Building Galactic Density Profiles

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The principal objective of this study is to provide a method to build galactic density profiles. The models developed in this study were tested against the zCosmos deep field galactic survey. The herein study suggests that light travel distances need to be converted into Euclidean distances in order to derive the galactic density profile of the survey which is the evolution of galactic density over time. In addition, the present study indicates an Ω_m of 0.19.

1 Introduction

The main purpose of the herein study is to provide a method to build galactic density profiles which requires the conversion of light travel distances (LTD) to Euclidean distances. The LTD is the distance traversed by a photon between the time it is emitted and the time it reaches the observer. In astronomical units, the Euclidean distance is defined as the equivalent distance that would be traversed by a photon between the time it is emitted and the time it reaches the observer if there were no expansion of the Universe.

The zCosmos deep field was used to derive the galatic density profile based on a sampling method, and to compute an estimate of the mean mass density of the Universe.

2 Mathematical development and methods

Galactic density profiles have been derived from the normalization of the galactic counts between redshift buckets by dividing by the corresponding sample volume. For the scenario with additive LTD, the LTDs were directly fed into the sampling volume formula eq. (2). For the scenario with a model of the motion of the photon in an expanding space, the Euclidean distances were fed into the sampling volume formula.

2.1 Method to build galactic density profiles

2.1.1 Normalisation of galactic counts

Let us consider an observer positioned at the center of a sphere of radius r and looking at a cone of sky in the z direction. The observer is counting galaxies within this cone, and measures the redshift for each object. A histogram of the galactic counts versus redshifts is obtained by counting the set of objects contained within each redshift bucket. This histogram is required to be normalised in order to obtain the density profile. Below is derived the expression of the sampling volume of the buckets, function of r_0 the lower radius of the sampling bucket, and Δr the radius width of the bucket. The sampling volume in spherical coordinates is described by the following integral:

$$V_{r_0 o, \Delta r} = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\theta_0} \sin\theta \, d\theta \, d\varphi \int_{r_0}^{r_o + \Delta r} r^2 \, dr. \quad (1)$$

By solving integral (1), the sampling volume for a spherical sampling ($\theta_0 = \pi$) is expressed as following:

$$V_{r_0, \Delta r} = \frac{4\pi}{3} \left((r_0 + \Delta r)^3 - r_0^3 \right), \quad (2)$$

where $V_{r_0, \Delta r}$ is the sampling volume for a given bucket, r_0 the lower radius of the bucket, and Δr the radius width of the bucket.

In order to use eq. (2), the galactic counts need to be converted into spherical values, by multiplying the counts by the sphere to survey solid angle ratio (η). Given the zCosmos survey spectroscopic area of 0.075 square degrees which is the solid angle, this ratio is the following:

$$\eta = \frac{4\pi \left(180/\pi \right)^2}{0.075} = 550'038. \quad (3)$$

The reported survey coverage area of the zCosmos-deep field is 1 deg2, [8]. However, what is required is the solid angle which is measured by the area of the survey projected in the plan described by the right ascension in degrees and $180/\pi * \sin(declination)$. Note that the sine of declination term is due to the Jacobian for spherical coordinates. The spectroscopic area obtained with this procedure is 0.075 deg2 (surface coverage in figure 1).

2.1.2 Conversion of redshifts to LTDs

Two approaches are available for converting the redshifts from observed galaxies into LTDs, one based on cosmological redshifts and the other one on dopplerian redshifts. First, let us introduce the method based on cosmological redshifts from the calculator of Wright [16] which uses a Lambda-CDM cosmology. The followings are generally assumed for this model: a flat Universe, with parameters: $\Omega_M = 0.27$, $\Omega_{vac} = 0.73$ and $H_o = 71$ [km s^{-1} Mpc^{-1}].
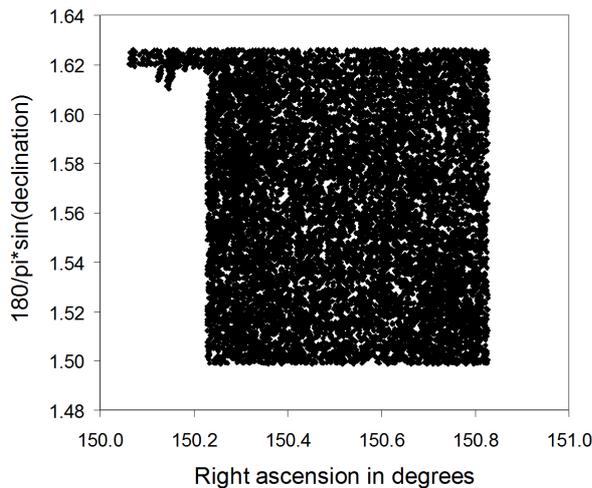
Fig. 1: Procedure to compute the spectroscopic area for the zCosmos survey as defined by the solid angle.

In the dopplerian redshift method, the relationship between redshifts and recession velocities is the following:

$$1 + z = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}. \tag{4}$$

From this equation, one may compute the recession velocity for a given redshift. Then the distance is computed as following:

$$distance = \frac{v}{H_o}. \tag{5}$$

From subsequent calculations an Ω_M of 0.19 was obtained which was used to derive the galactic density profile. Both methods give comparable distances with differences less than 5 % for redshifts up to 5.2 using $\Omega_M = 0.19$. The difference between dopplerian and cosmological redshifts is discussed by Bedran [2]. Historically, the first solution to compute distances from cosmological redshifts was obtained by Mattig [9] which is based on Friedmann equations of general relativity. Mattig equation with $q_o = 0.5$ also provides distances close to what is obtained using dopplerian redshifts; however, Mattig had to assume that conservation of mass is applicable to the Universe in his derivations which is a big bang cosmology. On the other hand, dopplerian redshifts do not require any assumption on the cosmology, and present the advantage that they also explain blueshifts that are being observed such as for Andromeda.

2.1.3 Sources of data

The zCosmos galactic survey Data Release DR1 was used [8].

2.2 Propagation of light in an expanding space

The main hypothesis for the development of a model for the propagation of light in an expanding space, is that the speed of light is frame-independent. Considering redshifts, this means that the relative movement of a light source does not change the speed of light emitted; however, it does add or subtract energy to the photon. In a dopplerian world, this change in energy level changes the frequency of the source of light, and not the speed. However, as space between the photon and the observer expands, this expansion is added to the overall distance the photon has to travel in order to reach the observer - in over words the speed of light is frame-independent with respect to the local space. This implies that there exists a distance for which the recession speed between the observer and the photon equals the speed of light, which is the Hubble sphere, and that recession speed can exceed than the speed of light for large distances. The frame-independent hypothesis for the speed of light has been established in the past with the experiment of Michelson-Morley [10]. Based on observations of double stars [14, 4] it was shown that the velocity of propagation of light does not depend on the velocity of motion of the body emitting the light.

As a consequence of the above, LTDs are not anymore additive, meaning that if we have three points aligned in space, the distance between the two extremes is not anymore equal to the sum of the two sub-segments as measured in LTDs.

Based on the above hypothesis, the Euclidean distance between the photon and the observer is described by the following differential equation:

$$\frac{dy}{dt} = -c + H_o \cdot c \cdot T, \tag{6}$$

where y is the Euclidean distance between the photon and the observer, T the LTD between the observer and the photon, c the celerity of light, and H_o the Hubble constant.

2.3 Conversion of light travel distances to Euclidean distances

Let us consider a photon initially situated at a Euclidean distance y_o from the observer and moving at celerity c in the direction of the observer. Let us say T is the initial LTD between the photon and the observer, and define the Hubble constant function of LTDs.

The differential equation describing the motion of the photon in the LTD framework is described by eq. (6). By taking a reference point in time in the past, and T_b be today time from this reference point, we get $T = T_b - t$. Hence, $dt = -dT$. Therefore, eq. (6) becomes:

$$\frac{dy}{dT} = c - Ho \cdot c \cdot T, \tag{7}$$

with boundary conditions $y(T) = y_o$ and $y(0) = 0$.

By integration from 0 to T, the following relationship relating Euclidean distances y to light travel distances T is obtained:

$$y = c \cdot T - \frac{c \cdot H_o \cdot T^2}{2}.$$ (8)

The corresponding horizon computed by setting $\frac{dy}{dT} = 0$ is $T_h = \frac{1}{H_o}$ which is the Hubble sphere.

2.4 The Hubble constant was determined with respect to LTDs

In general the literature refers to the Hubble constant measured with respect to LTDs. A common way to obtain the Hubble constant is based on standard candles with supernovae and cepheids [13, 1] and the Tully-Fisher relation [5]. Both the standard candle and Tully-Fisher method rely on the distance modulus. As shown below the distance modulus gives a measure of LTDs and not Euclidean distances.

Let us recall the derivation of the distance modulus. The magnitude as defined by [12] is:

$$m = -2.5 \log F + K,$$ (9)

where m is the magnitude, F the brightness or flux and K a constant. The absolute magnitude is defined as the apparent magnitude measured at 10 parsecs from the source.

Planck's law for the energy of the photon leads to a redshift correction to the distance modulus

$$E = \frac{h \cdot c}{\lambda},$$ (10)

where E is the energy of the photon, h the Planck's constant, and λ the light wavelength.

The ratio of observed to emitted energy flux is derived from eq. (10), leading to

$$\frac{E_{obs}}{E_{emit}} = \frac{\lambda_{emit}}{\lambda_{obs}} = \frac{1}{1+z}.$$ (11)

From geometrical considerations, the projected surface of the source of light on the receptor diminishes with a relationship proportional to the inverse of square distance from the source of light; hence, the following relationship is obtained for the brightness or flux:

$$F_{obs} \propto \frac{L_{emit}}{d^2} \cdot \frac{E_{obs}}{E_{emit}},$$ (12)

where L_{emit} is the emitted luminosity and d the distance to the source of light.

Combining eq. (9), (11) and (12), we obtain:

$$m = -2.5 \log \left(\frac{L_{emit}}{d^2 \cdot (1+z)} \right) + K.$$ (13)

And, because z is close to zero at 10 Parsec:

$$M = -2.5 \log \left(\frac{L_{emit}}{100} \right) + K,$$ (14)

where M is the absolute magnitude.

Hence, the distance modulus, eq. (13) minus (14) is:

$$m - M = -5 + 5 \log d + 2.5 \log(1+z),$$ (15)

with d in parsec and log means the logarithm to base 10.

The expansion of the Universe adds up to the Euclidean distance, and therefore the apparent magnitude of the source of light is fainter than if no expansion was present.

2.5 Evolution of the galactic density assuming no new galaxy formation

Assuming cosmological redshifts we have:

$$1 + z = \frac{a_o}{a_1},$$ (16)

where a_o and a_1 are respectively the present scale factor and the scale factor at z.

From the conservation of mass the density is proportional to the inverse of the cubic scale factor:

$$\rho \propto \frac{1}{a^3}.$$ (17)

Therefore, the model for the evolution of the density with respect to the present density is the following:

$$\rho_t = \rho_o \cdot (1+z)^3,$$ (18)

where ρ_t is the density in the past at redshift z and ρ_o is the present density.

3 Results

3.1 A flat density profile using Euclidean distances

Galactic density profiles have been derived for the two antagonistic scenarios respectively assuming that LTDs are additive, and with the propagation of light in an expanding space (figure 2). Note that the galactic density profiles obtained with cosmological redshifts and dopplerian redshifts are very similar. The highest redshift galaxies observed for the survey (z = 5.2) are very close to the Hubble sphere (which are at 13.65 Glyr) as calculated from cosmological redshifts with Ω_m=0.19.

The theoretical evolution of the galactic density with respect to the present density assuming no new formation of galaxies (figure 3) was computed assuming cosmological redshifts with eq. (18). Note that the first point in the galactic density profile is not representative of the average density as the sample volume is very small; hence, the measure represents the density in the neighbouring galactic cluster of the Milky Way (figure 2 and 3).
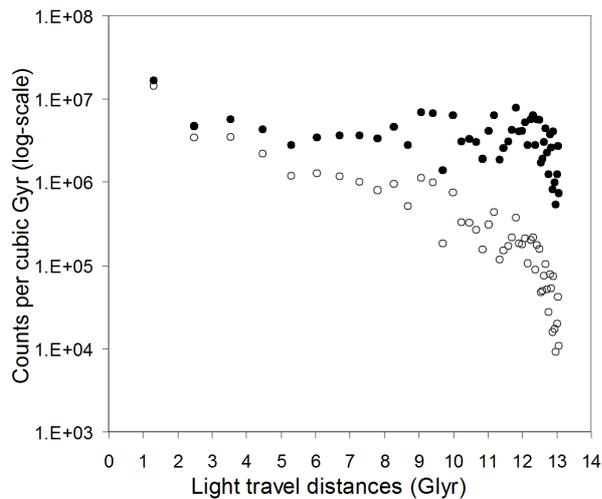
Fig. 2: Galactic density profile derived from the equivalent spherical sampling, where Glyr are billion light years from today. LTDs are obtained from redshift conversion with dopplerian redshifts. The blank dots indicate densities based on LTDs. The solid dots indicate densities obtained with Euclidean distances on the basis of dopplerian redshifts.
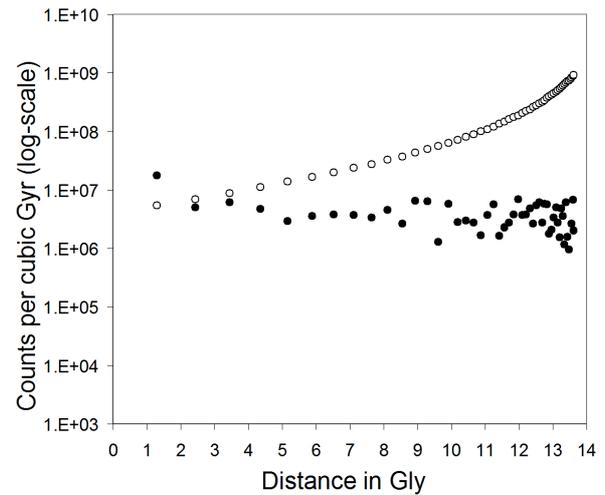

Fig. 3: Galactic density profile derived from the equivalent spherical sampling, where Gly are billion light years from today. LTDs are obtained from redshift conversion with cosmological redshifts (omega matter of 0.19).The solid dots indicate densities obtained with Euclidean distances on the basis of cosmological redshifts. The blank dots indicate the theoretical evolution of galaxies assuming that the survey is incomplete (with no new galaxy formation).

3.2 Estimation of Ω matter from galactic counts

The average galactic mass estimated from light deflection [15] is 1.7×10^{11} $M_\odot$. The Universe mean density is obtained by multiplying this figure with the average galactic count per cubic Glyr. Using dopplerian redshits the galactic count density is 4.6×10^6 counts per cubic Glyr, leading to a mean Universe density of 1.84×10^{-30} g/cm^3. Using a Hubble constant of 71 $km/s/Mpc$ and recent estimates of the gravitational constant of 6.67×10^{-8} $cm^3/g/sec^2$ [11], the critical density is estimated at 9.47×10^{-30} g/cm^3 (from $\rho_c = \frac{3H^2}{8\pi G}$). Therefore, the corresponding Ω_m equals to 0.19. Note that smaller values of the Hubble constant would lead to a higher Ω_m.

3.3 Estimation of the number of galaxies in the visible Universe

Another challenge is to estimate the number of galaxies in the visible Universe. Using the galactic density in the nearby Universe from figure 2 expressed per cubic Glyr LTD, and the volume of the sphere of radius 14 Gly LTD, the number of galaxies in the visible Universe is estimated at 175 billion. Gott et al. [6] estimated a number of galaxies in the visible Universe at about 170 billion based on the Sloan Digital Sky Survey luminosity function data using the Press-Schechter theory. Both figures are consistent with each other; however, the author believes that these figures need to be reviewed to account only for the Euclidean radius when computing the volume of the visible Universe. As the galactic density profile is flat, it is expected that the estimated number

of galaxies in the visible Universe is internally consistant with the bulk amount of galaxies observed in the survey converted to spherical values, i.e. multiplying the number of galaxies in the survey (10046 galaxies) by the sphere to survey solid angle ratio, which leads to 5.5 billion galaxies (see Table 1).

4 Discussion

A new approach is proposed in the present study to derive the galactic density profile which is based on the conversion of light travel distances to Euclidean distances. The method has been tested by computing the galactic density profiles based on the data from the zCosmos deep field survey.

In the scenario using LTDs with the sampling method, the galactic count per cubic Glyr grows according to a steep slope (figure 2), without accounting for the effect of the expansion which should add up to this growth. There is no explanation for such result - this scenario appears to be unrealistic. The scenario using Euclidean distances, shows a flat profile for the galactic counts per cubic Gyr (figure 3). However, there is still a gap between the computed galactic density profile and the theoretical evolution of galactic densities assuming no new galaxy formation. Leaving aside model bias, this gap may be interpreted as if galaxies grow in number over time. Another hypothesis is that the galactic survey is incomplete meaning that faint galaxies are left asside from the zCosmos survey at large distances, which would account for the missing galaxies causing the gap in figure 3. The theoretical density obtained by conservation of mass is too large by a factor

Table 1: Estimation of the number of galaxies in the visible Universe (radius 14 Glyr) using LTD distances and Euclidean distances.

	Radius of the visible Universe	Galactic density	Estimated number of galaxies
Using LTDs	14 Glyr	1.52×10^7 counts per cubic Glyr	175 billion
Using Euclidean distances with dopplerian redshifts	6.90 Glyr	4.60×10^6 counts per cubic Glyr	6.3 billion
Galaxy count of the survey converted to spherical values			5.5 billion

of order 200 at redshift 5.2. This discrepancy is unrealistically to large. Clearly more detailed work needs to be carried out to investigate this gap.

By applying conservation of mass, as we approach the singularity of the big bang, the Universe would have been so dense that it is difficult to explain how gravity did not prevent the early Universe from collapsing. A possibility is that the Hubble constant was much higher in the past leading to a higher critical density - cosmic inflation would still be necessary to overcome this issue. From the present study, the galactic density appears to be constant over time, which would corroborate the steady state cosmology of [3, 7]. The other condition being that the Hubble constant remains unchanged over time.

Acknowledgments

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