Dispersion of Own Frequency of Ion-Dipole by Supersonic Transverse Wave in Solid

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First, we predict an existence of transverse electromagnetic field formed by supersonic transverse wave in solid. This electromagnetic wave acquires frequency and speed of sound, and it propagates along of direction propagation of supersonic wave. We also show that own frequency of ion-dipole depends on frequency of supersonic transverse wave.

1 Introduction

In our latest paper [1], we investigated the light diffraction by supersonic longitudinal waves in crystal. In this respect, we predicted an existence of transverse electromagnetic field created by supersonic longitudinal waves in solid. This electromagnetic wave with frequency of ultrasonic field is moved by speed of supersonic field toward to direction propagation of sound. There was shown that the average Poynting vector of superposition field involves the intensities of the transverse electromagnetic and the optical fields which form the intensity of light diffraction. We considered a model of solid as lattice of ions and gas of free electrons. Each ion of lattice coupled with a point of lattice knot by spring, creating of ion dipole. The knots of lattice define a position dynamical equilibrium of each ion which is vibrated by own frequency Ω_0. Hence, we may argue that the presented new model of solid leads to the same results which may obtain for solid by one dimensional model of single atomic crystal representing as model of continuum elastic medium which is described by chain of ions [2]. The vibration of ion occurs near position of equilibrium corresponding to minimum of potential energy (harmonic approximation of nearing neighbors).

Thus, the existence of transverse electromagnetic field is an important factor for correction so called the Raman-Nath theory [3] and the theory of photo-elastic linear effect [4] which were based on a concept that acoustic wave generates a periodical distribution of refractive index in the coordinate-time space.

In this letter, we attempt to investigate a property of solid by under action of supersonic transverse wave. In this context, we find dispersion law for own frequency of ion-dipole which depends on frequency of supersonic transverse wave.

2 Formation of transverse electromagnetic field

Let's consider the coupled ions with points of lattice knots. These ions are vibrated by own frequency Ω_0 into ion-dipoles. We note that ion-dipole is differ from electron-ion dipole which was discussed within elementary dispersion theory [5]. Hence, we assume that property of springs of ion dipole and ion-electron one are the same. This assumption allows us to obtain a connection between own frequencies of electron ω_0 and ion Ω_0 by condition $\Omega_0 = \sqrt{\frac{q}{M}} = \omega_0 \sqrt{\frac{m}{M}}$ where q is the rigidity of spring; m is the mass of electron.

By under action of transverse acoustic wave, there is an appearance of vector displacement $\vec{u}$ of each ions in solid.

Consider the propagation of an ultrasonic transverse plane traveling wave in cubic crystal. Due to laws of elastic field for solid [6], the vector displacement $\vec{u}$ satisfies to condition which defines property of transverse supersonic field

$$\operatorname{div} \vec{u} = 0 \tag{1}$$

and is defined by wave-equation

$$\nabla^2 \vec{u} - \frac{1}{c_t^2} \frac{d^2 \vec{u}}{dt^2} = 0\,, \tag{2}$$

where c_t is the velocity of a transverse ultrasonic wave which is determined by elastic coefficients.

The simple solution of (2) in respect to $\vec{u}$ has a following form

$$\vec{u} = \vec{u}_0 \sin(Kx + \Omega t)\,, \tag{3}$$

where $\vec{u}_0$ is the amplitude of vector displacement; $K = \frac{\Omega\sqrt{\varepsilon}}{c}$ is the wave number of transverse electromagnetic wave.

Thus each ion acquires the dipole moment of ion $\vec{p} = -e\vec{u}$. Now, we may argue that there is a presence of transverse electromagnetic field with vector of electric field $\vec{E}$ due to displacement of ion:

$$M \frac{d^2 \vec{u}}{dt^2} + q\vec{u} = -e\vec{E}\,, \tag{4}$$

where $\vec{E}$ is the vector electric field which is induced by transverse ultrasonic wave; M is the mass of ion; the second term $q\vec{u}$ in left part represents as changing of quasi-elastic force which acts on ion.

Using of the operation div of the both part of (4) together with (1), we obtain a condition for Transverse electromagnetic wave

$$\operatorname{div} \vec{E} = 0\,. \tag{5}$$

Now, substituting solution $\vec{u}$ from (3) in (4), we find the vector transverse electric wave

$$\vec{E} = \vec{E}_0 \sin(Kx + \Omega t)\,, \tag{6}$$

where

$$\vec{E}_0 = \frac{M\left(\Omega_0^2 - \Omega^2\right)\vec{u}_0}{e} \tag{7}$$

is the amplitude of transverse electric field which acts on ion into ion dipole.

The ion dipole acquires a polarizability α, which is determined via total dipole moment:

$$\vec{P} = N\alpha\vec{E}\,. \tag{8}$$

where N is the concentration of ion dipoles.

In the presented theory, the vector electric induction $\vec{D}$ is determined as

$$\vec{D} = 4\pi\vec{P} + \vec{E}\,, \tag{9}$$

and

$$\vec{D} = \varepsilon\vec{E}\,, \tag{10}$$

where $\vec{P}{=}N_0\vec{p}$ is the total polarization created by ion-dipoles.

It is easy to find the dielectric respond ε of acoustic medium which takes a following form

$$\varepsilon = 1 + 4\pi N\alpha = 1 + \frac{4\pi N e^2}{M\left(\Omega_0^2 - \Omega^2\right)}\,, \tag{11}$$

This formulae is also obtained by model of ions chain [2].

The dielectric respond ε of acoustic medium is similar to optical one, therefore,

$$\sqrt{\varepsilon} = \frac{c}{c_t}\,, \tag{12}$$

where c is the velocity of electromagnetic wave in vacuum.

Thus, we found that transverse electric wave with frequency Ω is propagated by velocity c_t of ultrasonic transverse wave in the direction OX.

Furthermore, we present the Maxwell equations for electromagnetic field in acoustic medium with a magnetic permittivity $\mu = 1$:

$$\operatorname{curl}\vec{E} + \frac{1}{c}\frac{d\vec{H}}{dt} = 0\,, \tag{13}$$

$$\operatorname{curl}\vec{H} - \frac{1}{c}\frac{d\vec{D}}{dt} = 0\,, \tag{14}$$

$$\operatorname{div}\vec{H} = 0\,, \tag{15}$$

$$\operatorname{div}\vec{D} = 0 \tag{16}$$

where $\vec{E} = \vec{E}(\vec{r}, t)$ and $\vec{H} = \vec{H}(\vec{r}, t)$ are the vectors of local electric and magnetic fields in acoustic medium; $\vec{D} = \vec{D}(\vec{r}, t)$ is the local electric induction in the coordinate-time space; $\vec{r}$ is the coordinate; t is the current time in space-time coordinate system.

We search a solution of Maxwell equations by introducing the vectors of magnetic and electric fields by following way

$$\vec{H} = \operatorname{curl}\vec{A}\,, \tag{17}$$

where

$$\vec{E} = -\frac{d\vec{A}}{cdt}\,, \tag{18}$$

where $\vec{A}$ is the vector potential of electromagnetic wave.

After simple calculation, we reach to following equation for vector potential $\vec{A}$ of transverse electromagnetic field

$$\nabla^2\vec{A} - \frac{\varepsilon}{c^2}\frac{d^2\vec{A}}{dt^2} = 0 \tag{19}$$

with condition of plane transverse wave

$$\operatorname{div}\vec{A} = 0\,. \tag{20}$$

The solution of (24) and (25) may present by plane transverse wave with frequency Ω which is moved by speed c_t along of direction of unit vector $\vec{s}$:

$$\vec{A} = \vec{A}_0\cos\left(Kx + \Omega t\right)\,, \tag{21}$$

and

$$\vec{A}\cdot\vec{s} = 0\,, \tag{22}$$

where $K = \frac{\Omega\sqrt{\varepsilon}}{c}$ is the wave number of transverse electromagnetic wave; $\vec{s}$ is the unit vector in direction of wave normal; $\vec{A}_0$ is the vector amplitude of vector potential. In turn, at comparting (23) and (6), we may argue that the vector of wave normal $\vec{s}$ is directed along of axis OX ($\vec{s} = \vec{e}_x$), because the vector electric transverse wave $\vec{E}$ takes a form presented in (6):

$$\vec{E} = \vec{E}_0\sin\left(Kx + \Omega t\right)\,, \tag{23}$$

where

$$\vec{E}_0 = \frac{\Omega\vec{A}_0}{c}\,.$$

To find a connection between vector amplitude of electric field $\vec{E}_0$ and vector amplitude of acoustic field $\vec{u}_0$, we use of the law conservation energy. The average density energy w_a of ultrasonic wave is transformed by one w_t of transverse electromagnetic radiation. In turn, there is a condition $w_a = w_t$ where Thus,

$$w_a = MN\Omega^2u_0^2\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos^2\left(Kx + \Omega t\right)dt = \\ = \frac{MN\Omega^2u_0^2}{2}\,, \tag{24}$$

In analogy manner,

$$w_t = \frac{\varepsilon}{4\pi}\,E_0^2\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\sin^2\left(Kx + \Omega t\right)dt = \frac{\varepsilon}{8\pi}\,E_0^2\,. \tag{25}$$

Thus, at comparing (24) and (25), we arrive to an important expression which leads to foundation of dispersion law for own frequency of ion-dipole:

$$\frac{\varepsilon}{4\pi}\,E_0^2 = MN\Omega^2u_0^2\,, \tag{26}$$

where introducing meanings of $\vec{E}_0$ and ε from (7) and (11) into Eq.(26), we obtain a dispersion equation:

$$\left(\Omega_0^2 - \Omega^2\right)^2 + 2\Omega_p^2 \left(\Omega_0^2 - \Omega^2\right) - \Omega_p^2 \Omega^2 = 0\,, \qquad (27)$$

where $\Omega_p = \sqrt{\frac{4\pi N e^2}{M}} = \omega_p \sqrt{\frac{m}{M}}$ is the classic plasmon frequency of ion but ω_p is the plasmon frequency of electron. For solid $\omega_p \sim 10^{16}$ s^{-1}, therefore, at $\sqrt{\frac{m}{M}} \sim 10^{-2}$), it follows $\Omega_p \sim 10^{14}$ s^{-1}.

The solution of Eq.(27) in regard to own frequency of ion Ω_0 take following forms:

1. At $\Omega_0 \geqslant \Omega$

$$\Omega_0 = \sqrt{\Omega^2 - \Omega_p^2 + \sqrt{\Omega_p^4 + \Omega_p^2 \Omega^2}}\,. \qquad (28)$$

2. At $\Omega_0 \leqslant \Omega$

$$\Omega_0 = \sqrt{\Omega^2 - \Omega_p^2 - \sqrt{\Omega_p^4 + \Omega_p^2 \Omega^2}}\,. \qquad (29)$$

Now, consider following solutions of above presented equations:

1. At $\Omega \ll \Omega_p$, $\Omega_0 \geqslant \Omega$, we obtain $\Omega_0 \approx \sqrt{\frac{3}{2}}\Omega$ but at $\Omega_0 \leqslant \Omega$, it follows $\Omega_0 \approx \sqrt{\frac{1}{2}}\Omega$. This condition implies that we may consider model of solid as ideal gas of atoms at smaller Ω. 2. At $\Omega \gg \Omega_p$, $\Omega_0 \geqslant \Omega$ we obtain $\Omega_0 \approx \Omega + \frac{\Omega_p}{2}$ but at $\Omega_0 \leqslant \Omega$, it follows that $\Omega_0 \approx \Omega - \frac{\Omega_p}{2}$. 3. At $\Omega \approx \Omega_p$, $\Omega_0 \approx 2^{\frac{1}{4}}\Omega_p$.

In conclusion, we may note that the action of ultrasonic transverse wave in solid leads to new property as determination of own frequency of ion-dipole. This fact is useful because in the case of action of ultrasonic longitudinal wave in solid, the own frequency of ion-dipole can not be determined. However, knowledge of value of own frequency of ion-dipole allows us to calculate the intensity of sound by formulae (26) (at known meaning of intensity of transverse electromagnetic field excited by ultrasonic longitudinal wave in solid). In turn, it determines the resonance frequency ω_0 of optical light in solid because $\omega_0 = \Omega_0 \sqrt{\frac{M}{m}}$ due to condition that the rigidity of spring is the same for ion-dipole and electron-ion dipole. Thus, the action of ultrasonic transverse wave on solid may change an optical property of solid.

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