The Generalized Conversion Factor in Einstein's Mass-Energy Equation

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Einstein's September 1905 paper is origin of light energy-mass inter conversion equation ($L = \Delta mc^2$) and Einstein speculated $E = \Delta mc^2$ from it by simply replacing L by E. From its critical analysis it follows that $L = \Delta mc^2$ is only true under special or ideal conditions. Under general cases the result is $L \propto \Delta mc^2$ ($E \propto \Delta mc^2$). Consequently an alternate equation $\Delta E = Ac^2\Delta M$ has been suggested, which implies that energy emitted on annihilation of mass can be equal, less and more than predicted by $\Delta E = \Delta mc^2$. The total kinetic energy of fission fragments of U^{235} or Pu^{239} is found experimentally 20–60 MeV less than Q-value predicted by Δmc^2. The mass of particle Ds (2317) discovered at SLAC, is more than current estimates. In many reactions including chemical reactions $E = \Delta mc^2$ is not confirmed yet, but regarded as true. It implies the conversion factor than c^2 is possible. These phenomena can be explained with help of generalized mass-energy equation $\Delta E = Ac^2\Delta M$.

1 Introduction

Mass energy inter-conversion processes are the oldest in nature and constitute the basis of various phenomena. Before Einstein's work, Newton [1] stated that "Gross bodies and light are convertible into one another. . . ". Einstein derived light energy-mass inter-conversion equation for Newton's perception as $L = \Delta mc^2$. Before Einstein scientists such as S. Tolver Preston [2] Olinto De Pretto [3], Fritz Hasenohrl [4, 5] Frederick Soddi [6] contributed to the topic.

Einstein's derivation of $L = \Delta mc^2$ (from which Einstein speculated $E = \Delta mc^2$), is true under special conditions (where selective values of variables are taken). Under general conditions (when all possible values of parameters are taken) equations like $L = 0.0011\Delta mc^2$, $L = 0.999988\Delta mc^2$ etc. are obtained i.e. $L \propto \Delta mc^2$. Thus conversion factor other than c^2 is possible in Einstein's derivation. Further the generalized mass–energy equation $\Delta E = Ac^2\Delta M$, is derived, and $E = \Delta mc^2$ is special case of the former depending upon value of A (depends upon the characteristics conditions of the process). Thus apart from theoretical limitations, $E = \Delta mc^2$ has experimental limitations e.g. sometimes experimental results differ from it and in many cases it is not confirmed. Under such cases $\Delta E = Ac^2\Delta M$ is widely useful and applicable. The fission fragments result from U^{235} and Pu^{239} have total kinetic energy 20–60 MeV less than Q-value (200 MeV) of reaction predicted by $\Delta E = \Delta mc^2$ [7, 8, 9]. Palano [10] has confirmed that mass of particle Ds (2317) has been found more than current estimates based upon $\Delta E = \Delta mc^2$. Also $\Delta E = \Delta mc^2$ does not give consistent results in explaining the binding energy, as it violates the universal equality of masses of nucleons.

All these facts can be explained by $\Delta E = Ac^2\Delta M$ with value of A less or more than one. $\Delta E = \Delta mc^2$ is not confirmed in many processes such chemical reactions, atom bomb explosions, volcanic reactions etc. Whatever may be the case $\Delta E = Ac^2\Delta M$ is capable of explaining the phenomena. Thus conversion factor other than c^2 is possible, in Einstein's September 1905 derivation and confirmed experimentally also.

2 Einstein's light energy — mass equation $L = \Delta mc^2$ and its hidden aspects

Einstein [11] perceived that let there be a luminous body at rest in co-ordinate system (x, y, z). The system (ξ, η, ζ) is in uniform parallel translation w.r.t. system (x, y, z); and origin of system (ξ, η, ζ) moves along x-axis with relative velocity v. Let a system of plane light waves have energy ℓ relative to system (x, y, z), the ray direction makes angle ϕ with x-axis of the system (ξ, η, ζ). The quantity of light measured in system (ξ, η, ζ) has the energy [11, 12].

$$\ell* = \ell\,\frac{\left(1 - \frac{v}{c}\cos\phi\right)}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

Einstein has given Eq. (1) in his paper known as Special Theory of Relativity [12] and called Eq. (1) as Doppler principle for any velocities whatever.

Let E_0 and H_0 are energies in coordinate system (x, y, z) and system (ξ, η, ζ) before emission of light energy, further E_1 and H_1 are the energies of body in the both systems after it emits light energy. Thus Einstein wrote various equations as Energy of body in system (x, y, z)

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L; \tag{2}$$

Energy of body in system (ξ, η, ζ)

$$H_0 = H_1 + 0.5\beta\,L\left[\left(1 - \frac{v}{c}\cos\phi\right) + \left(1 + \frac{v}{c}\cos\phi\right)\right] \tag{3}$$

where $\beta = 1/[1 - v^2/c^2]^{1/2}$;

$$H_0 = H_1 + \beta L; \tag{4}$$

or

$$(H_0 - E_0) - (H_1 - E_1) = L\,(\beta - 1)\,. \tag{5}$$

Einstein calculated, kinetic energy of body before emission of light energy, $K_0(m_b v^2/2)$ and kinetic energy of body after emission of light energy, $K(m_a v^2/2)$ as

$$K_0 - K = L\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right) \tag{6}$$

Einstein considered the velocity in classical region thus applying binomial theorem,

$$K_0 - K = L\bigg(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + + \frac{15v^6}{48c^6} + \frac{105v^8}{384c^8} + \ldots - 1\bigg). \tag{7}$$

Further Einstein quoted [16] "Neglecting magnitudes of fourth and higher orders, we may place"

$$K_0 - K = L\,\frac{v^2}{2c^2} \tag{8}$$

$$M_b\frac{v^2}{2} - M_a\frac{v^2}{2} = L\,\frac{v^2}{2c^2} \tag{9}$$

or

$$L = (M_b - M_a)\,c^2 = \Delta mc^2, \tag{10}$$

or Mass of body after emission (M_a) = Mass of body before emission ($M_b - L/c^2$).

Now replacing L (light energy) by E (total energy or every energy) Einstein wrote

$$E = (M_b - M_a)\,c^2 = \Delta mc^2 \tag{11}$$

or Mass of body after emission (M_a) = Mass of body before emission ($M_b - E/c^2$).

Thus Einstein derived that conversion factor between mass and light energy is precisely equal to c^2, this aspect is elaborated by Fadner [13]. But Einstein's this derivation has been critically discussed by many such a Planck [14], Stark [15], Ives [16], Stanchel [17], Okun [18] and N. Hamdan [19] etc. At the same time in some references [20, 21] it is expressed that Einstein has taken hints to derive equation $E = \Delta mc^2$ and from existing literature without acknowledging the work of preceding scientists. Max Born [22] has expressed that Einstein should have given references of existing literature.

Thus Einstein's work on the topic has been critically analyzed by scientists since beginning, in views of its scientific and procedural aspects.

3 The conversion factor between mass-energy other than c^2 is also supported by Einstein's derivation under general conditions

As already mentioned Einstein's September 1905 derivation of $\Delta L = \Delta mc^2$ is true under special or ideal conditions (selected values of parameters is taken) only, this aspect is studied critically with details by the author [23–36] discussing those aspects which have not been raised earlier. Thus the value of conversion factor other than c^2 is also supported from Einstein's derivation under general conditions (all possible values of variables). The law or phenomena of interconversion of mass and energy holds good in all cases for all bodies and energies under all conditions.

In the derivation of $\Delta L = \Delta mc^2$ there are FOUR variables e.g.

(a) Number of waves emitted,

(b) l magnitude of light energy,

(c) Angle ϕ at which light energy is emitted and

(d) Uniform velocity, v.

Einstein has taken special values of parameters and in general for complete analysis the derivation can be repeated with all possible values of parameters i.e. under general conditions taking in account the momentum conservation (which is discussed in next sub-section).

(A) The body can emit large number of light waves but Einstein has taken only TWO light waves emitted by luminous body.

(B) The light waves emitted may have different magnitudes but Einstein has taken EQUAL magnitudes

(C) Body may emit large number of light waves of different magnitudes of energy making DIFFERENT ANGLES (other than 0° and 180°) assumed by Einstein.

(D) Einstein has taken velocity in classical region ($v \ll c$) has not at all used velocity in relativistic region. If velocity is regarded as in relativistic region (v is comparable with c), then equation for relativistic variation of mass with velocity i.e.

$$M_{rel} = \frac{M_{rest}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{12}$$

is taken in account. It must be noted that before Einstein's work this equation was given by Lorentz [37, 38] and firstly confirmed by Kaufman [39] and afterwards more convincingly by Bucherer [40]. Einstein on June 19, 1948 wrote a letter to Lincoln Barnett [41] and advocated abandoning relativistic mass and suggested that is better to use the expression for the momentum and energy of a body in motion, instead of relativistic mass.

It is strange suggestion as Einstein has used relativistic mass in his work including in the expression of relativistic kinetic energy [12] from which rest mass energy is derived.

(E) In addition Einstein has assumed that body remains at rest before and after emission of light energy. But the body may be at rest i.e. $v = 0$, velocity may be in classical region and velocity may be in relativistic region ($v \sim c$), the law of inter-conversion of mass and energy holds good under all conditions.

In electron-positron annihilation, the material particles are in motion before and after annihilation. In materialization of energy, a gamma ray photon is converted to electron positron pair, which move in opposite directions to conserve momentum. In nuclear fission and fusion particles remain in motion in the process of mass energy inter conversion. The thermal neutron which causes fission has velocity 2185 m/s.

4 $L \propto \Delta mc^2$ is mathematically consistent in Einstein's derivation, under general conditions

Under general conditions (all possible values of variables) the value of conversion factor other than c^2 can be easily justified mathematically in Einstein's derivation [23–36]. This aspect is not touched by the preceding authors [13–21].

(a) In Einstein's derivation if one wave is regarded as to form angle 0.5° rather than 0° then

$$H_0 = H_1 + 0.5\beta L \times \times \left[\left(1 - \frac{v}{c}\cos 0.5^\circ\right) + \left(1 - \frac{v}{c}\cos 180^\circ\right)\right], \quad (13)$$

or

$$H_0 = H_1 + \beta L \left(1 + 0.000018038\,\frac{v}{c}\right),$$

or

$$K_0 - K = 0.000019038\,lL\,\frac{v}{c} + L\,\frac{v^2}{2c^2},$$

$$\Delta m\,(M_b - M_a) = 0.000038077\,\frac{L}{cv} + \frac{L}{c^2}, \quad (14)$$

or

$$L = \frac{\Delta mc^2}{1141} = 0.000876\Delta mc^2, \quad (15)$$

$$\Delta L \propto \Delta mc^2.$$

Further, M_a (mass after emission of light energy) $= M_b$ (mass before emission of light energy): $0.000038077L/cv = L/c^2$ in (14).

According to Einstein if body emits two light waves of energy $0.5L$ each in opposite directions then decrease in mass is given by Eq. (10) i.e. $\Delta m = L/c^2$ and in this case decrease in mass is $(0.000038077L/cv + L/c^2)$ thus there is no definite value of decrease in mass in Einstein's derivation. In this case decrease in mass is more than as predicted by Einstein, hence again the conversion factor other than c^2 is confirmed i.e. $\Delta L \propto \Delta mc^2$. Like this there are many examples of this type.

(b) The central equation in Einstein's derivation is Eq. (1) and binomial theorem is equally applicable to it at any stage i.e. in the beginning or end. Einstein applied binomial theorem in the end and obtained $L = \Delta mc^2$, but the same equation is not obtained if binomial theorem is applied in the beginning. The binomial theorem is simply a mathematical tool and its application at any stage should not affect results i.e. make or mar equation $L = \Delta mc^2$.

The reason is that typical nature of derivation and Eq. (1) is different from other relativistic equations. The energy is scalar quantity and independent of direction but Eq. (1) is directional in nature due to angle ϕ. In contrast if binomial theorem is applied to Relativistic Kinetic Energy in the beginning or at the end then result is same i.e. classical form of kinetic energy $(m_{rest}v^2/2)$. So there is inconsistency in applications in this case.

Applying binomial theorem to Eq. (1) and repeating the calculations as Einstein did, altogether different results are obtained,

$$\ell^* = \ell\left(1 - \frac{v}{c}\cos\phi\right)\left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \ldots\right). \quad (16)$$

Here $v/c \ll 1$, hence v^2/c^2 and higher terms can be neglected. Thus

$$\ell^* = \ell\left(1 - \frac{v}{c}\cos\phi\right)$$

or

$$(H_0 - E_0) - (H_1 - E_1) = 0,$$

or

$$K_b - K_a = 0,$$

or

$$\frac{1}{2}\,M_b v^2 - \frac{1}{2}\,M_a v^2 = 0,$$

or

$$\text{Mass of body before emission } (M_b) = = \text{Mass of body after emission } (M_a). \quad (17)$$

Thus light energy is being emitted, but under this condition Einstein's this derivation does not provide any relationship (equality or proportionality) between mass annihilated and energy created. Similar is the situation if velocity $v = 0$. Hence Einstein's derivation gives decrease in mass of body equal to L/c^2 only under certain conditions. Thus in this case derivation is not valid.

Sr. No	Values of various parameters	Whether $L = \Delta mc^2$ or $L \propto \Delta mc^2$
1	$0.5L$, $0.5L$, $\phi = 0^\circ$, $\phi = 180^\circ$	$L = \Delta mc^2$
2	$0.5L$, $0.5L$, $\phi = 0.5^\circ$, $\phi = 180^\circ$	$L = \Delta mc^2/901$ or $L \propto \Delta mc^2$
3	$0.5001L$, $0.49999L$, $\phi = 0^\circ$, $\phi = 180^\circ$	$L = 0.9999988\Delta mc^2$ or $L \propto \Delta mc^2$
4	$0.5L$, $0.5L$, $\phi = 0^\circ$, $\phi = 180^\circ$ but $v = 0$	No relation between L and Δm
5	$0.5L$, $0.5L$, $\phi = 0^\circ$, $\phi = 180^\circ$ but Binomial Theorem is applied in beginning.	No relation between L and Δm
6	For other energies than light	Equations not considered

Table 1: Einstein's Sep 1905 derivation gives $L = \Delta mc^2$ under certain conditions and $L \propto \Delta mc^2$ under general conditions

(c) Let the body emits two light waves of slightly different energies i.e. $0.5001L$ and $0.4999L$ in opposite directions and other parameters remain the same as assumed by Einstein. In this case

$$H_0 = H_1 + 0.4999\beta L \left(1 - \frac{v}{c}\cos 0^\circ\right) + + 0.5001\beta L \left(1 - \frac{v}{c}\cos 180^\circ\right). \tag{18}$$

Now proceeding in the same way as Einstein did

$$K_0 - K = 0.0002\,L\,\frac{v}{c} + L\,\frac{v^2}{2c^2} \tag{19}$$

or

$\Delta m =$ Mass of body before emission$(M_b)-$ Mass of body after emission(M_a),

$$= 0.0004\,\frac{L}{cv} + \frac{L}{c^2} \tag{20}$$

or

$$M_a = 0.004\,\frac{L}{cv} - \frac{L}{c^2} + M_b$$

or

$$L = \frac{\Delta mc^2}{\left(0.0004\frac{c}{v} + 1\right)}.$$

The velocity v is in classical region, say 10 m/s,

$$L = \Delta mc^2 \left[0.000083\right], \tag{21}$$

$$\Delta L \propto \Delta mc^2.$$

Thus, $\Delta E \propto \Delta mc^2$. Hence conversion factor other than c^2 follows from Einstein's derivation under general conditions.

(d) Energy emitted in various reactions. In his September 1905 paper Einstein derived Eq. (10) i.e. $\Delta L = \Delta mc^2$ and then replaced L (light energy) by E (total energy) and speculated

$$\Delta E = \Delta mc^2. \tag{11}$$

In Eq. (11) E stands for all possible energies of the universe e.g.: (i) sound energy, (ii) heat energy, (iii) chemical energy, (iv) nuclear energy, (v) magnetic energy, (vi) electrical energy, (vii) energy emitted in form of invisible radiations, (viii) energy emitted in cosmological and astrophysical phenomena, (ix) energy emitted volcanic reactions, (x) energies co-existing in various forms etc., etc.

Now Eq. (1) i.e.

$$\ell^* = \ell\,\frac{\left(1 - \frac{v}{c}\cos\phi\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is put forth for light energy by Einstein in June 1905 paper (ℓ^* is light energy in moving frame), it is not meant for other possible energies as quoted above.

Einstein never justified Eq. (1) for all the energies cited above. The parameters used in Einstein's equation are defined for light energy only, not for all the energies. Thus by this derivation $L = \Delta mc^2$ is derived under special conditions for light energy only and replacing L by E in Eq. (10) is not justified.

There are evidences that Einstein worked hurriedly in other case also e.g. in theory of static universe the introduction of cosmological constant proved to be incorrect and Einstein accepted the mistake later as quoted by Gamow [42]. The various cases when $\Delta E \propto \Delta mc^2$ is justified are shown in Table 1.

5 Conservation of momentum in general cases

The momentum is conserved irrespective of the fact that body remains at rest or recoils or tends to recoil after emission of light energy [43]. The law of conservation of momentum can be used to calculate the velocity of recoil in this case also. Let the body of mass 10 kg emits two waves of energy in visible region of wavelength 5000Å it corresponds to energy $7.9512{\times}10^{-19}$ J. This energy is emitted in two waves i.e., as obvious, $0.5001L$ ($3.97639512{\times}10^{-19}$ J) and $0.4999L$ ($3.97480488{\times}10^{-19}$ J). Applying the conservation of momentum [43] the recoil velocity, recoil momentum and recoil kinetic energy comes out to be $-5.3{\times}10^{-32}$ m/s, $5.3{\times}10^{-31}$ kg×m/s and $1.404{\times}10^{-62}$ J respectively. This recoil velocity (V_r) will change the uniform velocity v as $V_r + v$, but it will not make any difference to final result of change in mass as in Eq. (21), due to negligible value of V_r [27]. Hence in the law of conservation of momentum is obeyed in this case also.

6 Experimental feasibility with conversion factors other than c^2

(a) Dirac [44] was one of the first physicists to suggest that, in connection with his theory of large numbers, fundamental dimensional constants may vary in time during the expansion of the universe. The idea of variation of the speed of light is suggested in various cosmological models [45, 46] and has been the subject of attention by physicists in investigations of extra dimensions, strings and branes [47]. Webb [48] has reported variations in fine structure constant over cosmological time scales and hence variations in c. This suggestion implies $\Delta E \propto \Delta mc^2$.
(b) Einstein has derived $L = \Delta mc^2$ (conversion factor between mass and energy is precisely equal to c^2) under the extremely special or ideal conditions , which are even difficult to attain practically. The work of scientists before Einstein also justifies $\Delta E \propto \Delta mc^2$.

This discussion does not confront with existing experimental situation but addresses those theoretical and experimental issues for which $\Delta E = \Delta mc^2$ is not analyzed yet. The mass energy inter-conversion equation, with conversion factor equal to c^2 i.e. $\Delta E = \Delta mc^2$ has been confirmed in nuclear physics and is also basis of nuclear physics. Even elementary units of atomic mass (1 amu) or and energy (eV) are based upon it. Thus it will remain standard in measurements as seven days in a week; its validity in this regard is not doubted at all.

The aim is to discuss experimentally those phenomena in which $\Delta E = \Delta mc^2$ is not applied yet. The mass energy conversion processes are weird in nature and all have not been at all studied in view of $\Delta E = \Delta mc^2$. The conversion factor other than c^2 is discussed for such elusive cases, not for those it is already confirmed. Hence there is no confrontation with the established experimental situation at all, but aim is to open a mathematical front ($\Delta E \propto \Delta mc^2$) for numerous experimentally unstudied phenomena in nature. This development can be discussed as below.

7 Most abundant chemical reactions

(i) **Unconfirmed chemical reactions.** When Einstein derived $E = \Delta mc^2$, chemical reactions were the most abundant sources of energy in nature. Till date $E = \Delta mc^2$ is not confirmed in the chemical reaction and the reason cited for this is that equipments are not enough sensitive [49, 50]. Consider burning of 1kg straw or paper or petrol in controlled way i.e. in such a way that masses, ashes, gases and energy produced can be estimated. Even if 0.001 kg or 1gm of matter is annihilated then energy equal to $9{\times}10^{13}$ J (can drive a truck of mass 1000 kg to distance of $9{\times}10^7$ km) will be produced. Until the equation is not confirmed in such reactions, then scientifically $E = \Delta mc^2$ may not be regarded as precisely true in such cases. It is equally possible that energy emitted may be less than predicted by $E = \Delta mc^2$ i.e. $E \propto \Delta mc^2$ is feasible, it is an open possibility unless ruled out.

Reactions in nuclear physics

(ii) **Less efficiency:** The efficiency of the nuclear weapons as well as nuclear reactors is far less than the theoretical value predicted by $E = \Delta mc^2$. Robert Serber (member of first American team entered Hiroshima and Nagasaki in September 1945 to assess loses), has indicated [51] that the efficiency of "Little Boy" weapon (U^{235}, 49 kg) that was used against Hiroshima was about 2% only. It is assumed that all the atoms don't undergo fission, thus material is wasted. But no such waste material is specifically measured quantitatively. Thus the waste material (nuclear reactor or weapon) must be measured and corresponding energy be calculated, and it must quantitatively explain that why efficiency is less. It may require the measurements of all types of energies (may co-exist in various forms) in the processes and experimental errors. Until such experiments are specifically conducted and $E = \Delta mc^2$ is confirmed, $\Delta E \propto \Delta mc^2$ is equally feasible.
(iii) **Less energy:** In laboratory it is confirmed [7, 52, 53] that using thermal neutrons the total kinetic energy (TKE) of fission fragments that result from of U^{235} and Pu^{239} is 20–60 MeV less than Q-value (200 MeV) of reaction predicted by $\Delta E = \Delta mc^2$. This observation is nearly four decades old. Bakhoum [7] has explained it on the basis of equation $H = mv^2$ (energy emitted is less than $E = \Delta mc^2$). Hence here $E \propto \Delta mc^2$ is justified.
(iv) **More mass:** Palano [10] has confirmed that mass of particle Ds (2317) has been found more than current estimates based upon $\Delta E = \Delta mc^2$. Thus in this case $E \propto \Delta mc^2$ is justified.
(v) **Binding energy and mass defect in deuteron:** There are two inherent observations [23, 28, 29] about nucleus: firstly, masses of nucleons are fundamental constants, i.e. they are the same universally (inside and outside the nucleus in all cases); and secondly nuclei possess Binding Energy ($BE = \Delta mc^2$) owing to a mass defect. To explain these observations, in the case of the deuteron ($BE = 2.2244$ MeV), the mass defect of nucleons must be 0.002388 amu or about 0.11854% of the mass of nucleons, i.e., nucleons must be lighter in the nucleus. This is not experimentally justified, as masses of nucleons are universal constants. Thus observations and predictions based upon $\Delta E = \Delta mc^2$ are not justified, hence $\Delta E \propto \Delta mc^2$ is equally feasible.

8 Mathematical form of extended equation

Until $E = \Delta mc^2$ is not precisely confirmed experimentally in ALL CASES , it is equally feasible to assume that the energy emitted may be less than $E = \Delta mc^2$ (or $E \propto \Delta mc^2$). It does not have any effect on those cases where $E = \Delta mc^2$ is confirmed, it simply scientifically stresses confirmation of $E = \Delta mc^2$ in all cases. Also when reactants are in bulk amount and various types of energies are simultaneously emitted and energies may co-exist. Thus both the possibilities are equally probable until one is not specifically ruled out. In view of weirdness in reactions emitting energy in universe, some theoretical inconsistencies in the derivation and non-availability of data, one can explore the second possibility even as a postulate. All the equations in science are regarded as confirmed when specifically justified in all experiments time and again. The reactions involving inter-conversion of mass and energy are utmost diverse, weird and new phenomena are being added regularly, thus $E = mc^2$ needs to be confirmed in all cases. Thus in general, in view of above proportionality it may be taken in account as

$$dE \propto c^2 dm\,.$$

The above proportionality $dE \propto c^2 dm$ can be changed into equation by introducing a constant of proportionality. The inception of proportionality constant is consistent with centuries old perception of constant of proportionality in physics since days of Aristotle and Newton. In second law of motion ($F = kma$) the value of constant of proportionality, k is always unity (like universal constant) i.e. $F = ma$. When more and more complex phenomena were studied or values of constants of proportionality were determined then it showed dependence on the inherent characteristics of the phenomena. In case constant of proportionality varies from one situation to other then it is known as co-efficient of proportionality e.g. co-efficient of thermal conductivity or viscosity etc. Thus removing the proportionality between dE and $c^2 dm$, we get

$$dE = A c^2 dm\,, \tag{22}$$

where$A$ is (a co-efficient) used to remove that sign of proportionality; it depends upon inherent characteristics of the processes in which conversion of mass to energy takes place and it is dimensionless. It has nature precisely like Hubble's constant (50 and 80 kilometers per second-Megaparsec, Mpc) or coefficient of viscosity (1.05×10^{-3} poise to 19.2×10^{-6} poise) or co-efficient of thermal conductivity (0.02 $\text{Wm}^{-1}\text{K}^{-1}$ to 400 $\text{Wm}^{-1}\text{K}^{-1}$) etc. Thus, in fact Hubble's constant may be regarded Hubble's variable constant or Hubble's coefficient, as it varies from one heavenly body to other. If "A" is equal to one, then we will get $dE = dmc^2$ i.e. same as Einstein's equation.

In Eq. (22) "A" is regarded as conversion factor as it describes feasibility and extent of conversion of mass into energy. For example out of bulk mass, the mass annihilated to energy is maximum in matter-antimatter annihilation, apparently least in chemical reactions, undetermined in volcanic reactions and cosmological reactions. It (the co-efficient A) depends upon the characteristic conditions of a particular process. It may be constant for a particular process and varies for the other depending upon involved parameters or experimental situation. Thus "A" cannot be regarded as universal constant, just like universal gravitational constant G and k in Newton's Second Law of Motion. The reason is that mass energy inter-conversion are the bizarre processes in nature and not completely studied.

Now consider the case that when mass is converted into energy. Let in some conversion process mass decreases from M_i(initial mass) to M_f (final mass), correspondingly energy increases from E_i (initial energy) to E_f (final energy). The Eq. (22) gives infinitesimally small amount of energy dE created on annihilation of mass dm. To get the net effect the Eq. (22) can be integrated similarly Einstein has obtained the relativistic form of kinetic energy in June 1905 paper [18]

$$\int dE = A c^2 \int dm\,,$$

$$\text{Initial limit of mass} = M_i,\ \text{Initial limit of Energy} = E_i\,,$$

$$\text{Final limit of mass} = M_f,\ \text{Final limit of Energy} = E_f\,.$$

Initially when mass of body is M_i, then E_i is the initial energy of the system. When mass (initial mass, M_i) is converted into energy by any process under suitable circumstances the final mass of system reduces to M_f. Consequently, the energy of system increases to E_f the final energy. Thus M_f and E_f are the quantities after the conversion. Hence, Eq. (22) becomes

$$E_f - E_i = A c^2 \left(M_f - M_i\right) \tag{23}$$

or

$$\Delta E = A c^2 \Delta m \tag{24}$$

$$\text{Energy evolved} = A c^2 \text{ (decrease in mass)}. \tag{25}$$

If the characteristic conditions of the process permit then whole mass is converted into energy i.e. after the reaction no mass remains ($M_f = 0$)

$$\Delta E = -\,A c^2 M_i \tag{26}$$

In this case energy evolved is negative implies that energy is created at the cost of annihilation of mass and the process is exo-energic nature (energy is emitted which may be in any form). Energy is scalar quantity having magnitude only, thus no direction is associated with it.

Thus the generalized mass-energy equivalence may be stated as

> "The mass can be converted into energy or vice-versa under some characteristic conditions of the process, but conversion factor may or may not always be c^2 (9×10^{16} m^2/s^2) or c^{-2}."

9 Applications of generalized mass energy inter conversion equation $\Delta E = A c^2 \Delta m$

(i) It is already mentioned in section (3) that if 0.001 kg or 1gm of matter is annihilated then energy equal to 9×10^{13} J (can drive a truck of mass 1000 kg to distance of 9×10^7 km) will be produced. Such or similar predictions are not experimentally confirmed and energy emitted can be found less than predictions.

Let the energy observed is 4.5×10^{13} J corresponding to mass annihilated 0.001 kg , then value of A from $\Delta E = A c^2 \Delta m$ will be 0.5 i.e.

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{4.5\times10^{13}\,\text{J}}{9\times10^{16}} = 0.5. \tag{27}$$

Thus in this case mass energy inter-conversion equation becomes

$$\Delta E = 0.5c^2 \Delta m\,. \tag{28}$$

(ii) Let the TKE of fission fragments of U^{235} and Pu^{239} is 175 MeV (as experimentally it is observed less), instead of expected 200 MeV. It can be explained with help of $\Delta E = A c^2 \Delta m$ with value of A is equal to 0.875 i.e.

$$A = \frac{\Delta E}{c^2}\Delta m = \frac{175}{200} = 0.875\,. \tag{29}$$

Thus energy of fission fragments of U^{235} and Pu^{239} is given by

$$\Delta E = 0.875c^2 \Delta m\,. \tag{30}$$

Thus value of A less than one is justified experimentally in this case.

(iii) The anomalous observation of excess mass of Ds(2317) can be understood with help of $\Delta E = A c^2 \Delta m$, as mass of the observed particle is found more [10] than predictions of $E = \Delta m c^2$. In this case value of A will be less than one. For understanding consider energy equal to 10^6 J is converted into mass, then corresponding mass must be 1.11×10^{-11} kg. We are considering the case that mass is found more than this. Let the mass be 1.12×10^{-11} kg. The value of A this case is 0.992, as calculated from $\Delta E = A c^2 \Delta m$ i.e.

$$A = \frac{10^6}{10.8\times10^5} = 0.992\,. \tag{31}$$

Thus in this mass energy inter conversion equation becomes

$$\Delta E = 0.992c^2 \Delta m \quad \text{or} \quad \Delta m = 1.008 \Delta E\,. \tag{32}$$

Thus corresponding to small energy more mass is emitted.

(vi) $\Delta E = A c^2 \Delta m$ is useful in explaining the binding energy (2.2244MeV or 3.55904×10^{-13} J), mass defect (0.002388 amu or 2.388×10^{-3} amu) and universal equality of mass of nucleons ($m_n = 1.008664$ amu, $m_p = 1.006082$ amu). Obviously neutron and protons contribute equally towards the mass defect (0.001194 amu), then mass of neutron inside nucleus must be 1.00747 amu (mass outside nucleus i.e. in Free State is 1.008664 amu). Similarly corresponding mass of proton in the nucleus must be 1.006082 amu (mass of proton outside nucleus 1.007274 amu). But decrease in mass of nucleons inside nucleus is not justified, as masses of nucleons are universally same [23, 28, 29].

Thus mass defect of deuteron must be **infinitesimally small**, only then masses of nucleons are same inside **nucleus and outside nucleus.** Also binding energy must be 2.2244 MeV as experimentally observed. Both these experimentally confirmed facts can be explained with help of $\Delta E = A c^2 \Delta m$.

Let in this case the mass defect is negligibly small i.e. 2.388×10^{-13} amu or 3.9653×10^{-40} kg. Then value of A (coefficient of proportionality or mass energy inter conversion coefficient) is 10^{10} i.e. for annihilation of infinitesimally small mass exceptionally large amount of energy is liberated. Thus

$$A = \frac{\Delta E}{c^2 \Delta m} = \frac{3.5634\times10^{-13}}{9\times10^{16}\times3.9653\times10^{-40}} = 10^{10}\,, \tag{33}$$

$$\Delta E = 10^{10} c^2 \Delta m\,. \tag{34}$$

(v) Webb [48] has reported results for time variability of the fine structure constant or Sommerfeld fine structure constant (α) using absorption systems in the spectra of distant quasars. The variation in magnitude of alpha has been observed as

$$\frac{\Delta\alpha}{\alpha} = \frac{(\alpha_{then} - \alpha_{now})}{\alpha_{now}} = -\,0.9\times10^{-5}. \tag{35}$$

According to CODATA currently accepted value of alpha (α_{now}) is 7.297352×10^{-3}. Hence from Eq. (35),

$$\alpha_{then} = 0.007296\,. \tag{36}$$

Now corresponding to the reduced value of α ($\alpha_{then} = 0.007296$) the the speed of light can be determined from equation

$$c_{then} = \frac{e^2}{2\alpha_{then}\varepsilon h} \tag{37}$$

as 2.994×10^8 m/s (where all terms have usual meanings). Currently accepted value of the speed of light is 2.99729×10^8 m/s.

To explain the energy emitted with this value of the speed of light is the value A ($\Delta E = A c^2 \Delta M$)

$$A = \frac{c^2}{c^2_{then}} = 1.001\,. \tag{38}$$

Thus in this case mass energy inter conversion equation becomes

$$\Delta E = 1.001c^2 \Delta m\,. \tag{39}$$

Hence $\Delta E \propto \Delta m c^2$ has both experimental and theoretical support, with emergence of new experimental data its significance will increase.

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