Geometry of Space-Time

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The geometry of the space-time is deduced from gravitational and electromagnetic fields. We have to state that Rainich's "already unified field theory" is the ground work of the proposed theory. The latter is deduced independently on Rainich. Rainich's analogies are brilliantly validated. His formulae are verified this way. Further reaching results and insights demonstrate that Rainich's theory is viable. In final result, we can formulate an enhanced equivalence principle. It is the equivalence of Newton's force with the Lorentz force.

To the memory of John Archibald Wheeler, who foresaw this simple idea.

1 The predecessor

George Yuri Rainich already saw the analogies of the electromagnetic with the gravitational field. Since Einstein's equivalence principle implies a geometric approach of gravitation [1], electromagnetism has to be geometry too. Not enough, Rainich also saw that the electromagnetic field tensor is performed from the congruences of two dual surfaces. It is the analogy of the curvature vector of the current path, performed from the main normal, see on generalized Frenet formulae in [2].

One can well pursue Rainich's way in his papers from 1923 to 1924. First, he tried to find a non-Riemannian geometry for the electromagnetic vacuum field [3]. Later, he saw that Riemannian geometry is sufficient to describe electromagnetism [4, 5]. Rainich's identities (also called algebraic Rainich conditions) are deduced without special techniques in [6]. Present paper provides a further derivation of Rainich's identities, additionally identifying the concrete geometry.

Since a full geometric approach precludes sources, Rainich concluded a central role of singularities. However, it is deduced in [7] that this role is commonly overestimated. The singularities pass for a bar to the geometric approach. It is shown in [7] that formal singularities are in areas (according to observer's coordinates), which are not locally imaged. The related boundaries specify the discrete values of the integration constants from field equations [7].

2 The derivation

The first precursor is to see in [8]. The derivation follows the steps according to the chapter "Geometric interpretation of the Ricci tensor — the Ricci main directions" in [2]. As well, we shall see that the space-time involves a vital difference to other manifolds.

The known source-free Einstein-Maxwell equations

$$R_{ik} = \kappa \left(\frac{1}{4}\, g_{ik} F_{ab} F^{ab} - F_{ia} F_k{}^a \right), \tag{1}$$

$$F^{ia}{}_{;a} = 0\,, \tag{2}$$

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \tag{3}$$

involve a special kind of Riemannian geometry, what is explained as follows.

The Ricci main directions (written in terms according to Eisenhart [2]) follow from

$$\det |R_{ik} + \rho g_{ik}| = 0 \tag{4}$$

with the solutions*

$$\rho_{|1} = \rho_{|4} = +\rho_0\,, \quad \rho_{|2} = \rho_{|3} = -\rho_0 \tag{5}$$

with

$${\rho_0}^2 = R_1{}^a R^1{}_a = R_2{}^a R^2{}_a = R_3{}^a R^3{}_a = R_4{}^a R^4{}_a\,, \tag{6}$$

what leads directly to Rainich's identities

$$R_i{}^a R^k{}_a = \delta_i^k {\rho_0}^2 = \frac{1}{4}\,\delta_i^k R_a{}^b R^a{}_b\,. \tag{7}$$

Characteristical are the two double-roots, that means: There are two dual surfaces of the congruences

$$e_{|1}{}^i e_{|4}{}^k - e_{|1}{}^k e_{|4}{}^i \quad \text{and} \quad e_{|2}{}^i e_{|3}{}^k - e_{|2}{}^k e_{|3}{}^i$$

with minimal and maximal mean Riemannian curvature. $e_{|1} \ldots e_{|4}$ are the vectors of an orthogonal quadruple (vierbein) in those "main surfaces". At single roots we had 4 main directions. But we will see that the main surfaces are a spe-

*Where ρ_0 has a negative value, what has to do with the special signature of the space-time.

ciality of the space-time. With the obtained solutions we get

$$\left.\begin{aligned} g_{ik} &= e_{|1\_i}e_{|1\_k} + e_{|2\_i}e_{|2\_k} + \\ &\quad + e_{|3\_i}e_{|3\_k} - e_{|4\_i}e_{|4\_k}\,, \\ \frac{R_{ik}}{\rho_0} &= -e_{|1\_i}e_{|1\_k} + e_{|2\_i}e_{|2\_k} + \\ &\quad + e_{|3\_i}e_{|3\_k} + e_{|4\_i}e_{|4\_k}\,. \end{aligned}\right\} \quad (8)$$

If we set

$$c_{|ik} = -c_{|ki} = F_{ab}e_{|i}{}^{a}e_{|k}{}^{b} \quad (9)$$

follows from elementary calculations

$$\left.\begin{aligned} &-\kappa\left((c_{|23})^2 + (c_{|14})^2\right) = 2\rho_0\,, \\ &c_{|12} = c_{|34} = c_{|13} = c_{|24} = 0\,. \end{aligned}\right\} \quad (10)$$

With it, the field tensor

$$\begin{aligned} F_{ik} = &-c_{|14}(e_{|1\_i}e_{|4\_k} - e_{|1\_k}e_{|4\_i}) + \\ &+ c_{|23}(e_{|2\_i}e_{|3\_k} - e_{|2\_k}e_{|3\_i}) \end{aligned} \quad (11)$$

is performed from the main surfaces. Rainich knew also these relations [4, 5].

3 Conclusions

Montesinos and Flores [9] deduce the electromagnetic energy-momentum tensor via Noether's theorem [10]. That means, the Ricci tensor must have just the form according to Eqn. (1). Therefore, the geometry with the main surfaces is necessary for the space-time. Since the electromagnetic field tensor is performed by the main surfaces, it is a curve parameter of the current path like the curvature vector (which is performed by the main normal, and is the geometric expression of both gravitation and accelerated motion), as Rainich already saw. We can formulate an enhanced equivalence principle this way. It is the equivalence of the Lorentz force with Newton's force. Because the test body means a current point on the path, i.e. all forces to the test body come from curve parameters.

Montesinos and Flores [9] derived a symmetric energy-momentum tensor from three different theories, with the result that sources have to vanish in each case. That means:

1. The Maxwell theory is sufficient, because it runs as demonstrated in [7], even also regarding quantization. Non-Riemannian ansatzes are not needed;
2. Any ansatz with distributed charges or masses is false in principle. This error was helpful in classical theories before Einstein, which were separately handled. Now, such error turns up to be counterproductive.

It appears inviting to specify metrics first via Eqn. (7) (see [6]), but this method has narrow limits. The electromagnetic integration constants (charge, magnetic momentum) come from Maxwell's equations. The geometric theory of fields [7] unifies electromagnetism with gravitation natural way.

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