LETTERS TO PROGRESS IN PHYSICS

Smarandache Spaces as a New Extension of the Basic Space-Time of General Relativity

Dmitri Rabounski

E-mail: rabounski@ptep-online.com

This short letter manifests how Smarandache geometries can be employed in order to extend the "classical" basis of the General Theory of Relativity (Riemannian geometry) through joining the properties of two or more (different) geometries in the same single space. Perspectives in this way seem much profitable: the basic space-time of General Relativity can be extended to not only metric geometries, but even to non-metric ones (where no distances can be measured), or to spaces of the mixed kind which possess the properties of both metric and non-metric spaces (the latter should be referred to as "semi-metric spaces"). If both metric and non-metric properties possessed at the same (at least one) point of a space, it is one of Smarandache geometries, and should be referred to as "Smarandache semi-metric space". Such spaces can be introduced according to the mathematical apparatus of physically observable quantities (chronometric invariants), if we consider a breaking of the observable space metric in the continuous background of the fundamental metric tensor.

When I was first acquainted with Smarandache geometries many years ago, I immediately started applying them, in order to extend the basic geometry of Einstein's General Theory of Relativity.

Naturally, once the General Theory of Relativity was established already in the 1910's, Albert Einstein stated that Riemannian geometry, as advised to him by Marcel Grossmann, was not the peak of excellence. The main advantage of Riemannian geometry was the invariance of the space metric and also the well-developed mathematical apparatus which allowed Einstein to calculate numerous specific effects, unknown or unexplained before (now, they are known as the effects of General Relativity). Thus, Einstein concluded, the basic spacetime of General Relativity would necessarily be extended in the future, when new experiments would overcome all the possibilities provided by the geometry of Riemannian spaces. Many theoretical physicists and mathematicians tried to extend the basic space-time of General Relativity during the last century, commencing in the 1910's. I do not survey all the results obtained by them (this would be impossible in so short a letter), but only note that they all tried to get another basic space, unnecessary Riemannian one, then see that effects manifest themselves in the new geometry. No one person (at least according to my information on this subject, perhaps incomplete) did consider the "mixed" geometries which could possess the properties of two or more (different in principle) geometries at the same point.

This is natural, because a theoretical physicist looks for a complete mathematical engine which could drive the applications to physical phenomena. What would have happened had there been no Bernhard Riemann, Erwin Christoffel, Tullio Levi-Civita, and the others; could Einstein have been enforced to develop Riemannian geometry in solitude from scratch? I think this would have been a "dead duck" after all. Einstein followed a very correct way when he took the well-approved mathematical apparatus of Riemannian geometry. Thus, a theoretical physicist needs a solid mathematical ground for further theoretical developments. This is why some people, when trying to extend the basis of General Relativity, merely took another space instead the four-dimensional pseudo-Riemannian space initially used by Einstein.

Another gate is open due to Smarandache geometries, which can be derived from any of the known geometries by the condition that one (or numerous, or even all) of its axioms is both true and violated in the space. This gives a possibility to create a sort of "mixed" geometries possessing the properties of two or more geometries in one. Concerning the extensions of General Relativity, this means that we can not refuse the four-dimensional pseudo-Riemannian space in place of another single geometry, but we may create a geometry which is common to the basic one, as well as one or numerous other geometries in addition to it. As a simplest example, we can create a space possessing the properties of both the curved Riemannian and the flat Euclidean geometries. So forth, we can create a space, every point of which possesses the common properties of Riemannian geometry and another geometry which is non-Riemannian.

Even more, we can extend the space geometry in such a way that the space will be particularly metric and particularly non-metric. In the future, I suggest we should refer to such spaces as semi-metric spaces. Not all semi-metric spaces manifest particular cases of Smarandache geometries. For example, a space wherein each pair of points is segregated from the others by a pierced point, i.e. distances can be determined only within diffeerential fragments of the space segregated by pierced points. This is undoubtely a semi-metric space, but is not a kind of Smarandache geometries. Contrarily, a space wherein at least one pair of points possesses both metric and non-metric properties at the same time is definitely that of Smarandache geometries. In the future, I suggest, we should refer to such spaces as *Smarandache semi-metric spaces*, or *ssm-spaces* in short.

Despite the seeming impossibility of joining metric and non-metric properties in "one package", Smarandache semi-metric spaces can easily be introduced even by means of "classical" General Relativity. The following is just one example of how to do it. Regularly, theoretical physicists are aware of the cases where the signature conditions of the space are violated. They argue that, because the violations produce a breaking of the space, the cases have not a physical meaning in the real world and, hence, should not be considered. Thus, when considering a problem of General Relativity, most theoretical physicists artificially neglect, from consideration, those solutions leading to the violated signature conditions and, hence, to the breaking of the space. On the other hand, we could consider these problems by means of the mathematical apparatus of chronometric invariants, which are physically observable quantities in General Relativity. In this way, we have to consider the observable (chronometrically invariant) metric tensor on the background of the fundamental (general covariant) metric tensor of the space. The signature conditions of the metrics are determined by different physical requirements. So, in most cases, the violated signature conditions of the observable metric tensor, i.e. breaking of the observable space, can appear in the continuous background of the fundamental metric tensor (and vice versa). This is definitely a case of Smarandache geometries. If a distance (i.e. a metric, even if non-Riemannian) can be determined on the surface of the space breaking, this is a metric space of Smarandache geometry. I suggest we should refer to such spaces as *Smarandache metric spaces*. However, if the space breaking is incapable of determining a distance inside it, this is a Smarandache semi-metric space: the space possesses both metric and non-metric properties at all points of the surface of the space breaking.

A particular case of this tricky situation can be observed in Schwarzschild spaces. There are two kinds of these: a space filled with the spherically symmetric gravitational field produced by a mass-point (the center of gravity of a spherical solid body), and a space filled with the spherically symmetric gravitational field produced by a sphere of incompressible liquid. Both cases manifest the most apparent metrics in the Universe: obviously, almost all cosmic bodies can be approximated by either a sphere of solid or a sphere of liquid. Such a metric space has a breaking along the spherical surface of gravitational collapse, surrounding the center of the gravitating mass (a sphere of solid or liquid). This space breaking originates in the singularity of the fundamental metric tensor. In the case of regular cosmic bodies, the radius of the space breaking surface (known as the gravitational radius, it is determined by the body's mass) is many orders smaller than the radius of such a body itself: it is 3 km for the Sun, and only 0.9 cm for the Earth. Obviously, only an extremely dense cosmic body can completely be located under its gravitational radius, thus consisting a gravitational collapsar (black hole). Meanwhile, the space breaking at the gravitational radius really exists inside any continuous body, close to its center of gravity. Contrary, the space breaking due to the singularity of the observable metric tensor is far distant from the body; the sphere of the space breaking is huge, and is like a planetary orbit. Anyhow, in the subspace inside the Schwarzschild space breaking, distances can be determined between any two points (but they are not those of the Schwarzschild space distances). Thus, when considering a Schwarzschild space without any breaking, as most theoretical physicists do, it is merely a kind of the basic space-time of General Relativity. Contrarily, being a Schwarzschild space considered commonly with the space breaking in it, as a single space, it is a kind of Smarandache metric spaces — a *Schwarzschild-Smarandache metric space*, which generalizes the basic space-time of General Relativity. Moreover, one can consider such a space breaking that no distance (metric) can be determined inside it. In this case, the common space of the Schwarzschild metric and the non-metric space breaking in it is a kind of Smarandache semi-metric spaces — a *Schwarzschild-Smarandache semi-metric space*, and is an actual semi-metric extension of the basic space-time of General Relativity.

So, we see how Smarandache geometries (both metric and semi-metric ones) can be a very productive engine for further developments in the General Theory of Relativity. Because the Schwarzschild metrics lead to consideration of the state of gravitational collapse, we may suppose that not only regular gravitational collapsars can be considered (the surface of a regular black hole possesses metric properties), but even a much more exotic sort of collapsed objects — a collapsar whose surface cannot be presented with metric geometries. Because of the absence of metricity, the surface cannot be inhabited by particles (particles, a sort of discrete matter, imply the presence of coordinates). Only waves can exist there. These are standing waves: in the metric theory, time cannot be introduced on the surface of gravitational collapse due to the collapse condition $g_{00} = 0$; the non-metric case manifests the state of collapse by the asymptopic conditions from each side of the surface, while time is not determined in the non-metric region of collapse as well. In other words, the non-metric surface of such a collapsar is filled with a system of standing waves, i.e. holograms. Thus, we should refer to such objects — the collapsars of a Schwarzschild-Smarandache semi-metric space — as *holographic black holes*. All these are in the very course of the paradoxist mathematics, whose motto is "impossible is possible".

Submitted on February 12, 2010 / Accepted on February 18, 2010