A Numerical Experiment with the Double Slit Geometry

Riadh H. Al Rabeh

Department of Mechanical Engineering, University of Basra, Iraq
Present address: Lydgate Close, Manningtree, Essex, UK
E-mail: alrabeh_rh@yahoo.com

Young's double slit experiment performed in 1801 was a milestone in the history of physics. The passing of light through two narrow slits creates interference patterns that sums up the diffraction patterns from each slit when separately uncovered. The experiment was later repeated by others using single photons, single electrons, atoms and even molecules producing similar effects. The present interpretation of the results is that photons and all other particles behave like waves and particles at the same time (the wave-particle duality principle). Further explanations were also given, including notions like particles can exist in more than one position at the same time and interfere with itself, and that the classical laws of physics are not applicable in an atomic scale. In this work we perform a numerical experiment in which a single charged particle is fired at a wall of (fixed) charged particles containing gaps to mimic slits, and collect the results over many events in time. Assuming only a classical inverse square relation to hold between the particles- including those of the wall, the results show clear diffraction and interference patterns indicating that the wave behaviour of the bullet particles arises simply from such interactions- hence providing a pure classical interpretation to the problem. That is; particles follow classical laws and produce waves only when interacting with each others. An analytical treatment of this subject is further required to remove the effects of a finite time step inherent in a numerical solution.

1 Introduction

The double slit experiment is considered an important milestone in the history of physics. It was first conducted by Thomas Young in 1801. In Young's experiment, light was made to pass through two narrow slits in an opaque barrier (wall) and collect on a photographic plate behind the barrier. The picture obtained with any one slit open, was that of diffraction in the form of one bright line in the middle of fading alternating dark and bright lines. When two slits are open, the picture changes into an interference pattern that can be explained by the addition of two diffraction patterns from the two slits separately. The double slit experiment was originally performed to settle the argument at the time of whether light- seen to travel along straight lines and reflect like being composed of particles (or corpuscles), and as suggested by Newton, or as waves like Huygens was advocating in his new theory for waves. The interference obtained were taken to favour the wave theory- since the effects of having particles should be producing only positive additions and no annihilation- as the slit experiment seemed to be suggesting [1].

As evidence from experiments in different fields and theoretical work started to accumulate in favour of the particle nature of light, there was a return to the slit experiment to be conducted this time using particles like electrons, neutrons, atoms and molecules [2, 3]. This is to establish if all particles do exhibit a wave-like behaviour as that of the photon particle. The results were again all positive prompting a new explanation to the results, namely that: particles have a dual particle-wave nature. Further tests were subsequently conducted using single photons, electrons and other particles fired one at a time. The interference pattern persisted in all these cases as well- prompting the conclusion that atomic scale particles do not obey the laws of classical mechanics [3–5]. In all these explanations however, the interaction between the barrier particles and those of the bullets are only taken to be of the go no-go relation with no regard to the possibility of some inverse square type forces being involved. Random scatter at the edges of the slits might have also been considered but thought not being capable of producing such consistent wave behaviour. The main thinking instead was concentrated on the interference pattern as being the result of an interaction between the bullet particles alone.

In this article we shall assume that the barrier particles do interact with the bullet particles through a simple inverse square relation. To do this we shoot a charged bullet particle at a wall composed of fixed and similarly or oppositely charged particles (with gaps to mimic the presence of slits). The path of the bullet particle is to be predicted by numerically integrating the equation of motion for a single path at a time and collect the paths over time. An interaction between the barrier particles and the bullet is a must of course, since otherwise there is no meaning to the word slit at all. The type of interaction however, is what is new in the present work. The results seem to show that an inverse square interaction is capable of producing the wave behaviour required to explain the results using pure classical laws and interpretations. A major drawback of the present numerical solution however, is that it is discrete and hence can be affected by the size of the time step. Further analytical treatment of the subject (in the light of the present results) will be needed before a concrete conclusion can be made on this matter. Such work is not expected to contradict the vivid wave and interference patterns observed in the numerical results.

To be able to cover two slits and to produce different wave patterns, the axial velocity of the bullet was changed in a systemic manner in the experiment and the vertical (transverse) component of the bullet velocity was changed randomly by a very small amount around zero. This allows the accumulating beam to cover both slits over time.

2 Theory

For Coulomb forces, the expression for the acceleration is given by;

$$a = \frac{d^2r}{dt^2} = \frac{k}{r^2}\,, \tag{1}$$

where $a = a(t)$, $r = r(t)$ are the acceleration and separation distances between any isolated pair of particles as a function of time t, and k is the coupling constant (negative for attractive, and positive for repulsive forces) and in which the masses and charges of all particles are unity. The magnitude of k is dependent on the type of interaction. For example, in the case of a repulsive Coulomb forces $k = 1/4\pi\varepsilon_0$, where ε_0 is the permittivity of empty space. In the case the number of interacting particles is small; the Coulomb forces by far dominate other forces as assumed here. As the interacting masses are points, there is no need to consider angular velocity, spin, angular momentum or any form of moments of forces on the particle. For a group of interacting particles, the net acceleration of particle j is given by;

$$\mathbf{a}_j = \frac{d\mathbf{v}_j}{dt} = \sum_i \frac{k_{ij}\mathbf{r}_{ij}}{r_{ij}^3}\,;\;\; r_{ij} = |\mathbf{r}_{ij}|,\;\; i,j = 1,2,\cdots N\,, \tag{2}$$

where $\mathbf{a}_j$ is the resultant acceleration, $\mathbf{v}$ is velocity, k_{ij} is the total coupling constant between particles i and j, and $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ is the vector from i to j positions and N is the total number of particles. Equation (2) is a set of simultaneous ode's that must be integrated once in order to find $\mathbf{v}_j(t)$ and again to find the position $\mathbf{r}_j(t)$ giving;

$$\mathbf{r}_j = \mathbf{r}_{j0} + (dt)\mathbf{v}_{j0} + (dt)^2 \sum_i \frac{\mathbf{r}_j - \mathbf{r}_i}{|\mathbf{r}_j - \mathbf{r}_i|^3}\,;\;\; j = 1:N;\;\; i \neq j. \tag{3}$$

If we know the initial position $\mathbf{r}_{j0}$, the initial velocity $\mathbf{v}_{j0}$, and the time step dt, we can find the new position of the bullet $\mathbf{r}_j$. This is to be repeated for different initial velocities and the resulting trajectories are collected over time and plotted. The values chosen for the various parameters do not necessarily correspond to particular physical values, but rather chosen to accentuate the resulting picture and make it clearer. The actual values used are given. A simple one step method is chosen for the integration as in equation (3) to avoid any erroneous contributions from any extra terms contained in a more refined integration procedure.

If we hope to produce results showing a wave behaviour using only inverse square relations, we should be able to show that this is possible in theory. In fact [6] states that the potential equation of motion becomes a spring like relation in the case of small displacements together with a large number of interacting particles. In the present case, we assume the wall particles are fixed in space, which is equivalent to a presence of a large number of particles in a small space making the group massive and well connected to resist the effects of the bullet particle approaching the barrier. We further confirm this in Fig. 1, where a spring type relation results from fixing two particles and allowing a third to experience a small displacement in the middle under an inverse square force. The algorithm needed to implement equation (3) is fairly straight forward as shown below;

```
Algorithm to compute the trajectory of a charged
particle fired at a wall containing slits and composed
of similarly (or oppositely) charged fixed particles.
Total number of particles nb=10 at position r(x,y),
velocity (vx,vy), acc.  (ax,ay)
and force (fx,fy)=acc.
For a fixed wall, x,y are calculated only
for the 1st particle.

ee=1e-100;X=[];Y=[];dt0=.01; v01=3.2; nb=12;
nbv=1:nb;x(nbv)=0;x=x';y=x;vx=x;
vy=x;kb(1:nb)=2e-3;kb(5:9)=0;
for ii=1:250;
y(1)=0; vy(1)=0.08*(rand-0.5); x(1)=-1;
vx(1)=v01;x(2:nb)=-0.25;
y(2:end)=0.002*((2:nb)-nb/2 -1);
for kk=1:100; for jj=1:nb; xj=x(jj); yj=y(jj);
vxj=vx(jj);vyj=vy(jj); xb=xj-x;yb=yj-y;
rb2=ee+xb.^ 2+yb.^ 2; rb=sqrt(rb2);
fb=kb'./rb2; fxb=fb.*xb./rb; fyb=fb.*yb./rb;
fx=sum(fxb);fy=sum(fyb);
ax=fx;ay=fy; dt=dt0;
if jj > 1;dt=0;end;
vxj=vxj+dt*ax; vyj=vyj+dt*ay; xj=xj+dt*vxj;
yj=yj+dt*vyj;x(jj)=xj; y(jj)=yj;
vx(jj)=vxj; vy(jj)=vyj;
end;
if abs(x(1)) > 1.5 | abs(y(1)) > 1;break;end;
X=[ X;x'];Y=[Y;y'];end;end;
figure(1);plot(X,Y);
```

The inner loop jj adds the forces over all the particles, then we advance in time in the kk loop to give one path (trajectory). The ii loop repeats this many times to arrive at the final picture. The rest of the algorithm is self explanatory.

3 Results

The values of the coupling constant k, (units m^3/s^2 as in (1)), the horizontal and vertical velocity components, the distances between slits and between the particles making the wall, the time step and other constants are clearly referenced in the al-

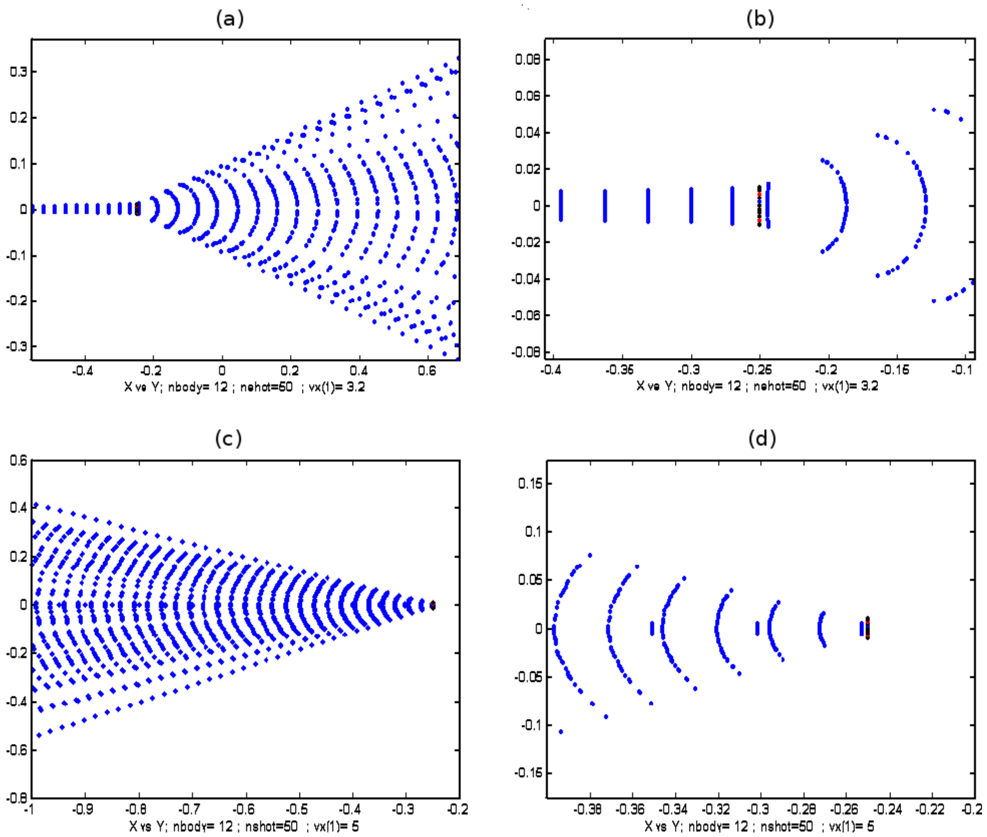

Fig. 2: Time collection of an electron fired (from left) against a fixed column barrier of loose electrons with a random small vel. component in the vertical direction. Total of 50 events are collected. Wave–front plane in (a), (b) changing to circular after the barrier. At another speed, the wave-front is completely reflected (b), (c) and also changed to circular.

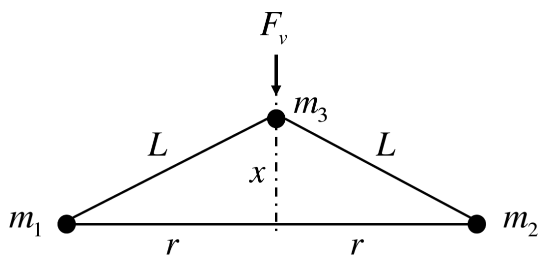

Fig. 1: A spring like force relation capable of producing a wave behavior can result from the interaction of particles under an inverse square relation. $F_{31} = F_{32} = k/r^2$; for small deflection x; $L = r$. $F_v = 2\frac{k}{r^2}\frac{x}{r} = 2\frac{k}{r^3}x = Kx$; k, r, K are constants. Therefore, force on m_3 is a spring type force.

gorithm given above. It is again stressed that the different constants are chosen so as to produce a clear picture rather than correspond to certain physical values. The main goal of this article is to show the wave phenomenon of diffraction and interference happening in a purely inverse square environment and with bullet particles that do not know of each other and hence never have a chance to interact as they exist in different times.

Fig. 2(a) shows a plot collecting 50 events and showing that what was originally a plane wave-front (elements of the front exist at different times) have been changed by the barrier to a circular wave-front as one would expect of a true wave. A magnified scale of the same is shown in the next figure. In Figs. 2(c),(d) the wave front is reflected completely as what could happen with real waves when the wavelength compared to the sparseness of the particle of the wall is of the correct order. In Fig. 3(a) few of the wall particles are assumed to be inert to mimic the presence of a slits. The result as expected is a superposition of two circular waves producing an interference pattern. It is seen that a single bullet collected over time is behaving like a true beam composed of many particles. The presence of the one barrier in all the shooting events is what unifies all the outputs and creates the observed effects.

4 Conclusions

The results shown indicate clearly that the passage of a bullet particle through a slit modifies its path and the wave-front composed of many particles, which need not exist at the same time, can change from plane to circular if the force between the barrier and the bullet particles is that of an inverse square type. In [6] and in Fig. 1 in this article, it is shown how a change from an inverse square to a spring relation can result

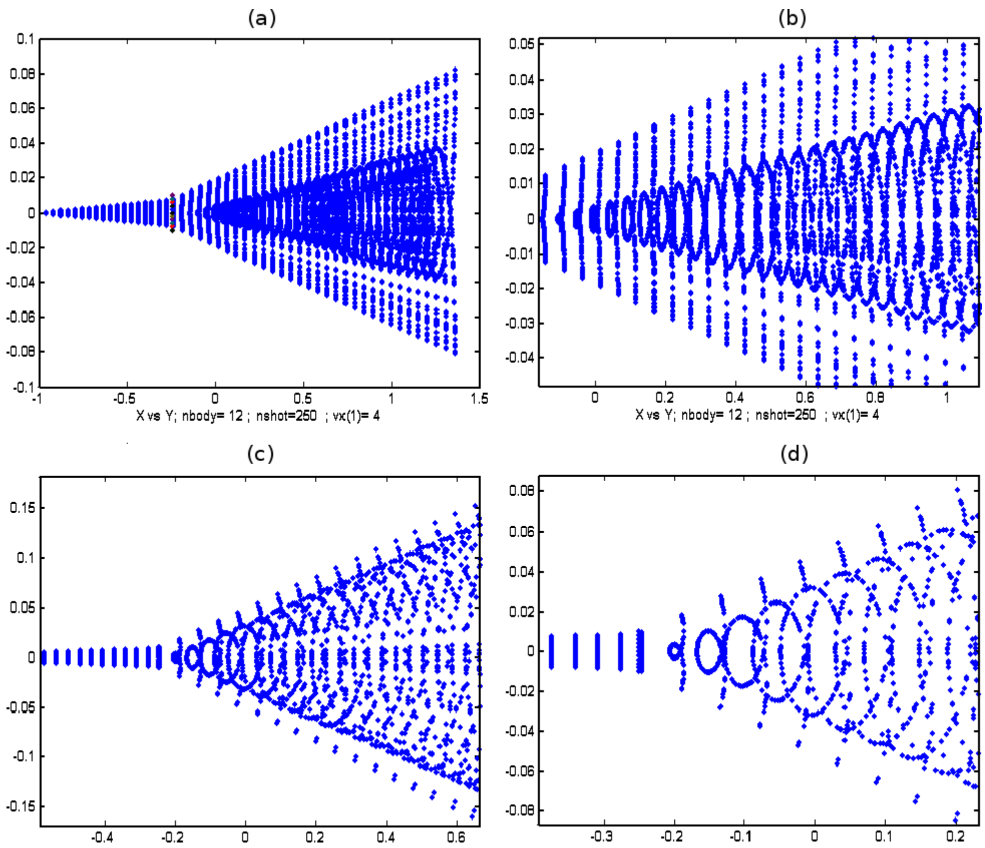

Fig. 3: Wall particles 5:9 (out of 11) are made neutral to mimic a slit. This causes two diffraction patterns interfering with each other. The last two (c), (d), are plotted using the algorithm given in this article with bullet horizontal speed $v_x = 3.2$.

in the case of large interacting particles as those of the barrier (and mimicked here by having fixed particles). This picture is equivalent to what happens in field theory in which a potential equation (resulting from inverse square relation) acquires wave solutions due to the presence of a boundary. This effect occurs in the case of waves in fluids and solids which are composed essentially of particles interacting under an inverse square environment.

The present results upholds the fact that particles behave like waves and particles, but differs in giving a more natural explanation that agrees with common logic and classical laws. It is difficult to believe at the end that classical laws that apply to planets composed of trillions of particles fail when considering few of them. The particle picture is simple to comprehend and can also afford to explain many of the relativistic and quantum findings in physics (see [7, 8] by this author for more on this).

For deeper understanding of the present results, it is useful to do a complementary theoretical analysis to overcome the finite time step effects inherent in any numerical solution. Further understanding of the problem may be achieved by using more elaborate particles where spin and moments are to be taken into consideration.

Submitted on February 09, 2010 / Accepted on February 16, 2010

References

1. Feynman, Richard P. The Feynman lectures on physics. *Addison-Wesley*, 1965, v. 3.
2. Jönsson C. Electron diffraction at multiple slits. *American Journal of Physics*, 1974, v. 4.
3. Nairz O., Arndt M., and Zeilinger A. Quantum interference experiments with large molecules. *American Journal of Physics*, 2003, v. 71.
4. Summhammer J., Rauch H., Tuppinger D. *Phys. Rev. A*, 1987, v. 36.
5. Greene B. The fabric of the cosmos: space, time, and the texture of reality. Vintage, 2000.
6. Landau L. D., Lifshitz E. M. Mechanics. Pergamon press, 1960.
7. Al Rabeh R. H. Primes, geometry and condensed matter. *Progress in Physics*, 2009, v. 3.
8. Al Rabeh R. H. New ideas for the extra dimensions and for deriving the basic laws of physics. *Progress in Physics*, 2010, v. 1.