On the Field of a Stationary Charged Spherical Source

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The equations of gravitation related to the field of a spherical charged source imply the existence of an interdependence between gravitation and electricity [5]. The present paper deals with the joint action of gravitation and electricity in the case of a stationary charged spherical source. Let m and ε be respectively the mass and the charge of the source, and let k be the gravitational constant. Then the equations of gravitation need specific discussion according as $|\varepsilon| < m\sqrt{k}$ (source weakly charged) or $|\varepsilon| = m\sqrt{k}$ or $|\varepsilon| > m\sqrt{k}$ (source strongly charged). In any case the curvature radius of the sphere bounding the matter possesses a strictly positive greatest lower hound, so that the source is necessarily an extended object. Pointwise sources do not exist. In particular, charged black holes do not exist.

1 Introduction

We recall that the field of an isotropic stationary spherical charged source is defined by solutions of the Einstein equations related to the stationary $\Theta(4)$-invariant metric

$$ds^2 = \big(f(\rho)\,dt + f_1(\rho)\,(x\,dx)\big)^2 - \left[(l_1(\rho))^2\,dx^2 + \frac{\big((l(\rho))^2 - (l_1(\rho))^2\big)}{\rho^2}\,(x\,dx)^2\right], \quad (1.1)$$

($\rho = \|x\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$, $l(0) = l_1(0)$). The functions of one variable $f(\rho)$, $f_1(\rho)$, $l_1(\rho)$, $l(\rho)$ are supposed to be C^∞ with respect to $\rho = \|x\|$ on the half-line $[0, +\infty[$ (or, possibly, on the entire real line $]-\infty, +\infty[$), but since the norm $\|x\|$ is not differentiable at the origin with respect to the coordinates x_1, x_2, x_3, these functions are not either. So, in general, the origin will appear as a singularity without physical meaning. In order to avoid the singularity, the considered functions must be smooth functions of the norm in the sense of the following definition.

Definition 1.1 *A function of the norm $\|x\|$, say $f(\|x\|)$, will be called smooth function of the norm, if:*

a). $f(\|x\|)$ is C^∞ on $\mathbb{R}^3 - \{(0,0,0)\}$ with respect to the coordinates x_1, x_2, x_3.

b). Every derivative of $f(\|x\|)$ with respect to the coordinates x_1, x_2, x_3 at the points $x \in \mathbb{R}^3 - \{(0,0,0)\}$ tends to a definite value as $x \to (0,0,0)$.

Remark 1.1 *In [3], [4] a smooth function of the norm is considered as a function C^∞ on $\mathbb{R}$. However this last characterisation neglects the fact that the derivatives of the function are not directly defined at the origin.*

The proof of the following theorem appears in [3].

Theorem 1.1 *$f(\|x\|)$ is a smooth function of the norm if and only if the function of one variable $f(u)$ is C^∞ on $[0, \infty[$ and its right derivatives of odd order at $u = 0$ vanish.*

This being said, a significant simplification of the problem results from the introduction of the radial geodesic distance

$$\delta = \int_0^\rho l(u)\,du = \beta(\rho), \quad (\beta(0) = 0),$$

which makes sense in the case of stationary fields.

Since $\beta(\rho)$ is a strictly increasing C^∞ function tending to $+\infty$ as $\rho \to +\infty$, the inverse function $\rho = \gamma(\delta)$ is also a C^∞ strictly increasing function of δ tending to $+\infty$ as $\delta \to +\infty$. So to the distance δ there corresponds a transformation of space coordinates:

$$y_i = \frac{\delta}{\rho}\,x_i = \frac{\beta(\rho)}{\rho}\,x_i, \quad (i = 1, 2, 3),$$

with inverse

$$x_i = \frac{\rho}{\beta(\rho)}\,y_i = \frac{\gamma(\delta)}{\delta}\,y_i, \quad (i = 1, 2, 3).$$

As shown in [4], these transformations involve smooth functions of the norm and since

$$x\,dx = \sum_{i=1}^{3} x_i\,dx_i = \frac{\gamma\gamma'}{\delta}\,(y\,dy)\,,$$

$$dx^2 = \sum_{i=1}^{3} dx_i^2 = \left(\frac{\gamma'^2}{\delta^2} - \frac{\gamma^2}{\delta^4}\right)(y\,dy)^2 + \frac{\gamma'^2}{\delta^2}\,dy^2$$

by setting

$$F(\delta) = f(\gamma(\delta)), \quad F_1(\delta) = f_1(\gamma(\delta))\,\frac{\gamma(\delta)\gamma'(\delta)}{\delta},$$

$$L_1(\delta) = l_1(\gamma(\delta))\,\frac{\gamma(\delta)}{\delta},$$

and taking into account that

$$L(\delta) = l(\gamma(\delta))\,\gamma'(\delta) = 1$$

we get the transformed metric:

$$ds^2 = \left(F dt + F_1\,(y dy)\right)^2 - \\ - \left(L_1^2 dy^2 + \frac{1 - L_1^2}{\delta^2}\,(y dy)^2\right). \quad (1.2)$$

Then $\delta = \|y\|$ and the curvature radius of the spheres $\delta = $ = const, is given by the function

$$G = G(\delta) = \delta L_1(\delta)\,.$$

Moreover, instead of $h = \rho f_1$, we have now the function

$$H = H(\delta) = \delta F_1(\delta)\,.$$

This being said, we recall [5] that, with respect to (1.1), the field outside the charged spherical source is defined by the equations

$$fl = c\,\frac{dg}{d\rho}\,,$$

$$\frac{dg}{d\rho} = l\sqrt{1 - \frac{2\mu}{g} + \frac{\nu^2}{g^2}} = l\,\frac{\sqrt{g^2 - 2\mu g + \nu^2}}{g}\,,$$

($\mu = \frac{km}{c^2}$, $\nu = \frac{\sqrt{k}}{c^2}|\varepsilon|$, $g^2 - 2\mu g + \nu^2 > 0$, where k is the gravitational constant, m and ε being respectively the mass and the charge of the source).

The function $h = \rho f_1$ does not appear in these equations. Every function $h = \rho f_1$ satisfying the required conditions of differentiability and such that $|h| \leqslant l$ is allowable.

We obtain a simpler system of equations if we refer to the metric (1.2). Then

$$F = c\frac{dG}{d\delta} = c\sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}}\,, \quad (1.3)$$

$$\frac{dG}{d\delta} = \sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}} = \frac{\sqrt{G^2 - 2\mu G + \nu^2}}{G}\,, \quad (1.4)$$

$$|H| \leqslant 1\,.$$

So our problem reduces essentially to the definition of the curvature radius $G(\delta)$ by means of the equation (1.4) the study of which depends on the sign of the difference

$$\nu^2 - \mu^2 = \frac{k}{c^4}\left(\varepsilon^2 - km^2\right).$$

A concise approach to this problem appeared first in the paper [1].

2 Source weakly charged ($\nu^2 < \mu^2$ or $|\varepsilon| < m\sqrt{k}$)

$G^2 - 2\mu G + \nu^2 = (G - \mu)^2 + \nu^2 - \mu^2$ vanishes for $G = $ $= \mu - \sqrt{\mu^2 - \nu^2}$ and $G = \mu + \sqrt{\mu^2 - \nu^2}$. Moreover $G^2 -$ $- 2\mu G + \nu^2 < 0$ if $\mu - \sqrt{\mu^2 - \nu^2} < G < \mu + \sqrt{\mu^2 - \nu^2}$ and $G^2 - 2\mu G + \nu^2 > 0$ if $G < \mu - \sqrt{\mu^2 - \nu^2}$ or $G > \mu +$ $+ \sqrt{\mu^2 - \nu^2}$. Since negative values of G are not allowed and since the solution must be topologically connected, we have to consider two cases according as

$$0 < G \leqslant \mu - \sqrt{\mu^2 - \nu^2}$$

or

$$\mu + \sqrt{\mu^2 - \nu^2} \leqslant G < +\infty.$$

The first case gives an unphysical solution, because G cannot be bounded outside the source. So, it remains to solve the equation (1.4) when G describes the half-line $\left[\,\mu + \sqrt{\mu^2 - \nu^2},\, +\infty\right[$. The value $\mu + \sqrt{\mu^2 - \nu^2}$ is the greatest lower bound of the values of G and is not reachable physically, because F vanishes, and hence the metric degenerates for this value. However the value $\mu + \sqrt{\mu^2 - \nu^2}$ must be taken into account for the definition of the mathematical solution. So, on account of (1.4) the function $G(\delta)$ is defined as an implicit function by the equation

$$\delta_0 + \int\limits_{\mu + \sqrt{\mu^2 - \nu^2}}^{G} \frac{u\,du}{\sqrt{u^2 - 2\mu u + \nu^2}} = \delta, \quad (\delta_0 = \text{const}),$$

or, after integration,

$$\delta_0 + \sqrt{G^2 - 2\mu G + \nu^2} + \\ + \mu \ln \frac{G - \mu + \sqrt{G^2 - 2\mu G + \nu^2}}{\sqrt{\mu^2 - \nu^2}} = \delta \quad (2.1)$$

with $G > \mu + \sqrt{\mu^2 - \nu^2}$.

We see that the solution involves a new constant δ_0 which is not defined classically. To given mass and charge there correspond many possible values of δ_0 depending probably on the size of the source as well as on its previous history, namely on its dynamical states preceding the considered stationary one. From the mathematical point of view, the determination of δ_0 necessitates an initial condition, for instance the value of the curvature radius of the sphere bounding the matter.

Let us denote by $E(G)$ the left hand side of (2.1). The function $E(G)$ is a strictly increasing function of G such that $E(G) \to +\infty$ as $G \to +\infty$. Consequently (2.1) possesses a unique strictly increasing solution $G(\delta)$ tending to $+\infty$ as $\delta \to +\infty$

The equation (2.1) allows to obtain two significant relations:

a) Since

$$\delta - G(\delta) = E(G) - G =$$

$$= \delta_0 + \mu \ln \frac{G - \mu + \sqrt{G^2 - 2\mu G + \nu^2}}{\sqrt{\mu^2 - \nu^2}} +$$

$$+ \sqrt{G^2 - 2\mu G + \nu^2} - G =$$

$$= \delta_0 + \mu \ln \frac{G - \mu + \sqrt{G^2 - 2\mu G + \nu^2}}{\sqrt{\mu^2 - \nu^2}} +$$

$$+ \frac{-2\mu + \frac{\nu^2}{G}}{1 + \sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}}} \to +\infty \quad \text{as} \quad G \to +\infty,$$

it follows that $\delta - G(\delta) \to +\infty$ as $\delta \to +\infty$.

b) Since

$$\frac{\delta}{G(\delta)} = \frac{E(G)}{G} = \frac{\delta_0}{G} + \sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}} +$$

$$+ \mu \frac{\ln G}{G} + \frac{\mu}{G} \ln \frac{1 - \frac{\mu}{G} + \sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}}}{\sqrt{\mu^2 - \nu^2}} \to 1 \quad \text{as} \quad G \to +\infty,$$

it follows that $\frac{\delta}{G(\delta)} \to 1$ as $\delta \to +\infty$.

Moreover from (2.1), it follows that the greatest lower bound $\mu + \sqrt{\mu^2 - \nu^2}$ of the values of $G(\delta)$ is obtained for $\delta = \delta_0$. The characteristics of the solution depend on the sign of δ_0.

Suppose first that $\delta_0 < 0$. Since function $G(\delta)$ is strictly increasing, we have $G(0) > \mu + \sqrt{\mu^2 - \nu^2}$, which is physically impossible, because the physical solution $G(\delta)$ vanishes for $\delta = 0$. Consequently there exists a strictly positive value δ_1 (the radius of the sphere bounding the matter) such that the solution is valid only for $\delta \geqslant \delta_1$. So, there exists no vacuum solution inside the ball $\|x\| < \delta_1$. In other words, the ball $\|x\| < \delta_1$ lies inside the matter.

Suppose secondly that $\delta_0 = 0$. Then

$$G(0) = \mu + \sqrt{\mu^2 - \nu^2} > 0,$$

which contradicts also the properties of the globally defined physical solution. Consequently there exists a strictly positive value δ_1 (the radius of the sphere bounding the matter) such that the solution is valid for $\delta \geqslant \delta_1$.

Suppose thirdly that $\delta_0 > 0$. Since

$$G(\delta_0) = \mu + \sqrt{\mu^2 - \nu^2}\,,$$

the derivative

$$G'(\delta_0) = \frac{\sqrt{\left(G(\delta_0)\right)^2 - 2\mu G(\delta_0) + \nu^2}}{G(\delta_0)}$$

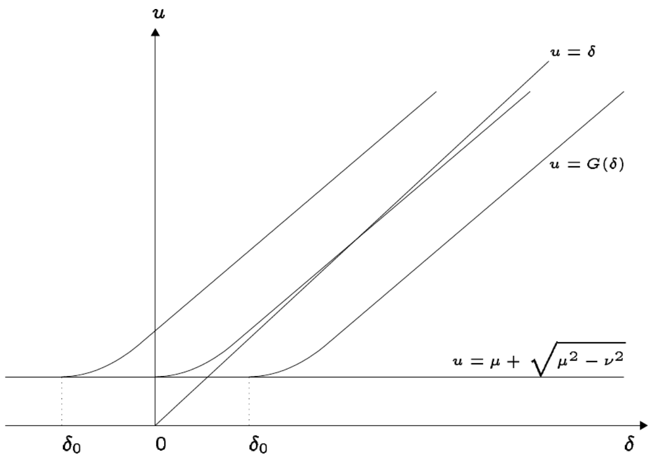

Fig. 1: Graph of G in the case where $\nu^2 < \mu^2$.

vanishes, so that $F(\delta_0) = c\,G'(\delta_0) = 0$. The vanishing of $F(\delta_0)$ implies the degeneracy of the spacetime metric for $\delta = \delta_0$ and since degenerate metrics have no physical meaning, there exists a value $\delta_1 > \delta_0$ such that the metric is physically valid for $\delta \geqslant \delta_1$. There exists no vacuum solution for $\delta \leqslant \delta_0$. The ball $\|x\| \leqslant \delta_0$ lies inside the matter.

From the preceding considerations it follows, in particular, that, whatever the case may be, a weakly charged source cannot be reduced to a point.

3 Source with $\mu^2 = \nu^2$ (or $|\varepsilon| = m\sqrt{k}$)

Since $\mu^2 = \nu^2$, we have $G^2 - 2\mu G + \nu^2 = (G - \mu)^2$, so that the equation (1.4) is written as

$$\frac{dG}{d\delta} = \frac{|G - \mu|}{G}\,.$$

Consider first the case where $G < \mu$. Then

$$\frac{dG}{d\delta} = \frac{G - \mu}{G} \quad \text{or} \quad \left(1 - \frac{\mu}{\mu - G}\right) dG = -\,d\delta\,,$$

whence

$$a_0 + G + \mu \ln\left(1 - \frac{G}{\mu}\right) = -\,\delta\,, \quad (a_0 = \text{const})\,.$$

If $G \to \mu$, then $\delta \to +\infty$, thus introducing a sphere with infinite radius and finite measure. This solution is unphysical. It remains to examine the case where $G > \mu$. Then

$$\left(1 + \frac{\mu}{G - \mu}\right) dG = d\delta\,,$$

whence

$$a_0 + G + \mu \ln\left(\frac{G}{\mu} - 1\right) = \delta\,, \quad (a_0 = \text{const})\,.$$

To the infinity of values of a_0 there correspond an infinity of solutions which results from one of them, for instance from

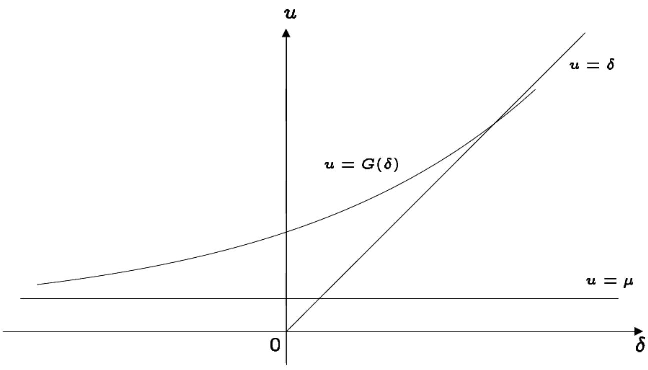

Fig. 2: Graph of G in the case where $\nu^2 = \mu^2$.

the solution obtained for $a_0 = 0$, by means of translations parallel to δ-axis.

For each value of a_0, we have $\delta \to +\infty$ as $G \to \mu$. The value μ is the unreachable greatest lower bound of the values of the corresponding solution $G(\delta)$ which is mathematically defined on the entire real line. If $\delta_1 > 0$ is the radius of the sphere bounding the matter, only the restriction of the solution to the half-line $[\delta_1, +\infty[$ is physically valid. In order to define the solution, we need the value of the corresponding constant a_0, the determination of which necessitates an initial condition, for instance the value $G(\delta_1)$. In any case, the values of $G(\delta)$ for $\delta \leqslant 0$ are unphysical.

Finally we remark that

$$\delta - G(\delta) \to +\infty \quad \text{and} \quad \frac{\delta}{G(\delta)} \to 1 \quad \text{as} \quad \delta \to +\infty.$$

4 Source strongly charged ($\nu^2 > \mu^2$ or $|\varepsilon| > m\sqrt{k}$)

Since $G^2 - 2\mu G + \nu^2 = (G - \mu)^2 + \nu^2 - \mu^2$, we have $G^2 - 2\mu G + \nu^2 > 0$ for every value of G. Regarding the function

$$\Phi(G) = 1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2} = \frac{G^2 - 2\mu G + \nu^2}{G^2},$$

we have

$$\Phi(G) \to +\infty \text{ as } G \to 0 \quad \text{and} \quad \Phi(G) \to 1 \text{ as } G \to +\infty.$$

On the other hand the derivative

$$\Phi'(G) = \frac{2}{G^2}\left(\mu - \frac{\nu^2}{G}\right),$$

vanishes for

$$G = \frac{\nu^2}{\mu} = \frac{\varepsilon^2}{mc^2}$$

and moreover

$$\Phi'(G) < 0 \quad \text{for} \quad G < \frac{\nu^2}{\mu},$$

$$\Phi'(G) > 0 \quad \text{for} \quad G > \frac{\nu^2}{\mu}.$$

It follows that the function $\Phi(G)$ is strictly decreasing on the interval $]\,0, \frac{\nu^2}{\mu}[$, strictly increasing on the half-line $[\frac{\nu^2}{\mu}, +\infty[$, so that

$$\Phi\left(\frac{\nu^2}{\mu}\right) = 1 - \frac{\mu^2}{\nu^2} = 1 - \left(\frac{m\sqrt{k}}{\varepsilon}\right)^2$$

is the minimum of $\Phi(G)$.

The behaviour of the solution on the half-line $[\frac{\nu^2}{\mu}, +\infty[$ is quite different from that on the interval $]\,0, \frac{\nu^2}{\mu}[$. Several arguments suggest that only the restriction of the solution to the half-line $[\frac{\nu^2}{\mu}, +\infty[$ is physically valid.

a) Let δ_0 be the radius of the spherical source. In order to prove that the restriction of the solution to $]\,0, \frac{\nu^2}{\mu}[$ is unphysical, we have only to prove that $G(\delta_0) \geqslant \frac{\nu^2}{\mu}$. We argue by contradiction assuming that $G(\delta_0) < \frac{\nu^2}{\mu}$. Since $G(\delta)$ is unbounded, there exists a value $\delta_1 > \delta_0$ such that $G(\delta_1) = \frac{\nu^2}{\mu}$. On the other hand, since $G(\delta)$ satisfies the equation (1.4), namely

$$\frac{dG}{d\delta} = \sqrt{1 - \frac{2\mu}{G} + \frac{\nu^2}{G^2}} = \sqrt{\Phi(G)},$$

the function

$$F = c\frac{dG}{d\delta} = c\sqrt{\Phi(G)}$$

is strictly decreasing on the interval $]\,0, \frac{\nu^2}{\mu}[$, and strictly increasing on the half-line $[\frac{\nu^2}{\mu}, +\infty[$. Such a behaviour of the important function F, which is involved in the law of propagation of light, is unexplained. We cannot indicate a cause compelling the function F first to decrease and then to increase outside the spherical source. The solution cannot be valid physically in both intervals $]\,0, \frac{\nu^2}{\mu}[$ and $[\frac{\nu^2}{\mu}, +\infty[$, and since the great values of G are necessarily involved in the solution, it follows that only the half-line $[\frac{\nu^2}{\mu}, +\infty[$ must be taken into account. The assumption that $G(\delta_0) < \frac{\nu^2}{\mu}$ is to be rejected.

b) The non-Euclidean (or, more precisely, non-pseudo-Euclidean) properties of the spacetime metric are induced by the matter, and this is why they become more and more apparent in the neighbourhood of the spherical source. On the contrary, when δ (or G) increases the spacetime metric tends progressively to a pseudo-Euclidean form. This situation is expressed by the solution itself. In order to see this, we choose a positive value b_1 and integrate the equation (1.4) in the half-line $[b_1, +\infty[$,

$$b_0 + \int_{b_1}^{G} \frac{u\,du}{\sqrt{u^2 - 2\mu u + \nu^2}} = \delta\,, \quad (b_0 = \text{const}),$$

and then writing down the explicit expression resulting from the integration, we find, as previously, that

$$\delta - G(\delta) \to +\infty \quad \text{and} \quad \frac{\delta}{G(\delta)} \to 1 \text{ as } \delta \to +\infty.$$

But, since

$$L_1(\delta) = \frac{G(\delta)}{\delta} \to 1 \quad \text{and} \quad F = c\sqrt{\Phi(G(\delta))} \to c$$

as $\delta \to +\infty$, the metric (1.2) tends effectively to a pseudo-Euclidean form as $\delta \to +\infty$. Now, if δ decreases, the non-Euclidean properties become more and more apparent, so that the minimum $c\sqrt{1-\frac{\mu^2}{\nu^2}}$ of F, obtained for $G = \frac{\nu^2}{\mu}$, is related to the "strongest non-Euclidean character of the metric". For values of G less than $\frac{\nu^2}{\mu}$, the behaviour of the mathematical solution becomes unphysical. In fact, the metric loses progressively its non-Euclidean properties, and, in particular, for $G = \frac{\nu^2}{2\mu}$, we have

$$\Phi\left(\frac{\nu^2}{2\mu}\right) = 1 - \frac{4\mu^2}{\nu^2} + \frac{4\mu^2}{\nu^2} = 1,$$

hence $F\left(\frac{\nu^2}{2\mu}\right) = c$ and $\frac{dG}{d\delta} = 1$.

On account of $G = \delta L_1$, the last condition implies

$$1 = \frac{dG}{d\delta} = L_1 + \delta\,\frac{dL_1}{d\delta}$$

and since we have to do physically with very small values of δ (in the neighbourhood of the origin), we conclude that

$$L_1\left(\frac{\nu^2}{2\mu}\right) \approx 1.$$

and since $F\left(\frac{\nu^2}{2\mu}\right) = c$, the metric is almost pseudo-Euclidean, a phenomenon inadmissible physically in the neighbourhood of the source. So we are led to reject the restriction of the mathematical solution to the interval $[\frac{\nu^2}{2\mu}, \frac{\nu^2}{\mu}[$. For values less than $\frac{\nu^2}{2\mu}$, the function $F(G)$ increases rapidly and tends to $+\infty$ as G decreases, so that the restriction of the mathematical solution to the interval $]\,0, \frac{\nu^2}{2\mu}[$ is also physically inadmissible. It follows that the restriction of the solution to the entire interval $]\,0, \frac{\nu^2}{\mu}[$ is unphysical.

c) Another argument supporting the above assertion is given in [2].

Let δ_1 be the radius of the spherical source and assume that $G(\delta_1) > \frac{\nu^2}{\mu}$. A radiation emitted radially from the sphere bounding the matter is redshifted, and its redshift at the points of a sphere $\|x\| = \delta$ with $\delta > \delta_1$ is given by the formula

$$Z(\delta,\delta_1) = -1 + \frac{F(G(\delta))}{F(G(\delta_1))} = -1 + \sqrt{\frac{\Phi(G(\delta))}{\Phi(G(\delta_1))}}\,.$$

Suppose δ fixed and let us examine the variation of $Z(\delta,\delta_1)$ considered as function of δ_1. If δ_1 (or $G(\delta_1)$) decreases, $Z(\delta,\delta_1)$ increases and tends to its maximum, obtained for $G(\delta_1) = \frac{\nu^2}{\mu}$,

$$\max Z(\delta,\delta_1) = -1 + \sqrt{\frac{\Phi(G(\delta))}{1-\frac{\mu^2}{\nu^2}}}\,.$$

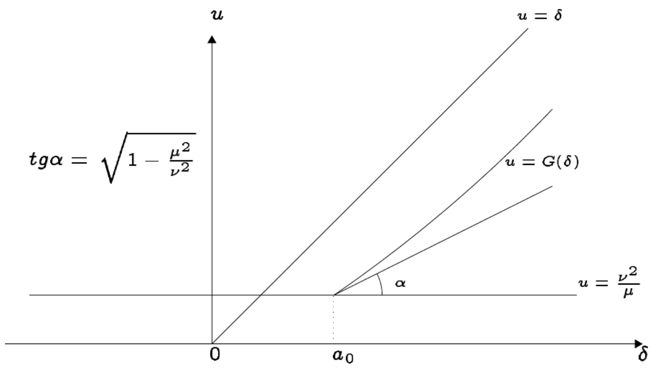

Fig. 3: Graph of G in the case where $\nu^2 > \mu^2$.

If $G(\delta_1)$ takes values less than $\frac{\nu^2}{\mu}$, the phenomenon is inverted: The redshift first decreases and then vanishes for a unique value $G(\delta_1) \in\,]\frac{\nu^2}{2\mu}, \frac{\nu^2}{\mu}[$ with $\Phi(G(\delta_1)) = \Phi(G(\delta))$. If $G(\delta_1)$ decreases further, instead of a redshift, we have a blueshift. This situation seems quite unphysical, inasmuch as the vanishing of the redshift depends on the position of the observer. In order to observe constantly a redshift, the condition $G(\delta_1) \geqslant \frac{\nu^2}{\mu}$ is necessary.

From the preceding considerations we conclude that the value

$$\frac{\nu^2}{\mu} = \frac{\varepsilon^2}{mc^2}$$

is the greatest lower bound of the curvature radius $G(\delta)$ outside the spherical strongly charged source. In particular, the curvature radius of the sphere bounding the matter is $\geqslant \frac{\varepsilon^2}{mc^2}$, so that a strongly charged source cannot be reduced to a point. Our study does not exclude the case where the solution $G(\delta)$ attains its greatest lower bound, namely the case where the curvature radius of the sphere bounding the matter is exactly equal to $\frac{\varepsilon^2}{mc^2}$. So, in order to take into account all possible cases, the equation (1.4) must be integrated as follows

$$a_0 + \int\limits_{\nu^2/\mu}^{G} \frac{u\,du}{\sqrt{u^2 - 2\mu u + \nu^2}} = \delta\,, \quad (a_0 = \text{const})\,.$$

If $a_0 \leqslant 0$, there exists a value $\delta_1 > 0$ such that the solution is valid only for $\delta \geqslant \delta_1$.

If $a_0 > 0$, the solution is valid for $\delta \geqslant a_0$, only if the sphere bounding the matter has the curvature radius $\frac{\nu^2}{\mu}$. Otherwise there exists a value $\delta_1 > a_0$ such that the solution is valid for $\delta \geqslant \delta_1$.

The expression $\frac{\varepsilon^2}{mc^2}$ is also known in classical electrodynamics, but in the present situation it appears on the basis of new principles and with a different signification. Consider, for instance, the case of the electron. Then $\frac{|\varepsilon|}{m\sqrt{k}} = 2.02\times10^{21}$, so that the electron is strongly charged, and, from the point of view of the classical electrodynamics, is a spherical object with radius $\frac{\varepsilon^2}{mc^2} = 2.75\times10^{-13}$ cm.

Regarding the present theory, we can only assert that, if the electron is a stationary spherical object, then it is a non-Euclidean ball such that the value 2.75×10^{-13} cm is the greatest lower bound of the possible values of the curvature radius of the sphere bounding it. The radius of the electron cannot be deduced from the present theory.

The proton is also strongly charged with $\frac{|\varepsilon|}{m\sqrt{k}} = 1.1\times10^{18}$. The corresponding value $\frac{\varepsilon^2}{mc^2} = 1.5\times10^{-16}$ cm is less than that related to the electron by a factor of the order 10^{-3}. So, if the proton is assumed to be spherical and stationary, it is not reasonable to accept that this value represents its radius. This last is not definable by the present theory.

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