A Unified Theory of Interaction: Gravitation, Electrodynamics and the Strong Force

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A unified model of gravitation and electromagnetism is extended to derive the Yukawa potential for the strong force. The model satisfies the fundamental characteristics of the strong force and calculates the mass of the pion.

1 Introduction

A unified theory of interaction, as it is generally understood, implies a description of the four fundamental forces — gravitation, electromagnetism, the strong interaction and the weak force — in terms of a single mathematical formulation. It has been shown [1, 2, 3] that a unified model of gravitation and electromagnetism can be derived by starting from a Lagrangian for gravitation,

$$L = -m_0(c^2 + v^2) \exp R/r \,, \tag{1}$$

where

m_0 = *gravitational rest mass* of a test body moving at velocity $\mathbf{v}$ in the vicinity of a massive, central body of mass M,

γ = $1/\sqrt{1 - v^2/c^2}$,

R = $2GM/c^2$ is the Schwarzschild radius of the central body.

This Lagrangian characterizes the *dynamics* of a system.

Applying the canonical equations of motion, the following conservation equations follow:

$$E = mc^2 e^{R/r} = \text{total energy} = \text{constant}\,, \tag{2}$$
$$L^2 \equiv M^2 e^{2R/r} = \text{constant}, \tag{3}$$
$$L_z \equiv M_z e^{R/r} = e^{R/r} m_0 r^2 \sin^2\theta \, \dot{\phi}, \tag{4}$$
$$= z \text{ component of } \mathbf{L} = \text{constant},$$

where $m = m_0/\gamma^2$ and

$$\mathbf{M} = (\mathbf{r} \times m_0 \mathbf{v}) \tag{5}$$

is the total angular momentum of the test body.

The *kinematics* of the system is determined by assuming the local and instantaneous validity of special relativity (SR). This leads to a Lagrangian characterizing the kinematics of the system,

$$L = -\tilde{m}_0 c^2 \sqrt{1 - v^2/c^2} \, \exp(r_e/r) \,. \tag{6}$$

giving the following conservation equations:

$$E_e = \tilde{m} c^2 e^{r_e/r} = \text{constant}, \tag{7}$$
$$L^2 \equiv M^2 \exp(2r_e/r) = \text{constant}, \tag{8}$$
$$L_z \equiv M_z \exp(r_e/r) = \text{constant}, \tag{9}$$

where

$$r_e = R/2\,, \tag{10}$$
$$\tilde{m} = \gamma \tilde{m}_0, \tag{11}$$
$$\mathbf{M} = \mathbf{r} \times \tilde{m} \mathbf{v}\,. \tag{12}$$

For the hydrogen atom, $R = $ Schwarzschild radius of the proton, $r_e = $ classical electron radius $= R/2 = -e^2/\tilde{m}_0 c^2$, while $\tilde{m}_0$ is the *relativistic* or *kinematical* rest mass of the electron and $\mathbf{M}$ is the total angular momentum of the orbiting electron.

We also note that

$$E_e = \tilde{E} \, e^{r_e/r}\,, \tag{13}$$

where $\tilde{E} = \tilde{m}c^2$ is the total relativistic energy.

The common factor between the gravitational and electromagnetic interactions is the radius constant, $R = 2r_e$. These two radii are related in terms of electromagnetic masses $\tilde{m}$ by $N_p \approx 10^{40}$, one of the numbers of Dirac's Large Number Hypothesis (LNH).

2 Basic properties of nuclear interaction

Any theory of the strong interaction must satisfy certain basic properties of the force. They are:

(1) the force is charge independent,

(2) it only acts over a range $\sim 10^{-13}$ cm,

(3) the form of its potential is

$$-\frac{Q^2}{r} \exp(-r/r_q)\,,$$

(4) where the coupling constant $Q^2/\hbar c \sim 1\text{--}15$,

(5) r_q is related to the mass of a pion by $r_q \sim h/m_\pi c$.

The above items describe the fundamental properties of the strong force and we shall limit ourselves to showing how these are accommodated in our model.

3 Derivation of an energy relation for the strong interaction

The energy equation (2) can be rearranged in a unique form for $r \approx R$ as follows:

$$E = mc^2 \exp(R/r),$$
$$\approx mc^2(1 + R/r),$$
$$= mc^2(r/R + 1)R/r,$$
$$\approx mc^2 R/r \exp(r/R). \tag{14}$$

The mathematical condition for the approximate equality of (2) and (14) is found by equating the two equations:

$$\exp(R/r) \cong R/r \exp(r/R)$$
$$\Rightarrow R/r \cong \exp\left[(R^2 - r^2)/rR\right]. \tag{15}$$

The approximate equality of the two exponential forms therefore holds uniquely for $r^2 \cong R^2$.

Repeating the above procedure for the electromagnetic energy (7) we find

$$E_e \approx \tilde{m}c^2 r_e/r \exp(r/r_e). \tag{16}$$

We rewrite the classical electron radius r_e as

$$\tilde{m}_{e0}c^2 = -e^2/r_e, \tag{17}$$

where we now write $\tilde{m}_{e0}$ for the electromagnetic rest mass of the electron.

Substituting (17) in (16) gives

$$E \approx -\tilde{m}c^2 \left(\frac{e^2}{\tilde{m}_{e0}c^2}\right) \frac{1}{r} \exp\left[r/(-e^2/\tilde{m}_{e0}c^2)\right]. \tag{18}$$

Defining

$$r_q = |r_e|, \tag{19}$$

(18) can be written as

$$E \approx -\tilde{m}c^2 \frac{r_q}{r} \exp(-r/r_q) \tag{20}$$

$$= -\frac{Q^2}{r} \exp(-r/r_q), \tag{21}$$

where Q^2 is defined as

$$Q^2 = \tilde{m}c^2 r_q = \tilde{E} r_q. \tag{22}$$

Eq.(21) has the form of the Yukawa potential. The corresponding gravitational form is given by (14).

3.1 Model for the strong interaction

It was seen that a Yukawa-type potential exists at $r = R$ for gravitational interaction as well as at $r = r_e = R/2$ for electromagnetic interaction. The two related energy equations are respectively (14) and (16). Since our model postulates the concurrent action of gravitation and electromagnetism we have to find a model for the nuclear force that reconciles both these equations simultaneously.
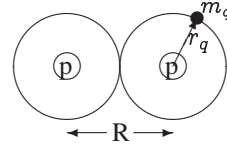

Fig. 1: Model of a deuteron. Two protons are separated at a distance R from each other. A particle of mass $\tilde{m}_q$ and charge $-e$ moves in a figure eight pattern alternatively about each of them at a radius of $r = r_q = |r_e|$ from each proton.

Consider the model of a deuteron depicted in Figure 1.

The two protons are bound by a gravitational force according to the energy given by (2). Each proton moves in the gravitational field of the other, with the total kinetic energy expressed in terms of their reduced mass. The form of this energy is not relevant at this stage. At the same time, a charged particle of mass $\tilde{m}_q$ moves at a radius of $r = r_q$ alternately about each proton, causing alternative conversions from proton to neutron and vice-versa. Only this hybrid form simultaneously and uniquely satisfies both the conditions for the two Yukawa-type potentials. This is possible, as can be seen from Figure 1, because $R = 2r_q$.

We provisionally call the charged, orbiting particle a q-particle.

3.2 Determination of the mass $\tilde{m}_q$

The mass $\tilde{m}_q$ cannot be determined independently without using some boundary condition. For gravitation, the Newtonian form in the weak-field limit was used, and for electromagnetism the condition for bound motion was applied. Both conditions are derived from observation. In this case we apply the experimental value for Q^2 and assume

$$\frac{Q^2}{\hbar c} \approx 1. \tag{23}$$

The q particle orbiting the protons spends half of its period about each proton. In considering the proton-q particle electromagnetic interaction, we must therefore assume that the mass $\tilde{m}_q$ is spread over both protons. Its electromagnetic energy $\tilde{E}$ is therefore equal to $\tilde{m}_q c^2/2$ for a single proton-q particle interaction.

Applying this condition to (22) and using (17) we get

$$Q^2 = \tilde{E} r_q,$$
$$= \frac{1}{2} \tilde{m}_q c^2 \frac{e^2}{\tilde{m}_{e0} c^2},$$
$$= \frac{\tilde{m}_q \alpha \hbar c}{2 \tilde{m}_{e0}}, \tag{24}$$

where $\alpha = e^2/\hbar c$ is the fine-structure constant.

The condition $Q^2/\hbar c = 1$ then yields

$$\tilde{m}_q = \frac{2 \tilde{m}_{e0}}{\alpha}. \tag{25}$$

The mass $\widetilde{m}_q$ is therefore equal to the mass of the π^- meson, namely

$$\widetilde{m}_q = 274\,\widetilde{m}_{e0} = \widetilde{m}_\pi\,. \tag{26}$$

We henceforth refer to the q particle as the π^- meson or pion, and use $\widetilde{m}_\pi$ for $\widetilde{m}_q$, and $\widetilde{m}_0$ for $\widetilde{m}_{e0}$.

3.3 Comparison with characteristics of the strong interaction

In Section 2 we listed the characteristics of the strong interaction. Comparing these with the results of our model we find:

1. The attractive force between the nucleons is gravitational and therefore charge independent. It must be remembered that the gravitational force acts on the *gravitational masses* of the protons, which are reduced to the magnitude of the electromagnetic masses by the LNH factor;

2. The strong interaction appears in its unique form at $r = = R = 2\,r_q \approx 10^{-13}$ cm;

3. The Yukawa potential is given by (21);

4. The value of the coupling constant had to be assumed to calculate the mass of the orbiting particle;

5. The expression for r_q follows from (17), (25) and $\alpha = e^2/\hbar c$:

$$r_q = \frac{e^2}{\widetilde{m}_0 c^2} = \frac{2 e^2}{\alpha\,\widetilde{m}_\pi c^2} = \frac{2\hbar}{\widetilde{m}_\pi c}\,. \tag{27}$$

4 Discussion

The above derivations are in accord with Yukawa's model of nucleon interaction through the exchange of mesons. Eq.(21) confirms the experimental result that nuclear forces only act in the region $r \cong r_q \cong 10^{-13}$ cm. Conversely, forces that only manifest in this region are describable by the Yukawa potential, which is a unique form for *both* the gravitational and electrodynamic energy equations in this region. In terms of our unified model it implies that nuclear forces only appear different from the gravitational force because experimental observations at 10^{-13} cm confirm the form of the Yukawa potential.

One of the main obstacles to the unification of gravity and the strong force has been the large difference in their coupling constants. The foregoing derivations overcomes this difficulty by the special form of the energy equations at distances close to the Schwarzschild radius.

Since the strong force appears to be a special form of gravity at small distances it explains why the strong force, like gravity, is attractive. The occurrence of repulsion at the core of the nucleus is presently little understood and if this is to be explained in terms of our model one would have to look at the form of the general energy equation in the region $r < r_q$.

It was previously shown [2, 3] how gravitational and electromagnetic energies could respectively be expressed as a power series in R/r or r_e/r. However, the form of (21) shows that this cannot be done for the energy arising from nuclear forces since $r \approx r_q$.

Our analysis of the three fundamental forces shows that the forces are all manifestations of one fundamental force, manifesting as universal gravitation. Electrodynamics arises as a kinematical effect and the nuclear force as a particular form at a distance equal to the classical electron radius. The weak force is not yet accommodated in this model, but analogously it is expected to be described by the energies of (2) and (7) in the region $r < R$.

References

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