left(\frac{\partial F_i}{\partial x^k} + \frac{\partial F_k}{\partial x^i} - 2\Delta^m_{ik}F_m\right) = \\ &\qquad = \frac{\kappa}{2}\left(\rho c^2 h_{ik} + 2U_{ik} - U h_{ik}\right) \end{aligned}\right\} \qquad (17)$$

$$
\left.
\begin{aligned}
& -2\omega^2 - 2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r} + 2\omega\left(\cos\varphi-\sin\varphi\right)v_r + \left(\cos\varphi+\sin\varphi\right)v_{tr} + \left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r} + \\
& + \left(\cos^2\varphi-\sin^2\varphi\right)\frac{v_r v_\varphi}{r} + \cos\varphi\sin\varphi\left(v_r^2-\frac{v_\varphi^2}{r^2}\right) - v_r^2 - \frac{v_\varphi^2}{r^2} = -\kappa\rho c^2 \\
& \frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right) + \left(\cos\varphi-\sin\varphi\right)\left(\frac{v_\varphi}{r^2}-\frac{v_{r\varphi}}{r}\right)\right] = -\kappa J^1 \\
& \frac{1}{2}\left[\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_\varphi}{r^3}-\frac{v_{r\varphi}}{r^2}\right) - \left(\cos\varphi-\sin\varphi\right)\frac{v_{rr}}{r}\right] = -\kappa J^2 \\
& \frac{1}{2}\left(v_{rr}+\frac{v_r}{r}+\frac{v_{\varphi\varphi}}{r^2}\right) = -\kappa J^3 \\
& v_r^2 + \frac{v_\varphi^2}{2r^3} + 3\omega^2 + \frac{2GM}{r^3} + 2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r} - 2\omega\left(\cos\varphi-\sin\varphi\right)v_r + \left(\cos\varphi+\sin\varphi\right)v_{tr} - \\
& - \left(\cos^2\varphi-\sin^2\varphi\right)\frac{v_r v_\varphi}{r} - \cos\varphi\sin\varphi\left(v_r^2-\frac{v_\varphi^2}{r^2}\right) = \kappa U_{11} \\
& \frac{r^2}{2}\left[\frac{v_r v_\varphi}{r^2} + \left(\cos\varphi+\sin\varphi\right)\frac{v_{t\varphi}}{r^2} + \left(\cos\varphi-\sin\varphi\right)\frac{v_{tr}}{r}\right] = \kappa U_{12} \\
& \frac{1}{2}\left[2\omega\frac{v_\varphi}{r} + v_{tr} + \left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi^2}{r^2} - \left(\cos\varphi-\sin\varphi\right)\frac{v_r v_\varphi}{r}\right] = \kappa U_{13} \\
& 2\omega^2 + 2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r} - 2\omega\left(\cos\varphi-\sin\varphi\right)v_r + \left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r} + \frac{v_r^2}{2} + \frac{v_\varphi^2}{r^2} - \\
& - \left(\cos^2\varphi-\sin^2\varphi\right)\frac{v_r v_\varphi}{r} + \cos\varphi\sin\varphi\left(\frac{v_\varphi^2}{r^2}-v_r^2\right) = \kappa\,\frac{U_{22}}{r^2} \\
& \frac{r^2}{2}\left[\frac{v_{t\varphi}}{r^2} - 2\omega\frac{v_r}{r} - \left(\cos\varphi+\sin\varphi\right)\frac{v_r v_\varphi}{r^2} + \left(\cos\varphi-\sin\varphi\right)\frac{v_r^2}{r}\right] = \kappa U_{23} \\
& \frac{v_r^2}{2} + \frac{v_\varphi^2}{2r^2} = \kappa U_{33}
\end{aligned}
\right\}
\qquad (18)
$$

where $\rho = \frac{T_{00}}{g_{00}}$, $J^i = \frac{c\,T_0^i}{\sqrt{g_{00}}}$, and $U^{ik} = c^2 T^{ik}$ are the respective projections of the energy-momentum tensor $T_{\alpha\beta}$ of distributed matter on the right side of the equations: ρ is the density of the energy of the matter field, J^i is the density of the field momentum, and U^{ik} is the stress-tensor of the field.

We substitute here the formulae obtained for the space of the Earth. In this deduction we take into account the weightlessness condition $\omega^2 r^2 = \frac{GM}{r}$. (This is because we calculate the equations for a satellite-bound observer.) We also apply the condition $\rho c^2 = U$, which is specific to any electromagnetic field; so we mean only an electromagnetic field distributed in the space. As a result, after some algebra, we obtain the projected Einstein equations for the Earth space filled with a background field of matter. The resulting Einstein equations, the system of 10 equations with partial derivatives, are given in formula (18).

(Obvious substitutions such as $\cos^2\varphi - \sin^2\varphi = \cos 2\varphi$ and $\cos\varphi\sin\varphi = \frac{1}{2}\sin 2\varphi$ can be used herein.)

We are looking for a solution of the scalar Enstein equation, the first equation of the system (18). In other words, we are looking for the density of the field's energy, ρ, which originates in the Earth, expressed through the physical properties of the space of the Earth (which decrease with distance from the Earth as well).

As seen, the quantity ρ is expressed through the distribution function of the linear velocity of the space rotation v (see the first equation of the system), which are unknown yet. A great help to us is that fact that we have only an electromagnetic field distributed in the space. This means that with use of the condition $\rho c^2 = U$ we equalize ρc^2 and U taken from the Einstein equations (18) so that we get an equation containing the distribution functions of v without the properties of matter (an electromagnetic field, in our case). With such an equation, we find a specific correlation between the distribution functions.

First we calculate is the trace of the stress-tensor of distributed matter

$$U = U_{11} + \frac{U_{22}}{r^2} + U_{33} \qquad (19)$$

which comes from the 5th, 8th, and 10th equations of the

$$\left.\begin{aligned}&(\cos\varphi-\sin\varphi)\left(\frac{v_{tr\varphi}}{r}-\frac{v_{t\varphi}}{r^2}\right)+\omega\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_{r\varphi}}{r}-\frac{v_{\varphi}}{r^2}\right)-\omega\left(\cos\varphi-\sin\varphi\right)v_{rr}+2v_r v_{rr}+\frac{v_\varphi v_{r\varphi}}{r^2}+\\&-\frac{v_\varphi^2}{r^3}+\left(\cos\varphi+\sin\varphi\right)v_{trr}-\frac{1}{2}\cos 2\varphi\left(\frac{v_\varphi v_{rr}}{r}+\frac{v_r v_{r\varphi}}{r}-\frac{v_r v_\varphi}{r^2}\right)+\frac{1}{2}\sin 2\varphi\left(\frac{v_\varphi v_{r\varphi}}{r^2}-\frac{v_\varphi^2}{r^3}-v_r v_{rr}\right)=0\\&(\cos\varphi+\sin\varphi)\left(\frac{v_{tr\varphi}}{r^2}-\frac{v_{t\varphi}}{r^3}\right)+\left(\cos\varphi-\sin\varphi\right)\left(\frac{v_{t\varphi\varphi}}{r^3}+\frac{v_{tr}}{r^2}\right)+\omega\left(\cos\varphi+\sin\varphi\right)\left(\frac{v_r}{r^2}+\frac{v_{\varphi\varphi}}{r^3}\right)+\\&+\omega\left(\cos\varphi-\sin\varphi\right)\left(\frac{v_\varphi}{r^3}-\frac{v_{r\varphi}}{r^2}\right)+\frac{v_\varphi v_{\varphi\varphi}}{r^4}+\frac{v_r v_{r\varphi}}{r^2}+\frac{1}{2}\cos 2\varphi\left(\frac{v_\varphi^2}{r^4}-\frac{v_r^2}{r^2}-\frac{v_r v_{\varphi\varphi}}{r^3}-\frac{v_\varphi v_{r\varphi}}{r^3}\right)+\\&+\frac{1}{2}\sin 2\varphi\left(\frac{2v_r v_\varphi}{r^3}+\frac{v_\varphi v_{\varphi\varphi}}{r^4}-\frac{v_r v_{r\varphi}}{r^2}\right)=0\end{aligned}\right\} \qquad (24)$$

Einstein equations (18). We obtain

$$\begin{aligned}\kappa U&=4\omega^2+4\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-\\&-4\omega\left(\cos\varphi-\sin\varphi\right)v_r+2v_r^2+\frac{2v_\varphi^2}{r^2}+\\&+\sin 2\varphi\left(\frac{v_\varphi^2}{r^2}-v_r^2\right)-\cos 2\varphi\,\frac{v_r v_\varphi}{r}+\\&+\left(\cos\varphi+\sin\varphi\right)v_{tr}+\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}\,.\end{aligned} \qquad (20)$$

Equalizing it to $\kappa\rho c^2$ of the first equation of the Einstein equations (18), we obtain

$$\begin{aligned}&2\omega^2+2\omega\left(\cos\varphi+\sin\varphi\right)\frac{v_\varphi}{r}-2\omega\left(\cos\varphi-\sin\varphi\right)v_r+\\&+v_r^2+\frac{v_\varphi^2}{r^2}+\frac{1}{2}\sin 2\varphi\left(\frac{v_\varphi^2}{r^2}-v_r^2\right)-\cos 2\varphi\,\frac{v_r v_\varphi}{r}+\\&+2\left(\cos\varphi+\sin\varphi\right)v_{tr}+2\left(\cos\varphi-\sin\varphi\right)\frac{v_{t\varphi}}{r}=0\,.\end{aligned} \qquad (21)$$

Thus we have all physically observable components of $T_{\alpha\beta}$ expressed in only the physical observable properties of the space. Substituting the components into the conservation law for the common field of distributed matter in the space, we look for the formulae of the distribution functions of the space rotation velocity v.

The conservation law $\nabla_\sigma T^{\alpha\sigma}=0$, expressed in terms of the physical observed quantities*, is [3, 4]

$$\left.\begin{aligned}&\frac{^*\partial\rho}{\partial t}+D\rho+\frac{1}{c^2}\,D_{ij}U^{ij}+\\&\qquad+\left(^*\nabla_i-\frac{1}{c^2}\,F_i\right)J^i-\frac{1}{c^2}\,F_iJ^i=0\\&\frac{^*\partial J^k}{\partial t}+2\left(D_i^k+A_{i\cdot}^{\cdot k}\right)J^i+\\&\qquad+\left(^*\nabla_i-\frac{1}{c^2}\,F_i\right)U^{ik}-\rho F^k=0\end{aligned}\right\} \qquad (22)$$

*The asterisk denotes the chronometrically invariant differential operators, e.g. $\frac{^*\partial}{\partial t}=\frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i}=\frac{\partial}{\partial x^i}+\frac{1}{c^2}\,v_i\frac{^*\partial}{\partial t}$; see [3, 4].

which, under the specific conditions of our problem, become

$$\left.\begin{aligned}&\frac{\partial J^i}{\partial x^i}+\frac{\partial\ln\sqrt{h}}{\partial x^i}J^i=0\\&\frac{\partial J^k}{\partial t}+2A_{i\cdot}^{\cdot k}J^i+\frac{\partial U^{ik}}{\partial x^i}+\Delta_{im}^kU^{im}+\\&\qquad+\frac{\partial\ln\sqrt{h}}{\partial x^i}U^{ik}-\rho F^k=0\end{aligned}\right\} \qquad (23)$$

The first, a scalar equation of conservation, means $\nabla_i J^i=0$, i.e. the flow of the common field of distributed matter is conserved in the space of the Earth. The second, a vector equation of conservation, after substituting the components of J^i and U^{ik} from the Einstein equations (18), and also A_{ik} (14) and Δ_{kn}^i (15), give the system (24) of two non-linear differential equations with partial derivatives with respect to v (while the third equation vanishes becoming the identity "zero equals zero").

The exact solution of the system, i.e. a function which when substituted into the equations makes them identities, is

$$v=T(t)\,re^{i\varphi}, \qquad (25)$$

where i is the imaginary unit, while T is a function of time (its dimension is sec^{-1}).

Substituting the derivatives

$$\left.\begin{aligned}&v_r=Te^{i\varphi}\,,\qquad v_\varphi=irTe^{i\varphi}\,,\quad v_t=\dot{T}re^{i\varphi}\\&v_{t\varphi}=i\dot{T}re^{i\varphi},\quad v_{tr}=Te^{i\varphi}\end{aligned}\right\} \qquad (26)$$

into (21), we obtain, after transformations,

$$T_t\left(i+1\right)+\omega T\left(i-1\right)-\frac{iT^2}{2}+\omega^2=0\,, \qquad (27)$$

where $\dot{T}_t=\frac{\partial T}{\partial t}$. We obtain, for the real part of the equation

$$\dot{T}-\omega T+\omega^2=0\,, \qquad (28)$$

which is a linear differential equation of the first order

$$\dot{T}+f(t)T=g(t)\,, \qquad (29)$$

whose exact solution is

$$T = e^{-F}\left(T_0 + \int_{t_0=0}^{t} g(t)\, e^{F} dt\right), \tag{30}$$

$$F(t) = \int f(t)\, dt\,. \tag{31}$$

Substituting $f = -\omega$, $g = -\omega^2$ and integrating the resulting expression within the limits from t to $t_0 = 0$, we obtain the solution for the real part of the function $T(t)$:

$$T(t) = e^{\omega t}\left(T_0 - \omega\right) + \omega\,, \tag{32}$$

where T_0 is the initial value of T.

The imaginary part of the (27) satisfies the differential equation

$$T_t + \omega T - \frac{1}{2}\, T^2 = 0\,, \tag{33}$$

which is Bernoulli's equation

$$T_t + f T^2 + g T = 0\,, \tag{34}$$

where $f = -\frac{1}{2}$ and $g = \omega$ are constant coefficients. Such a Bernoulli equation has the solution

$$\frac{1}{T} = E(t) \int \frac{f dt}{E(t)}\,, \qquad E(t) = e^{\int g\, dt}\,. \tag{35}$$

Integrating this expression, we obtain

$$T(t) = \frac{2\omega}{1 + Ce^{\omega t}}\,, \tag{36}$$

which is the imaginary part of T. Here C is a constant of integration. Assuming the initial value $t_0 = 0$, we obtain

$$C = \frac{2\omega}{T_0} - 1\,, \tag{37}$$

where T_0 is the initial value of T. Because, by definition $v = T r e^{i\varphi}$ (25), T has a dimension of $\sec^{-1}$, we consider T_0 to be the initial frequency of the vibrations of the distributed matter (background).

So we obtain the final formula for the imaginary part of the solution for T:

$$T(t) = \frac{2\omega T_0}{T_0 + \left(2\omega - T_0\right) e^{\omega t}}\,. \tag{38}$$

We therefore write the full solution for T as a complex function, which is

$$T(t) = e^{\omega t}\left(T_0 - \omega\right) + \omega + i\, \frac{2\omega T_0}{\left(2\omega - T_0\right) e^{\omega t} + T_0}\,. \tag{39}$$

We see that the imaginary part of T is zero if $T_0 = 0$. Hence the imaginary part of T originates in the presence of the initial non-zero value of T.

Assuming $T_0 = 0$, we obtain: the full solution for T has only the real solution

$$T = \omega\left(1 - e^{\omega t}\right) \tag{40}$$

when $T_0 = 0$. Substituting this solution into the expression for ρc^2, i.e. the first equation of the system (18), and taking into account the geometrization condition 21 we have obtained for electromagnetic field, we obtain the real component of the density of the energy, which is

$$\rho c^2 = \frac{3\omega}{\varkappa}\left(\omega - T\right) = \frac{3\omega^2}{\varkappa}\left[1 - \left(1 - e^{\omega t}\right)\right]. \tag{41}$$

This is the final formula for the observable density of the energy $W = \rho c^2$ of distributed matter in the space of the Earth, where the matter is represented by an electromagnetic field which originates in the Earth, with an additional component due to the complete rotation of the Earth's space.

2.3 Calculation of the density of the Earth microwave background at the COBE orbit and at the L2 point

We simplify formula (41) according to the assumptions of our problem. The quantity $\omega = \sqrt{GM_{\oplus}/R^3}$, the frequency of the rotation of the Earth space for an observer existing in the weightless state, takes its maximum numerical value at the equator of the Earth's surface, where $\omega = 1.24\times10^{-3}\ \sec^{-1}$. Obviously, the numerical value of ω decreases with altitude above the surface of the Earth. Since ω is a small value, we expand $e^{\omega t}$ into the series

$$e^{\omega t} \approx 1 + \omega t + \frac{1}{2}\,\omega^2 t^2 + \ldots \tag{42}$$

where we omit the higher order terms from consideration. As a result, we obtain, for the density of the energy of distributed matter (41) in the space of the Earth (we mean an electromagnetic field originating in the Earth as above),

$$\rho c^2 = \frac{3\omega^2}{\varkappa}\,, \tag{43}$$

where $\omega = \sqrt{GM_{\oplus}/R^3}$. (In derivation of this formula we neglected the orders of ω higher than ω^2.) It should be noted that the quantity ω is derived from the weightless condition in the space, depending on the mass of the Earth $M_{\oplus}$, and the distance R from the centre of the Earth.

Because microwave radiation is related to an electromagnetic field, our theoretical result (43) is applicable to a microwave background originating from the Earth.

Now, with formula (43), we calculate the ratio between the density of the energy of the Earth microwave background at the L2 point ($R_{\rm L2} = 1.5$ million km) and at the COBE orbit ($R_{\rm COBE} = 6{,}370 + 900 = 7{,}270$ km)

$$\frac{\rho_{\rm L2}}{\rho_{\rm COBE}} = \frac{R^3_{\rm COBE}}{R^3_{\rm L2}} \simeq 1.1\times10^{-7}. \tag{44}$$

At the altitude of a U2 aeroplane (25 km altitude, which almost coincides with the location at the Earth's surface (within the framework of the precision of our calculation), we have $R_{\rm U2} = 6{,}370 + 25 = 6{,}395$ km. So, we obtain the ratio between the density of the Earth microwave background at the L2 point, at the COBE orbit, and that at the U2 altitude is

$$\frac{\rho_{\rm L2}}{\rho_{\rm U2}} = \frac{R^3_{\rm U2}}{R^3_{\rm L2}} \simeq 7.8\times10^{-8}, \quad \frac{\rho_{\rm COBE}}{\rho_{\rm U2}} = \frac{R^3_{\rm U2}}{R^3_{\rm COBE}} \simeq 0.68. \tag{45}$$

We see, concerning a microwave background field which originates in the Earth (the Earth microwave background), that a measurement of the background by an absolute instrument will give almost the same result at the position of a U2 aeroplane and the COBE satellite. However, at the L2 point (as far as 1.5 million km from the Earth, the point of location of the WMAP satellite and the planned PLANCK satellite), PLANCK, with its ability to function as an absolute instrument, should sense only $\sim 10^{-7}$ of the field registered either by the U2 aeroplane or by the COBE satellite.

3 The anisotropy of the Earth microwave background in the COBE orbit and at the L2 point

It is also important to understand what is the anisotropy of the Earth microwave background due to a drift of the whole space of the Earth which would one observe at the COBE orbit and at the L2 point. We solve this problem by using the equations of motion of free light-like particles (photons), which are mediators transferring electromagnetic radiation, including those in the microwave region. When treating the photons which originate in the Earth's field (the Earth microwave background, for instance), the equations of motion should manifest an anisotropy in the directions of motion of the photon due to the presence of a linear drift in the Earth's space as a whole, relative to the source of another field such as the common field of a compact group of galaxies or that of the Universe as a whole [1, 2] (a weak microwave background).

The equations of motion of free particles are the *geodesic equations*.

A light-like free particle, e.g. a free photon, moves along isotropic geodesic trajectories whose four-dimensional equations are [3, 4]

$$\frac{dK^\alpha}{d\sigma} + \Gamma^\alpha_{\mu\nu} K^\mu \frac{dx^\nu}{d\sigma} = 0\,, \tag{46}$$

where $K^\alpha = \frac{\Omega}{c}\frac{dx^\alpha}{d\sigma}$ is the four-dimensional wave vector of the photon (the vector satisfies the condition $K_\alpha K^\alpha = 0$), while Ω is the proper cyclic frequency of the photon. The three-dimensional observable interval equals the interval of observable time $d\sigma = cd\tau$ along isotropic trajectories, so $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$. In terms of the physical observable quantities, the isotropic geodesic equations are represented by their projections on the time line and spatial section of an observer [1, 2]

$$\left.\begin{aligned} &\frac{d\Omega}{d\tau} - \frac{\Omega}{c^2}\,F_i\,c^i + \frac{\Omega}{c^2}\,D_{ik}\,c^i c^k = 0\,,\\ &\frac{d}{d\tau}\left(\Omega c^i\right) + 2\Omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k - \\ &\qquad\qquad - \Omega F^i + \Omega\Delta^i_{kn}c^k c^n = 0\,, \end{aligned}\right\} \tag{47}$$

where $c^i = \frac{dx^i}{d\tau}$ is the three-dimensional vector of the observable velocity of light (the square of c^i satisfies $c_k c^k = c^2$ in the fixed spatial section of the observer). The first of the equations (the scalar equation) represents the law of energy for the particle, while the vectorial equation is the three-dimensional equation of its motion.

We apply the isotropic geodesic equations to the space metric (9), which includes a linear drift of the reference space in the z-direction with a velocity v. Because the dipole-fit velocity of the Earth, extracted from the experimentally obtained anisotropy of the microwave background, is only $\mathrm{v} = 365\pm18$ km/sec, we neglect the relativistic square in the metric (9) so that it is

$$\begin{aligned} ds^2 &= \left(1 - \frac{2GM}{c^2 r} - \frac{\omega^2 r^2}{c^2} + \frac{2v\mathrm{v}}{c^2}\right)c^2 dt^2 + \\ &+ \frac{2v\left(\cos\varphi + \sin\varphi\right)}{c}\,cdtdr + \\ &+ \frac{2r\left[v\left(\cos\varphi - \sin\varphi\right) - \omega r\right]}{c}\,cdtd\varphi + \frac{2v}{c}\,cdt\,dz - \\ &- \left(1 + \frac{2GM}{c^2 r}\right)dr^2 + \frac{2v\mathrm{v}\left(\cos\varphi + \sin\varphi\right)}{c^2}\,drdz - r^2 d\varphi^2 + \\ &+ \frac{2r\mathrm{v}\left[v\left(\cos\varphi - \sin\varphi\right) - \omega r\right]}{c^2}\,d\varphi dz - \left(1 - \frac{2v\mathrm{v}}{c^2}\right)dz^2, \end{aligned} \tag{48}$$

We use the metric with the approximation specific to an observer located on board the COBE satellite or the WMAP satellite: the observer exists in the weightless state, so $\omega^2 r^2 = \frac{GM}{r}$; the linear velocity v of the Earth's space rotation doesn't depend on the z-coordinate, the direction of the drift of the whole space. We neglect the terms $\frac{v^2}{c^2}$ and also higher order terms, but retain the term $\frac{v\mathrm{v}}{c^2}$ which takes into account the drift of the whole space of the Earth: the value of v is determined in the weightless state of the observer; it is $\simeq 7.9$ km/sec close to the surface of the Earth, and hence we have, near the surface, $\frac{v^2}{c^2} \approx 7\times10^{-10}$ and $\frac{v\mathrm{v}}{c^2} \approx 3\times10^{-8}$. Both values decrease with distance (altitude) from the Earth's surface, but the term $\frac{v\mathrm{v}}{c^2}$ remains two orders higher than $\frac{v^2}{c^2}$. We also neglect $\frac{GM}{c^2 r}$ which is $\approx 10^{-9}$ at the Earth's surface.

Due to the fact that the terms $\frac{v\mathrm{v}}{c^2}$ are small corrections in the metric (48), it is easy to show that the exact solution of the conservation equations $v = T(t)re^{i\varphi}$, obtained earlier in the framework of such a metric without a drift of the whole space (10), satisfies the present metric (48) where the drift is taken into account.

Using the solution for $T(t)$ (40), and expanding $e^{\omega t}$ into series $e^{\omega t} \approx 1 + \omega t + \ldots$, we obtain

$$T = -\omega^2 t\,, \tag{49}$$

then

$$v = -\omega^2 t r\, e^{i\varphi}. \tag{50}$$

We assume φ to be small. We calculate the observable characteristics of the Earth space where the drift of the whole space is taken into account, i.e. the space of the metric (48). Using the components of the fundamental metric tensor $g_{\alpha\beta}$ taken from the metric (48), we obtain

$$\left.\begin{aligned} v_1 &= \omega^2 t r e^{i\varphi} (\cos\varphi + \sin\varphi) \\ v_2 &= \omega r^2 \left[\omega t e^{i\varphi} (\cos\varphi - \sin\varphi) + 1\right] \\ v_3 &= \omega^2 r t e^{i\varphi} \end{aligned}\right\} \tag{51}$$

$$\left.\begin{aligned} F_1 &= -\omega^2 r e^{i\varphi} (\cos\varphi + \sin\varphi) + \omega^2 \mathrm{v} t e^{i\varphi} \\ F_2 &= -\omega^2 r^2 e^{i\varphi} (\cos\varphi - \sin\varphi) - i\omega^2 r \mathrm{v} t e^{i\varphi} \\ F_3 &= -\omega^2 r e^{i\varphi} \end{aligned}\right\} \tag{52}$$

$$\left.\begin{aligned} &A_{12} = \omega r \left[1 + \frac{\omega t}{2}(1 - i)\right] \\ &A_{23} = \frac{i\omega^2 t r e^{i\varphi}}{2}\,, \qquad A_{13} = \frac{\omega^2 t e^{i\varphi}}{2} \end{aligned}\right\} \tag{53}$$

$$\left.\begin{aligned} &h_{11} = 1\,, \quad h_{13} = \frac{\omega^2 \mathrm{v} t r (\cos\varphi + \sin\varphi)\, e^{i\varphi}}{c^2} \\ &h_{22} = r^2, \quad h_{23} = \frac{\omega r^2 \mathrm{v} \left[\omega t e^{i\varphi} (\cos\varphi - \sin\varphi) + 1\right]}{c^2} \\ &h_{33} = 1 - \frac{2\omega^2 \mathrm{v} t r e^{i\varphi}}{c^2} \\ &h = r^2 \left(1 + \frac{2\omega^2 \mathrm{v} t r e^{i\varphi}}{c^2}\right) \\ &h^{11} = 1\,, \quad h^{13} = -\frac{\omega^2 \mathrm{v} t r (\cos\varphi + \sin\varphi)\, e^{i\varphi}}{c^2} \\ &h^{22} = \frac{1}{r^2}\,, \quad h^{23} = -\frac{\omega \mathrm{v} \left[\omega t e^{i\varphi} (\cos\varphi - \sin\varphi) + 1\right]}{c^2} \\ &h^{33} = 1 + \frac{2\omega^2 \mathrm{v} t r e^{i\varphi}}{c^2} \end{aligned}\right\} \tag{54}$$

Because the components h_{13} and h_{23} of the tensor h_{ik} depend on the time coordinate t, we obtain two non-zero components of the tensor of the space deformation D_{ik}

$$\left.\begin{aligned} D_{13} &= \frac{\omega^2 r \mathrm{v} (\cos\varphi + \sin\varphi)\, e^{i\varphi}}{2c^2} \\ D_{23} &= \frac{\omega^2 r^2 \mathrm{v} (\cos\varphi - \sin\varphi)\, e^{i\varphi}}{2c^2} \\ D_{33} &= \frac{\omega^2 r \mathrm{v} e^{i\varphi}}{c^2} \end{aligned}\right\} \tag{55}$$

the scalar $D = h^{ik} D_{ik}$ is

$$D = \frac{\omega^2 r \mathrm{v} e^{i\varphi}}{c^2}. \tag{56}$$

We now calculate the chronometric Christoffel symbols of the second kind

$$\left.\begin{aligned} &\Delta^1_{22} = -r\,, \quad \Delta^1_{23} = \frac{\omega^2 r \mathrm{v} t (i-1)}{2c^2} - \frac{\omega r \mathrm{v}}{c^2} \\ &\Delta^1_{33} = \frac{\omega^2 \mathrm{v} t e^{i\varphi}}{c^2} \\ &\Delta^2_{12} = \frac{1}{r}\,, \quad \Delta^2_{13} = \frac{\omega^2 \mathrm{v} t (1-i)}{2c^2 r} + \frac{\omega \mathrm{v}}{c^2 r} \\ &\Delta^2_{33} = \frac{i\omega^2 \mathrm{v} t e^{i\varphi}}{c^2 r} \\ &\Delta^3_{11} = \frac{\omega^2 \mathrm{v} t (\cos\varphi + \sin\varphi)\, e^{i\varphi}}{c^2} \\ &\Delta^3_{12} = \frac{\omega^2 r \mathrm{v} t (i+1)\, e^{2i\varphi}}{2c^2}\,, \quad \Delta^3_{13} = -\frac{\omega^2 \mathrm{v} t e^{i\varphi}}{c^2} \\ &\Delta^3_{22} = \frac{i\omega^2 r^2 \mathrm{v} t (\cos\varphi - \sin\varphi)\, e^{i\varphi}}{c^2} \\ &\Delta^3_{23} = \frac{i\omega^2 r \mathrm{v} t e^{i\varphi}}{c^2} \end{aligned}\right\} \tag{57}$$

We use the above characteristics of the Earth's space to write the isotropic geodesic equations (47) in component form. We neglect the terms proportional to $\frac{1}{c^2}$ in the equations. Besides, in the framework of our asumptions, the differential with respect to proper time τ, i.e.

$$\frac{d}{d\tau} = \frac{{}^*\partial}{\partial t} + \mathrm{v}^i \frac{{}^*\partial}{\partial x^i}\,, \tag{58}$$

can be removed with the regular partial derivative $\frac{d}{d\tau} = \frac{\partial}{\partial t}$. (The starred derivatives become the regular derivatives, and also the observable velocity of light c^i doesn't depend on the z coordinate in our case where the whole space has a drift in the z direction.)

The vectorial isotropic geodesic equations, written in component notation, are

$$\left.\begin{aligned} &\frac{dc^1}{d\tau} + 2\left(D^1_k + A^{\cdot 1}_{k\cdot}\right) c^k - F^1 + \Delta^1_{22} c^2 c^2 + \\ &\qquad + 2\Delta^1_{23} c^2 c^3 + \Delta^1_{33} c^3 c^3 = 0 \\ &\frac{dc^2}{d\tau} + 2\left(D^2_k + A^{\cdot 2}_{k\cdot}\right) c^k - F^2 + 2\Delta^2_{12} c^1 c^2 + \\ &\qquad + 2\Delta^2_{13} c^1 c^3 + \Delta^2_{33} c^3 c^3 = 0 \\ &\frac{dc^3}{d\tau} + 2\left(D^3_k + A^{\cdot 3}_{k\cdot}\right) c^k - F^3 + \Delta^3_{11} c^1 c^1 + 2\Delta^3_{12} c^1 c^2 + \\ &\qquad + 2\Delta^3_{13} c^1 c^3 + \Delta^3_{22} c^2 c^2 + 2\Delta^3_{23} c^2 c^3 = 0 \end{aligned}\right\} \tag{59}$$

and after substituting the observable characteristics of the space, take the form (60–62), where dot denotes differentiation with respect to time.

$$\ddot{r} - 2\omega r\left[1 + \frac{\omega t\,(1-i)}{2}\right]\dot{\varphi} - \omega^2 e^{i\varphi}\left[t - \frac{\mathrm{v}r\,(\cos\varphi + \sin\varphi)}{c^2}\right]\dot{z} + \omega^2\left[r\,(\cos\varphi + \sin\varphi) - \mathrm{v}t\right]e^{i\varphi} - \\ - r\dot{\varphi}^2 + \frac{2\omega r\mathrm{v}}{c^2}\left[\frac{\omega t(i-1)}{2} - 1\right]\dot{\varphi}\dot{z} + \frac{\omega^2\mathrm{v}te^{i\varphi}}{c^2}\,\dot{z}^2 = 0\,, \tag{60}$$

$$\ddot{\varphi} + \frac{2\omega}{r}\left[1 + \frac{\omega t\,(1-i)}{2}\right]\dot{r} - \frac{\omega^2 e^{i\varphi}}{r}\left[it - \frac{\mathrm{v}r\,(\cos\varphi - \sin\varphi)}{c^2}\right]\dot{z} + \frac{\omega^2}{r}\left[r\,(\cos\varphi - \sin\varphi) + i\mathrm{v}t\right]e^{i\varphi} + \\ + \frac{2}{r}\,\dot{r}\dot{\varphi} - \frac{2\omega\mathrm{v}}{c^2 r}\left[\frac{\omega t(i-1)}{2} - 1\right]\dot{r}\dot{z} - \frac{i\omega^2\mathrm{v}t}{c^2}\,\dot{z}^2 = 0\,, \tag{61}$$

$$\ddot{z} + \omega^2 e^{i\varphi}\left[t + \frac{\mathrm{v}r\,(\cos\varphi + \sin\varphi)}{c^2}\right]\dot{r} + \omega^2 re^{i\varphi}\left[it + \frac{\mathrm{v}r\,(\cos\varphi - \sin\varphi)}{c^2}\right]\dot{\varphi} + \frac{2\omega^2 r\mathrm{v}e^{i\varphi}}{c^2}\,\dot{z} + \\ + \omega^2 re^{i\varphi} + \frac{\omega^2\mathrm{v}te^{i\varphi}\,(\cos\varphi + \sin\varphi)}{c^2}\,\dot{r}^2 + \frac{\omega^2 r\mathrm{v}t\,(i+1)\,e^{2i\varphi}}{c^2}\,\dot{r}\dot{\varphi} + \frac{2\omega^2\mathrm{v}te^{i\varphi}}{c^2}\,\dot{r}\dot{z} + \\ + \frac{i\omega^2 r^2\mathrm{v}t\,(\cos\varphi - \sin\varphi)\,e^{i\varphi}}{c^2}\,\dot{\varphi}^2 + \frac{2i\omega^2 r\mathrm{v}te^{i\varphi}}{c^2}\,\dot{\varphi}\dot{z} = 0, \tag{62}$$

$$\dot{r}^2 + \frac{2\omega^2 r\mathrm{v}t\,(\cos\varphi + \sin\varphi)\,e^{i\varphi}}{c^2}\,\dot{r}\dot{z} + r^2\dot{\varphi}^2 + \frac{2\omega r^2\mathrm{v}\left[\omega te^{i\varphi}\,(\cos\varphi - \sin\varphi) + 1\right]}{c^2}\,\dot{\varphi}\dot{z} + \\ + \left(1 - \frac{2\omega^2 r\mathrm{v}te^{i\varphi}}{c^2}\right)\dot{z}^2 = c^2. \tag{63}$$

The space-time interval ds along isotropic geogesics satisfies the condition $ds^2 = 0$. This condition, in the terms of physical observed quantities, implies constancy of the square of the three-dimensional observable velocity of light $c_i c^i = h_{ik}\, c^i c^k = c^2$ along the trajectory. This condition, for the metric (48), takes the form (63).

A system of the differential equations (60–63) describes the motion of light-like particles completely, in the given space-time of the metric (48).

Earlier in this study we considered only the real part $v = T(t)\, r\, e^{i\varphi}$ of the solution of the conservation equations in an electromagnetic field. Because we study the motion of photons in such an electromagnetic field (in the sample of a microwave background) we only use the real solution in the system of the equations (60–63). After the function $v = T(t)\, r\, e^{i\varphi}$ is substituted into (60–63), we have, after transformations, the formulae (64–67) (see Page 93).

We assume that a light-like signal (photon) of the Earth microwave radiation moves along the radial direction r. Because the space of the Earth at the location of a satellite (the space of the weightless state) rotates with an angular velocity ω which depends upon r, we have $\dot{\varphi} = 0$. Two satellites which measure the Earth microwave background are located at the altitudes $r_1 = 900$ km and $r_2 = 1.5$ million km respectively. Calculation of $\omega^2 = \frac{GM_\oplus}{r^3}$, where $M_\oplus = 6\times 10^{27}$g is the mass of the Earth, gives the values: $\omega_1 = 10^{-3}$ sec^{-1} and $\omega_2 = 3.5\times 10^{-6}$ sec^{-1}. Because both values are small, we use $\cos\varphi \simeq 1 + \omega t$ and $\sin\varphi \simeq \omega t$. Substituting these into the system of equations (64–67), and neglecting the terms of order higher than ω^2 (and also the other higher order terms), we obtain, finally,

$$\ddot{r} - \omega^2\left(t - \frac{r\mathrm{v}}{c^2}\right)\dot{z} + \omega^2\,(r - \mathrm{v}t) + \frac{\omega^2\mathrm{v}t}{c^2}\,\dot{z}^2 = 0\,, \tag{68}$$

$$\ddot{\varphi} + 2\omega\left(1 + \frac{2\omega t}{2}\right)\frac{\dot{r}}{r} + \frac{\omega^2\mathrm{v}}{c^2}\,\dot{z} + 4\omega^2 + 2\omega\frac{\dot{r}}{r} + \\ + \frac{2\omega\mathrm{v}\left(1 + \frac{\omega t}{2}\right)}{c^2 r}\,\dot{r}\dot{z} = 0\,, \tag{69}$$

$$\ddot{z} + \omega^2\left(t + \frac{r\mathrm{v}}{c^2}\right)\dot{r} + \frac{2\omega^2\mathrm{v}r}{c^2}\,\dot{z} + \omega^2 r + \\ + \frac{\omega^2\mathrm{v}t}{c^2}\,\dot{r}^2 + \frac{2\omega^2\mathrm{v}t}{c^2}\,\dot{r}\dot{z} = 0\,, \tag{70}$$

$$\dot{r}^2 + \frac{2\omega^2 r\mathrm{v}t}{c^2}\,\dot{r}\dot{z} + \frac{2\omega^2 r^2\mathrm{v}}{c^2}\,\dot{z} + \\ + \left(1 - \frac{2\omega^2 r\mathrm{v}t}{c^2}\right)\dot{z}^2 = c^2. \tag{71}$$

We do choose the coordinate axes so that the z-axis is directed along the motion of the Earth, in common with its own electromagnetic field, relative to the source of another feld such as the common feld of a compact group of galaxies or that of the Universe as a whole (a weak microwave background). We also assume, for simplicity, that the orbit of the satellite, on board of which an observer is located, lies in the plane orthogonal to the z-direction. In such a case, we have $\dot{z}_0 = 0$. We obtain, assuming $\dot{z}_0 = 0$,

$$\dot{r}_0^2 = c^2, \tag{72}$$

$$\ddot{r} - \omega^2 \left[t\cos\varphi - \frac{r\mathrm{v}\left(1+\cos 2\varphi+\sin 2\varphi\right)}{c^2}\right]\dot{z} - 2\omega r\left(1+\frac{\omega t}{2}\right)\dot{\varphi} + 2\omega^2 r\left(1+\cos 2\varphi+\sin 2\varphi\right) + \\ + \omega^2 \mathrm{v} t\cos\varphi - r\dot{\varphi}^2 - \frac{2\omega r\mathrm{v}\left(\frac{\omega t}{2}+1\right)}{c^2}\dot{\varphi}\dot{z} + \frac{\omega^2 \mathrm{v} t\cos\varphi}{c^2}\dot{z}^2 = 0\,, \tag{64}$$

$$\ddot{\varphi} + 2\omega\left(1+\frac{\omega t}{2}\right)\frac{\dot{r}}{r} + \frac{\omega^2}{r}\left[t\sin\varphi + \frac{\mathrm{v}r\left(1+\cos 2\varphi-\sin 2\varphi\right)}{c^2}\right]\dot{z} + 2\omega^2\left(1+\cos 2\varphi-\sin 2\varphi\right) - \\ - \frac{\omega^2}{r}\mathrm{v}t\sin\varphi + \frac{2\dot{r}\dot{\varphi}}{r} + \frac{2\omega\mathrm{v}\left(\frac{\omega t}{2}+1\right)}{c^2 r}\dot{r}\dot{z} = 0\,, \tag{65}$$

$$\ddot{z} + \omega^2\left[t\cos\varphi + \frac{r\mathrm{v}\left(1+\cos 2\varphi+\sin 2\varphi\right)}{c^2}\right]\dot{r} - 2\omega^2 r\left[2t\sin\varphi - \frac{r\mathrm{v}\left(1+\cos 2\varphi-\sin 2\varphi\right)}{c^2}\right]\dot{\varphi} + \\ + \frac{2\omega^2 r\mathrm{v}\cos\varphi}{c^2}\dot{z} + \omega^2 r\cos\varphi + \frac{\omega^2\mathrm{v}t\left(1+\cos 2\varphi+\sin 2\varphi\right)}{2c^2}\dot{r}^2 + \frac{\omega^2\mathrm{v}t\left(\cos 2\varphi-\sin 2\varphi\right)}{c^2}\dot{r}\dot{\varphi} + \\ + \frac{2\omega^2\mathrm{v}t\cos\varphi}{c^2}\dot{r}\dot{z} + \frac{2\omega^2 r^2\mathrm{v}t\left(1-\cos 2\varphi-\sin 2\varphi\right)}{c^2}\dot{\varphi}^2 - \frac{2\omega^2 r\mathrm{v}t\sin\varphi}{c^2}\dot{\varphi}\dot{z} = 0\,, \tag{66}$$

$$\dot{r}^2 + \frac{2\omega^2 r\mathrm{v}t\left(1+\cos 2\varphi+\sin 2\varphi\right)}{c^2}\dot{r}\dot{z} + r^2\dot{\varphi}^2 + \frac{2\omega r^2\mathrm{v}\left[\frac{\omega t}{2}\left(1+\cos 2\varphi-\sin 2\varphi\right)+1\right]}{c^2}\dot{\varphi}\dot{z} + \\ + \left(1-\frac{2\omega^2 r\mathrm{v}t\cos\varphi}{c^2}\right)\dot{z}^2 = c^2. \tag{67}$$

hence we assume $\dot{r} \simeq c$. So we have $r \simeq ct$. Substituting these into the equation of motion of a photon in the z-direction (70), and taking the weightless condition into account, we obtain the equation of motion in the z direction for a photon associated with the Earth's electromagnetic field, the Earth microwave background in particular. The equation is

$$\ddot{z} + \frac{2GM_{\oplus}}{c^2 t^2}\left(1+\frac{\mathrm{v}}{c}\right) = 0\,. \tag{73}$$

Integrating the equation with the conditions $\dot{z}_0 = 0$ and $r \simeq ct$ taken into account, we obtain

$$\dot{z} = \frac{2GM_{\oplus}}{cr}\left(1+\frac{\mathrm{v}}{c}\right) = \dot{z}' + \Delta\dot{z}', \tag{74}$$

where the first term shows that such a photon, initially launched in the r-direction in the rotating space (gravitational field) of the Earth, is carried into the z-direction by the rotation of the space of the Earth. The second term shows carriage into the z-direction due to the motion of the Earth in this direction relative to another source such as a local group of galaxies or the whole Universe.

Denoting the first term in this formula as $\dot{z}' = \frac{2GM_{\oplus}}{cr}$ and the second term as $\Delta\dot{z}' = \frac{2GM_{\oplus}\mathrm{v}}{c^2 r}$, we obtain the relative carriage of the three-dimensional vector of the light velocity from the initial r-direction to the z-direction, due to the motion of the Earth, as

$$\frac{\Delta\dot{z}'}{\dot{z}'} = \frac{\mathrm{v}}{c}\,. \tag{75}$$

Such a relative carriage of a photon radiated from the Earth's surface, applied to the field of photons of the Earth microwave background radiated in the radial directions, reveals the anisotropy associated with the dipole component of the background.

Such a relative carriage of a photon, associated with the Earth's electromagnetic field, into the z-direction, doesn't depend on the path travelled by such a photon in the radial direction r from the Earth. This means that the anisotropy associated with the dipole component of the Earth microwave background shouldn't be dependent on altitude: it should be the same be it measured on board a U2 aeroplane (25 km), at the orbit of the COBE satellite (900 km), and at the L2 point (the WMAP satellite and PLANCK satellite, 1.5 million km from the Earth).

4 Comparing the theoretical results to experimental data. Conclusions

We have obtained, from General Relativity, two fundamental results:

- A microwave background which originates in the Earth (the EMB) decreases with altitude, such that the density of the energy of this background at the height of the COBE satellite (900 km) is just 0.68 times less that that at the height of a U2 aeroplane (25 km). The energy of the background at the L2 point (which is up to 1.5 million km from the Earth) is only $\sim 10^{-7}$ that experienced at the location either of a U2 aeroplane or of the COBE satellite;
- The anisotropy of the Earth microwave background,

due to the fast motion of the Earth relative to the source of another field, which isn't connected to the Earth but located in depths of the cosmos, does not depend on the position relative to the Earth's surface. The dipole anisotropy is therefore independent of altitude; the anisotropy will be the same be it measured at the altitude of a U2 aeroplane (25 km), the COBE satellite (900 km), or the WMAP satellite located at the L2 point (1.5 million km).

These purely theoretical conclusions, from General Relativity, cause us to consider an Earth origin of the microwave background, the monopole 2.7 K component of which was discovered in 1965 by Penzias and Wilson, in a ground-based observation [6], while the dipole 3.35 mK component was first observed in 1969 by Conklin, also via a ground-based observation [7], then studied by Henry [8], Corey [9], and also Smoot, Gorenstein, and Muller, who organized a stratosphere observation on board a U2 aeroplane [11]. (See the history of the observations in detail in Lineweaver's paper [10].)

There are many problems in the observation of the microwave background. The monopole component, at low frequencies, is easy to observe at the Earth's surface [6]. The dipole component is best observed at the altitude of a U2 aeroplane [11], at the altitude of 900 km (the COBE satellite) and also at 1.5 million km (the WMAP satellite located at the L2 point) where its anisotropy is clearly indicated [12–17]. Conversely, the monopole observed on Earth and in COBE orbit, has yet to be recorded at the L2 point: the WMAP satellite has only differential instruments on board, which are able to indicate only the anisotropy of the background, not its absolute value.

On the other hand, as shown by Robitaille [18–22], such a phenomenology of the observations has a clear explanation as an Earth microwave background which originates not in a cosmic source, but the oceans of the Earth, which produce microwave signals, in particular, with an apparent temperature of 2.7 K. Besides, as pointed out in [21, 23], the observed anisotropy of the microwave background can be explained as a relativistic effect of the motion of the observer, in common with the source of the background (the Earth), relative to the source of a noise microwave field, which has no specific temperature, and a source of which is located in depths of the cosmos (i.e. the distance from the many sources).

According to our theory, which supports the phenomenology of the Earth microwave background, proposed by Robitaille [18–22], we have four new specific terms, namely:

1. The EMB (the Earth Microwave Background);
2. The EMBM (the monopole associated with the Earth Microwave Background);
3. The EMBD (the dipole associated with the Earth Microwave Background);
4. The EMBA (the anisotropy of the Earth Microwave Background, associated with the dipole).

The PLANCK satellite (which has an absolute instrument on board), will soon be launched to the L2 point, on 31st July 2008, and should find an experimental verification of our theory.

Acknowledgement

We are very grateful to Pierre-Marie Robitaille for consultations and valuable comments. We also are thankful to Delyan Zhelyazov for a few typing mistakes found in the formulae when this issue was sent to print.

Submitted on August 29, 2007 / Accepted on September 06, 2007
First published online on September 08, 2007
Online version corrected on February 11, 2008

References

1. Borissova L. Preferred spatial directions in the Universe: a General Relativity approach. *Progress in Physics*, 2006, v. 4, 51–58.
2. Borissova L. Preferred spatial directions in the Universe. Part II. Matter distributed along orbital trajectories, and energy produced from it. *Progress in Physics*, 2006, v. 4, 59–64.
3. Zelmanov A. L. Chronometric invariants and co-moving coordinates in the general relativity theory. *Doklady Acad. Nauk USSR*, 1956, v.107 (6), 815–818.
4. Zelmanov A. L. Chronometric invariants. American Research Press, Rehoboth (NM), 2006.
5. Schouten J. A. und Struik D. J. Einführung in die neuren Methoden der Differentialgeometrie. Noordhoff, Groningen, 1938.
6. Penzias A. A. and Wilson R. W. A measurement of excess antenna temperature at 4080 Mc/s. *Astrophys. J.*, 1965, v. 1, 419–421.
7. Conklin E. K. Velocity of the Earth with respect to the Cosmic Background Radiation. *Nature*, 1969, v. 222, 971.
8. Henry P. S. Isotropy of the 3 K Background. *Nature*, 1971, v. 231, 516–518.
9. Corey B. E. and Wilkinson D. T. A measurement of the Cosmic Microwave Background Anisotropy at 19 GHz. *Bulletin of the American Astronomical Society*, 1976, v. 8, 351.
10. Lineweaver C.H. The CMB Dipole: The most recent measurement and some history. In *Microwave Background Anistropies. Proceedings of the XVIth Moriond Astrophysics Meeting*, Les Arcs, Savoie, France, March 16th-23rd, 1996, F. R. Bouchet, R. Gispert, B. Guilderdoni, and J.T.T. Van, eds., Gif-sur-Yvette: Editions Frontieres, 1997; (see also arXiv: astro-ph/9609034).
11. Smoot G. F., Gorenstein M. V. and Muller R. A. Detection of anisotropy in the Cosmic Blackbody Radiation. *Phys. Rev. Lett.*, 1977, v. 39, 898–901.
12. Boggess N.W., et al. The COBE mission: its design and perfomance two years after launch. *Astrophys. J.*, 1992, v. 397, 420–429.
13. Fixsen D.J., et al. The Cosmic Microwave Background spectrum from the full COBE FIRAS data set. *Astrophys. J.*, 1996, v. 473, 576–587.

14. Smoot G.F., et al. Preliminary results from the COBE differential microwave interferometers: large angular scale isotropy of the Cosmic Microwave Background. *Astrophys. J.*, 1991, v. 371, L1–L5.
15. Page L., et al. The optical design and characterization of the Microwave Anisotropy Probe. *Astrophys. J.*, 2003, v. 585, 566–586.
16. Bennett C.L., et al. The Microwave Anisotropy Probe mission. *Astrophys. J.*, 2003, v. 583(1), 1–23.
17. Bennett C.L., et al. First-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: preliminary maps and basic results. *Astrophys. J. Suppl. Ser.*, 2003, v. 148(1), 1–27.
18. Robitaille P.-M. WMAP: a radiological analysis. *Progress in Physics*, 2007, v. 1, 3–18.
19. Robitaille P.-M. On the origins of the CMB: insight from the COBE, WMAP and Relikt-1 satellites. *Progress in Physics*, 2007, v. 1, 19–23.
20. Robitaille P.-M. On the Earth Microwave Background: absorption and scattering by the atmosphere. *Progress in Physics*, 2007, v. 3, 3–4.
21. Robitaille P.-M. and Rabounski D. COBE and the absolute assignment of the CMB to the Earth. *2007 APS March Meeting*, Denver, Colorado, Monday–Friday, March 5–9, 2007, http://meetings.aps.org/link/BAPS.2007.MAR.L20.7
22. Robitaille P.-M. On the nature of the microwave background at the Lagrange 2 Point. Part I. *Progress in Physics*, 2007, v. 4, 74–83.
23. Rabounski D. The relativistic effect of the deviation between the CMB temperatures obtained by the COBE satellite. *Progress in Physics*, 2007, v. 1, 19–21.