On the Cold Big Bang Cosmology

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We solve the general relativity (GR) field equations under the cosmological scope via one extra postulate. The plausibility of the postulate resides within the Heisenberg indeterminacy principle, being heuristically analysed throughout the appendix. Under this approach, a negative energy density may provide the positive energy content of the universe via fluctuation, since the question of conservation of energy in cosmology is weakened, supported by the known lack of scope of the Noether's theorem in cosmology. The initial condition of the primordial universe turns out to have a natural cutoff such that the temperature of the cosmological substratum converges to the absolute zero, instead of the established divergence at the very beginning. The adopted postulate provides an explanation for the cosmological dark energy open question. The solution agrees with cosmological observations, including a 2.7K CMBT prediction.

1 Revisiting the Theoretical Assumptions

The study of the dynamics of the entire universe is known as Cosmology [1–3]. The inherent simplicity in the mathematical treatment of the Cosmology, although the entire universe must be under analysis, should be recognized as being due to Copernicus. Indeed, since the primordial idea permeating the principle upon which the simplicity arises is just an extension of the copernican revolution*: the cosmological principle. This extension, the cosmological principle, just asseverates we are not in any sense at a privileged position in our universe, implying that the average large enough scale† *spatial* properties of the physical universe are the same from point to point at a given cosmological instant. Putting these in a mathematical jargon, one says that the large enough scale spatial geometry at a given cosmological instant t is exactly the same in spite of the position of the observer at some point belonging to this t-sliced three-dimensional universe or, equivalently, that the spatial part of the line element of the entire universe is the same for all observers. Hence, the simplicity referred above arises from the very two principal aspects logically encrusted in the manner one states the cosmological principle:

- The lack of a privileged physical description of the universe at a t-sliced large enough scale $\Rightarrow$ large enough scale $\Rightarrow$ one neglects all kind of known physical interactions that are unimportant on the large enough scales $\Rightarrow$ remains gravity;

- The lack of a privileged physical description of the universe at a t-sliced large enough scale $\Rightarrow$ large enough scale $\Rightarrow$ one neglects local irregularities of a global t-sliced substratum representing the t-sliced universe $\forall$

cosmological instants t $\Rightarrow$ substratum modelled as a fluid without t-sliced spatially localized irregularities $\Rightarrow$ homogeneous and isotropic t-sliced‡ fluid.

One shall verify the t-local characteristic of the the cosmological principle, i.e., that non-privileged description does not necessarily hold on the global time evolution of that t-sliced spacelike hypersurfaces. In other words, two of such t-sliced hypersurfaces at different instants would not preserve the same aspect, as experimentally asseverated by the expansion of the universe. Hence, some further assumption must be made regarding the time evolution of the points belonging to the t-sliced spacelike hypersurfaces:

- The particles of the cosmological fluid are encrusted in spacetime on a congruence of timelike geodesics from a point in the past, i.e., the substratum is modelled as a perfect fluid.

Hence, the following theoretical ingredients are available regarding the above way in which one mathematically construct a cosmological model:

Gravity modeled by Einstein's General Relativity field equations (in natural units):

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{1}$$

Homogeneity is mathematically translated by means of a geometry (metric) that is the same from point to point, spatially speaking. Isotropy is mathematically translated by means of a lack of privileged directions, also spatially speaking. These two characteristics easily allow one to consider spaces equipped with constant curvature K. From a differential geometry theorem, Schur's, a n-dimensional space $\mathbb{R}^n$, $n \geq 3$, in which a η-neighbourhood has isotropy $\forall$ points

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*Copernicus told us that the Earth is not the center of our planetary system, namely the solar system, pushing down the historical button leading to the collapse of the established *anthropocentric status quo*.

†One must understand large enough scale as being that of cluster of galaxies.

‡One shall rigorously attempt to the fact: the isotropy and homogeneity are t-sliced referred, i.e., these two properties logically emerging from the cosmological principle hold upon the entire fluid at t, holding spatially at t, i.e., homogeneity and isotropy are spatial properties of the fluid. Regarding the time, one observer can be at an own proper τ-geodesic...

belonging to it, has constant curvature K throughout η. Since we are considering, spatially, global isotropy, then K is constant everywhere. Hence, one defines the Riemann tensor:

$$R_{abcd} = K \left(g_{ac}g_{bd} - g_{ad}g_{bc} \right), \qquad (2)$$

spatially speaking.

As indicated before, homogeneity and isotropy are spatial properties of the geometry. Time evolution, e.g.: expansion, can be conformally agreed with these two spatial properties logically emerging from the cosmological principle in terms of Gaussian normal coordinates. Mathematically, the spacetime cosmological metric has the form:

$$ds^2 = dt^2 - [a(t)]^2 \, d\sigma^2. \qquad (3)$$

Since spatial coordinates for a spatially fixed observer do not change, $ds^2 = dt^2 \Rightarrow g_{tt} = 1$.

Regarding the spatial part of the line element, the Schwarzschild metric is spherically symmetric, a guide to our purposes. From the Scharzschild metric (signature $+ - - -$):

$$ds^2 = e^{2\nu(r)}dt^2 - e^{2\lambda(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2, \qquad (4)$$

one easily writes down the spatial part of the spacetime cosmological metric:

$$d\sigma^2 = e^{2f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2. \qquad (5)$$

One straightforwardly goes through the tedious calculation of the Christoffel symbols and the components of the Ricci tensor, finding:

$$e^{2f(r)} = \frac{1}{1 - Kr^2}. \qquad (6)$$

Absorbing constants* by the scale factor in eqn. (3), one normalizes the curvature constant K, namely $k \in \{-1; 0; +1\}$. Hence, the cosmological spacetime metric turns out to be in the canonical form:

$$ds^2 = dt^2 - [a(t)]^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right). \qquad (7)$$

Now, regarding the fluid substratum, one sets in co-moving coordinates ($dt/d\tau = 1$, $u^\mu = (1; 0; 0; 0)$):

$$T^\mu{}_\nu = 0, \; \mu \neq \nu; \;\; T^0{}_0 = \rho; \;\; T^\mu{}_\mu = -p, \; \text{for } \mu \in \{1; 2; 3\} \quad (8)$$

since the particles in the fluid are clusters of galaxies falling together with small averaged relative velocities compared with the cosmological dynamics, where the substratum turns out to be averaged described by an average substratum density ρ and by an average substratum pressure p.

The Einstein tensor in eqn. (1), $G_{\mu\nu}$, is related to the Ricci tensor $R_{\mu\nu} = R^\gamma{}_{\mu\gamma\nu}$ (the metric contraction of the curvature tensor (Riemann tensor)), to the Ricci scalar $R = R^\mu{}_\mu$ (the

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*Defining $r' = \sqrt{|K|}\, r$, one straightforwardly goes through...

metric contraction of the Ricci tensor) and to the metric $g_{\mu\nu}$ itself:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \qquad (9)$$

The curvature tensor $R^\alpha{}_{\beta\gamma\delta}$ is obtained via a metric connection, the Christoffel $\Gamma^\alpha{}_{\beta\delta}$ symbols in our case of non-torsional manifold:

$$R^\alpha{}_{\beta\gamma\delta} = \partial_\gamma \Gamma^\alpha{}_{\beta\delta} - \partial_\delta \Gamma^\alpha{}_{\beta\gamma} + \Gamma^\epsilon{}_{\beta\delta}\Gamma^\alpha{}_{\epsilon\gamma} - \Gamma^\epsilon{}_{\beta\gamma}\Gamma^\alpha{}_{\epsilon\delta}, \qquad (10)$$

where the metric connection is obtained, in the present case, from the Robertson-Walker cosmological spacetime geometry given by eqn. (7) (from which one straightforwardly obtains the metric coefficients of the diagonal metric tensor in the desired covariant or contravariant representations) via:

$$\Gamma^\alpha{}_{\beta\gamma} = g^{\alpha\delta}\Gamma_{\delta\beta\gamma}, \qquad (11)$$

being the metric connection (Christoffel symbols) of the first kind $\Gamma_{\delta\beta\gamma}$ given by:

$$\Gamma_{\delta\beta\gamma} = \frac{1}{2} \left(\frac{\partial g_{\beta\gamma}}{\partial x^\delta} + \frac{\partial g_{\gamma\delta}}{\partial x^\beta} - \frac{\partial g_{\delta\beta}}{\partial x^\gamma} \right). \qquad (12)$$

These set of assumptions under such mathematical apparatus lead one to the tedious, but straightforward, derivation, via eqn. (1), of the ordinary differential cosmological equations emerging from the relation between the Einstein's tensor, $G_{\mu\nu}$, the Robertson-Walker spacetime cosmological metric of the present case, $g_{\mu\nu}$ via eqn. (7), and the stress-energy tensor, $T_{\mu\nu}$ via metric contraction of the eqn. (8) (signature $+ - - -$):

$$\frac{\dot{R}^2 + kc^2}{R^2} = \frac{8\pi G}{3c^2} \left(\rho + \tilde{\rho} \right); \qquad (13)$$

$$\frac{2R\ddot{R} + \dot{R}^2 + kc^2}{R^2} = -\frac{8\pi G}{c^2} \left(p + \tilde{p} \right), \qquad (14)$$

where we are incorporating the cosmological constant Λ through the energy density and the pressure of the vacuum: $\tilde{\rho}$ and $\tilde{p}$, respectively. One also must infer we are no more working with natural units. The scale factor becomes $R(t)$, and one must interpret it as the magnification length scale of the cosmological dynamics, since $R(t)$ turns out to be length. This measures how an unitary length of the pervading cosmological substratum at t_0 becomes stretched as the universe goes through a time evolution from t_0 to t. One should not literally interpret it as an increase of the distance between two points, e.g., in a case of expansion, a stretched stationary wavelength connecting two cosmological points at a t_0-sliced spacelike substratum would remain stationarily connecting the very same two points after the stretched evolution to the respective t-sliced spacelike substratum, but less energetically.

2 A Cold Beginning?

Applying the following conservation criteria:

$$\nabla_\mu T^\mu{}_t = \partial_\mu T^\mu{}_t + \Gamma^\mu{}_{\mu\nu} T^\nu{}_t - \Gamma^\nu{}_{\mu t} T^\mu{}_\nu = 0, \qquad (15)$$

one finds via the diagonal stress-energy tensor (see eqn. (8)), the metric connection (see eqs. (11) and (12)) and the space-time cosmological geometry of the present case (eqn. (7)):

$$\frac{\partial}{\partial t}(\rho + \tilde\rho) + 3\frac{\dot R}{R}(\rho + \tilde\rho + p + \tilde p) = 0. \qquad (16)$$

eqn. (16) is the first law of thermodynamics applied to our substratum (including vacuum), since, despite of geometry, a spatial slice of the substratum has volume $\alpha(k)\,[R(t)]^3$, density $(\rho(t) + \tilde\rho)^*$ and energy $(\rho(t) + \tilde\rho)\,\alpha(k)\,[R(t)]^3$, implying that $dE + pdV = 0$ turns out to be eqn. (16). $\alpha(k)$ is the constant that depends on geometry (open, $k = -1$; flat, $k = 0$; closed, $k = 1$) to give the correct volume expression of the mentioned spatial slice of the t-sliced cosmological substratum.

Now, we go further, considering the early universe as being dominated by radiation. In the ultrarelativistic limit, the equation of state is given by:

$$\rho - 3p = 0. \qquad (17)$$

Putting this equation of state in eqn. (16) and integrating, one obtains the substratum pressure as a function of the magnification scale R:

$$4\ln\|R\| + \ln\|p\| = C' \;\Rightarrow\; \|p\| = \frac{e^{C'}}{R^4} \;\Rightarrow\; p = \pm\frac{C^+}{R^4}, \qquad (18)$$

where $C^+ \geq 0$ is a constant of integration. In virtue of eqn. (18), eqn. (14) is rewritten in a total differential form:

$$2R\dot R d\dot R + \left(\dot R^2 + kc^2 \pm \frac{8\pi G}{c^2}\frac{C^+}{R^2} + \frac{8\pi G}{c^2}\tilde p R^2\right) dR = 0. \qquad (19)$$

Indeed, eqn. (19) is a total differential of a constant $\lambda(R, \dot R) = constant$:

$$d\lambda(R, \dot R) = \frac{\partial \lambda(R, \dot R)}{\partial \dot R} d\dot R + \frac{\partial \lambda(R, \dot R)}{\partial R} dR = 0, \qquad (20)$$

since:

$$\frac{\partial \lambda(R, \dot R)}{\partial \dot R} = 2R\dot R \;\Rightarrow\; \frac{\partial^2 \lambda(R, \dot R)}{\partial R\,\partial \dot R} = 2\dot R; \qquad (21)$$

$$\frac{\partial \lambda(R, \dot R)}{\partial R} = \dot R^2 + kc^2 \pm \frac{8\pi G}{c^2}\frac{C^+}{R^2} + \frac{8\pi G}{c^2}\tilde p R^2 \;\Rightarrow\; \qquad (22)$$

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*One shall remember the cosmological principle: on average, for large enough scales, at t-sliced substratum, the universe has the same aspect in spite of the spatial localization of the observer in the t-slice $\Rightarrow \rho = \rho(t)$. Also, since Λ is constant, $\tilde\rho$ and $\tilde p$ are constants such that $\tilde\rho + \tilde p = 0$.

$$\frac{\partial^2 \lambda(R, \dot R)}{\partial \dot R\,\partial R} = 2\dot R \;\therefore\; \frac{\partial^2 \lambda(R, \dot R)}{\partial R\,\partial \dot R} = \frac{\partial^2 \lambda(R, \dot R)}{\partial \dot R\,\partial R} = 2\dot R. \qquad (23)$$

Integrating, one has:

$$\int \partial \lambda(R, \dot R) = \int 2R\dot R\,\partial \dot R = 2R \int \dot R\,d\dot R + h(R) \;\therefore\; \qquad (24)$$

$$\lambda(R, \dot R) = R\dot R^2 + h(R), \qquad (25)$$

where $h(R)$ is a function of R. From eqs. (22) and (25):

$$\frac{\partial}{\partial R}\lambda(R, \dot R) = \dot R^2 + kc^2 \pm \frac{8\pi G}{c^2}\frac{C^+}{R^2} + \frac{8\pi G}{c^2}\tilde p R^2 \Rightarrow$$

$$h(R) = \int\left(kc^2 \pm \frac{8\pi G}{c^2}\frac{C^+}{R^2} + \frac{8\pi G}{c^2}\tilde p R^2\right) dR \;\therefore\; \qquad (26)$$

$$h(R) = kc^2 R \mp \frac{8\pi G}{c^2}\frac{C^+}{R} + \frac{8\pi G}{3c^2}\tilde p R^3. \qquad (27)$$

Putting this result from eqn. (27) in eqn. (25):

$$\lambda(R, \dot R) = R\dot R^2 + kc^2 R \mp \frac{8\pi G}{c^2}\frac{C^+}{R} + \frac{8\pi G}{3c^2}\tilde p R^3 = constant \qquad (28)$$

is the general solution of the total differential equation eqn. (19). Dividing both sides of eqn. (28) by $R^3 \neq 0$:

$$\frac{\lambda(R, \dot R)}{R^3} = \frac{\dot R^2 + kc^2}{R^2} \mp \frac{8\pi G}{c^2}\frac{C^+}{R^4} + \frac{8\pi G}{3c^2}\tilde p, \qquad (29)$$

using the eqn. (13), one obtains:

$$\frac{\lambda(R, \dot R)}{R^3} = \frac{8\pi G}{c^2}\left(\frac{\rho}{3} \mp \frac{C^+}{R^4}\right) + \frac{8\pi G}{3c^2}(\tilde\rho + \tilde p) \;\therefore\; \qquad (30)$$

$$\lambda(R, \dot R) = constant = 0, \qquad (31)$$

in virtue of eqns. (17), (18) and $\tilde\rho + \tilde p = 0$ for the background vacuum. Of course, the same result is obtained from eqn. (13), since this equation is a constant of movement of eqn. (14), being eqn. (16) the connection between the two. Neglecting the vacuum contribution in relation to the ultrarelativistic substratum, one turns back to the eqn. (28), set the initial condition $R = R_0$, $\dot R = 0$, at $t = 0$, obtaining for the substratum pressure:

$$p(R) = k\frac{c^4 R_0^2}{8\pi G R^4}, \qquad (32)$$

and for the magnification scale velocity:

$$\dot R^2 = -kc^2\left(1 - \frac{R_0^2}{R^2}\right). \qquad (33)$$

Now, robustness[†] requires an open universe with $k = -1$. Hence, the locally flat substratum energy is given by[‡]:

$$E^+ = -4\pi R^3 p(R) \Rightarrow R_0 = -\frac{2GE_0^+}{kc^4}, \qquad (34)$$

[†]For, $\dot R^2 \in \mathbb{R}$ in eqn. (33) with $R \geq R_0$.

[‡]The Hawking-Ellis dominant energy condition giving the positive energy, albeit the expansion dynamics obtained via eqn. (32).

in virtue of eqn. (32) and the initial condition $E^+ = E_0^+$, $R = R_0$ at $t = 0$. Returning to eqn. (33), one obtains the magnification scale velocity:

$$\dot{R} = c \sqrt{1 - \frac{4G^2 \left(E_0^+\right)^2}{c^8 R^2}}, \tag{35}$$

giving $\dot{R} \to c$ as $R \to \infty$. Rewriting eqn. (35), one obtains the dynamical Schwarzschild horizon:

$$R = \frac{2G}{c^4} \frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}}. \tag{36}$$

We will not use the eqn. (34) (now you should read the appendix to follow the following argument) to obtain the energy from the energy density and volume for $t \neq 0$, since we do not handle very well the question of the conservation of energy in cosmology caused by an inherent lack of application of the Noether's theorem. In virtue of the adopted initial conditions, an initial uncertainty R_0 related to the initial spatial position of an arbitrary origin will be translated to a huge uncertainty R at the actual epoch. Indeed, one never knows the truth about the original position of the origin, hence the uncertainty grows as the universe enlarge. The primordial energy from which the actual energy of the universe came from was taken as E_0^+ at the beginning. This amount of energy is to be transformed over the universe evolution, giving the present amount of the universe, i.e., the energy of an actual epoch t-sliced hypersurface of simultaneity. But this energy at each instant t of the cosmological evolution turns out to be the transformed primordial indeterminacy E_0^+, since E_0^+ is to be obtained via the Heisenberg indeterminacy principle. In other words, we argue that the energetic content of the universe at any epoch is given by the inherent indeterminacy caused by the primordial indeterminacy. At any epoch, one may consider a copy of all points pertaining to the same hypersurface of simultaneity but at rest, i.e., an instantaneous non-expanding copy of the expanding instantaneous hypersurface of simultaneity. Related to an actual R indeterminacy of an origin in virtue of its primordial R_0 indeterminacy, one has the possibility of an alternative shifted origin at R. This shifted origin expands with $\dot{R}$ in relation to that non-expanding instantaneous copy of the universe at t. Since the primordial origin was considered to encapsulate the primordial energy E_0^+, this energy at the shifted likely alternative origin should be $E_0^+/\sqrt{1 - \dot{R}^2/c^2}$, since, at R, a point expands with $\dot{R}$ in relation to its non-expanding copy. We postulate:

- The actual energy content of the universe is a consequence of the increasing indeterminacy of the primordial era. Any origin of a co-moving reference frame within the cosmological substratum has an inherent indeterminacy. Hence, the indeterminacy of the energy content of the universe may create the impression that

the universe has not enough energy, raising illusions as dark energy and dark matter speculations. In other words, since the original source of energy emerges as an indeterminacy, we postulate this indeterminacy continues being the energy content of the observational universe: $\delta E(t) = E^+(t) = E_0^+/\sqrt{1 - \dot{R}^2/c^2}$.

This result is compatible with the Einstein field equations. The compatibility is discussed within the appendix. In virtue of this interpretation, eqn. (36) has the aspect of the Schwarzchild radius, hence the above designation. The t-instantaneous locally flat spreading out rate of dynamical energy at t-sliced substratum is given by the summation over the ν-photonic frequencies:

$$\dot{R} \frac{d}{dR} \left(\frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}} \right) =$$

$$= \frac{8\pi^2 R^2 h}{c^2} \int_0^\infty \frac{\nu^3}{\exp\left(h\nu/k_B T\right) - 1} d\nu = \frac{8\pi^6 k_B^4 R^2}{15 c^2 h^3} T^4, \tag{37}$$

where k_B is the Boltzmann constant, h the Planck constant and T the supposed rapid thermodynamically equilibrated t-sliced locally flat instantaneous cosmological substratum temperature. Now, setting, in virtue of the Heisenberg principle:

$$\frac{E_0^+ R_0}{c} \stackrel{(34)}{\approx} h \Rightarrow \left(E_0^+\right)^2 = \frac{hc^5}{2G}, \tag{38}$$

one obtains, in virtue of eqn. (37):

$$T^4 = \frac{15 c^7 h^3}{16\pi^6 G k_B^4} \frac{1}{R^2} \sqrt{1 - \frac{2Gh}{c^3 R^2}}. \tag{39}$$

Hence, the temperature of the cosmological substratum vanishes[*] at $t = 0$, rapidly reaching the maximum $\approx 10^{32} K$, and assintotically decreasing to zero again as $t \to \infty$.

Indeed. $R_0 = R(t = 0) = \sqrt{2Gh/c^3}$, in virtue of eqs. (34) and (38), giving $T^4(R_0) = T^4(t = 0) = 0$. Also, the maximum temperature is $T \approx 10^{32} K$, from eqn. (39), occuring when $R = R_{max} = \sqrt{3/2}R_0 = \sqrt{3Gh/c^3}$, as one obtains by $dT^4/dR = 0$ with $d^2T^4/dR^2 < 0$. Below[†], one infers these properties of eqn. (39).

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[*]We argue there is no violation of the third law of thermodynamics, since one must go from the future to the past when trying to reach the absolute zero, violating the second law of thermodynamics. At $t = 0$, one is not reaching the absolute zero since there is no past before the beginning of the time. To reach the absolute zero, in an attempt to violate the Nernst principle, one must go from the past to the future.

[†]The eqn. (39) is simply rewritten to plot the graph, i.e.: $T_{max}^4 = \left(5\sqrt{3}\,c^{10}h^2\right)/\left(48\pi^6 G^2 k_B^4\right)$ and, as obtained before, $R_0 = \sqrt{2Gh/c^3}$.

$$(T/T_{max})^4 = \left(3\sqrt{3}/2\right)\sqrt{1 - 1/x^2}/x^2$$

$$x \equiv R/R_0$$
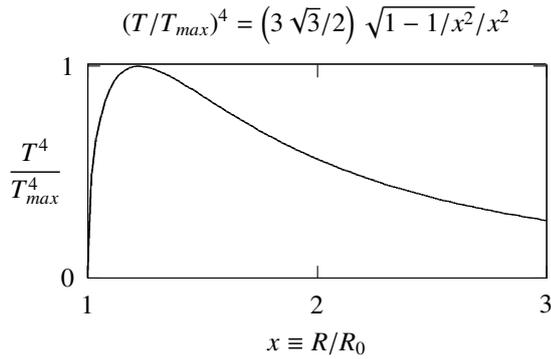
Now, one puts the result of eqn. (38) in eqn. (35) and integrates:

$$\int_{(2Gh/c^3)^{1/2}}^{R} \frac{R}{\sqrt{R^2 - 2Gh/c^3}}\, dR = c\int_0^t d\tau, \qquad (40)$$

obtaining:

$$t = \frac{1}{c}\sqrt{R^2 - 2Gh/c^3} \Rightarrow t(R_{max}) = \sqrt{\frac{Gh}{c^5}} \approx 10^{-43}\,\text{s}, \quad (41)$$

for the elapsed time from $t = 0$ to the instant in which the substratum temperature reaches the maximum value $T \approx 10^{32}$ K. The initial acceleration, namely the explosion/ignition acceleration at $t = 0$ of the substratum is obtained from eqn. (35):

$$\ddot{R} = \dot{R}\frac{d\dot{R}}{dR} = \frac{4G^2\left(E_0^+\right)^2}{c^6 R^3} \overset{(38)}{=} \frac{2Gh}{cR^3} \quad \therefore \qquad (42)$$

$$\ddot{R}\left(R = R_0 = \sqrt{2Gh/c^3}\right) = \sqrt{\frac{c^7}{2Gh}} \approx 10^{51}\,\text{m/s}^2. \qquad (43)$$

An interesting calculation is the extension of the eqn. (39) formula to predict the actual temperature of the universe. Since $2Ghc^{-3}R^{-2} \ll 1$ for actual stage of the universe, eqn. (39) is approximately given by:

$$T^4 \approx \frac{15c^7 h^3}{16\pi^6 G k_B^4}\frac{1}{R^2} \Rightarrow R^2 \approx \frac{15c^7 h^3}{16\pi^6 G k_B^4}\frac{1}{T^4}. \qquad (44)$$

Also, for actual age of the universe, eqn. (41) is approximately given by:

$$t \approx \frac{R}{c} \overset{(44)}{=} \sqrt{\frac{15c^5 h^3}{16\pi^6 G k_B^4}\frac{1}{T^2}} \quad \therefore \qquad (45)$$

$$T_{\text{Now}}^2 = \sqrt{\frac{15c^5 h^3}{16\pi^6 G k_B^4}}\; t_{\text{Now}}^{-1} = 5.32\times 10^{20} t_{\text{Now}}^{-1}\left(\text{K}^2\text{s}\right). \quad (46)$$

Before going further on, one must remember we are not in a radiation dominated era. Hence, the left-hand side and the right-hand side of eqn. (37) must be adapted for this situation.

The left-hand accomplishes the totality of spreading out energy in virtue of cosmological dynamics. It equals the right-hand side in an ultrarelativistic scenario. But, as the universe evolves, the right-hand side becomes a fraction of the totality of spreading out energy. Rigorously, as the locally flatness of the t-sliced substratum increases, one multiplies both sides of eqn. (37) by $(4/c)\times\left(1/4\pi R^2\right)$ and obtains the t-sliced instantaneously spreading out enclosed energy density. Hence the right-hand side of eqn. (37) turns out to be multiplied by the ratio between the total cosmological density* ρ_c and the radiation density ρ_r. Hence, eqn. (46) is rewritten:

$$\sqrt{\frac{\rho_c}{\rho_r}}\; T_{\text{Now}}^2 = 5.32\times 10^{20} t_{\text{Now}}^{-1}\left(\text{K}^2\text{s}\right). \qquad (47)$$

The actual photonic density is $\rho_r = 4.7\times 10^{-31}$ kg/m^3 and the actual total cosmological density is $\rho_c = 1.3\times 10^{-26}$ kg/m^3. For the reciprocal age of universe, t_{Now}^{-1} in eqn. (47), one adopts the Hubble's constant, for open universe, $H = t_{\text{Now}}^{-1} = 2.3\times 10^{-18}$ s^{-1}. Hence, by eqn. (47), one estimates the actual temperature of the universe:

$$T_{\text{Now}}^2 = \sqrt{\frac{4.7\times 10^{-31}}{1.3\times 10^{-26}}}\times 5.32\times 10^{20}\times 2.3\times 10^{-18}\;\text{K}^2 \;\therefore\; (48)$$

$$T_{\text{Now}} = 2.7\,\text{K}, \qquad (49)$$

very close to the CMB temperature.

3 Appendix

From eqns. (17) and (32):

$$\rho = 3p = -\frac{3c^4 R_0^2}{8\pi G}\frac{1}{R^4} \Rightarrow E_\rho = -\frac{c^4 R_0^2}{2G}\frac{1}{R}, \qquad (50)$$

since $k = -1$; E_ρ is the energy (negative) obtained from volume and ρ. From eqn. (34), $R_0^2 = 4G^2(E_0^+)^2/c^8$. Hence, eqn. (50) is rewritten:

$$E_\rho = -\frac{2G}{c^4}\left(E_0^+\right)^2\frac{1}{R}. \qquad (51)$$

With the eqn. (36), we reach:

$$E_\rho = -E_0^+\sqrt{1 - \dot{R}^2/c^2}. \qquad (52)$$

This negative energy arises from the adopted negative pressure solution. But, its fluctuation is positive:

$$\delta E_\rho = \frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}}\frac{\dot{R}\,\delta\dot{R}}{c^2}, \qquad (53)$$

since both, $\dot{R}$ and $\delta\dot{R}$, are positive within our model (see eqn. (40)). Let δt be the time interval within this fluctuation process. Multiplying both sides of the eqn. (53) by δt, we obtain:

$$\delta E_\rho\,\delta t = \frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}}\left(\dot{R}\delta\dot{R}/c^2\right)\delta t. \qquad (54)$$

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*Actually, the critical one, since observations asseverate it.

The above relation must obey the Heisenberg indeterminacy principle, and one may equivalently interpret it under the following format:

$$\delta E_\rho \, \delta t = \frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}} \, (\delta t)^* \approx h, \qquad (55)$$

An energy indeterminacy having the magnitude of the actual cosmological energy content carries an indeterminacy $\delta \dot{R} \approx c$ about the magnification scale velocity $\dot{R}$ with $\dot{R} \approx c$. For such an actual scenario in which $\dot{R} \approx c$ (see eqn. (35) with $R \to \infty$), we have:

$$\delta t \approx (\delta t)^* \Rightarrow \delta E_\rho \big|_{R_0}^{\infty} = E^+ = \frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}}, \qquad (56)$$

if* $\dot{R} \to c$. Now, let's investigate the primordial time domain $t \approx 0$. To see this, we rewrite $\dot{R}\delta\dot{R}$ within the eqn. (54). Firstly, from eqn. (35):

$$\dot{R} = c\sqrt{1 - R_0^2/R^2} \Rightarrow \dot{R}\,\delta\dot{R} = \frac{c^2 R_0^2}{R^3}\,\delta R, \qquad (57)$$

where $R_0 = \sqrt{2Gh/c^3}$ as obtained before. Within the primordial time domain $t \approx 0$, we have $R \approx R_0$ and $\delta R \approx R_0$, as discussed before. Hence, the eqn. (57) reads:

$$\dot{R}\,\delta\dot{R} \approx c^2. \qquad (58)$$

if $t \approx 0$. Back to the eqn. (54) we obtain again:

$$\delta t \approx (\delta t)^* \Rightarrow \delta E_\rho \big|_{\approx R_0} = E^+ = \frac{E_0^+}{\sqrt{1 - \dot{R}^2/c^2}}, \qquad (59)$$

if $t \approx 0$. This justify the use of $E^+ = E_0^+/\sqrt{1 - \dot{R}^2/c^2}$ within our postulate, emerging from the positive fluctuation of the negative energy E_ρ obtained from volume and the negative energy density ρ stated via the fluid state equation, eqn. (17), and entering within the field equations.

Acknowledgements

References

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*Eqn. (56) holds from $t > 10^{-43}$ seconds, as one easily verify from eqn. (35).