Charged Polaritons with Spin 1

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We present a new model for metal which is based on the stimulated vibration of independent charged Fermi-ions, representing as independent harmonic oscillators with natural frequencies, under action of longitudinal and transverse elastic waves. Due to application of the elastic wave-particle principle and ion-wave dualities, we predict the existence of two types of charged Polaritons with spin 1 which are induced by longitudinal and transverse elastic fields. As result of presented theory, at small wavenumbers, these charged polaritons represent charged phonons.

1 Introduction

In our recent paper [1], we proposed a new model for dielectric materials consisting of neutral Fermi atoms. By the stimulated vibration of independent charged Fermi-atoms, representing as independent harmonic oscillators with natural frequencies by actions of the longitudinal and transverse elastic waves, due to application of the principle of elastic wave-particle duality, we predicted the lattice of a solid consists of two types of Sound Boson-Particles with spin 1, with finite masses around 500 times smaller than the atom mass. Namely, we had shown that these lattice Sound-Particles excite the longitudinal and transverse phonons with spin 1. In this context, we proposed new model for solids representing as dielectric substance which is different from the well-known models of Einstein [2] and Debye [3] because: 1), we suggest that the atoms are the Fermi particles which are absent in the Einstein and Debye models; 2), we consider the stimulated oscillation of atoms by action of longitudinal and transverse lattice waves which in turn consist of the Sound Particles.

Thus, the elastic lattice waves stimulate the vibration of the fermion-atoms with one natural wavelength, we suggested that ions have two independent natural frequencies by under action of a longitudinal and a transverse wave. Introduction of the application of the principle of elastic wave-particle duality as well as the model of hard spheres we found an appearance of a cut off in the spectrum energy of phonons which have spin 1 [1].

In this letter, we treat the thermodynamic property of metal under action of the ultrasonic waves. We propose a new model for metal where the charged Fermi-ions vibrate with natural frequencies Ω_l and Ω_t, by under action of longitudinal and transverse elastic waves. Thus, we consider a model for metal as independent charged Fermi-ions of lattice and gas of free electrons or free Frölich-Schafroth charged bosons (singlet electron pairs) [4]. Each charged ion is coupled with a point of lattice knot by spring, creating an ion dipole [5,6]. The lattice knots define the equilibrium positions of all ions which vibrate with natural frequencies Ω_l and Ω_t, under action of longitudinal and transverse elastic fields

which in turn leads to creation of the transverse electromagnetic fields moving with speeds c_l and c_t. These transverse electromagnetic waves describe the ions by the principle of ion-wave duality [7]. Using the representation of the electromagnetic field structure of one ion with ion-wave duality in analogous manner, as it was presented in a homogenous medium for an electromagnetic wave [8], we obtain that the neutral phonons cannot be excited in such substances as metals, they may be induced only in dielectric material [1]. In this respect, we find the charged polaritons with spin 1 which are always excited in a metal, and at small wavenumbers, they represent as charged phonons.

2 New model for metal

The Einstein model of a solid considers the solid as gas of N atoms in a box with volume V. Each atom is coupled with a point of the lattice knot. The lattice knots define the dynamical equilibrium position of each atom which vibrates with natural frequency Ω_0. The vibration of atom occurs near equilibrium position corresponding to the minimum of potential energy (harmonic approximation of close neighbors). We presented the model of ion-dipoles [5,6] which represents ions coupled with points of lattice knots. It differs from the Einstein model of solids where the neutral independent atoms are considered in lattice knots, these ions are vibrating with natural frequencies Ω_l and Ω_t forming ion-dipoles by under action longitudinal and transverse ultrasonic lattice fields.

Usually, matters are simplified assuming the transfer of heat from one part of the body to another occurs very slowly. This is a reason to suggest that the heat exchange during times of the order of the period of oscillatory motions in the body is negligible, therefore, we can regard any part of the body as thermally insulated, and there occur adiabatic deformations. Since all deformations are supposed to be small, the motions considered in the theory of elasticity are small elastic oscillations. In this respect, the equation of motion for elastic continuum medium [9] represents as

$$\varrho\ddot{\vec{u}} = c_t^2\nabla^2\vec{u} + (c_l^2 - c_t^2)\, grad\, div\, \vec{u}, \tag{1}$$

where $\vec{u} = \vec{u}(\vec{r}, t)$ is the vectorial displacement of any particle in the solid; c_l and c_t are, respectively, the velocities of a longitudinal and a transverse ultrasonic wave.

We shall begin by discussing a plane longitudinal elastic wave with condition *curl* $\vec{u} = 0$ and a plane transverse elastic wave with condition *div* $\vec{u} = 0$ in an infinite isotropic medium. In this respect, the vector displacement $\vec{u}$ is the sum of the vector displacements of a longitudinal u_l and of a transverse ultrasonic wave u_t:

$$\vec{u} = \vec{u}_l + \vec{u}_t. \tag{2}$$

In turn, the equations of motion for a longitudinal and a transverse elastic wave take the form of the wave-equations:

$$\nabla^2 \vec{u}_l - \frac{1}{c_l^2} \frac{d^2 \vec{u}_l}{dt^2} = 0, \tag{3}$$

$$\nabla^2 \vec{u}_t - \frac{1}{c_t^2} \frac{d^2 \vec{u}_l}{dt^2} = 0. \tag{4}$$

It is well known, in quantum mechanics, a matter wave is determined by electromagnetic wave-particle duality or de Broglie wave of matter [7]. We argue that in analogous manner, we may apply the elastic wave-particle duality. This reasoning allows us to present a model of elastic field as the Bose-gas consisting of the Sound Bose-particles with spin 1 having non-zero rest masses which are interacting with each other. In this respect, we may express the vector displacements of a longitudinal u_l and of a transverse ultrasonic wave u_t via the second quantization vector wave functions of Sound Bosons as

$$\vec{u}_l = C_l \left(\phi(\vec{r}, t) + \phi^+(\vec{r}, t) \right) \tag{5}$$

and

$$\vec{u}_t = C_t \left(\psi(\vec{r}, t) + \psi^+(\vec{r}, t) \right), \tag{6}$$

where C_l and C_t are unknown constant normalization coefficients; $\vec{\phi}(\vec{r}, t)$ and $\vec{\phi}^+(\vec{r}, t)$ are, respectively, the second quantization wave vector functions for one Sound-Particle, corresponding to the longitudinal elastic wave, at coordinate $\vec{r}$ and time t; $\vec{\psi}(\vec{r}, t)$ and $\vec{\psi}^+(\vec{r}, t)$ are, respectively, the second quantization wave vector functions for one Sound-Particle, corresponding to the transverse elastic wave, at coordinate $\vec{r}$ and time t:

$$\vec{\phi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}, \sigma} \vec{a}_{\vec{k}, \sigma} e^{i(\vec{k}\vec{r} + kc_l t)} \tag{7}$$

$$\vec{\phi}^+(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}, \sigma} \vec{a}_{\vec{k}, \sigma}^+ e^{-i(\vec{k}\vec{r} + kc_l t)} \tag{8}$$

and

$$\vec{\psi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}, \sigma} \vec{b}_{\vec{k}, \sigma} e^{i(\vec{k}\vec{r} + kc_t t)} \tag{9}$$

$$\vec{\psi}^+(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}, \sigma} \vec{b}_{\vec{k}, \sigma}^+ e^{-i(\vec{k}\vec{r} + kc_t t)}, \tag{10}$$

where $\vec{a}_{\vec{k}, \sigma}^+$ and $\vec{a}_{\vec{k}, \sigma}$ are, respectively, the Bose vector-operators of creation and annihilation for one free longitudinal Sound Particle with spin 1, described by a vector $\vec{k}$ whose direction gives the direction of motion of the longitudinal wave; $\vec{b}_{\vec{k}, \sigma}^+$ and $\vec{b}_{\vec{k}, \sigma}$ are, respectively, the Bose vector-operators of creation and annihilation for one free transverse Sound Particle with spin 1, described by a vector $\vec{k}$ whose direction gives the direction of motion of the transverse wave.

In this respect, the vector-operators $\vec{a}_{\vec{k}, \sigma}^+$, $\vec{a}_{\vec{k}, \sigma}$ and $\vec{b}_{\vec{k}, \sigma}^+$, $\vec{b}_{\vec{k}, \sigma}$ satisfy the Bose commutation relations as:

$$\left[\hat{a}_{\vec{k}, \sigma}, \hat{a}_{\vec{k}', \sigma'}^+ \right] = \delta_{\vec{k}, \vec{k}'} \cdot \delta_{\sigma, \sigma'}$$

$$[\hat{a}_{\vec{k}, \sigma}, \hat{a}_{\vec{k}', \sigma'}] = 0$$

$$[\hat{a}_{\vec{k}, \sigma}^+, \hat{a}_{\vec{k}', \sigma'}^+] = 0$$

and

$$\left[\hat{b}_{\vec{k}, \sigma}, \hat{b}_{\vec{k}', \sigma'}^+ \right] = \delta_{\vec{k}, \vec{k}'} \cdot \delta_{\sigma, \sigma'}$$

$$[\hat{b}_{\vec{k}, \sigma}, \hat{b}_{\vec{k}', \sigma'}] = 0$$

$$[\hat{b}_{\vec{k}, \sigma}^+, \hat{b}_{\vec{k}', \sigma'}^+] = 0.$$

Thus, as we see the vector displacements of a longitudinal u_l and of a transverse ultrasonic wave u_t satisfy the wave-equations of (3) and (4) because they have the following forms due to application of (5) and (6):

$$\vec{u}_l = \frac{C_l}{\sqrt{V}} \sum_{\vec{k}, \sigma} \left(\vec{a}_{\vec{k}, \sigma} e^{i(\vec{k}\vec{r} + kc_l t)} + \vec{a}_{\vec{k}, \sigma}^+ e^{-i(\vec{k}\vec{r} + kc_l t)} \right) \tag{11}$$

and

$$\vec{u}_t = \frac{C_t}{\sqrt{V}} \sum_{\vec{k}, \sigma} \left(\vec{b}_{\vec{k}, \sigma} e^{i(\vec{k}\vec{r} + kc_t t)} + \vec{b}_{\vec{k}, \sigma}^+ e^{-i(\vec{k}\vec{r} + kc_t t)} \right). \tag{12}$$

In this context, we may emphasize that the Bose vector operators $\vec{a}_{\vec{k}, \sigma}^+$, $\vec{a}_{\vec{k}, \sigma}$ and $\vec{b}_{\vec{k}}^+, \sigma$ and $\vec{b}_{\vec{k}}, \sigma$ communicate with each other because the vector displacements of a longitudinal $\vec{u}_l$ and a transverse ultrasonic wave $\vec{u}_t$ are independent, and in turn, satisfy the condition of a scalar multiplication $\vec{u}_l \cdot \vec{u}_t = 0$.

Consequently, the Hamiltonian operator $\hat{H}$ of the system, consisting of the vibrating Fermi-ions with mass M, is represented in the following form:

$$\hat{H} = \hat{H}_l + \hat{H}_t, \tag{13}$$

where

$$\hat{H}_l = \frac{MN}{V} \int \left(\frac{d\vec{u}_l}{dt}\right)^2 dV + \frac{NM\Omega_l^2}{V} \int (\vec{u}_l)^2 dV \tag{14}$$

and

$$\hat{H}_t = \frac{MN}{V} \int \left(\frac{d\vec{u}_t}{dt}\right)^2 dV + \frac{NM\Omega_t^2}{V} \int (\vec{u}_t)^2 dV, \tag{15}$$

where Ω_l and Ω_t are, respectively, the natural frequencies of the atom through action of the longitudinal and transverse elastic waves.

To find the Hamiltonian operator $\hat{H}$ of the system, we use the formalism of Dirac [10]:

$$\frac{d\vec{u}_l}{dt} = \frac{ic_l C_l}{\sqrt{V}} \sum_{\vec{k},\sigma} k \left(\vec{a}_{\vec{k},\sigma} e^{ikc_l t} - \vec{a}_{-\vec{k},\sigma}^{+} e^{-ikc_l t}\right) e^{i\vec{k}\vec{r}} \tag{16}$$

and

$$\frac{d\vec{u}_t}{dt} = \frac{ic_t C_t}{\sqrt{V}} \sum_{\vec{k},\sigma} k \left(\vec{b}_{\vec{k},\sigma} e^{ikc_t t} - \vec{b}_{-\vec{k},\sigma}^{+} e^{-ikc_t t}\right) e^{i\vec{k}\vec{r}}, \tag{17}$$

which by substituting into (14) and (15), using (11) and (12), gives the reduced form of the Hamiltonian operators $\hat{H}_l$ and $\hat{H}_t$:

$$\hat{H}_l = \sum_{\vec{k},\sigma} \left(\frac{2MNC_l^2 c_l^2 k^2}{V} + \frac{2MNC_l^2\Omega_l^2}{V}\right)\vec{a}_{\vec{k},\sigma}^{+} \vec{a}_{\vec{k},\sigma} -$$

$$- \sum_{\vec{k},\sigma} \left(\frac{2MNC_l^2 c_l^2 k^2}{V} - \frac{2MNC_l^2\Omega_l^2}{V}\right)\left(a_{\vec{k},\sigma}\vec{a}_{-\vec{k},\sigma} + a_{-\vec{k}}^{+}\vec{a}_{\vec{k},\sigma}^{+}\right) \tag{18}$$

and

$$\hat{H}_t = \sum_{\vec{k},\sigma} \left(\frac{2MNC_t^2 c_t^2 k^2}{V} + \frac{2MNC_t^2\Omega_t^2}{V}\right)\vec{b}_{\vec{k},\sigma}^{+} \vec{b}_{\vec{k},\sigma} -$$

$$- \sum_{\vec{k},\sigma} \left(\frac{2MNC_t^2 c_t^2 k^2}{V} - \frac{2MNC_t^2\Omega_t^2}{V}\right)\left(b_{\vec{k},\sigma}\vec{b}_{-\vec{k},\sigma} + b_{-\vec{k}}^{+}\vec{b}_{\vec{k},\sigma}^{+}\right), \tag{19}$$

where the normalization coefficients C_l and C_t are defined by the first term of right side of (18) and (19) which represent the kinetic energies of longitudinal Sound Particles $\frac{\hbar^2 k^2}{2m_l}$ and transverse Sound Particles $\frac{\hbar^2 k^2}{2m_t}$ with masses m_l and m_t, respectively. Therefore we suggest to find C_l and C_t:

$$\frac{2MNC_l^2 c_l^2 k^2}{V} = \frac{\hbar^2 k^2}{2m_l} \tag{20}$$

and

$$\frac{2MNC_t^2 c_t^2 k^2}{V} = \frac{\hbar^2 k^2}{2m_t}, \tag{21}$$

which in turn determine

$$C_l = \frac{\hbar}{2c_l \sqrt{m_l\rho}} \tag{22}$$

and

$$C_t = \frac{\hbar}{2c_t \sqrt{m_t\rho}}, \tag{23}$$

where $\rho = \frac{MN}{V}$ is the density of solid.

As we had shown in [1], at absolute zero $T = 0$, the Fermi ions fill the Fermi sphere in momentum space. Thus, there are two type Fermi atoms by the value of its spin z-component $\mu = \pm\frac{1}{2}$ with the boundary wave number k_f of the Fermi, which, in turn, is determined by a condition:

$$\frac{V}{2\pi^2} \int_0^{k_f} k^2 dk = \frac{N}{2},$$

where N is the total number of Fermi-ions in the solid. This reasoning together with the model of hard spheres claims the important condition to introduce the boundary wave number $k_f = \left(\frac{3\pi^2 N}{V}\right)^{\frac{1}{3}}$ coinciding with k_l and k_t. Then, there is an important condition $k_f = k_l = k_t$ which determines a relationship between natural oscillator frequencies

$$k_f = \frac{\Omega_l}{c_l} = \frac{\Omega_t}{c_t}. \tag{24}$$

3 Charged Polaritons

In papers [5, 6], we demostrated the so-called transformation of longitudinal and transverse elastic waves into transverse electromagnetic fields with vectors of the electric waves $\vec{E}_l$ and $\vec{E}_t$, corresponding to the ion displacements $\vec{u}_l$ and $\vec{u}_t$, respectively. In turn, the equations of motion are presented in the following forms [5, 6]:

$$M\frac{d^2\vec{u}_l}{dt^2} + M\Omega_l^2\vec{u}_l = -e\vec{E}_l \tag{25}$$

and

$$M\frac{d^2\vec{u}_t}{dt^2} + M\Omega_t^2\vec{u}_t = -e\vec{E}_t. \tag{26}$$

The vector of the electric waves $\vec{E}_l$ and $\vec{E}_t$ are defined by substitution of the meaning of $\vec{u}_l$ and $\vec{u}_t$ from (11) and (12), respectively, into (25) and (26):

$$\vec{E}_l(\vec{r},t) = \frac{C_l}{e\sqrt{V}} \sum_{\vec{k},\sigma} \gamma_{\vec{k},l}\left(\vec{a}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}+kc_lt)} + \vec{a}^+_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}+kc_lt)}\right) \quad (27)$$

and

$$\vec{E}_t(\vec{r},t) = \frac{C_t}{e\sqrt{V}} \sum_{\vec{k},\sigma} \gamma_{\vec{k},t}\left(\vec{b}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}+kc_lt)} + \vec{b}^+_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}+kc_lt)}\right), \quad (28)$$

where

$$\gamma_{\vec{k},l} = M\left(k^2c_l^2 - \Omega_l^2\right) \quad (29)$$

and

$$\gamma_{\vec{k},t} = M\left(k^2c_t^2 - \Omega_t^2\right). \quad (30)$$

On the other hand, by action of the longitudinal and transverse ultrasonic waves on the charged ion [5, 6], these ultrasonic waves are transformed into transverse electromagnetic fields with electric wave vectors $\vec{E}_l$ and $\vec{E}_t$ which in turn describe the de Broglie wave of charged ions expressed via electric $\vec{E}_l(\vec{r},t)$ and $\vec{E}_t(\vec{r},t)$ fields of one ion-wave particle in homogeneous medium. In fact, these electric $\vec{E}_l(\vec{r},t)$ and $\vec{E}_t(\vec{r},t)$ fields satisfy the Maxwell's equations in dielectric medium:

$$curl\, \vec{H}_l - \frac{\varepsilon_l}{c}\frac{d\vec{E}_l}{dt} = 0 \quad (31)$$

$$curl\, \vec{E}_l + \frac{1}{c}\frac{d\vec{H}_l}{dt} = 0 \quad (32)$$

$$div\, \vec{E}_l = 0 \quad (33)$$

$$div\, \vec{H}_l = 0 \quad (34)$$

and

$$curl\, \vec{H}_t - \frac{\varepsilon_t}{c}\frac{d\vec{E}_t}{dt} = 0 \quad (35)$$

$$curl\, \vec{E}_t + \frac{1}{c}\frac{d\vec{H}_t}{dt} = 0 \quad (36)$$

$$div\, \vec{E}_t = 0 \quad (37)$$

$$div\, \vec{H}_t = 0 \quad (38)$$

with

$$\sqrt{\varepsilon_l} = \frac{c}{c_l} \quad (39)$$

and

$$\sqrt{\varepsilon_l} = \frac{c}{c_l}, \quad (40)$$

where $\vec{H}_l = \vec{H}_l(\vec{r},t)$ and $\vec{H}_t = \vec{H}_t(\vec{r},t)$ are, respectively, the local magnetic fields, corresponding to longitudinal and transverse ultrasonic waves, depending on space coordinate $\vec{r}$ and time t; ε_l and ε_t are, respectively, the dielectric constants for transverse electric fields $\vec{E}_l(\vec{r},t)$ and $\vec{E}_t(\vec{r},t)$ corresponding to longitudinal and transverse ultrasonic waves; c is the velocity of electromagnetic wave in vacuum; $\mu = 1$ is the magnetic susceptibility.

When using Eqs. (31–40) and results of letter [8], we may present the transverse electric fields $\vec{E}_l(\vec{r},t)$ and $\vec{E}_t(\vec{r},t)$ by the quantization forms:

$$\vec{E}_l(\vec{r},t) = \frac{A_l}{\sqrt{V}} \sum_{\vec{k}}\left(\vec{c}_{\vec{k}}e^{i(\vec{k}\vec{r}+kc_lt)} + \vec{c}^+_{\vec{k}}e^{-i(\vec{k}\vec{r}+kc_lt)}\right) \quad (41)$$

and

$$\vec{E}_t(\vec{r},t) = \frac{A_t}{\sqrt{V}} \sum_{\vec{k}\neq0}\left(\vec{d}_{\vec{k}}e^{i(\vec{k}\vec{r}+kc_lt)} + \vec{d}^+_{\vec{k}}e^{-i(\vec{k}\vec{r}+kc_lt)}\right), \quad (42)$$

where A_l and A_t are the unknown constants which are found as below; $\vec{c}^+_{\vec{k}}, \vec{d}^+_{\vec{k}}$ and $\vec{c}_{\vec{k}}, \vec{d}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation of electric fields of one ion-wave particle with wave vector $\vec{k}$ which are directed along of the wave normal $\vec{s}$ or $\vec{k} = k\vec{s}$. These Bose vector-operators $\vec{E}_l(\vec{r},t)$ and $\vec{E}_t(\vec{r},t)$ are directed to the direction of the unit vectors $\vec{l}$ and $\vec{t}$ which are perpendicular to the wave normal $\vec{s}$; $\hat{N}$ is the operator total number of charged ions.

In this context, we indicate that the vector-operators $\vec{c}^+_{\vec{k},\sigma}$, $\vec{c}_{\vec{k},\sigma}$ and $\vec{d}^+_{\vec{k},\sigma}, \vec{d}_{\vec{k},\sigma}$ satisfy the Bose commutation relations as:

$$\left[\hat{c}_{\vec{k},\sigma}, \hat{c}^+_{\vec{k}',\sigma'}\right] = \delta_{\vec{k},\vec{k}'} \cdot \delta_{\sigma,\sigma'}$$

$$[\hat{c}_{\vec{k},\sigma}, \hat{c}_{\vec{k}',\sigma'}] = 0$$

$$[\hat{c}^+_{\vec{k},\sigma}, \hat{c}^+_{\vec{k}',\sigma'}] = 0$$

and

$$\left[\hat{d}_{\vec{k},\sigma}, \hat{d}^+_{\vec{k}',\sigma'}\right] = \delta_{\vec{k},\vec{k}'} \cdot \delta_{\sigma,\sigma'}$$

$$[\hat{d}_{\vec{k},\sigma}, \hat{d}_{\vec{k}',\sigma'}] = 0$$

$$[\hat{d}^+_{\vec{k},\sigma}, \hat{d}^+_{\vec{k}',\sigma'}] = 0.$$

Comparing (41) with (27) and (42) with (28), we get

$$\vec{d}_{\vec{k},\sigma} = \frac{eA_l}{C_l\gamma_{\vec{k},l}}\vec{c}_{\vec{k},\sigma} \quad (43)$$

and

$$\vec{b}_{\vec{k},\sigma} = \frac{eA_t}{C_t \gamma_{\vec{k},t}} \vec{d}_{\vec{k},\sigma}. \tag{44}$$

Now, substituting $\vec{a}_{\vec{k},\sigma}$ and $\vec{b}_{\vec{k},\sigma}$ into (18) and (19), we obtain the reduced form of the Hamiltonian operators $\hat{H}_l$ and $\hat{H}_l$ which are expressed via terms of the electric fields of the ion-wave particle:

$$\hat{H}_l = \sum_{\vec{k},\sigma} \frac{e^2 A_l^2}{\gamma_{\vec{k},l}^2} \left[\left(\frac{2MNc_l^2 k^2}{V} + \frac{2MN\Omega_l^2}{V} \right) \vec{c}_{\vec{k},\sigma}^{+} \vec{c}_{\vec{k},\sigma} - \right.$$
$$\left. - \left(\frac{MNc_l^2 k^2}{V} - \frac{MN\Omega_l^2}{V} \right) \left(\vec{c}_{-\vec{k},\sigma} \vec{c}_{\vec{k},\sigma} + \vec{c}_{\vec{k},\sigma}^{+} \vec{c}_{-\vec{k},\sigma}^{+} \right) \right] \tag{45}$$

and

$$\hat{H}_t = \sum_{\vec{k},\sigma} \frac{e^2 A_t^2}{\gamma_{\vec{k},l}^2} \left[\left(\frac{2MNc_t^2 k^2}{V} + \frac{2MN\Omega_t^2}{V} \right) \vec{d}_{\vec{k},\sigma}^{+} \vec{d}_{\vec{k},\sigma} - \right.$$
$$\left. - \left(\frac{MNc_t^2 k^2}{V} - \frac{MN\Omega_t^2}{V} \right) \left(\vec{d}_{-\vec{k},\sigma} \vec{d}_{\vec{k},\sigma} + \vec{d}_{\vec{k},\sigma}^{+} \vec{d}_{-\vec{k},\sigma}^{+} \right) \right]. \tag{46}$$

To evaluate the energy levels of the operators $\hat{H}_l$ (45) and $\hat{H}_t$ (46) within the diagonal form, we use a transformation of the vector-Bose-operators:

$$\vec{c}_{\vec{k},\sigma} = \frac{\vec{l}_{\vec{k},\sigma} + L_{\vec{k}} \vec{l}_{-\vec{k},\sigma}^{+}}{\sqrt{1 - L_{\vec{k}}^2}} \tag{47}$$

and

$$\vec{d}_{\vec{k},\sigma} = \frac{\vec{t}_{\vec{k},\sigma} + M_{\vec{k}} \vec{t}_{-\vec{k},\sigma}^{+}}{\sqrt{1 - M_{\vec{k}}^2}}, \tag{48}$$

where $L_{\vec{k}}$ and $M_{\vec{k}}$ are, respectively, the real symmetrical functions of a wave vector $\vec{k}$.

Consequently,

$$\hat{H}_l = \sum_{k<k_f,\sigma} \varepsilon_{\vec{k},l} \vec{l}_{\vec{k},\sigma}^{+} \vec{l}_{\vec{k},\sigma} \tag{49}$$

and

$$\hat{H}_t = \sum_{k<k_f,\sigma} \varepsilon_{\vec{k},t} \vec{t}_{\vec{k},\sigma}^{+} \vec{t}_{\vec{k},\sigma} \tag{50}$$

at

$$L_{\vec{k}}^2 = \frac{\frac{2MNc_l^2 k^2}{V} + \frac{2MN\Omega_l^2}{V} - \varepsilon_{\vec{k},l}}{\frac{2MNc_l^2 k^2}{V} + \frac{2MN\Omega_l^2}{V} + \varepsilon_{\vec{k},l}}$$

$$M_{\vec{k}}^2 = \frac{\frac{2MNc_t^2 k^2}{V} + \frac{2MN\Omega_l^2}{V} - \varepsilon_{\vec{k},t}}{\frac{2MNc_t^2 k^2}{V} + \frac{2MN\Omega_l^2}{V} + \varepsilon_{\vec{k},t}}.$$

Hence, we infer that the Bose-operators $\vec{l}_{\vec{k},\sigma}^{+}$, $\vec{l}_{\vec{k},\sigma}$ and $\vec{t}_{\vec{k},\sigma}^{+}$, $\vec{t}_{\vec{k},\sigma}$ are, respectively, the vector of "creation" and the vector of "annihilation" operators of charged polaritons with spin 1 with the energies:

$$\varepsilon_{\vec{k},l} = \frac{4e^2 \rho c_l A_l^2 \Omega_l k}{\gamma_{\vec{k},l}^2} \tag{51}$$

and

$$\varepsilon_{\vec{k},t} = \frac{4e^2 \rho c_t A_t^2 \Omega_t k}{\gamma_{\vec{k},t}^2}. \tag{52}$$

Hence, we note that these polaritons are charged because the Hamiltonian contains the square of charge, e^2. This picture is similar to the Coulomb interaction between two charges.

Obviously, at small wave numbers $k \ll \frac{\Omega_l}{c_l}$ and $k \ll \frac{\Omega_t}{c_t}$, these charged polaritons are presented as charged phonons with energies:

$$\varepsilon_{\vec{k},l} \approx \hbar k v_l \tag{53}$$

and

$$\varepsilon_{\vec{k},t} \approx \hbar k v_t, \tag{54}$$

where $v_l = \frac{4\rho c_l e^2 A_l^2}{\hbar M^2 \Omega_l^3}$ and $v_t = \frac{4\rho c_t e^2 A_t^2}{\hbar M^2 \Omega_t^3}$ are, respectively, the velocities of charged phonons with spin 1 corresponding to the longitudinal and transverse acoustic fields. To find the unknown constants A_l^2 and A_t^2, we suggest that $v_l = c_l$ and $v_t = c_t$ as it was presented in [1]. This suggestion leads to the results obtained in [1] and in turn presented in Debye's theory. Thus, when choosing $A_l^2 = \frac{\hbar M^2 \Omega_l^3}{4\rho e^2}$ and $A_t^2 = \frac{\hbar M^2 \Omega_t^3}{4\rho e^2}$, the energies of charged polaritons represent as

$$\varepsilon_{\vec{k},l} = \frac{\hbar \Omega_l^4 c_l k}{\left(k^2 c_l^2 - \Omega_l^2 \right)^2} \tag{55}$$

and

$$\varepsilon_{\vec{k},t} = \frac{\hbar \Omega_t^4 c_t k}{\left(k^2 c_t^2 - \Omega_t^2 \right)^2}, \tag{56}$$

which at large wave numbers $k \gg \frac{\Omega_l}{c_l}$ and $k \gg \frac{\Omega_t}{c_t}$, and taking into account (24), leads to the following form for the energies of charged polaritons:

$$\varepsilon_{\vec{k},l} = \frac{\hbar k_f^4 c_l}{k^3}. \tag{57}$$

In fact, the stimulated vibration of ions by elastic waves lead to the formation of the charged polaritons with spin 1.

Thus, we predicted the existence of a new type of charged quasiparticles in nature. On the other hand, we note that the quantization of elastic fields is fulfilled for the new model of metals. In analogous manner, as it was presented in [1], we may show that the acoustic field operator does not commute with its momentum density.

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