Potential Energy Surfaces of the Even-Even $^{230\text{-}238}$U Isotopes

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Nuclear structure of $^{230\text{-}238}$U isotopes hav been studied in the frame work of the interacting boson approximation model $(IBM-1)$. The contour plot of the potential energy surfaces, $V(\beta,\gamma)$, shows that all nuclei are deformed and have rotational characters, $SU(3)$. Levels energy spectra belonging to the gsb, β , γ bands, electromagnetic transition rates $B(E1)$ and $B(E2)$, quadrupole moment Q_0, deformation parameterare β_2 and the strength of the electric monopole transitions $X(E0/E2)$ are calculated. The calculated values are compared with the available theoretical and experimental data and show reasonable agreement.

1 Introduction

The observation of a large quadrupole moments to $^{230\text{-}238}$U isotopes had led to the suggestion that these nuclei might be deformed and have to be confirmed by the measurement of their nuclear properties as well as the observation of their rotational band structures. It is noticed that the level schemes of uranium isotopes are characterized by the existence of two bands of opposite parity and lay in the region of octupole deformations. The primary evidence for this octupole deformaton comes from the parity-doublet bands, fast electric transition $(E1)$ between the negative and positive parity bands and the low-lying 1^-, 0_2^+ and 2_2^+ excitation energy states. Many authors have studied $^{230\text{-}238}$U isotopes theoretically using different models. The relativistic Mean Field Model has employed [1–4] to obtain the densities of the cluster and daughter nuclei. Also, a systematic α-decay properties of the even-even heavy and superheavy nuclei have been investigated. The energy of the deformed nuclei in the actinide region has been determined in the frame work of the macroscopic — microscopic approach. The Yukawa folding procedure has used [5] together with the Liquid Drop Model [6].

The properties of the states of the alternating parity bands in actinides are analyzed within the Cluster Model. The model has been used successfully in calculating levels energy, quadrupole moments and half-lives of cluster radioactivity. A comparison was mad between the predicted data [7–13] and the calculated values by other models and show good agreement.

The band heads, energy spacings within bands and a number of interband as well as intraband $B(E2)$ transition rates are well reproduced [14] for all actinide nuclei using the Exactly Separable Davidson (ESD) solution of the Bohr Hamiltonian.

The potential energy surfaces are calculated [15] to ^{230}U using the most advanced asymmetric two-center shell model that are added to the Yukawa-plus-exponential model.

Until now scarce informations are available about the actinide region in general and this is due to the experimental difficulties associated with this mass region. In the present article we used the Interacting Boson Model $(IBM-1)$ which is a theoretical model and differ than all the previous models used with the actinid nuclei. The aim of the present work is to process calculation for the follows:

1. For the potential energy surfaces, $V(\beta,\gamma)$, for all $^{230\text{-}238}$U nuclei;
2. For levels energy;
3. For the electromagnetic transition rates $B(E1)$ and also calculation for $B(E2)$;
4. For the electric quadrupole moment Q_0;
5. For the deformation parameter β_2;
6. For the strength of the electric monopole transitions $X(E0/E2)$.

2 (IBA-1) model

2.1 Level energies

The IBA-1 model was applied to the positive and negative parity low-lying states in even-even $^{230\text{-}238}$U isotopes. The proton, π, and neutron, ν, bosons are treated as one boson and the system is considered as an interaction between s-bosons and d-bosons. Creation $(s^\dagger d^\dagger)$ and annihilation $(s\tilde{d})$ operators are for s and d bosons. The Hamiltonian [16] employed for the present calculation is given as:

$$\begin{aligned} H &= EPS\cdot n_d + PAIR\cdot(P\cdot P) + \\ &+ \frac{1}{2}\,ELL\cdot(L\cdot L) + \frac{1}{2}\,QQ\cdot(Q\cdot Q) + \\ &+ 5OCT\cdot(T_3\cdot T_3) + 5HEX\cdot(T_4\cdot T_4)\,, \end{aligned} \tag{1}$$

nucleus	EPS	$PAIR$	ELL	QQ	OCT	HEX	$E2SD(eb)$	$E2DD(eb)$
^{230}U	0.2000	0.000	0.005	−0.0150	0.0000	0.0000	0.2060	−0.6094
^{232}U	0.2000	0.000	0.0050	−0.0150	0.0000	0.0000	0.1890	−0.5591
^{234}U	0.2000	0.0000	0.0044	−0.0150	0.0000	0.0000	0.1782	−0.5271
^{236}U	0.2000	0.0000	0.0055	−0.0150	0.0000	0.0000	0.1720	−0.5088
^{238}U	0.2000	0.0000	0.0057	−0.0150	0.0000	0.0000	0.1630	−0.4822

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

where

$$P\cdot p=\frac{1}{2}\left[\begin{array}{l}\left\{(s^{\dagger}s^{\dagger})_0^{(0)}-\sqrt{5}(d^{\dagger}d^{\dagger})_0^{(0)}\right\}x\\ \left\{(ss)_0^{(0)}-\sqrt{5}(\tilde{d}\tilde{d})_0^{(0)}\right\}\end{array}\right]_0^{(0)}, \tag{2}$$

$$L\cdot L=-10\sqrt{3}\left[(d^{\dagger}\tilde{d})^{(1)}x\,(d^{\dagger}\tilde{d})^{(1)}\right]_0^{(0)}, \tag{3}$$

$$Q\cdot Q=\sqrt{5}\left[\begin{array}{l}\left\{(S^{\dagger}\tilde{d}+d^{\dagger}s)^{(2)}-\frac{\sqrt{7}}{2}(d^{\dagger}\tilde{d})^{(2)}\right\}x\\ \left\{(s^{\dagger}\tilde{d}++\tilde{d}s)^{(2)}-\frac{\sqrt{7}}{2}(d^{\dagger}\tilde{d})^{(2)}\right\}\end{array}\right]_0^{(0)}, \tag{4}$$

$$T_3\cdot T_3=-\sqrt{7}\left[(d^{\dagger}\tilde{d})^{(2)}x\,(d^{\dagger}\tilde{d})^{(2)}\right]_0^{(0)}, \tag{5}$$

$$T_4\cdot T_4=3\left[(d^{\dagger}\tilde{d})^{(4)}x\,(d^{\dagger}\tilde{d})^{(4)}\right]_0^{(0)}. \tag{6}$$

In the previous formulas, n_d is the number of boson; $P\cdot P$, $L\cdot L$, $Q\cdot Q$, $T_3\cdot T_3$ and $T_4\cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and $PAIR$, ELL, QQ, OCT, HEX is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator [16] employed in this study is given by:

$$T^{(E2)}=E2SD\cdot(s^{\dagger}\tilde{d}+d^{\dagger}s)^{(2)}+ +\frac{1}{\sqrt{5}}E2DD\cdot(d^{\dagger}\tilde{d})^{(2)}. \tag{7}$$

The reduced electric quadrupole transition rates between $I_i\to I_f$ states are given by

$$B(E_2,I_i-I_f)=\frac{[<I_f\parallel T^{(E_2)}\parallel I_i>]^2}{2I_i+1}. \tag{8}$$

3 Results and discussion

3.1 The potential energy surface

The potential energy surfaces [17], $V(\beta,\gamma)$, for uranium isotopes as a function of the deformation parameters β and γ have been calculated using :

$$\begin{aligned}&E_{N_{\Pi}N_{\nu}}(\beta,\gamma)=<N_{\pi}N_{\nu};\beta\gamma\,|H_{\pi\nu}|N_{\pi}N_{\nu};\beta\gamma>=\\&=\zeta_d(N_{\nu}N_{\pi})\beta^2(1+\beta^2)+\beta^2(1+\beta^2)^{-2}\times\\&\times\left\{kN_{\nu}N_{\pi}[4-(\bar{X}_{\pi}\bar{X}_{\nu})\beta\cos 3\gamma]\right\}+\\&+\left\{[\bar{X}_{\pi}\bar{X}_{\nu}\beta^2]+N_{\nu}(N_{\nu}-1)\left(\frac{1}{10}c_0+\frac{1}{7}c_2\right)\beta^2\right\},\end{aligned} \tag{9}$$

where

$$\bar{X}_{\rho}=\left(\frac{2}{7}\right)^{0.5}X_{\rho}\quad\rho=\pi\text{ or }\upsilon. \tag{10}$$

The calculated potential energy surfaces, $V(\beta,\gamma)$, for uranium series of isotopes are presented in Fig. 1 and Fig. 2. It shows that all nuclei are deformed and have rotational-like characters. The two wells on both oblate and prolate sides are not equal but the prolate is deeper in all nuclei.. The energy and electromagnetic transition rates are calculated cosidering uranium series of isotopes a rotational-like nuclei.

3.2 Energy spectra

IBA-1 model has been used in calculating the energy of the positive and negative parity low -lying levels of uranium series of isotopes. In many deformed actinide nuclei the negative parity bands have been established and these nuclei are considered as an octupole deformed. A simple means to examine the nature of the band is to consider the ratio R which for octupole band, $R\succ 1$, and defined as [18]:

$$R=\frac{E(I+3)-E(I-1)_{NPB}}{E(I)-E(I-2)_{GSB}}. \tag{11}$$

In the present calculations all values of R for uranium series of isotopes are $\succ$ 1, and we treated them as octupole deformed nuclei.

A comparison between the experimental spectra [19–23] and our calculations, using values of the model parameters given in Table 1 for the ground and octupole bands, are illustrated in Fig. 3. The agreement between the calculated levels energy and their correspondence experimental values for all uranium nuclei are reasonable, but slightly higher especially for the higher excited states. We believe this is due to the change of the projection of the angular momentum which

I_i^+ I_f^+	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
0_1 Exp. 2_1	9.70(12)	10.0(10)	10.66(20)	11.61(15)	12.09(20)
0_1 Theor. 2_1	9.7128	10.0163	10.6479	11.6506	12.1143
2_1 0_1	1.9426	2.0033	2.1296	2.3301	2.4229
2_2 0_1	0.0107	0.0113	0.0104	0.0095	0.0081
2_2 0_2	1.2419	1.3677	1.5411	1.7598	1.8855
2_3 0_1	0.0190	0.0131	0.0099	0.0082	0.0066
2_3 0_2	0.0027	0.0095	0.0131	0.0144	0.0139
2_3 0_3	0.0245	0.0085	0.0031	0.0013	0.0007
2_4 0_3	0.7577	0.8679	1.0291	1.2308	1.3730
2_4 0_4	0.0508	0.0415	0.1309	0.0710	0.0022
4_1 2_1	2.7740	2.8443	3.0182	3.3014	3.4336
4_1 2_2	0.0699	0.0480	0.0352	0.0276	0.0213
4_1 2_3	0.0046	0.0019	0.0010	0.0007	0.0005
6_1 4_1	3.0183	3.0849	3.2707	3.5790	3.7256
6_1 4_2	0.0706	0.0532	0.0412	0.0333	0.0260
6_1 4_3	0.0128	0.0066	0.0039	0.0026	0.0018
8_1 6_1	3.0670	3.1387	3.3335	3.6548	3.8121
8_1 6_2	0.0618	0.0503	0.0415	0.0351	0.0381
8_1 6_3	0.0201	0.0117	0.0073	0.0049	0.0034
10_1 8_1	2.9919	3.0827	3.2910	3.6237	3.7930
10_1 8_2	0.0510	0.0439	0.0383	0.0340	0.0280

Table 2: Values of the theoretical reduced transition probability, $B(E2)$ (in $e^2\,b^2$).

I_i^- I_f^+	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
1_1 0_1	0.1353	0.1602	0.1824	0.2071	0.2294
1_1 0_2	0.0531	0.0512	0.0492	0.0475	0.0449
3_1 2_1	0.2509	0.2811	0.3075	——	——
3_1 2_2	0.0811	0.0763	0.0711	——	——
3_1 2_3	0.0013	0.0002	0.0000	——	——
5_1 4_1	0.3628	0.3913	——	——	——
5_1 4_2	0.0862	0.0831	——	——	——
5_1 4_3	0.0020	0.0006	——	——	——
7_1 6_1	0.4809	0.5064	——	——	——
7_1 6_2	0.0816	0.0811	——	——	——
9_1 8_1	0.6043	0.6267	——	——	——
9_1 8_2	0.0736	0.0749	——	——	——

Table 3: Values of the theoretical reduced transition probability, $B(E1)$ (in $\mu\,e^2 b$).

$nucleus$	^{230}U	^{232}U	^{234}U	^{236}U	^{238}U
Q_0	9.920	10.020	10.340	10.800	11.020
β_2	0.263	0.264	0.272	0.282	0.286

Table 4: The calculated electric quadrupole moment Q_0 and deformation parameter β_2.

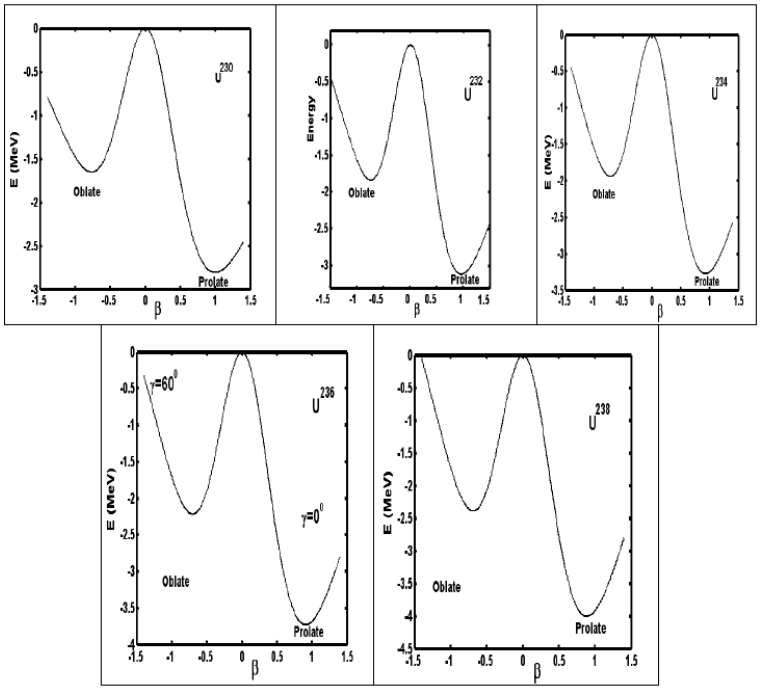

Fig. 1: Potential energy surfaces for $^{230\text{-}238}$U nuclei at $\gamma = 0^\circ$ (prolate) and 60° (oblate).

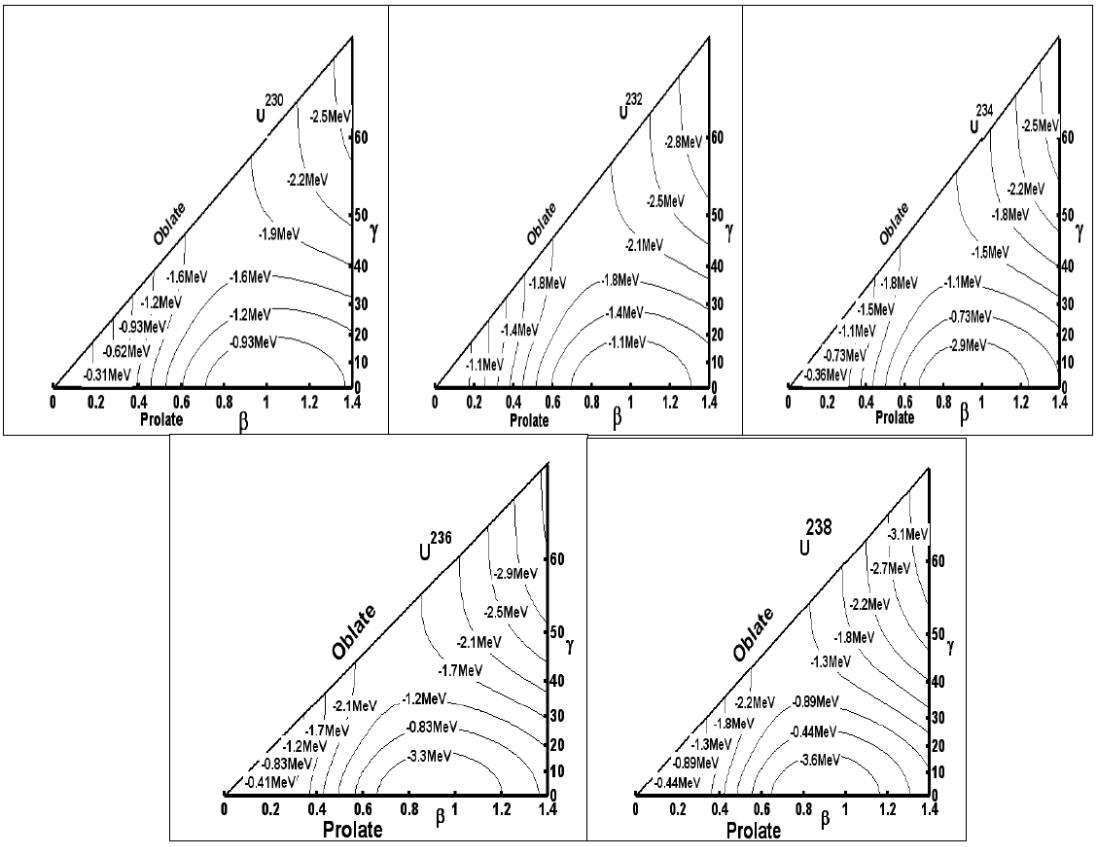

Fig. 2: Contour plot of the potential energy surfaces for $^{230\text{-}238}$U nuclei.

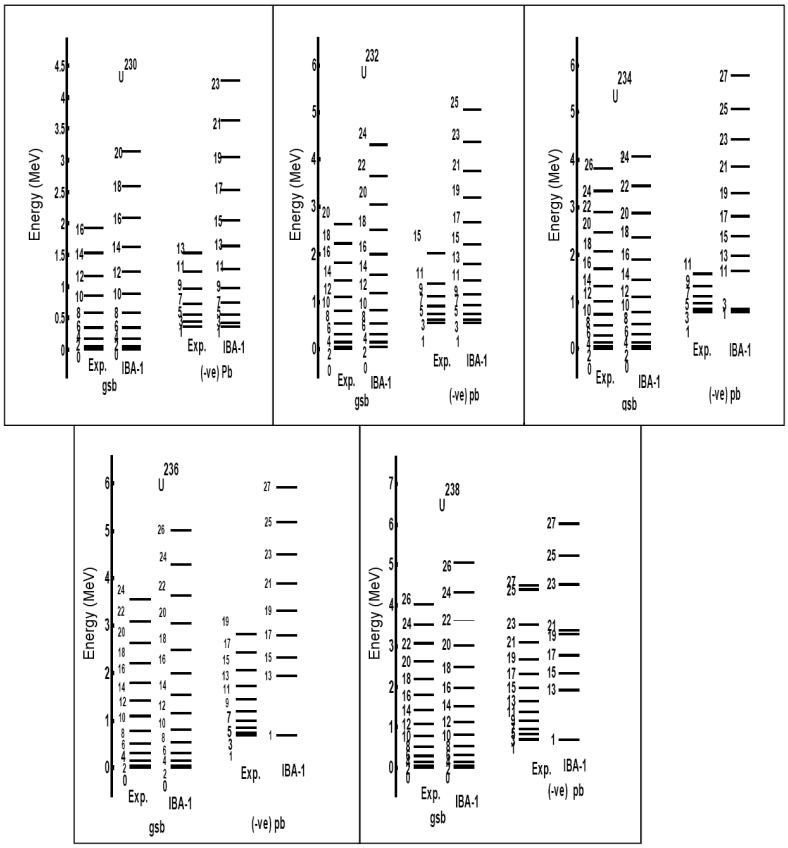

Fig. 3: Comparison between experimental (Exp.) [19–23] and theoretical (IBA-1) energy levels in $^{230\text{-}238}$U.

is due to band crossing and octupole deformation. From γ-bands [24] octupole deformation deformation has observed at $I = 14$ (for ^{232}U), $I = 10$ (for ^{234}U), $I = 15$ (for ^{236}U) and $I = 10$ (for ^{238}U) respectively.

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E2)$ or $B(E1)$ for these series of nuclei. The only measured $B(E2, 0_1^+ \to 2_1^+)$'s are presented, in Table 2 for comparison with the calculated values. The parameters $E2SD$ and $E2DD$ used in the present calculations are displayed in Table 1.

The calculated [equations 12, 13] electric quadrupole moment Q_0 and deformation parameter β_2 are given in Table 4. It is clear that both values are increasing with the increase of the neutron number of uranium isotopes.

$$Q_0 = \left[\frac{16\pi B(E_2)_{exp.}}{5}\right]^{1/2}, \qquad (12)$$

$$\beta_2 = \frac{[B(E_2)_{exp.}]^{1/2}}{\frac{3ZR_0^2}{4\pi}} \qquad (13)$$

3.3 Electric monopole transitions

The electric monopole transitions, $E0$, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. The strength of the electric monopole transitions, $X_{if'f}(E0/E2)$, [25] are calculated using equations (14, 15) and presented in Table 5.

$$X_{if'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_{'f})}, \qquad (14)$$

$$X_{if'f}(E0/E2) = (2.54{\times}10^9)\, A^{3/4} \times \times \frac{E_\gamma^5(\mathrm{MeV})}{\Omega_{KL}}\, \alpha(E2)\, \frac{T_e(E0, I_i - I_f)}{T_e(E_2, I_i - I_{'f})}. \qquad (15)$$

3.4 Conclusions

The IBA-1 model has been applied successfully to $^{230\text{-}238}$U isotopes and we have got:

1. The ground state and octupole bands are successfully reproduced;
2. The potential energy surfaces are calculated and show rotational behavior to $^{230\text{-}238}$U isotopes where they are mainly prolate deformed nuclei;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;

I_i^+	I_f^+	$I_{f'}^+$	^{230}U	^{232}U	^{234}U	^{226}U	^{238}U
0_2	0_1	2_1	0.660	0.560	0.001	0.920	1.470
0_3	0_1	2_1	13.370	1.300	15.910	0.282	—
0_3	0_1	2_2	2.400	0.410	3.000	1.960	212.500
0_3	0_1	2_3	3.510	0.280	2.420	1.240	0.520
0_3	0_2	2_1	0.620	0.590	0.660	0.500	—
0_3	0_2	2_2	0.110	0.180	0.120	0.001	1.500
0_3	0_2	2_3	0.180	0.130	0.100	0.001	3.720
0_4	0_1	2_2	1.960	7.750	0.001	0.230	7.250
0_4	0_1	2_3	1.320	0.250	—	0.250	0.190
0_4	0_1	2_4	32.660	0.330	1.000	0.170	0.460
0_4	0_2	2_2	—	0.020	0.0000	0.060	3.250
0_4	0_2	2_3	—	0.750	—	0.070	0.080
0_4	0_2	2_4	—	—	0.000	0.100	0.2000
0_4	0_3	2_1	0.330	—	—	24.000	19.000
0_4	0_3	2_2	0.020	0.080	0.110	0.330	4.750
0_4	0_3	2_3	0.010	2.750	—	0.360	0.130
0_4	0_3	2_4	0.330	—	17.000	0.520	0.300

Table 5. Theoretical $X_{if'f}$ ($E0/E2$) ratios for $E0$ transitions in Ra isotopes.

4. Electric quadrupole moment Q_0 are calculated;
5. Deformation parameter β_2 are calculated.

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