An Explantion of Hubble Redshift due to the Global Non-Holonomity of Space

Dmitri Rabounski

E-mail: rabounski@ptep-online.com

In General Relativity, the change of the energy of a freely moving photon should be the solution to the scalar equation of the isotropic geodesic equations, which manifests the work produced on the photon being moved along the path. I solved the equation in terms of physical observables (Zelmanov, *Physics Doklady*, 1956, v. 1, 227–230), and in the large scale approximation, i.e. with gravitation and deformation neglected in the space, while supposing the isotropic space to be globally non-holonomic (the time lines are non-orthogonal to the spatial section, a condition manifested by the rotation of the space). The solution is $E = E_0 \exp(-\Omega^2 a\, t/c)$, where Ω is the angular velocity of the space (it meets the Hubble constant $H_0 = c/a = 2.3 \times 10^{-18}\ \mathrm{s}^{-1}$), a is the radius of the Universe, $t = r/c$ is the time of the photon's travel. So a photon loses energy with distance due to the work against the field of the space non-holonomity. According to the solution, the redshift should be $z = \exp(H_0\, r/c) - 1 \approx H_0\, r/c$. This solution explains both the redshift $z = H_0\, r/c$ observed at small distances and the non-linearity of the empirical Hubble law due to the exponent (at large r). The ultimate redshift, according to the theory, should be $z = \exp(\pi) - 1 = 22.14$.

In this short thesis, I show how the Hubble law, including its non-linearity with distance, can be deduced directly from the equations of the General Theory of Relativity.

In General Relativity, the change of the energy of a freely moving photon should be the solution to the scalar equation of isotropic geodesics, which is also known as the equation of energy and manifests the work produced on the photon being moved along the path. In terms of physically observable quantities — chronometric invariants (Zelmanov, 1944), which are the respective projections of four-dimensional quantities onto the time line and spatial section of a given observer — the isotropic geodesic equations are presented with two projections onto the time line and spatial section, respectively [1–3]

$$\left.\begin{aligned}
\frac{d\omega}{d\tau} - \frac{\omega}{c^2}\, F_i c^i + \frac{\omega}{c^2}\, D_{ik}\, c^i c^k = 0 \\[2mm]
\frac{d(\omega c^i)}{d\tau} - \omega F^i + 2\omega\left(D_k^i + A_{k\cdot}^{\cdot i}\right)c^k + \omega\triangle_{nk}^i\, c^n c^k = 0
\end{aligned}\right\},$$

where ω is the proper frequency of the photon, $d\tau$ is the interval of physically observable time, c^i is the vector of the observable velocity of light ($c_k c^k = c^2$), F_i is the gravitational inertial force, A_{ik} is the angular velocity of the space rotation due to the non-holonomity of space (the non-orthogonality of the time lines to the spatial section), D_{ik} is the deformation of space, $\triangle_{nk}^i$ are the three-dimensional Christoffel symbols. Integration of the scalar equation should give a function $E = E(t)$, where $E = \hbar\omega$ is the proper energy of the photon. However, integration of time in a Riemannian space is not a trivial task. This is because the observable interval of time $d\tau = \sqrt{g_{00}}\, dt - \frac{1}{c^2}\, v_i dx^i$ depends on the gravitational potential along the path, on the linear velocity $v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}$ of the rotation of space (due to the non-holonomity of it), and on the displacement dx^i of the observer with respect to his coordinate net during the measurement in process. The result of integration depends on the integration path, so time is not integrable in a general case. We consider the "large scale approximation", where distances are close to the curvature radius of the Universe; so gravitation and deformation are neglected in the space ($g_{00} = 1$ and $D_{ik} = 0$, respectively), and the observer is resting with respect to his coordinate net ($dx^i = 0$). In such a case, integration of time is allowed, and is simple as $d\tau = dt$. We also suppose the isotropic space, the "home space" of photons, to be globally non-holonomic ($v_i \neq 0$). With these, the gravitational inertial force F_i, losing the gravitational potential $\mathrm{w} = c^2\left(1 - \sqrt{g_{00}}\right) = 0$, consists of only the second term, which is due to the space non-holonomity

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right) \simeq -\frac{\partial v_i}{\partial t}\,.$$

We consider a single photon travelling in the x-direction ($c^1 = c,\ c^2 = c^3 = 0$). With the "large scale approximation" in a globally non-holonomic isotropic space, and assuming the linear velocity of the space rotation to be $v_1 = v_2 = v_3 = v$, and be stationary, i.e. $\frac{\partial v}{\partial t} = B = \mathrm{const}$, the scalar equation of isotropic geodesics for such a photon takes the form

$$\frac{dE}{dt} = -\frac{B}{c}\, E\,.$$

This is a simplest uniform differential equation of the 1st order, like $\dot{y} = -ky$, so that $\frac{dy}{y} = -kdt$ or $d(\ln y) = -kdt$. It solves as $\ln y = -kt + \ln C$, so we obtain $y = y_0 e^{-kt}$. As a result, the scalar equation of isotropic geodesics (the equation of energy), in the "large scale approximation" in the globally non-holonomic space, gives the solution for the photon's energy (frequency) and the redshift $z = \frac{\omega_0 - \omega}{\omega}$ as depending on the distance $r = ct$ travelled from the observer

$$E = E_0 e^{-kt}, \qquad z = e^{kt} - 1,$$

such that at small distances of the photon's travel, i.e. with the exponent $e^x = 1 + x + \frac{1}{2}x^2 + \ldots \simeq 1 + x$, takes the form

$$E \simeq E_0 (1 - kt), \qquad z \simeq kt,$$

where $k = \frac{1}{c}B = \frac{1}{c}\frac{\partial v}{\partial t} = \text{const}$. Thus, according to our calculation based on the General Theory of Relativity, a photon being moved in a non-holonomic space loses its proper energy/frequency due to the work produced by it against the field of the space non-holonomity (or the negative work produced by the field on the photon).

It is obvious that, given a stationary non-holonomity of the isotropic space, we can express k through the angular velocity Ω and the curvature radius $a = \frac{c}{H_0}$ of the isotropic space connected to our Metagalaxy (we suppose this is a constant curvature space of sperical geometry), as

$$k = \frac{1}{c}\Omega^2 a,$$

where H_0 is the Hubble constant. So for the galaxies located at a distance of $r \simeq 630$ Mpc* (the redshift observed on them is $z \simeq 0.16$) we obtain

$$\Omega = \sqrt{\frac{zc}{at}} = \sqrt{\frac{zc^2}{ar}} \simeq 2.4 \times 10^{-18} \text{ sec}^{-1},$$

that meets the Hubble constant $H_0 = 72 \pm 8 \times 10^5$ cm/sec$\times$Mpc $= 2.3 \pm 0.3 \times 10^{-18}$ sec^{-1} (according to the Hubble Space Telescope data, 2001 [4]).

With these we arrive at the following law

$$E = E_0 e^{-\frac{H_0 r}{c}}, \qquad z = e^{\frac{H_0 r}{c}} - 1,$$

as a purely theoretical result obtained from our solution to the scalar equation of isotropic geodesics. At small distances of the photon's travel, this law becomes

$$E \simeq E_0 \left(1 - \frac{H_0 r}{c}\right), \qquad z \simeq \frac{H_0 r}{c}.$$

As seen, this result provides a complete theoretical ground to the linear Hubble law, empirically obtained by Edwin Hubble for small distances, and also to the non-linearity of the Hubble law observed at large distances close to the size of the Metagalaxy (the non-linearity is explained due to the

---

*1 parsec $= 3.0857 \times 10^{18}$ cm $\simeq 3.1 \times 10^{18}$ cm.

exponent in our solution, which is sufficient at large r).

Then, proceeding from our solution, we are able to calculate the ultimate redshift, which is allowed in our Universe. It is, according to the exponential law,

$$z_{\max} = e^\pi - 1 = 22.14.$$

In the end, we calculate the linear velocity of the rotation of the isotropic space, which is due to the global non-holonomity of it. It is $\breve{v} = \Omega a = H_0 a = c$, i.e. is equal to the velocity of light. I should note, to avoid misunderstanding, that this linear velocity of rotation is attributed to the isotropic space, which is the home of isotropic (light-like) trajectories specific to massless light-like particles (e.g. photons). It isn't related to the non-isotropic space of sub-light-speed trajectories, which is the home of mass-bearing particles (e.g. galaxies, stars, planets). In other words, our result doesn't mean that the visible space of cosmic bodies rotates at the velocity of light, or even rotates in general. The space of galaxies, stars, and planets may be non-holonomic or not, depending on the physical conditions in it.

A complete presentation of this result will have been held at the *April Meeting 2009 of the American Physical Society* (May 2–5, Denver, Colorado) [5], and also published in a special journal on General Relativity and cosmology [6].

References

1. Zelmanov A.L. Chronometric invariants and accompanying frames of reference in the General Theory of Relativity. *Soviet Physics Doklady*, MAIK Nauka/Interperiodica, distributed by Springer, 1956, v. 1, 227–230 (translated from *Doklady Academii Nauk USSR*, 1956, v. 107, no. 6, 815–818).

2. Zelmanov A.L. Chronometric invariants: on deformations and the curvature of accompanying space. Translated into English from the preprint of 1944 (Sternberg Astron. Inst., Moscow), American Research Press, Rehoboth (NM), 2006, 232 pages.

3. Zelmanov A.L. On the relativistic theory of an anisotropic inhomogeneous universe. Translated from the manuscript of 1957, *The Abraham Zelmanov Journal*, 2008, v. 1, 33–63 (originally presented, in Russian, at the *6th Soviet Meeting on Cosmogony*, Moscow, 1959).

4. Freedman W.L., Madore B.F., Gibson B.K., Ferrarese L., Kelson D.D., Sakai S., Mould J.R., Kennicutt R.C.Jr., Ford H.C., Graham J.A., Huchra J.P., Hughes S.M.G., Illingworth G.D., Macri L.M., Stetson P.B. Final results from the Hubble Space Telescope Key Project to measure the Hubble constant. *Astrophys. Journal*, 2001, v. 553, issue 1, 47–72.

5. Rabounski D. Hubble redshift due to the global non-holonomity of space. *2009 APS April Meeting*, May 2–5, 2009, Denver, Colorado.

6. Rabounski D. Hubble redshift due to the global non-holonomity of space. *The Abraham Zelmanov Journal*, 2009, v. 2, 11–17.