On the Origin of the Dark Matter/Energy in the Universe and the Pioneer Anomaly

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Einstein's special relativity is a theory rich of paradoxes, one of which is the recently discovered *Relativistic Invariant Mass Paradox*. According to this Paradox, the relativistic invariant mass of a galaxy of moving stars exceeds the sum of the relativistic invariant masses of the constituent stars owing to their motion relative to each other. This excess of mass is the mass of *virtual matter* that has no physical properties other than positive relativistic invariant mass and, hence, that reveals its presence by no means other than gravity. As such, this virtual matter is the dark matter that cosmologists believe is necessary in order to supply the missing gravity that keeps galaxies stable. Based on the Relativistic Invariant Mass Paradox we offer in this article a model which quantifies the anomalous acceleration of Pioneer 10 and 11 spacecrafts and other deep space missions, and explains the presence of dark matter and dark energy in the universe. It turns out that the origin of dark matter and dark energy in the Universe lies in the Paradox, and that the origin of the Pioneer anomaly results from neglecting the Paradox. In order to appreciate the physical significance of the Paradox within the frame of Einstein's special theory of relativity, following the presentation of the Paradox we demonstrate that the Paradox is responsible for the extension of the kinetic energy theorem and of the additivity of energy and momentum from classical to relativistic mechanics. Clearly, the claim that the acceleration of Pioneer 10 and 11 spacecrafts is anomalous is incomplete, within the frame of Einstein's special relativity, since those who made the claim did not take into account the presence of the Relativistic Invariant Mass Paradox (which is understandable since the Paradox, published in the author's 2008 book, was discovered by the author only recently). It remains to test how well the Paradox accords with observations.

1 Introduction

Einstein's special relativity is a theory rich of paradoxes, one of which is the *Relativistic Invariant Mass Paradox*, which was recently discovered in [1], and which we describe in Section 5 of this article. The term mass in special relativity usually refers to the rest mass of an object, which is the Newtonian mass as measured by an observer moving along with the object. Being observer's invariant, we refer the Newtonian, rest mass to as the *relativistic invariant mass*, as opposed to the common *relativistic mass*, which is another name for energy, and which is observer's dependent. Lev B. Okun makes the case that the concept of relativistic mass is no longer even pedagogically useful [2]. However, T. R. Sandin has argued otherwise [3].

As we will see in Section 5, the Relativistic Invariant Mass Paradox asserts that the resultant relativistic invariant mass m_0 of a system S of uniformly moving N particles exceeds the sum of the relativistic invariant masses m_k, $k = 1, \dots, N$, of its constituent particles, $m_0 > \sum_{k=1}^{N} m_k$, since the contribution to m_0 comes not only from the masses m_k of the constituent particles of S but also from their speeds relative to each other. The resulting excess of mass in the resultant relativistic invariant mass m_0 of S is the mass of virtual matter that has no physical properties other than positive relativistic invariant mass and, hence, that reveals itself by no means other than gravity. It is therefore naturally identified as the *mass* of *virtual* dark matter that the system S possesses. The presence of dark matter in the universe in a form of virtual matter that reveals itself only gravitationally is, thus, dictated by the Relativistic Invariant Mass Paradox of Einstein's special theory of relativity. Accordingly, (i) the fate of the dark matter particle(s) theories as well as (ii) the fate of their competing theories of modified Newtonian dynamics (MOND [4]) are likely to follow the fate of the eighteenth century phlogiston theory and the nineteenth century luminiferous ether theory, which were initiated as *ad hoc* postulates and which, subsequently, became obsolete.

Dark matter and dark energy are *ad hoc* postulates that account for the observed missing gravitation in the universe and the late time cosmic acceleration. The postulates are, thus, a synonym for these observations, as C. Lämmerzahl, O. Preuss and H. Dittus had to admit in [5] for their chagrin. An exhaustive review of the current array of dark energy theories is presented in [6].

The Pioneer anomaly is the anomalous, unmodelled acceleration of the spacecrafts Pioneer 10 and 11, and other spacecrafts, studied by J. D. Anderson *et al* in [7] and summarized by S. G. Turyshev *et al* in [8]. In [7], Anderson *et al* compared the measured trajectory of a spacecraft against its theoretical trajectory computed from known forces acting on the spacecraft. They found the small, but significant discrepancy known as the anomalous, or unmodelled, acceleration directed approximately towards the Sun. The inability to explain the Pioneer anomaly with conventional physics has contributed to the growing interest about its origin, as S. G. Turyshev, M. M. Nieto and J. D. Anderson pointed out in [9]. It is believed that no conventional force has been overlooked [5] so that, seemingly, new physics is needed. Indeed, since Anderson *et al* announced in [7] that the Pioneer 10 and 11 spacecrafts exhibit an unexplained anomalous acceleration, numerous articles appeared with many plausible explanations that involve new physics, as C. Castro pointed out in [10].

However, we find in this article that no new physics is needed for the explanation of both the presence of dark matter/energy and the appearance of the Pioneer anomaly. Rather, what is needed is to cultivate the Relativistic Invariant Mass Paradox, which has recently been discovered in [1], and which is described in Section 5 below.

Accordingly, the task we face in this article is to show that the Relativistic Invariant Mass Paradox of Einstein's special relativity dictates the formation of dark matter and dark energy in the Universe and that, as a result, the origin of the Pioneer anomaly stems from the motions of the constituents of the Solar system relative to each other.

2 Einstein velocity addition vs. Newton velocity addition

The improved way to study Einstein's special theory of relativity, offered by the author in his recently published book [1], enables the origin of the dark matter/energy in the Universe and the Pioneer anomaly to be determined. The improved study rests on analogies that Einsteinian mechanics and its underlying hyperbolic geometry share with Newtonian mechanics and its underlying Euclidean geometry. In particular, it rests on the analogies that Einsteinian velocity addition shares with Newtonian velocity addition, the latter being just the common vector addition in the Euclidean 3-space $\mathbb{R}^3$.

Einstein addition $\oplus$ is a binary operation in the ball $\mathbb{R}^3_c$ of $\mathbb{R}^3$,

$$\mathbb{R}^3_c = \{\mathbf{v} \in \mathbb{R}^3 : \|\mathbf{v}\| < c\} \tag{1}$$

of all relativistically admissible velocities, where c is the speed of light in empty space. It is given by the equation

$$\mathbf{u}\oplus\mathbf{v} = \frac{1}{1+\dfrac{\mathbf{u}\cdot\mathbf{v}}{c^2}}\left\{\mathbf{u} + \frac{1}{\gamma_{\mathbf{u}}}\mathbf{v} + \frac{1}{c^2}\frac{\gamma_{\mathbf{u}}}{1+\gamma_{\mathbf{u}}}(\mathbf{u}\cdot\mathbf{v})\mathbf{u}\right\} \tag{2}$$

where $\gamma_{\mathbf{u}}$ is the gamma factor

$$\gamma_{\mathbf{v}} = \frac{1}{\sqrt{1-\dfrac{\|\mathbf{v}\|^2}{c^2}}} \tag{3}$$

in $\mathbb{R}^3_c$, and where $\cdot$ and $\|\ \|$ are the inner product and norm that the ball $\mathbb{R}^3_c$ inherits from its space $\mathbb{R}^3$. Counterintuitively, Einstein addition is neither commutative nor associative.

Einstein gyrations $\mathrm{gyr}[\mathbf{u},\mathbf{v}] \in Aut(\mathbb{R}^3_c,\oplus)$ are defined by the equation

$$\mathrm{gyr}[\mathbf{u},\mathbf{v}]\mathbf{w} = \ominus(\mathbf{u}\oplus\mathbf{v})\oplus(\mathbf{u}\oplus(\mathbf{v}\oplus\mathbf{w})) \tag{4}$$

for all $\mathbf{u},\mathbf{v},\mathbf{w} \in \mathbb{R}^3_c$, and they turn out to be automorphisms of the Einstein groupoid $(\mathbb{R}^3_c,\oplus)$. We recall that a groupoid is a non-empty space with a binary operation, and that an automorphism of a groupoid $(\mathbb{R}^3_c,\oplus)$ is a one-to-one map f of $\mathbb{R}^3_c$ onto itself that respects the binary operation, that is, $f(\mathbf{u}\oplus\mathbf{v}) = f(\mathbf{u})\oplus f(\mathbf{v})$ for all $\mathbf{u},\mathbf{v} \in \mathbb{R}^3_c$. To emphasize that the gyrations of the Einstein groupoid $(\mathbb{R}^3_c,\oplus)$ are automorphisms of the groupoid, gyrations are also called gyroautomorphisms.

Thus, $\mathrm{gyr}[\mathbf{u},\mathbf{v}]$ of the definition in (4) is the gyroautomorphism of the Einstein groupoid $(\mathbb{R}^3_c,\oplus)$, generated by $\mathbf{u},\mathbf{v} \in \mathbb{R}^3_c$, that takes the relativistically admissible velocity $\mathbf{w}$ in $\mathbb{R}^3_c$ into the relativistically admissible velocity $\ominus(\mathbf{u}\oplus\mathbf{v})\oplus(\mathbf{u}\oplus(\mathbf{v}\oplus\mathbf{w}))$ in $\mathbb{R}^3_c$.

The gyrations, which possess their own rich structure, measure the extent to which Einstein addition deviates from commutativity and associativity as we see from the following identities [1, 11, 12]:

$\mathbf{u}\oplus\mathbf{v} = \mathrm{gyr}[\mathbf{u},\mathbf{v}](\mathbf{v}\oplus\mathbf{u})$	Gyrocommutative Law
$\mathbf{u}\oplus(\mathbf{v}\oplus\mathbf{w}) = (\mathbf{u}\oplus\mathbf{v})\oplus\mathrm{gyr}[\mathbf{u},\mathbf{v}]\mathbf{w}$	Left Gyroassociative
$(\mathbf{u}\oplus\mathbf{v})\oplus\mathbf{w} = \mathbf{u}\oplus(\mathbf{v}\oplus\mathrm{gyr}[\mathbf{u},\mathbf{v}]\mathbf{w})$	Right Gyroassociative
$\mathrm{gyr}[\mathbf{u},\mathbf{v}] = \mathrm{gyr}[\mathbf{u}\oplus\mathbf{v},\mathbf{v}]$	Left Loop Property
$\mathrm{gyr}[\mathbf{u},\mathbf{v}] = \mathrm{gyr}[\mathbf{u},\mathbf{v}\oplus\mathbf{u}]$	Right Loop Property

Einstein addition is thus regulated by its gyrations so that Einstein addition and its gyrations are inextricably linked. Indeed, the Einstein groupoid $(\mathbb{R}^3_c,\oplus)$ forms a group-like mathematical object called a *gyrocommutative gyrogroup* [13], which was discovered by the author in 1988 [14]. Interestingly, Einstein gyrations are just the mathematical abstraction of the relativistic *Thomas precession* [1, Sec. 10.3].

The rich structure of Einstein addition is not limited to its gyrocommutative gyrogroup structure. Einstein addition admits scalar multiplication, giving rise to the Einstein gyrovector space. The latter, in turn, forms the setting for the Beltrami-Klein ball model of hyperbolic geometry just as vector spaces form the setting for the standard model of Euclidean geometry, as shown in [1].

Guided by the resulting analogies that relativistic mechanics and its underlying hyperbolic geometry share with classical mechanics and its underlying Euclidean geometry, we are able to present analogies that Newtonian systems of particles share with Einsteinian systems of particles in Sections 3 and 4. These analogies, in turn, uncover the Relativistic Invariant Mass Paradox in Section 5, the physical significance of which is illustrated in Section 6 in the frame of Einstein's special theory of relativity. Finally, in Sections 7 and 8 the Paradox reveals the origin of the dark matter/energy in the Universe as well as the origin of the Pioneer anomaly.

3 Newtonian systems of particles

In this section we set the stage for revealing analogies that a Newtonian system of N particles and an Einsteinian system of N particles share. In this section, accordingly, as opposed to Section 4, $\mathbf{v}_k$, $k = 0, 1, \ldots, N$, are Newtonian velocities in $\mathbb{R}^3$, and m_0 is the Newtonian resultant mass of the constituent masses m_k, $k = 1, \ldots, N$ of a Newtonian particle system S.

Accordingly, let us consider the following well known classical results, (6) – (8) below, which are involved in the calculation of the Newtonian resultant mass m_0 and the classical center of momentum (CM) of a Newtonian system of particles, and to which we will seek Einsteinian analogs in Section 4. Thus, let

$$S = S(m_k, \mathbf{v}_k, \Sigma_\mathbf{0}, N)\,, \qquad \mathbf{v}_k \in \mathbb{R}^3 \tag{5}$$

be an isolated Newtonian system of N noninteracting material particles the k-th particle of which has mass m_k and Newtonian uniform velocity $\mathbf{v}_k$ relative to an inertial frame $\Sigma_\mathbf{0}$, $k = 1, \ldots, N$. Furthermore, let m_0 be the resultant mass of S, considered as the mass of a virtual particle located at the center of mass of S, and let $\mathbf{v}_0$ be the Newtonian velocity relative to $\Sigma_\mathbf{0}$ of the Newtonian CM frame of S. Then,

$$1 = \frac{1}{m_0}\sum_{k=1}^{N} m_k \tag{6}$$

and

$$\left.\begin{aligned} \mathbf{v}_0 &= \frac{1}{m_0}\sum_{k=1}^{N} m_k\mathbf{v}_k \\ \mathbf{u}+\mathbf{v}_0 &= \frac{1}{m_0}\sum_{k=1}^{N} m_k(\mathbf{u}+\mathbf{v}_k) \end{aligned}\right\}, \tag{7}$$

$\mathbf{u}, \mathbf{v}_k \in \mathbb{R}^3$, $m_k > 0$, $k = 0, 1, \ldots, N$. Here m_0 is the Newtonian mass of the Newtonian system S, supposed concentrated at the center of mass of S, and $\mathbf{v}_0$ is the Newtonian velocity relative to $\Sigma_\mathbf{0}$ of the Newtonian CM frame of the Newtonian system S in (5).

It follows from (6) that m_0 in (6) – (7) is given by the Newtonian resultant mass equation

$$m_0 = \sum_{k=1}^{N} m_k\,. \tag{8}$$

The derivation of the second equation in (7) from the first equation in (7) is immediate, following (i) the distributive law of scalar-vector multiplication, and (ii) the simple relationship (8) between the Newtonian resultant mass m_0 and its constituent masses m_k, $k = 1, \ldots, N$.

4 Einsteinian systems of particles

In this section we present the Einsteinian analogs of the Newtonian expressions (5) – (8) listed in Section 3. The presented analogs are obtained in [1] by means of analogies that result from those presented in Section 2.

In this section, accordingly, as opposed to Section 3, $\mathbf{v}_k$, $k = 0, 1, \ldots, N$, are Einsteinian velocities in $\mathbb{R}^3_c$, and m_0 is the Einsteinian resultant mass, yet to be determined, of the masses m_k, $k = 1, \ldots, N$, of an Einsteinian particle system S.

In analogy with (5), let

$$S = S(m_k, \mathbf{v}_k, \Sigma_\mathbf{0}, N), \qquad \mathbf{v}_k \in \mathbb{R}^3_c \tag{9}$$

be an isolated Einsteinian system of N noninteracting material particles the k-th particle of which has invariant mass m_k and Einsteinian uniform velocity $\mathbf{v}_k$ relative to an inertial frame $\Sigma_\mathbf{0}$, $k = 1, \ldots, N$. Furthermore, let m_0 be the resultant mass of S, considered as the mass of a virtual particle located at the center of mass of S (calculated in [1, Chap. 11]), and let $\mathbf{v}_0$ be the Einsteinian velocity relative to $\Sigma_\mathbf{0}$ of the Einsteinian center of momentum (CM) frame of the Einsteinian system S in (9). Then, as shown in [1, p. 484], the relativistic analogs of the Newtonian expressions in (6) – (8) are, respectively, the following Einsteinian expressions in (10) – (12),

$$\left.\begin{aligned} \gamma_{\mathbf{v}_0} &= \frac{1}{m_0}\sum_{k=1}^{N} m_k\gamma_{\mathbf{v}_k} \\ \gamma_{\mathbf{u}\oplus\mathbf{v}_0} &= \frac{1}{m_0}\sum_{k=1}^{N} m_k\gamma_{\mathbf{u}\oplus\mathbf{v}_k} \end{aligned}\right\} \tag{10}$$

and

$$\left.\begin{aligned} \gamma_{\mathbf{v}_0}\mathbf{v}_0 &= \frac{1}{m_0}\sum_{k=1}^{N} m_k\gamma_{\mathbf{v}_k}\mathbf{v}_k \\ \gamma_{\mathbf{u}\oplus\mathbf{v}_0}(\mathbf{u}\oplus\mathbf{v}_0) &= \frac{1}{m_0}\sum_{k=1}^{N} m_k\gamma_{\mathbf{u}\oplus\mathbf{v}_k}(\mathbf{u}\oplus\mathbf{v}_k) \end{aligned}\right\}, \tag{11}$$

$\mathbf{u}, \mathbf{v}_k \in \mathbb{R}^3_c$, $m_k > 0$, $k = 0, 1, \ldots, N$. Here m_0,

$$m_0 = \sqrt{\left(\sum_{k=1}^{N} m_k\right)^2 + 2\sum_{\substack{j,k=1\\ j<k}}^{N} m_j m_k(\gamma_{\ominus\mathbf{v}_j\oplus\mathbf{v}_k} - 1)} \tag{12}$$

is the relativistic invariant mass of the Einsteinian system S, supposed concentrated at the relativistic center of mass of S

(calculated in [1, Chap. 11]), and $\mathbf{v}_0$ is the Einsteinian velocity relative to Σ_0 of the Einsteinian CM frame of the Einsteinian system S in (9).

5 The relativistic invariant mass paradox of Einstein's special theory of relativity

In analogy with the Newtonian resultant mass m_0 in (8), which follows from (6), it follows from (10) that the Einsteinian resultant mass m_0 in (10)–(11) is given by the elegant Einsteinian resultant mass equation (12), as shown in [1, Chap. 11].

The Einsteinian resultant mass equation (12) presents a Paradox, called the *Relativistic Invariant Mass Paradox*, since, in general, this equation implies the inequality

$$m_0 > \sum_{k=1}^{N} m_k \tag{13}$$

so that, paradoxically, the invariant resultant mass of a system may exceed the sum of the invariant masses of its constituent particles.

The paradoxical invariant resultant mass equation (12) for m_0 is the relativistic analog of the non-paradoxical Newtonian resultant mass equation (8) for m_0, to which it reduces in each of the following two special cases:

(i) The Einsteinian resultant mass m_0 in (12) reduces to the Newtonian resultant mass m_0 in (8) in the limit as $c \to \infty$; and

(ii) The Einsteinian resultant mass m_0 in (12) reduces to the Newtonian resultant mass m_0 in (8) in the special case when the system S is rigid, that is, all the internal motions in S of the constituent particles of S relative to each other vanish. In that case $\ominus\mathbf{v}_j \oplus \mathbf{v}_k = \mathbf{0}$ so that $\gamma_{\ominus\mathbf{v}_j\oplus\mathbf{v}_k} = 1$ for all $j, k = 1, N$. This identity, in turn, generates the reduction of (12) to (8).

The second equation in (11) follows from the first equation in (11) in full analogy with the second equation in (7), which follows from the first equation in (7) by the distributivity of scalar multiplication and by the simplicity of (8). However, while the proof of the latter is simple and well known, the proof of the former, presented in [1, Chap. 11], is lengthy owing to the lack of a distributive law for the Einsteinian scalar multiplication (see [1, Chap. 6]) and the lack of a simple relation for m_0 like (8), which is replaced by (12). Indeed, the proof of the former, that the second equation in (11) follows from the first equation in (11), is lengthy, but accessible to undergraduates who are familiar with the vector space approach to Euclidean geometry. However, in order to follow the proof one must familiarize himself with a large part of the author's book [1] and with its "gyrolanguage", as indicated in Section 2.

It is therefore suggested that interested readers may corroborate numerically (using a computer software like MATLAB) the identities in (10)–(12) in order to gain confidence in their validity, before embarking on reading several necessary chapters of [1].

6 The physical significance of the paradox in Einstein's special theory of relativity

In this section we present two classically physical significant results that remain valid relativistically owing to the Relativistic Invariant Mass Paradox, according to which the relativistic analog of the classical resultant mass m_0 in (8) is, paradoxically, the relativistic resultant mass m_0 in (12).

To gain confidence in the physical significance that results from the analogy between

(i) the Newtonian resultant mass m_0 in (8) of the Newtonian system S in (5) and

(ii) the Einsteinian invariant resultant mass m_0 in (12) of the Einsteinian system S in (9)

we present below two physically significant resulting analogies. These are:

(1) *The Kinetic Energy Theorem* [1, p. 487]: According to this theorem,

$$K = K_0 + K_1\,, \tag{14}$$

where

(i) K_0 is the relativistic kinetic energy, relative to a given observer, of a virtual particle located at the relativistic center of mass of the system S in (9), with the Einsteinian resultant mass m_0 in (12); and

(ii) K_1 is the relativistic kinetic energy of the constituent particles of S relative to its CM; and

(iii) K is the relativistic kinetic energy of S relative to the observer.

The Newtonian counterpart of (14) is well known; see, for instance, [15, Eq. (1.55)]. The Einsteinian analog in (14) was, however, unknown in the literature since the Einsteinian resultant mass m_0 in (12) was unknown in the literature as well till its first appearance in [1]. Accordingly, Oliver D. Johns had to admit for his chagrin that "The reader (of his book; see [15, p. 392]) will be disappointed to learn that relativistic mechanics does not have a theory of collective motion that is as elegant and complete as the one presented in Chapter 1 for Newtonian mechanics."

The proof that m_0 of (12) is compatible with the validity of (14) in Einstein's special theory of relativity is presented in [1, Theorem 11.8, p. 487].

(2) *Additivity of Energy and Momentum*: Classically, energy and momentum are additive, that is, the total energy and the total momentum of a system S of particles is, respectively, the sum of the energy and the sum of momenta of its constituent particles. Consequently,

also the resultant mass m_0 of S is additive, as shown in (8). Relativistically, energy and momentum remain additive but, consequently, the resultant mass m_0 of S is no longer additive. Rather, it is given by (12), which is the relativistic analog of (8).

The proof that m_0 of (12) is compatible with the additivity of energy and momentum in Einstein's special theory of relativity is presented in [1, pp. 488–491].

Thus, the Einsteinian resultant mass m_0 in (12) of the Einsteinian system S in (9) is the relativistic analog of the Newtonian resultant mass m_0 in (8) of the Newtonian system S in (5). As such, it is the Einsteinian resultant mass m_0 in (12) that is responsible for the extension of the validity of (14) and of the additivity of energy and momentum from classical to relativistic mechanics.

However, classically, mass is additive. Indeed, the Newtonian resultant mass m_0 equals the sum of the masses of the constituent particles, $m_0 = \sum_{k=1}^{N} m_k$, as we see in (8). Relativistically, in contrast, mass is not additive. Indeed, the Einsteinian resultant mass m_0 may exceed the sum of the masses of the constituent particles, $m_0 \geqslant \sum_{k=1}^{N} m_k$, as we see from (12). Accordingly, from the relativistic viewpoint, the resultant mass m_0 in (12) of a galaxy that consists of stars that move relative to each other exceeds the sum of the masses of its constituent stars. This excess of mass reveals its presence only gravitationally and, hence, we identify it as the mass of dark matter. Dark matter is thus virtual matter with positive mass, which reveals its presence only gravitationally. In particular, the dark mass m_{dark} of the Einsteinian system S in (9), given by (16) below, is the mass of virtual matter called the dark matter of S. To contrast the real matter of S with its virtual, dark matter, we call the former *bright* (or, luminous, or, baryonic) matter. The total mass m_0 of S, which can be detected gravitationally, is the composition of the bright mass m_{bright} of the real, bright matter of S, and the dark mass m_{dark} of the virtual, dark matter of S. This mass composition, presented in (15)–(17) in Section 7 below, quantifies the effects of dark matter.

7 The origin of the dark matter

Let

$$m_{bright} = \sum_{k=1}^{N} m_k \tag{15}$$

and

$$m_{dark} = \sqrt{2 \sum_{\substack{j,k=1 \\ j<k}}^{N} m_j m_k (\gamma_{\ominus \mathbf{v}_j \oplus \mathbf{v}_k} - 1)} \tag{16}$$

so that the Einsteinian resultant mass m_0 in (12) turns out to be a composition of an ordinary, bright mass m_{bright} of real matter and a dark mass m_{dark} of virtual matter according to the equation

$$m_0 = \sqrt{m_{bright}^2 + m_{dark}^2} \tag{17}$$

The mass m_{bright} in (15) is the Newtonian resultant mass of the particles of the Einsteinian system S in (9). These particles reveal their presence gravitationally, as well as by radiation that they may emit and by occasional collisions.

In contrast, the mass m_{dark} in (16) is the mass of virtual matter in the Einsteinian system S in (9), which reveals its presence only gravitationally. In particular, it does not emit radiation and it does not collide. As such, it is identified with the dark matter of the Universe.

In our expanding universe, with accelerated expansion [16], relative velocities between some astronomical objects are significantly close to the speed of light c. Accordingly, since gamma factors $\gamma_{\mathbf{v}}$ approach ∞ when their relative velocities $\mathbf{v} \in \mathbb{R}_c^3$ approach the speed of light, it follows from (16) that dark matter contributes an increasingly significant part of the mass of the universe.

8 The origin of the dark energy

Under different circumstances dark matter may appear or disappear resulting in gravitational attraction or repulsion. Dark matter increases the gravitational attraction of the region of each stellar explosion, a supernova, since any stellar explosion creates relative speeds between objects that were at rest relative to each other prior to the explosion. The resulting generated relative speeds increase the dark mass of the region, thus increasing its gravitational attraction. Similarly, relative speeds of objects that converge into a star vanish in the process of star formation, resulting in the decrease of the dark mass of a star formation region. This, in turn, decreases the gravitational attraction or, equivalently, increases the gravitational repulsion of any star formation inflated region. The increased gravitational repulsion associated with star formation results in the accelerated expansion of the universe, first observed in 1998; see [6, p. 1764], [17] and [18, 19]. Thus, according to the present special relativistic dark matter/energy model, the universe accelerated expansion is a late time cosmic acceleration that began at the time of star formation.

9 The origin of the Pioneer anomaly

The Einsteinian resultant mass m_0 of our Solar system is given by the composition (17) of the bright mass m_{bright} and the dark mass m_{dark} of the Solar system. The bright mass m_{bright} of the Solar system equals the sum of the Newtonian masses of the constituents of the Solar system. Clearly, it is time independent. In contrast, the dark mass m_{dark} of the Solar system stems from the speeds of the constituents of the Solar system relative to each other and, as such, it is time dependent.

The Pioneer 10 and 11 spacecrafts and other deep space missions have revealed an anomalous acceleration known as the Pioneer anomaly [7, 8]. The Pioneer anomaly, described in the introductory section, results from an unmodelled acceleration, which is a small constant acceleration on top of which there is a smaller time dependent acceleration. A brief summary of the Pioneer anomaly is presented by K. Tangen, who asks in the title of [20]: "Could the Pioneer anomaly have a gravitational origin?"

Our answer to Tangen's question is affirmative. Our dark matter/energy model, governed by the Einsteinian resultant mass m_0 in (15) – (17), offers a simple, elegant model that explains the Pioneer anomaly. The motion of any spacecraft in deep space beyond the Solar system is determined by the Newtonian law of gravity where the mass of the Solar system is modelled by the Einsteinian resultant mass m_0 in (17) rather than by the Newtonian resultant mass m_0 in (8). It is the contribution of the dark mass m_{dark} to the Einsteinian resultant mass m_0 in (15) – (17) that generates the Pioneer anomaly.

Ultimately, our dark matter/energy model, as dictated by the paradoxical Einsteinian resultant mass m_0 in (12), will be judged by how well the model accords with astrophysical and astronomical observations. Since our model is special relativistic, only uniform velocities are allowed. Hence, the model can be applied to the solar system, for instance, under the assumption that, momentarily, the solar system can be viewed as a system the constituents of which move uniformly.

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