The Asymptotic Approach to the Twin Paradox

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The argument of twins' asymmetry, essentially put forward in the common solution of the Twin Paradox, is revealed to be inoperative in some asymptotic situations in which the noninertial effects are insignificant. Consequently the respective solution proves itself as unreliable thing and the Twin Paradox is re-established as an open problem which require further investigations.

1 Introduction

Undoubtely that, in connection with Special Relativity, one of the most disputed subjects was (and still remaining) the so-called the *Twin Paradox*. Essentially this paradox consists in a contradiction between *time-dilatation* (relativistic transformations of time intervals) and the simple belief in symmetry regarding the ageing degrees of two relatively moving twins. The idea of time-dilattation is largely agreed in scientific literature (see [1–5] and references therein) as well as in various (more or less academic) media. However the experimental convincingness of the respective idea still remains a subject of interest even in the inverstigations of the last decades (see for examples [6–9]).

It is notable the fact that, during the last decades, the disputes regarding the Twin Paradox were diminished and disimulated owing to the *common solution* (CS), which seems to be accredited with a great and unimpeded popularity. In the essence, CS argues that the twins are in completely asymmetric ageing situations due to the difference in the noninertial effects which they feel. Such noninertial effects are connected with the nonuniform motion of only one of the two twins. Starting from the mentioned argumentation, without any other major and credible proof, CS states that the Twin Paradox is nothing but an apparent and fictitious problem.

But even in the situations considered by CS a kind of symmetry between the twins can be restored if the noninertial effects are adequately managed. Such a management is possible if we take into account an asymptotic situation when the motions of the traveling twin is prevalently uniform or, in addition, the nonuniform motions are symmetrically present for both of the twins. Here we will see that the existence of the mentioned asymptotic situations have major consequences/implications for the reliability of CS. Our search is done in the Special Relativity approach (without appeals to General Relativity). This is because we consider such an approach to be sufficiently accurate/adequate for the situations under discussion.

In the end we shall conclude that the existence of the alluded asymptotic situations invalidate the CS and restores the Twin Paradox as a real (non-apparent, non-fictitious) and open problem which requires further investigations.

2 Asymptotic situations in which the noninertial effects are insignificant

In order to follow our project let us reconsider, in a quantitative manner, the twins arrangement used in CS. We consider two twins A and B whose proper reference frames are K_A and K_B respectively. The situations of the two twins A and B are reported in comparison with an inertial reference frame K.

2.1 Discussions about an asymptotic asymmetric situation

Within the framework of a first approach, we consider the twin A remaining at rest in the coordinate origin O of the frame K while the twin B moves forth and back along the positive part of the x-axis of K. The motion of B passes throgh the points O, M, N and P whose x-positions are: $x_O = 0$, $x_M = D$, $x_N = D + L$, $x_P = 2D + L$. The motion starts and finishes at O, while P is a turning point — i.e. the velocity of B is zero at O and P . Along the segments OM and NP the motion is nonuniform (accelerated or decelerated) with a time t dependent velocity $v(t)$. On the other hand, along the segment MN, the motion is uniform with the velocity of $v_0 = \text{const}$. In the mentioned situation K_A coincides with K, while K_B moves (nonuniformly or uniformly) with respect to K. The time variables describing the degrees of ageeing of the two twins will be indexed respective to A and B. Also the mentioned time variables will be denoted respective to τ and t as they refer to the proper (intrinsic) time of the considered twin or, alternatively, to the time measured (estimed) in the reference frame of the other twin. The infinitesimal or finite intervals of τ and t will be denoted by $d\tau$ and dt respectively by $\Delta\tau$ and Δt.

With the menioned specifications, according to the relativity theory, for the time interval from the start to the finish of the motion of the twin B, one can write the relations

$$\Delta\tau_A = (\Delta t_B)_n + (\Delta t_B)_u \ , \tag{1}$$

$$\Delta\tau_B = \int\limits_{(\Delta t_B)_n} \sqrt{1 - \frac{v^2\,(t_B)}{c^2}} \cdot dt_B + (\Delta t_B)_u \sqrt{1 - \frac{v_0^2}{c^2}} \ . \tag{2}$$

In these equations the indices n and u refer to the nonuniform respectively uniform motions, while c denotes the light velocity. In (2) it was used the fact (accepted in the relativity theory [10]) that instantaneously, at any moment of time, an arbitrarily moving reference frame can be considered as inertial. Because $v(t_B) \leqslant v_0 \leqslant c$, from (1) and (2) a formula follows:

$$\Delta\tau_A > \Delta\tau_B\,. \tag{3}$$

On the other hand, in the framework of a simple conception (naive belief) these two twins must be in symmetric ageing when B returns at O. This means that, according to the respective conception, the following supposed relation (s.r) have to be taken into account

$$\Delta\tau_A = \Delta\tau_B \qquad \text{(s.r.)}\,. \tag{4}$$

Moreover, for the same simple conception, by invoking the relative character of the twins' motion, the roles of A and B in (3) might be (formally) inverted. Then one obtains another supposed relation, namely

$$\Delta\tau_A < \Delta\tau_B \qquad \text{(s.r.)}\,. \tag{5}$$

This obvious disagreement between the relativistic formula (3) and the supposed relations (4) and (5) represents just the Twin Paradox.

For resolving of the Twin Paradox, CS invokes [1–3] (as essential and unique argument) the assertion that the twins ageing is completely asymmetric. The respective assertion is argued with an idea that, in the mentioned arrangement of twins, B feels non-null noninertial effects during its nonuniform motions, while A, being at rest, does not feel such effects. Based on the alluded argumentation, without any other major and credible proof, CS rejects the supposed relations (4) and (5) as unfounded and fictitious. Then, according to CS only the relativistic formula (3) must be regarded as a correct relation. Consequently CS inferes the conclusion: the Twin Paradox is nothing but a purely and apparent fictitious problem.

But now we have to notify the fact that CS does not approach any discussion on the comparative importance (significance) in the Twin Paradox problem of the respective nonuniform and uniform motions. Particularly, it is not taken into discussions the asymptotic situatios where, comparatively, the effects of the noninertial motions become insignificant. Or, it is clear that, as it is considered by CS, the asymmetry of the twins is generated by the nonuniform motions, while the uniform motions have nothing to do on the respective asymmetry. That is why we discuss that the alluded comparative importance is absolutely necessary. Moreover such a discussion should refer (in a quantitative manner) to the comparative value/ratio of L and D. This is because

$$(\Delta t_B)_u = \frac{2L}{v_0}\,, \tag{6}$$

while, on the other hand, $(\Delta t_B)_n$ depends on D, — e.g. when the nonuniformity of B motions is caused by constant forces, the relativity theory gives

$$(\Delta t_B)_n = \frac{4v_0 D}{c^2\left(1-\sqrt{1-\frac{v_0^2}{c^2}}\,\right)}\,. \tag{7}$$

Then with the notation $\eta = \frac{D}{L}$ one obtains

$$\frac{(\Delta t_B)_n}{(\Delta t_B)_u} = \eta\,\frac{2v_0^2}{c^2\left(1-\sqrt{1-\frac{v_0^2}{c^2}}\,\right)} \approx 4\eta \quad (\text{for } v_0 \ll c)\,. \tag{8}$$

This means that, in the mentioned circumstances, the ratio $\eta = \frac{D}{L}$ has a property which gives a quantitative description to the comparative importance (significance) of the respective nonuniform and uniform motions. It is natural to consider η as the bearing the mentioned property in the circumstances that are more general than those refered in (7) and (8). That is why we will conduct our discussions in terms of the parameter η.

Specification: The quantities D and v_0 are considered as bing nonnull and constant, while L is regarded as an adjustable quantity. So we can consider situations where $\eta \ll 1$ or even where $\eta \to 0$.

Now let us discuss the cases where $\eta \ll 1$. In such a case the twin B moves predominantly uniform, and the noninertiel effects on it are prevalently absent. The twins' positions are prevalently symmeric or even become asymptotically symetric when $\eta \to 0$. That is why we regard/denote the respective cases as asymptotic situations. In such situations the role of the accelerated motions (and of associated noninertial effects) becomes insignifiant (negligible).

These just alluded situations should be appreciated by consideration (prevalently or even asymptotically) of Einstein's posulate of relativity, which states [3] that the inertial frames of references are equivalent to each other, and they cannot be distinguished by means of investigation of physical phenomena. Such an appreciation materializes itself in the relations

$$\left.\begin{aligned} &\Delta\tau_A \approx \Delta\tau_B\,, \qquad (\eta \ll 1) \\ &\lim_{\eta\to 0}\Delta\tau_A = \lim_{\eta\to 0}\Delta\tau_B \end{aligned}\right\}. \tag{9}$$

Also, from (1) and (2) one obtains

$$\Delta\tau_B \approx \Delta\tau_A\sqrt{1-\frac{v_0^2}{c^2}} < \Delta\tau_A\,, \qquad (\eta \ll 1)\,. \tag{10}$$

By taking into account the mentioned Einstein postulate in (10), the roles of A and B might be inverted and one finds

$$\Delta\tau_A \approx \Delta\tau_B\sqrt{1-\frac{v_0^2}{c^2}} < \Delta\tau_B\,, \qquad (\eta \ll 1)\,. \tag{11}$$

Note that, in the framework of the discussed case, the relations (9) and (11) are not supposed (or fictitious) pieces as (4) and (5) are, but they are true formulae like (10). This means that for $\eta \ll 1$ the mentioned arrangement of the twins leads to a set of incompatible relations (9)–(11). Within CS the respective incompatibility cannot be avoided by any means.

2.2 Discusions about an asymptotic completely symmetric situation

Now let us consider a new arrangement of the twins as follows. The twin B preserves exactly his situation previously presented. On the other hand, the twin A moves forward and backward in the negative part of the x-axis in K, symmetric to as B moves with respect to the point O. All the mentioned notations remain unchanged as the above. Evidently that, in the framework of the new arrangement, the situations of these two twins A and B, as well as their proper frames K_A and K_B, are completely symmetric with respect to K. From this fact, for the time intervals between start and finish of the motions, it results directly the relation

$$\Delta\tau_A = \Delta\tau_B\,. \qquad (12)$$

In addition, for asymptotic situations where $\eta \ll 1$, one obtains

$$\Delta\tau_A = \Delta\tau_B \approx \frac{2L}{v_0}\sqrt{1-\frac{v_0^2}{c^2}}\,, \qquad (\eta \ll 1)\,. \qquad (13)$$

On the other hand, by taking into account Einstein's postulate of relativity, similarly to the relations (10) and (11) for the new arrangement in the asymptotic situations (i.e., where $\eta \ll 1$ and the noninertial effects are insignificant), one finds

$$\Delta\tau_B \approx \Delta\tau_A\sqrt{1-\frac{w_0^2}{c^2}} < \Delta\tau_A\,, \qquad (\eta \ll 1)\,, \qquad (14)$$

$$\Delta\tau_A \approx \Delta\tau_B\sqrt{1-\frac{w_0^2}{c^2}} < \Delta\tau_B\,, \qquad (\eta \ll 1)\,, \qquad (15)$$

with

$$w_0 = \frac{2v_0}{1+\frac{v_0^2}{c^2}}\,. \qquad (16)$$

It should be noted that fact that, with respect to the relativity theory, the relations (12), (14) and (15) are true formulae: they are not supposed and/or fictitious. On the other hand, one finds that the mentioned relations are incompatible to each other. The respective incompatibility cannot be resolved or avoided in a rational way by CS whose solely major argument is the asymmetry of the twins.

3 Some final comments

The above analysed facts show that, in the mentioned "asymptotic situations", the noninertial effects are insignificant for the estimation of the time intervals evaluated (felt) by the two twins. Consequently in such situations the inertial-noninertial asymmetry between such two twins cannot play a significant role. Therefore the respective asymmetry cannot be considered a reliable proof in the resolving of the Twin Paradox. This means that the CS loses its essential (and solely) argument. So, the existence of the above mentioned asymptotic situations appears as a true incriminating test for CS.

Regarding to its significance and implications, the mentioned test has to be evaluated/examined concurently with the "approvingly ilustrations" invoked and preached by the supporters of CS. At this point it seems to be of some profit to remind the Feynmann's remark [11] that, in fact, a conception/theory is invalidated (proved to be wrong) by the real and irrefutable existence of a single incriminating test, indifferently of the number of approving illustrations. Some scientists consider that such a test must be only of experimental but not of theoretical nature. We think that the role of such tests can be played also by theoretical consequences rigurously derived from a given conception. So thinking, it is easy to see that for CS the existence of the above discussed asymptotic situations has all the characteristics of an irrefutable incriminating test. The respecrtive existence invalidate the CS which must be abandoned as a wrong and unreliable approach of the Twin Paradox.

But even if CS is abandoned the incompatibility regardig the relations (9)–(11) or/and the formulae (12), (14) and (15) remains as an unavoidable and intriguing fact. Then what is the significance and importance of the respective fact? We think that it restores the Twin Paradox as an authentic unsolved problem which is still waiting for further investigations. Probably that such investigations will involve a large variety of facts/arguments/opinions.

In connection to the alluded further investigations the following first question seems to be non-trivially interesting: can the investigations on the Twin Paradox be done in a credible manner withot troubling the Special Theory of Relativity? If a negative answer, a major importance goes to the second question: can the Twin Paradox, restored as mentioned above, be an incriminating test for the Special Theory of Relativity, in the sense of the previously noted Feynmann's remark, or not? Can the second question be connected to the "sub-title" of the volume mentioned in the reference [9], or not?

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