Spooky Action at a Distance or Action at a Spooky Distance?

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The paper demonstrates that the non-locality and non-reality of the quantum world are direct consequences of the concept of uncertainty. It is also shown that the analysis of states in the phase space entails the operator formalism of wave mechanics. While being well known that the uncertainty principle is a consequence of the commutation rules of operators, the paper shows that the reverse path is also possible; i.e. the uncertainty equations entails themselves the operators and wave equations of energy and momentum. The same theoretical approach has been eventually extended to infer significant results of the special relativity.

1 Introduction

Einstein never liked the weirdness and the conceptual limit of the quantum mechanics due to its probabilistic character; for instance, he disliked the incomplete knowledge about position and momentum of a particle, about all components of angular momentum and so forth. Paradoxically, just his theory of the specific heat and its explanation of the photoelectric effect were the strongest support to the energy quantization early introduced by Plank to explain the black body radiation. In fact to the quantum theory we owe not only the ability to explain weird experimental data, e.g. the dual wave/particle behavior of matter and the tunnel effect, but also important discoveries like the laser, the transistor and the superconductivity. Further experimental evidences recently obtained compelled however accepting besides its weird character other aspects even more counterintuitive of quantum behavior. Mostly important are in this respect the non-localism and non-realism: according to the former, exchange of information is allowed even between particles separated by a superluminal distance; according to the latter, the experimental measurements do not reveal preexisting properties of particles but concur to define themselves the measured properties. The EPR gedanken experiment [1] tried to overcome the conceptual incompleteness of quantum mechanics by hypothesizing "hidden variables" in the wave function, i.e. variables not accessible to experimental evidence but able to improve our extent of knowledge and to overcome the difficulty of a "spooky action at a distance" between correlated couples of particles. Yet, several experiments were able to exclude the existence of hidden variables while demonstrating instead non-local effects [2, 3]. The theoretical apparatus of quantum mechanics acknowledges the non-local behavior of the quantum particles through the concept of entanglement [4, 5]. This term was early introduced by Schrodinger [6] to describe the possibility of correlating quantum systems even though spatially separated; the most controversial point concerns of course the difficulty arising from the requirements of relativity. Even today the concept of entanglement has different interpretations: the most acknowledged point of view is the quantum superposition of states, according which two correlated particles share a single quantum state until a measurement is carried out. The quantum mechanics is founded on a set of mathematical rules, which however do not incorporate themselves since the beginning the non-locality and non-reality in its fundamental conceptual structure, in order to include and rationalize per se these effects. For this reason the EPR paper appears legitimate from a rational point of view, although in fact wrong from a physical point of view; indeed a separate theoretical tool, the Bell inequality [7], was necessary to evidence the inconsistency of the EPR attempt [8, 9]: the predictions of local realism on which is based the Bell inequality conflict with the results obtained in various experiments, e.g. [10, 11, 12]. It is worth noticing that no theoretical foundation of the wave mechanics can be considered really general without containing inherently the non-realism and non-localism of the quantum world. It is therefore interesting to examine in this respect the approach followed in previous papers [13, 14], where results consistent with that of wave mechanics have been inferred exploiting the following equations only

$$\Delta x \Delta p_x = n\hbar = \Delta\varepsilon\Delta t. \qquad (1,1)$$

The second equality is consequence of the first one defining formally $\Delta t = \Delta x/v_x$ and $\Delta\varepsilon = \Delta p_x v_x$, where v_x is the average velocity with which any particle travels through Δx; the equalities share the common number n of allowed states. The equations (1,1) do not require any assumption about the ranges, about the motion of the particle and even about its wave/corpuscle nature; this latter will be inferred as a corollary in section 6. The present paper aims to contribute some ideas about how to regard the non-locality and non-reality uniquely according to eqs. (1,1). For reasons that will be clear below, it is useful to introduce shortly in section 2 the way of exploiting these equations to infer the quantum angular momentum; the remarks at the end of this section, which has a preliminary worth, are essential to discuss subsequently the weirdness of the quantum world. Although the angular momentum has been already introduced in [13], its elucidation is so straightforward and elementary that it deser-

ves being shortly sketched here; in doing so, indeed, it introduces reference concepts that will be further developed in the following sections 3 and 4 that concern the non-reality and non-locality. Eventually, the connection between quantum theory and special relativity is also sketched in sections 5 and 7; the link between eqs. (1,1) and the operator formalism of wave mechanics is discussed in section 6.

2 The non-relativistic angular momentum

The non-relativistic quantization of the classical angular momentum M^2 and of one of its components M_w along an arbitrary direction defined by the unit vector $\mathbf{w}$ starts from the classical scalar $\mathbf{r} \times \mathbf{p} \cdot \mathbf{w}$; here $\mathbf{r}$ is the radial distance of any particle from the origin O of an arbitrary reference system R and $\mathbf{p}$ its momentum. For instance, this could be the case of an electron in the field of a nucleus centered in O. As introduced in [15], the positions

$$\mathbf{r} \rightarrow \Delta\mathbf{r} \qquad \mathbf{p} \rightarrow \Delta\mathbf{p} \tag{2,1}$$

enable the number l of quantum states to be calculated as a function of the ranges $\Delta\mathbf{r}$ and $\Delta\mathbf{p}$ of all local distances and momenta physically allowed to the particle. These ranges only, and not the random local values $\mathbf{r}$ and $\mathbf{p}$ themselves, are considered in the following. The first step yields $M_w = (\Delta\mathbf{r} \times \Delta\mathbf{p}) \cdot \mathbf{w} = (\mathbf{w} \times \Delta\mathbf{r}) \cdot \Delta\mathbf{p}$ and so $M_w = \Delta\mathbf{I} \cdot \Delta\mathbf{p}$, where $\Delta\mathbf{I} = \mathbf{w} \times \Delta\mathbf{r}$. If $\Delta\mathbf{p}$ and $\Delta\mathbf{I}$ are orthogonal, then $M_w = 0$; else, writing $\Delta\mathbf{I} \cdot \Delta\mathbf{p}$ as $(\Delta\mathbf{p} \cdot \Delta\mathbf{I}/\Delta I)\,\Delta I$ with $\Delta I = |\Delta\mathbf{I}|$, the component $\pm\Delta p_I = \Delta\mathbf{p} \cdot \Delta\mathbf{I}/\Delta I$ of $\Delta\mathbf{p}$ along $\Delta\mathbf{I}$ yields $M_w = \pm\Delta I \Delta p_I$. In turn this latter equation yields according to eqs. (1,1) $M_w = \pm l\hbar$, being l the usual notation for the number of states of the angular momentum; l is positive integer including zero. As expected, M_w is not a single valued function because of the uncertainties initially postulated for $\mathbf{r}$ and $\mathbf{p}$. One component of $\mathbf{M}$ only, e.g. along the z-axis, is knowable; repeating the same approach for the y and x components would trivially mean changing $\mathbf{w}$. Just this conclusion suggests that the average values $< M_x^2 >$, $< M_y^2 >$ and $< M_z^2 >$ should be equal; so the quantity of physical interest to describe the properties of quantum angular momentum is l, as a function of which M^2 is indeed inferred as well. Let us calculate these average components over the possible states summing $(l\hbar)^2$ from $-L$ to $+L$, where L is an arbitrary maximum value of l. Being by definition $< M_i^2 >= \sum_{l_i=-L}^{l_i=L} (\hbar l)^2/(2L+1)$, one finds $M^2 = \sum_{i=1}^{3} < M_i^2 >= L(L+1)\hbar^2$. Note that the mere physical definition of angular momentum is enough to find quantum results completely analogous to that of wave mechanics; any local detail of motion, like that of electron "orbit" around the nucleus, is utterly unnecessary. The quantization of the classical values appears merely introducing the delocalisation ranges into the definition of angular momentum and then exploiting eqs. (1,1). The reason of it is evident: after the steps (2,1), the unique information available comes from the uncertainty ranges of coordinates and momentum, rather than from the local values of these latter; then the quantities thereafter calculated concern the number of allowed states only, which have in fact the same physical meaning of the quantum number defined by the solution of the pertinent wave equation. An analogous approach shows that the non-relativistic hydrogenlike energy levels depend on a further integer n because of the radial uncertainty equation $\Delta p_\rho \Delta\rho = n\hbar$ of an electron from the nucleus [13]; again, even without specifying any local detail of motion, the numbers of states l and n related to the angular and radial uncertainties of the electron in the field of nucleus correspond to the respective quantum numbers that characterize the energy levels. This preliminary introduction on how to exploit eqs. (1,1) was included in the present paper to emphasize several points useful in the following, i.e.: (i) the replacements (2,1) that allow to exploit eqs. (1,1) are enough to plug the classical physical definition $\mathbf{r} \times \mathbf{p}$ of angular momentum into the quantum world; (ii) no hypothesis is necessary about the geometrical properties of motion of the particle nor about its wave/matter nature to infer the quantum result; (iii) trivial algebraic manipulations replace the solution of the pertinent wave equation; (iv) the information inferred through eqs. (1,1) only is fully consistent with that of the wave mechanics; (v) the local momentum and distance between the particles concerned in the "orbiting" system do not play any role in determining l; (vi) as found elsewhere, [15, 17], the number of allowed states plays actually the role of the quantum numbers of the operator formalism of wave mechanics; (vii) the amount of information accessible for the angular momentum is not complete like that expected in the classical physics; (viii) eqs. (1,1) rule out "a priori" any possibility of "hidden variables" that could in principle enhance our knowledge about M_w and M^2 in order to obtain a more complete description of the orbiting quantum system.

It is worth mentioning that the validity of the point (i) has been checked and extended in the papers [13, 14] also to more complex quantum systems like many electron atoms/ions and diatomic molecules. The fact that eqs. (1,1) efficiently replace the standard approach of wave mechanics has central interest for the topics introduced in following sections, especially as concerns the very important point (viii). In principle one could not exclude that the wave function, from which is extracted all physical information allowed about the quantum systems, could actually contain hidden variables; indeed this chance, reasonably suspected in the famous EPR paper, has been excluded later thanks to a separate theoretical tool only, the Bell inequality. In the present approach, instead, the quantization of angular momentum is more "transparent" in that it explicitly displays variables and steps that lead to the quantum result; in other words, the present approach excludes any possibility of hidden variables because it works with actual quantities inherent the mere definition of angular momentum only. In conclusion the present section aimed mostly to ensure that sensible results are obtained regarding the uncertainty as a fundamental principle of nature itself, rather than

as a by-product of the operator formalism of wave mechanics. It is necessary however to better understand eqs. (1,1). To ascertain "a posteriori" that these equations work well has no heuristic worth. Therefore, after having checked their validity, the remainder of the paper starts from a step behind them, i.e. to highlight the more profound physical basis rooted in the concept of space-time uncertainty.

3 Non-realism and non-localism of eqs. (1,1)

Let us introduce a reference system R to define the ranges of eqs. (1,1). In the simplest 1D case, R is represented by an arbitrary axis where are defined two coordinates x_o and x_t with respect to an arbitrary origin O: the former describes the position of the range $\Delta x = x_t - x_o$ with respect to O, the latter describes its size. The postulated arbitrariness of size makes Δx consistent with the local coordinate x_o in the limit case $x_t \to x_o$ and with any other coordinate if is also allowed the limit size $\Delta x \to \infty$. If neither boundary coordinate is time dependent, then the section 2 and the papers [15, 16] show that this is all we need to know to define an observable physical property of the concerned quantum system: indeed, with the help of an analogous reasoning for the momentum range, this approach is enough to find the number of allowed states i.e. the quantum numbers that define the eigenvalues of the observable. If instead x_o and x_t are in general time dependent, then Δx expands or shrinks as a function of time, while possibly shifting with respect to O too, depending on how are mutually related the displacements of x_o and x_t. Actually the paper [15] shows that such a detailed information about how both of them displace with respect to O is physically redundant; all we need to know is the resulting $\Delta\dot{x}$ only. If Δx is an empty range, the chance of displacement in principle possible for x_o and x_t entails the presence of a force field within Δx; in the absence of a particle delocalized in it, however, this conclusion has a self-contained worth only that concerns a property of the the range itself in R. Instead consequences of physical interest are expected when a free particle is possibly therein delocalized; first of all because this presence requires itself highlighting the physical meaning of x_o and x_t to justify why these boundary coordinates, although remaining in principle completely arbitrary, can in fact include all values of dynamical variables allowed to the particle. Assume for instance two infinite potential barriers at x_o and x_t: if the size of the delocalization range changes from Δx_1 to Δx_2 during the time range $\Delta t = t_2 - t_1$, it means that necessarily the properties of the particle are affected during Δt as well; at the time t_1 the particle was constrained bouncing within Δx_1 with average frequency $\nu_1 = v_x \Delta x_1^{-1}$, at the time t_2 with average frequency $\nu_2 = v'_x \Delta x_2^{-1}$. The average displacement velocity v_x of the particle has been regarded different at the times t_1 and t_2 for sake of generality; however this fact is not essential, since $\Delta x_2 \neq \Delta x_1$ is enough to ensure $\nu_2 \neq \nu_1$. Hence the deformation of Δx as a function of time entails changing average displacement velocity, bouncing frequency of the particle and thus its momentum as well. To draw such a conclusion two essential elements have implemented the initial definition of delocalization range: the presence of a particle and the size change of Δx. Since however no assumption has been made about times and range sizes, nor about v_x and v'_x, these properties do not define themselves any state allowed to the particle; nothing about arbitrary range sizes, frequencies and velocities can be related to an integer number. Despite the intuitive fact that the particle dynamics has changed, n still appears unexplainable. This conclusion is important because, for the reasons introduced in section 2, just n entails the chance of measuring a physical observable of the particle. Overcoming this indeterminacy requires thus a further condition or constraint on ν_1 and ν_2, e.g. on the change of energy or momentum of the particle during the aforesaid time range. In effect, this condition is a crucial step to allow the transition from an unphysical "virtual" state towards an observable state: if for instance to define n concur the values of momentum or energy related to ν_1 and ν_2, then the sought number of states should correspondingly represent just the allowed eigenvalues of momentum or energy of the particle. The fact that a unique range is inadequate to define n, justifies reasonably the idea of introducing a further range ancillary to Δx able to represent in R the values of a second dynamical variable. Apart from this intuitive conclusion, it is necessary to explain why two arbitrary ranges of allowed dynamical variables are necessary to define the sought observable state of the particle. A reasonable idea is to examine the concept itself of measurement process. It is known that this concept is replaced in quantum mechanics by that of interaction, whose effect is to perturb the early state of the particle under test. The dynamical variables of the unperturbed free particle in R represent the initial boundary condition as a function of which is determined the effect of the interaction between particle and observer. Let the intensity of the local perturbation, whatever it might be, depend in general on the current local position and momentum of the particle; then the observer records an outcome somehow related to the boundary condition describing the particle before the measurement process. Since however the initial dynamical variables were unknown, they remain unpredictable and unknown after the measurement process as well; any correlation between initial and final state of the particle is impossible, simply because the former is in fact undefined. Renouncing "a priori" to know the local values of conjugate dynamical variables compels thus introducing ranges of their allowed values. Despite the lack of information about the sought correlation and kind of interaction, let us show that even so the concept of measurement allows defining the number of states, which in fact makes actual the properties of the particles. Regard to this purpose the aforesaid x_o and x_t respectively as coordinates of the particle before and after the measurement process; in agreement with eqs. (1,1), both are random, unknown and unpredictable, whereas du-

ring the interaction even intermediate values are expected to fall between these extremal boundaries. Considerations analogous to $x_t - x_o$ hold also for the conjugate momentum range $p_t - p_o$, whose boundary values p_o and p_t are related to the momentum of the particle before and after the measurement process. However $x_t - x_o$ and $p_t - p_o$, although fulfilling the requirements of both measurement process and eqs. (1,1), cannot be directly related themselves to Δx and Δp_x; the former are indeed uncorrelated and thus still unable to justify n, the central aim of the present discussion. Let us introduce thus the probabilities Π_x and Π_{p_x} that the values of both dynamical variables change during the measurement process in such a way that

$$x_t - x_o \rightarrow measurement \rightarrow \Delta x$$

$$p_t - p_o \rightarrow measurement \rightarrow \Delta p_x$$

where the usual notations Δx and Δp_x refer to ranges compliant with eqs. (1,1). This suggests writing

$$\Pi_x = \Delta x/(\Delta x + \Delta x'), \quad \Pi_{p_x} = \Delta p_x/(\Delta p_x + \Delta p'_x), \qquad (3,1)$$

where $\Delta x'$ and $\Delta p'_x$ are ancillary ranges consistent with the conditions $\Pi_x \rightarrow 0$ for $\Delta x \rightarrow 0$ and $\Pi_x \rightarrow 1$ for $\Delta x \rightarrow \infty$; analogous considerations hold of course for the momentum probability too. By definition therefore $\Delta x' > 0$ and $\Delta p'_x > 0$, in agreement with the idea that all ranges in the present model are positive. The physical meaning of $\Delta x'$ and $\Delta p'_x$ appears noting that initially, i.e. before defining n, space delocalization and momentum ranges are unrelated. Let us regard then $\Delta x + \Delta x' = x_t - x_o$ and $\Delta p_x + \Delta p'_x = p_t - p_o$ as the unperturbed early ranges, whose respective final sizes are just Δx and Δp_x of eqs. (1,1). So eqs. (3,1) concern the probability that the particle is eventually in Δx resulting after the measurement driven perturbation of the early $\Delta x + \Delta x'$, whereas an analogous explanation holds of course for Π_{p_x} as well. The total probability $\Pi_n = \Pi_x \Pi_{p_x}$ for space delocalization and momentum ranges fulfilling eqs. (1,1) is thus

$$\Pi_n = \Delta x \Delta p_x/(\Delta x \Delta p_x + \Delta x \Delta p'_x + \Delta p_x \Delta x' + \Delta p'_x \Delta x'). \quad (3,2)$$

In eq. (3,2) Π_n is expressed as a function of Δx and Δp_x that will bring us to eqs. (1,1) although starting from initial larger ranges still unrelated, whence the notation. First of all note that eq. (3,2) requires $(\Delta x/\sqrt{\Pi_n})(\Delta p_x/\sqrt{\Pi_n}) > \Delta x' \Delta p'_x$. Since all ranges appearing in this inequality are arbitrary, the left hand side can be shortly written as $\delta x \delta p_x$ whatever the specific values of $\Pi_x \neq 0$ and $\Pi_{p_x} \neq 0$ might be; these last positions are straightforward consequences of the previous considerations. Second, also note that the probability of quantum interest is the square root $\sqrt{\Pi_n} = \sqrt{\Pi_x \Pi_{p_x}}$ of that defined classically as ratio between favorable and total chances; this point will be further concerned in section 6. Third, by definition the product of ranges at right hand side of the inequality cannot be made equal to zero; this would contradict the concept of uncertainty, which must hold for any ranges of any size not simultaneously vanishing. So $\delta x \delta p_x > 0$ requires the existence of a value $const' > 0$ such that

$$\delta x \delta p_x > const' \qquad \Rightarrow \qquad \delta \varepsilon \delta t > const'. \qquad (3,3)$$

The second equation is obtained from the first likewise as in eqs. (1,1). This is in effect the uncertainty principle with the value of $const'$ of the order of the Plank constant; this inequality is then direct consequence of the probabilistic definition of eqs. (3,1) and supports the idea that the perturbation induced by the measurement process shrinks the initial uncorrelated ranges $\Delta x + \Delta x'$ and $\Delta p_x + \Delta p'_x$ to the correlated ones Δx and Δp_x of eqs. (1,1). The fact that eqs. (3,3) concern by definition observable states ensures that effectively $\sqrt{\Pi_n} \neq 0$. Eventually, together with eq. (3,2) must in principle exist also the probability

$$\Pi'_n = 1 - \Pi_n. \qquad (3,4)$$

Note that eq. (3,2) admits in principle $\Delta x' << \Delta x$ and $\Delta x' >> \Delta x$, together with analogous features of $\Delta p'_x$; so both limit probabilities can tend to 0 or to 1. Thus it is possible to regard eq. (3,2) as the effective chance of getting an eigenvalue from the measurement process and eq. (3,4) as that of not getting any eigenvalue. Both account for well known outcomes of wave mechanics, e.g.: (i) eq. (3,4) accounts for eigenvalues that actually do not exist, see for instance the previous conclusions about the x and y components of angular momentum once having determined M_z; (ii) when a quantum states is described by a superposition of several eigenfunctions, several eigenvalues exist whose respective actual occurrence is probabilistic, and so on. These chances must be inferred case by case when exploiting eqs. (1,1) through specific reasonings like that of section 2. The physical meaning of $\sqrt{\Pi_n}$ will also be shortly discussed in the next section 6; so eqs. (3,2) and (3,4) do not deserve further comments here. Now instead let us pose a question before proceeding on: why just shrinking and not expanding further the initial unrelated ranges? Apart from ther fact that the ranges are by definition all positive, the second chance would mean $\Delta x + \Delta x'$ and $\Delta p_x + \Delta p'_x$ defined by negative $\Delta x'$ and $\Delta p'_x$, which in turn would exclude the possibility of defining the probabilities Π_x and Π_{p_x} themselves. Besides this inconsistency, a plain consideration further clarifies the question. The measurement process tries to determine a physical property. Expanding the early unrelated ranges would mean decreasing our degree of knowledge about the particle, whose dynamical variables would oscillate within wider ranges of possible values; if so, the concept of measurement would be itself an oxymoron. Shrinking the early ranges, instead, is the best compromise offered by the nature to us during what we call "measurement process": while being forbidden the exact local values of the classical physics we must content ourselves,

at least, of reduced ranges of values for conjugate dynamical variables to which correspond however numbers of states. We must accept therefore the probabilities of eqs. (3,1) as the best we can get from a measurement process; this is what tells us the Heisenberg inequality just obtained from our probabilistic knowledge of the reality around us. To proceed further exploit again the arbitrariness of all ranges so far introduced in order to rewrite eq. (3,2) in various possible ways. In the first way $\Pi = \Delta x\Delta p_x/(\Delta x''\Delta p_x'')$, being $\Delta x''\Delta p_x'' \geq \Delta x\Delta p_x$ the sum of all addends at denominator. This suggests that $\Delta x\Delta p_x = \alpha const$, where *const* is a constant and α a parameter to be defined consistently with the actual product of the resulting uncertainties. Indeed this position allows writing in general

$$\Delta x''\Delta p_x'' = \alpha'' const, \quad \Delta x'''\Delta p_x''' = \alpha''' const \tag{3,5}$$

and so forth, depending on the values of the range products at left hand side. Let for instance be $\alpha''' \leq \alpha''$; eliminating *const* from these equations one finds $\Delta x'''\Delta p_x'''/(\Delta x''\Delta p_x'') = \alpha'''/\alpha''$ i.e. the sought form of Π_n. A further possibility of rewriting eq. (3,2) is $\Pi_n = \Delta x\Delta p_x/(4\Delta x^{\S}\Delta p_x^{\S})$ in the particular case where all terms at denominator of eq. (3,2) are equal to that here indicated with the unique notation $\Delta x^{\S}\Delta p_x^{\S}$; there is indeed no reason to discard also this chance, which must be therefore included in our definition of Π_n. Eventually, another consequence of the arbitrariness in defining $\Delta x'$ and thus $\Delta x''$ and $\Delta x'''$ of eqs. (3,5) must be taken into account: $\Delta x'$ could have been even rewritten itself as $\Delta x' = \Delta x^{\S} + \Delta x^{\S\S} + \cdots$, with several addends again arbitrary; in this case the number of addends at denominator of eq. (3,2) would have been any integer n rather than 4. All these requirements are easily included in the definition of Π_n simply putting $\alpha \equiv n$, so that eqs. (3,5) read $\Delta x''\Delta p_x'' = n''const$ and so forth with n arbitrary integer; in other words, n corresponds to the arbitrary number of possible subdivisions of the early ranges induced by the measurement process. This result effectively leads to both eqs. (1,1), which merely specify the value of *const* as that of $\hbar$. Note eventually that dividing more and more the initial interval $\Delta x'$ into an increasing number of intervals $\Delta x^{\S}$, $\Delta x^{\S\S}$, ... means considering smaller and smaller sized ranges, to which corresponds an increasing number n; since a smaller and smaller range actually tends to the limit of a local coordinate better and better defined, one realizes that $n \to \infty$ corresponds to the deterministic limit of the classical physics. Once more, the same holds for the other ranges. Since eqs. (1,1) are adequate to describe the existence of eigenvalues, one concludes that the measurement process is in fact consistent with the existence of experimental observables despite the initial uncertainties of both dynamical variables. Note that the reasoning above did not exploit any specific feature of the momentum; in other words, instead of the momentum range the reasoning could have identically exploited directly the perturbation of the velocity v_x of the particle under observation, i.e. a velocity range. The question about why we have in fact introduced just the momentum is irrelevant, as it rests merely on the particular choice of the physical dimension of *const*; regarding this latter as a product $const^{\S}m$, involving m times another constant, one would still find eqs. (3,5) with the form $\Delta x''\Delta p_x'' = n''const^{\S}m$ i.e. $\Delta x''\Delta v_x'' = n''const^{\S}$. Two further considerations are instead by far more relevant. The first is that eqs. (1,1) compel regarding any observable as the consequence of the measurement process itself, rather than as intrinsic feature of matter; no pre-existing state, and thus n, was indeed definable for the particle before the measurement. The conclusion that n characterizing the eigenvalues is consequence of the measurement process, rules the realism out of the quantum world. The second relevant feature of eqs.(1,1), which clearly appears recalling the results of section 2, concerns the localism. The particular example of the angular momentum has been introduced before any further consideration of central interest for the purposes of the present paper just to show that the local dynamical variables do not play any role in determining the observable properties of reality around us, as the experimental properties we measure are related to the eigenvalues and thus to the number of allowed states only. So the local values of dynamical variables become unphysical once accepting eqs. (1,1) to formulate quantum problems: nothing measurable corresponds to the local values. Hence, in lack of local information, the concept of distance is unphysical itself in the quantum world. For instance, in [15] the Newton and Coulomb forces between two interacting masses or charges have been inferred replacing the dependence on their classical distance x_{12}^{-2} with the dependence on Δx^{-2}: according to eqs. (1,1), the space range includes all possible local distances between the interacting particles whose coordinates fall within Δx. Regarded from this point of view, the EPR paradox is unphysical itself: it is impossible to define a superluminal distance conflicting with the exchange of information about the spin orientation of two particles arbitrarily apart each other. Whatever their distance might be, a range Δx including both of them certainly exists because its size is by definition arbitrary. Once regarding two particles within Δx, however, the concept of their local distance fails together with that of the respective local coordinates; in principle nobody knows or can measure how far they might actually be. For this reason it would be appropriate to describe the EPR gedankenexperiment as an action at a spooky distance, instead of a spooky action at a distance. Moreover the concept of entanglement appears itself implicitly inherent the present approach, as even particles at superluminal distance must behave consistently with their chance of being anywhere and thus of exchanging information as if they would actually be at very short distance. In this respect, just the quantum entanglement is itself the best demonstration of the correctness of the present point of view based exclusively on the eqs. (1,1), which thus exclude "a priori" both realism and localism from the quantum world; all this clearly appears in section 2. Also the Aharonov-Bohm effect is immediately understandable in the

frame of the present reasoning: an electrically charged particle is affected by an electro-magnetic field even when it is confined in a region where both electric and magnetic fields are zero. Actually it is here *and* there just like a wave propagating through, and thus filling, all available delocalization range. The previous considerations show indeed that regarding a quantum particle here *or* there is physically illusory; assigning a specific location is an idea arbitrarily and incorrectly extrapolated from the classical physics to the quantum world.

4 The Bell inequality

At this point, the exposition brings unavoidably into the mind the Bell inequality. The non-locality and non-reality of the results inferred from eqs. (1,1) suggest emphasizing the connection between the considerations of section 3 and the Bell inequality. To highlight this link let us rewrite the eqs. (1,1) as

$$\frac{\Delta x}{\Delta x_1}\frac{\Delta p_x}{\Delta p_1} = n, \qquad \frac{\Delta t}{\Delta t_1}\frac{\Delta \varepsilon}{\Delta \varepsilon_1} = n, \qquad n \geq 1, \tag{4,1}$$

where the subscript "1" means $n = 1$. In this way $\hbar$ does no longer appear explicitly in the expression of the number of states. Eqs. (4,1) appear therefore as an appropriate starting point to examine the relationship between eqs. (1,1) and Bell inequality, which has indeed general character not specifically related to the quantum theory. Considering for sake of brevity the first equation only (the second is indeed its straightforward consequence) and taking the logarithms of both sides one finds

$$\log\left(\frac{\Delta x}{\Delta x_1}\right) + \log\left(\frac{\Delta p_x}{\Delta p_1}\right) \geq 0. \tag{4,2}$$

This equation presents a formal analogy with the Bell-like inequality, [9]

$$N(A, B_{\rm n}) + N(B, C_{\rm n}) \geq N(A, C_{\rm n}), \tag{4,3}$$

where the subscript "n" stands for "not". Its demonstration is amazingly simple. Whatever the properties A, B and C might represent, the inequality $N(A, B_{\rm n}, C) + N(A_{\rm n}, B, C_{\rm n}) \geq 0$ expressing the sum of the respective numbers of occurrences/non-occurrences possible for A, B and C is self-evident. Add to both sides the sum $N(A, B_{\rm n}, C_{\rm n}) + N(A, B, C_{\rm n})$ expressing further numbers of occurrences/non-occurrences possible for B and C and note that terms like $N(A, B_{\rm n}, C) + N(A, B_{\rm n}, C_{\rm n})$ read actually $N(A, B_{\rm n})$; the notation emphasizes a resulting term no longer distinguished according to either property C, i.e. the sum including both chances allowed for C with the same A and $B_{\rm n}$ discriminates in fact the occurrences/non-occurrences of A and B only. So one infers immediately the inequality (4,3) that can be more expressively rewritten as

$$N_n(A, B_{\rm n}) + (N_n(B, C_{\rm n}) - N_n(A, C_{\rm n})) \geq 0 \tag{4,4}$$

with notations N_n for reasons that will be clear soon. Comparing the inequalities (4,2) and (4,4) requires emphasizing first of all what "not" stands for. In eqs. (3,1) the ranges $\Delta x'$ and $\Delta p'_x$ additional to Δx and Δp_x have been introduced to define the probability Π_x that after the measurement interaction the particle delocalization is described by Δx and no longer by $\Delta x + \Delta x'$, while an analogous idea holds also for Π_{p_x}; as we have shown, just the probabilities that both initial ranges shrink to new ranges fulfilling eqs. (1,1) entail the numbers of states n and thus the existence of the respective eigenvalues. This suggests that B and $B_{\rm n}$ describe respectively the chances of leaving the initial delocalization range unchanged or not after the perturbation induced by the observer, whereas C and $C_{\rm n}$ concern in an analogous way the momentum ranges of the particle. As regards A, it represents the existence of an eigenvalue of the particle; of course $A_{\rm n}$ means that delocalization and momentum ranges of the particle remain unchanged and so unrelated, thus not corresponding to any number of states. The notation N_n relates thus the inequality (4,4) to any possible eigenvalue. For instance: since n requires that are verified both favorable probabilities (3,1), it is reasonable to think that the various probabilities P_n corresponding to eq. (4,4) fulfill also the condition

$$P_n(A, B_{\rm n})P_n(A, C_{\rm n}) + P_n(A_{\rm n}, B)P_n(A_{\rm n}, C) = 1. \tag{4,5}$$

In effect, it is possible to normalize eq. (4,4) be means of an appropriate numerical factor in order to express the various numbers N_n of occurrences/non-occurrences through their respective probabilities P_n for one particle only. The first addend of eq. (4,5) represents the probability of getting an eigenvalue as a consequence of the measurement process, the second does not; in fact this idea was already introduced through the probabilities Π_n and Π'_n of eqs. (3,2) and (3,4). The sum of both chances that correspond to the Bell-like inequality

$$P_n(A, B_{\rm n}) + P_n(B, C_{\rm n}) - P_n(A, C_{\rm n}) \geq 0$$

must be of course equal to 1 in eq. (4,5). Let us try now to correlate term by term eqs. (4,2) and (4,4); the latter concerns directly the numbers of occurrences/non-occurrences leading to the n-th number of states allowed for one particle. This correlation yields

$$\Delta x = \Delta x_1 \exp\left(N_n(A, B_{\rm n})\right),$$

$$\Delta p_x = \Delta p_1 \exp\left(N_n(B, C_{\rm n}) - N_n(A, C_{\rm n})\right).$$

To verify if these equations can be simultaneously fulfilled, let us multiply them side by side; recalling that by definition $\Delta x_1 \Delta p_1 = \hbar$, one obtains

$$n = \exp\left(Q_n\right),$$

$$Q_n = N_n(A, B_{\rm n}) + N_n(B, C_{\rm n}) - N_n(A, C_{\rm n}) \geq 0. \tag{4,6}$$

So the result is that n must be equal just to the exponential of the number Q_n of occurrences/non-occurrences of the Bell-like inequality. It is clear however that in general the first equation (4,6) is false. Even admitting the chance that it is effectively verified for one among the possible numbers of states, say $n^{\S}$, by an appropriate value $Q_{n^{\S}}$, what about other numbers of states like for instance $n^{\S} - 1$ or $n^{\S} + 1$? It is clear that a hypothesis should be made on the respective $Q_{n^{\S}-1}$ and $Q_{n^{\S}+1}$. However the Bell-like inequality (4,3) does not prospect itself any indication about such a hypothesis, which therefore would require an "ad hoc" assumption valid for all arbitrary integers n progressively increasing from 1 by steps of 1 until to infinity. Note in this respect that the impossibility of eqs. (1,1) to fulfil the Bell-like inequality is in fact due to the quantization of n; if this latter could take any non-quantized value, then eq. (4,6) would be fulfilled in principle whatever Q_n might be. Hence is just the quantization of the eigenvalues that makes itself non-real and non-local the quantum world. In effect for $n \to \infty$ the number n approximates better and better a continuous variable of the classical physics, whence the realism and localism of the macroscopic classical world.

5 Uncertainty and special relativity

After having justified why the uncertainty ranges of position and momentum entail non-locality and non-reality, remains the concept of time and energy uncertainty to be better explained in the frame of such a conceptual context. Consider that also the time measurement requires a macroscopic apparatus, whose outcome is nothing else but the time of the observer. The question arises: is the observer time coincident with that of the particle? This question can be answered considering first that during the measurement process eqs. (1,1) apply to different reference systems, about which no hypothesis is made. Suppose that eqs. (1,1) refer to the particle; we must rewrite them as $\Delta x' \Delta p'_x = n'\hbar = \Delta\varepsilon' \Delta t'$ for the observer. Let R and R' be the respective reference systems; in both cases the ranges are completely arbitrary by definition, as concerns their sizes and analytical form. For instance it is not possible to establish if $\Delta x = x_o + v_x \Delta t$ or if $\Delta x = \sqrt{x_o^2 + (v_x \Delta t)^2}$ or anything else. The same holds also for the momentum range and for the energy range. Moreover n and n' are not assigned values, rather they are mere notations to indicate any integer unspecified and unspecifiable. So n and n' remain indistinguishable despite any integer of either reference system might turn into a different integer in the other reference system. Hence the arbitrariness of the analytical form of the ranges does not contradict the validity of eqs. (1,1) in different reference systems despite the chance of their possible size changes; the uncertainty equations (1,1) hold identically in R and in R', regardless of whether they refer to particle and observer in the respective reference systems. So, whatever the sizes of Δx of the particle and $\Delta x'$ of the observer might be, in principle eqs. (1,1) do not require that the time ranges Δt and $\Delta t'$ coincide. Recall now that the time range was introduced in section 1 to infer eqs. (1,1) through the positions $\Delta t = \Delta x / v_x$, which thus requires analogously $\Delta t' = \Delta x' / v'_x$, and note that both signs are allowed for the velocity components v_x and v'_x defined in R and R'. This means that with respect to the origin O of R we expect $\Delta x \pm v_x \Delta t = 0$ depending on whether the particle moves leftwards or rightwards. A possible position to summarize into a unique equation these chances regardless of either sign of v_x is $\Delta x^2 - v_x^2 \Delta t^2 = 0$; to this result corresponds of course an analogous expression in R', i.e. $\Delta x'^2 - v'^2_x \Delta t'^2 = 0$. Hence it is possible to write

$$\Delta x'^2 - v'^2_x \Delta t'^2 = 0 = \Delta x^2 - v_x^2 \Delta t^2. \qquad (5,1)$$

Both v_x and v'_x are reminiscent of the respective reference systems where they have been initially defined. Since no constraint is required for these velocities, both arbitrary by definition, the last equation allows replacing v_x and v'_x with any other values of velocity still defined in R and R'; so

$$\Delta x'^2 - v''^2_x \Delta t'^2 = \delta s^2_{v'',v'''} = \Delta x^2 - v'''^2_x \Delta t^2 \quad \delta s^2_{v'',v'''} \neq 0. \qquad (5,2)$$

Being unchanged the delocalization range sizes at right hand side, the interval $\delta s^2_{v'',v'''}$ is no longer equal to zero once having replaced v_x^2 with v'''^2_x; yet this does not hinder that this interval is still equal to the expression at left hand side if v'_x is replaced by another appropriate velocity v''_x also defined in R'; thus remains unchanged the analytical form of eqs. (5,1) and (5,2). In this way we have found a unique interval $\delta s^2_{v'',v'''}$ common to both reference systems R and R'. Yet this result is not a property of an interval defined by uncertainty ranges only, as it involves the presence of a particle through its displacement velocity; however it is interesting the fact that $\delta s^2_{v',v'''}$ does not require specific values of v''^2_x and v'''^2_x, which are indeed arbitrary like the ranges themselves. In the paper [15], was identified a velocity invariant in any reference system, called $v_x^{\max}$, i.e. the maximum average velocity with which any particle can displace in any Δx. This suggest the chance of expressing eqs. (5,2) just through this velocity, which will be called from now on c. If in particular we replace v''^2_x and v'''^2_x with c, then

$$\Delta x'^2_c - c^2 \Delta t'^2_c = \delta s^2_c = \Delta x_c^2 - c^2 \Delta t_c^2 \quad \delta s_c \neq 0. \qquad (5,3)$$

This result contains new delocalization ranges that can be chosen in order to generalize the previous result; this can be certainly done in agreement with this appropriate choice of the velocity, to which refers indeed the subscript c. In general eq. (5,3) holds for δs_c not necessarily equal to zero and represents a real step onwards with respect to eq. (5,2) because of the peculiar property of c, which is defined regardless of a specific reference system. The only quantities that depend on R are Δx_c and Δt_c that define δs_c regardless of the presence itself of any kind of particle thanks to the universal character of c. In conclusion, the present discussion allowed to find a

relationship that describes the form of an interval invariant in R and R', thus in any other reference system. Since this result has been obtained from eqs. (1,1), it is also compliant with the requirements of non-locality and non-reality previously introduced. The interval rule is a fundamental statement of special relativity, for instance it allows to infer the Lorentz transformations of space, time, momentum and energy [18]. However, apart from the formal analogy, the ranges introduced here have fully quantum physical meaning, i.e. they are uncertainty ranges; instead the ranges of relativity have the deterministic character of classical physics, i.e. they are defined as a function of selected local coordinates in principle exactly known. Therefore eq. (5,3) shows that even the relativity can be made compliant with the requirements of the quantum world provided that the local dynamical variables be discarded as done here and the macroscopic deterministic ranges take the physical meaning of uncertainty ranges. This crucial step, although abstractly simple, is certainly non-trivial as concerns the different way of regarding the conceptual basis of relativity. The next considerations concern just the consequences of this conclusion. From eq. (5,3) and according to eqs. (1,1) one infers, omitting for simplicity the subscripts c and x from now on but still intending that v is a component of average velocity along an arbitrary axis,

$$\frac{c^2\Delta t'^2}{c^2\Delta t^2} = \frac{(v/c)^2 - 1}{(v'/c)^2 - 1}, \qquad v = \Delta x/\Delta t, \qquad v' = \Delta x'/\Delta t'. \tag{5,4}$$

Putting in this equation $c \to \infty$, i.e. in the non-relativistic limit, $\Delta t' \to \Delta t$; as expected, without a finite light speed one finds the absolute time of Newton. Suppose now R and R' displacing each other at constant rate V such that in either of them, say in R, the particle is at rest. In the particular case $v = 0$, therefore, v' is just the rate V with which R displaces with respect to R'; of course it is also identically possible to put $v' = 0$, in which case $v = -V$. Since we have two equivalent ways to regard v and v', let us exploit for instance the first chance to find the transformation properties of the time range and the second chance for the space range; in the latter case it is convenient to put in eq. (5,3) $\delta s_c = 0$ to infer directly $c\Delta t_c = \Delta x_c$ and $c\Delta t'_c = \Delta x'_c$. One finds then

$$\Delta t' = \Delta t\Big(1 - (V/c)^2\Big)^{-1/2}, \qquad \Delta x'_c = \Delta x_c\Big(1 - (V/c)^2\Big)^{1/2}. \tag{5,5}$$

Actually the subscript c could have been omitted in the second equation; being arbitrary both time ranges of eq. (5,3), it holds in fact for any Δx and $\Delta x'$. The relevant remark is however that to time dilation corresponds length contraction in the primed reference system. It is also immediate to find the expressions of momentum and energy of a free particle. Let us consider first the following equalities obtained from eqs. (1,1) in the particular case $n = 1$

$$\Delta p^{(v)}\Delta x^{(v)} = \Delta t^{(v)}\Delta\varepsilon^{(v)} = \Delta t^{(c)}\Delta\varepsilon^{(c)} = \hbar,$$

$$\Delta t^{(c)} = \Delta t_{\min}, \qquad \Delta\varepsilon^{(c)} = \Delta\varepsilon_{\max}.$$

The superscripts emphasize the values taken by the velocity v in the various cases; the subscripts emphasize that when $v = c$ the traveling time is minimum whereas $\Delta\varepsilon$ is maximum, both consistently with $\hbar$ and with the arbitrary $\Delta p^{(v)}$ and $\Delta x^{(v)}$ describing a slower massive particle. These positions are important as they compel specifying how, in a given reference system, $\Delta p^{(v)}$ and $\Delta\varepsilon^{(v)}$ scale with respect to $\Delta p^{(c)}$ and $\Delta\varepsilon^{(c)}$ when $v < c$. Since $\Delta\varepsilon^{(c)} = cp_2^{(c)} - cp_1^{(c)}$, then $\varepsilon^{(c)} = cp^{(c)}$ by definition; here $\varepsilon^{(c)}$ and $p^{(c)}$ are random local values of energy and momentum within their own uncertainty ranges. For a slower massive particle $\Delta t^{(v)}$ and $\Delta\varepsilon^{(v)}$ scale like c/v and v/c with respect to $\Delta t^{(c)}$ and $\varepsilon^{(c)}$; hence, according to the former equality, $\varepsilon^{(v)} = \varepsilon^{(c)}v/c$ requires $p^{(v)}$ scaling with respect to $p^{(c)}$ like $cp^{(v)} = \varepsilon^{(c)}v/c$, i.e. $p^{(v)} = \varepsilon^{(c)}v/c^2$. Being $p^{(v)}$ and $\varepsilon^{(c)}$ random local quantities within the respective ranges, the functional relationship between any possible value of momentum and energy must be

$$p = \varepsilon v/c^2. \tag{5,6}$$

Momentum and energy of a free particle are constants both in classical physics and in special relativity. However eq. (5,6) is here a quantum result, which therefore must be accordingly handled. Let us admit that during a short time range δt even the energy of a free particle is allowed to fluctuate randomly by $\delta\varepsilon$. Eq. (5,6) is thus exploited to calculate the link between $\delta\varepsilon$ and related values of δp and δv during the time transient where the fluctuation allows the particle moving in altered way. Differentiating eq. (5,6) one finds $\delta\varepsilon = c^2\delta p/v - p(c/v)^2\delta v$: once having fixed p and v, this result defines the functional dependence of $\delta\varepsilon$ upon arbitrary δp and $\delta v = v_2 - v_1$ defined by two arbitrary values v_1 and v_1. Summing $\delta\varepsilon$ and eq. (5,6) one finds $\varepsilon + \delta\varepsilon = c^2(p + \delta p)/v - \varepsilon\delta v/v$. Note now that in general $\delta p\delta x = n\hbar$ reads identically $(\delta p)^2 = n\hbar\delta p/\delta x$, whereas in an analogous way $(\delta\varepsilon)^2 = n\hbar\delta\varepsilon/\delta t$. Regard in this way just the new ranges $\varepsilon + \delta\varepsilon$ and $p + \delta p$; putting $\delta x = v\delta t$ and replacing in the last expression to calculate $\delta(\varepsilon + \delta\varepsilon)/\delta t$, one finds

$$(n\hbar)^{-1}(\Delta\varepsilon)^2 = (n\hbar)^{-1}(\Delta pc)^2 - \varepsilon\delta\omega, \tag{5,7}$$

$$\Delta\varepsilon = \varepsilon + \delta\varepsilon, \qquad \Delta p = p + \delta p.$$

The last addend results because $v/\delta x$ has physical dimensions of a frequency ω, so that $\delta v/\delta x = \omega_2 - \omega_1$. Since $n\hbar\omega\delta\varepsilon = \delta(\varepsilon n\hbar\omega) - \varepsilon\delta(n\hbar\omega)$, replacing this identity in the last equation one finds $(\Delta\varepsilon)^2 = (\Delta pc)^2 + n\hbar\omega\delta\varepsilon - \delta(\varepsilon n\hbar\omega)$. Let us specify this result via the position

$$n\hbar\omega = \delta\varepsilon \tag{5,8}$$

which yields also $(\Delta\varepsilon)^2 - (\Delta pc)^2 = (\delta\varepsilon)^2 - \delta(\varepsilon\delta\varepsilon)$. At left hand side appear terms containing the ranges $\varepsilon + \delta\varepsilon$ and $p + \delta p$ only, at right hand side the ranges $\delta\varepsilon$ and δp only; so it is reasonable

to expect that the last equation splits into two equations linked by a constant energy ε_o

$$(\Delta\varepsilon)^2 - (\Delta pc)^2 = \varepsilon_o^2 = (\delta\varepsilon)^2 - \delta(\varepsilon\delta\varepsilon).$$

Indeed ε_o agrees with both of them just because it does not depend upon neither of them. Trivial manipulations show that the first equation yields

$$p = \pm\frac{\varepsilon_o v/c^2}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}}, \qquad \varepsilon = \pm\frac{\varepsilon_o}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}}, \quad (5,9)$$

$$r_p = 1 + \frac{\delta p}{p}, \qquad r_\varepsilon = 1 + \frac{\delta\varepsilon}{\varepsilon}.$$

As expected, eq. (5,6) results fulfilled even during the transient. The value of the constant ε_o is immediately found through the following boundary condition consequence of eq. (5,6)

$$\lim_{v\to 0}\frac{p}{v} = \frac{\varepsilon_{rest}}{c^2} = m. \qquad (5,10)$$

Then $\varepsilon_o^2 = \varepsilon_{rest}^2$. Eqs. (5,9) hold during the time transient allowing $\delta\varepsilon$; before and after that transient one must put $\delta\varepsilon = 0$ and $\delta p = 0$ which yields the "standard" Einstein momentum and energy of the particle, which are of course

$$\varepsilon_{Ein}^2 = c^2 p_{Ein}^2 + \varepsilon_{rest}^2, \qquad \varepsilon_{rest} = mc^2, \qquad (5,11)$$

$$p_{Ein} = \pm\frac{mv}{\sqrt{1-(v/c)^2}}, \qquad \varepsilon_{Ein} = \pm\frac{mc^2}{\sqrt{1-(v/c)^2}}.$$

It is easy now to calculate the energy and momentum gaps $\varepsilon - \varepsilon_{Ein}$ and $p - p_{Ein}$ during the time transient δt as a function of $\delta p/p$ and $\delta\varepsilon/\varepsilon$ as follows

$$\frac{mv}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}} - \frac{mv}{\sqrt{1-(v/c)^2}} = \frac{\hbar}{\delta l}, \qquad (5,12)$$

$$\frac{mc^2}{\sqrt{r_\varepsilon^2 - r_p^2(v/c)^2}} - \frac{mc^2}{\sqrt{1-(v/c)^2}} = \frac{\hbar}{\delta t}.$$

These equations, which are nothing else but the uncertainty equations of the fluctuation gaps, will be commented and exploited in section 7. The chance of obtaining the eqs. (5,6), (5,10) and (5,11) could be reasonably expected; in the paper [15] it was shown that eqs. (1,1) only are enough to infer the following corollaries: (i) equivalence of all inertial reference systems in describing the physical laws, (ii) existence of a maximum average displacement rate allowed for any particle in its delocalization range and (iii) invariance in all reference systems of such a maximum velocity. These corollaries are in fact the basic statements of special relativity. Five further remarks are crucial in this respect: (i) the mass m is not introduced here as the familiar concept of everyday common experience, rather the mass is inferred itself as a consequence of the uncertainty; (ii) the analytical expressions of energy and momentum have been obtained without need of any hypothesis additional to eqs. (1,1); (iii) the most representative formulas of special relativity are here obtained as straightforward consequences of the quantum uncertainty through trivial algebraic manipulations of eqs. (1,1) only; (iv) eqs. (5,11) are typical expressions of particle behaviour of matter, eq. (5,8) involves instead the wave behavior of matter too, because the frequency ω is a typical property of waves; unifying both properties into a unique equation leads to the well known relativistic formulas; (v) uncertainty ranges only appear in formulas coincident with that, well known, of the special relativity.

Note in this respect that the Einstein deterministic approach excludes the random fluctuation of velocity, energy and momentum, which is a typical quantum phenomenon; here instead the well known eqs. (5,11) are particular cases only of the more general eqs. (5,9) taking into account the possibility of fluctuations, in agreement with the fact that here the Einstein intervals here are actually quantum uncertainty ranges. Just this last statement opens the way to further considerations, carried out in section 7. Before exploiting the results of the present section, however, the next section 6 will concern a further topic previously introduced: the possibility of defining uncertainty sub-ranges included in larger ranges. The aim is to clarify the physical meaning of such a further way to regard the quantum uncertainty.

6 Uncertainty and operator formalism of wave mechanics

It is well known that the uncertainty principle is a consequence of the operator formalism of wave mechanics. This section aims to emphasize that the reverse path is also possible: here we show how to infer the momentum and energy wave equations starting from eqs. (1,1). This result is nontrivial: it emphasizes that the fundamental basis of the present theoretical approach leads also to the early wave equations from which has been developed the modern formulation of quantum mechanics. The uncertainty inherent Δx does not prevent to define in principle the probability $\Pi = \Pi(x,t)$ that the particle be in an arbitrary sub-range δx inside the total range

$$\frac{\delta x}{\Delta x} = \Pi, \qquad \delta x = x - x_o, \qquad \delta x \le \Delta x, \qquad (6,1)$$

provided that hold for δx the same uncertainty features of Δx; so no hypothesis is made about δx. Moreover x and x_o are both arbitrary and unknown likewise that of Δx; there is no chance of defining width or location of δx within Δx or distinguishing δx with respect to any other possible sub-range. In general Π is expected to depend on space coordinate and time; yet we consider first the explicit dependence of Π on x only, i.e. t is regarded as fixed parameter in correspondence to which are examined the properties of Π as a function of

x. Regard the width of δx variable, with x current coordinate and x_o constant. The couples of coordinates defining Δx and Δp_x are instead considered fixed. Eqs. (6,1) yield

$$\frac{1}{\Delta x} = \frac{\partial \Pi}{\partial x}, \qquad \Pi = \Pi(x, t). \tag{6,2}$$

Let Π and $1 - \Pi$ be the chances for the particle to be or not within δx and be n_+ and n_- the arbitrary numbers of states consistent with the respective probabilities. Putting

$$\delta x \Delta p = n_+ \hbar, \qquad (\Delta x - \delta x)\Delta p = n_- \hbar, \qquad n_+ + n_- = n, \tag{6,3}$$

then $n_+/n + n_-/n = 1$; also, eq. (6,3) yields the identity

$$(1 - \Pi)\Pi \Delta p^2 = n_- n_+ \hbar^2 \left(\frac{\partial \Pi}{\partial x}\right)^2. \tag{6,4}$$

Putting $n_+ n_- = n' + n''$, where n' and n'' are further arbitrary integers, eq. (6,4) splits as follows

$$\Pi \Delta p^2 = n' \hbar^2 \left(\frac{\partial \Pi}{\partial x}\right)^2, \tag{6,5a}$$

$$\Pi^2 \Delta p^2 = -n'' \hbar^2 \left(\frac{\partial \Pi}{\partial x}\right)^2. \tag{6,5b}$$

Since n_+ and n_- are by definition positive, at least one among n' and n'' or even both must be positive. Consider separately the possible signs of n' and n''.

Case (i) $n' > 0$ and $n'' < 0$. Eqs. (6,5) read also $\delta x \Delta p = (n'/n)\hbar$ and $\delta x^2 \Delta p^2 = |n''| \hbar^2$ because of eqs. (6,1) and (6,2). Moreover multiplying both sides of the latter by $|n''|$ and both sides of the former by $n^{\S} n/n'$, with $n^{\S}$ arbitrary integer, one finds

$$\delta x'' \Delta p = n'' \hbar, \qquad \delta x^{\S} \Delta p = n^{\S} \hbar,$$

where $\delta x'' = \sqrt{|n''|}\delta x$ and $\delta x^{\S} = (n^{\S} n/n')\delta x$. Also, $(n'/n)^2 = |n''|$ and $\Pi = |n''|/n'$. These results are mutually consistent for any integers at right hand sides, because are arbitrary not only n' and n'' but also δx; indeed the new uncertainty equations have an analogous form and physical meaning. Hence eqs. (6,5) do not exclude each other and are both acceptable; yet they are both formally analogous also to the initial eq. (1,1), the only difference being the size of their space uncertainty ranges only. In conclusion, being the sizes arbitrary by definition, this combination of signs of n' and n'' does not entails anything new with respect to eq. (1,1), and thus has no physical interest.

Case (ii) $n' < 0$ and $n'' > 0$. The right hand sides of both eqs. (6,5) have negative sign, so neither of them can have the same physical meaning of the initial eq. (1,1); they read $\Pi = -|n'|/n^2$ and $\Pi^2 = -n''/n^2$ because of eq. (6,2). Yet the result $\Pi = n''/|n'| = -|n'|/n^2$ is clearly absurd, so also this combination of signs has no physical interest.

Case (iii) $n' > 0$ and $n'' > 0$. Eqs. (6,5) are now physically different, because their ratio would entail Π negative. Thus these equations cannot be combined together, because of their different ways to describe the particle delocalized in Δx; they must be considered separately. Eq. (6,5a) is conceptually analogous to eq. (1,1); eq. (6,5b) excludes eq. (6,2) and admits the solution $\Pi = A' \exp(\pm i(x - x_o)\Delta p/\hbar \sqrt{n''})$, being A' the integration constant. Rewriting $\Pi = A \exp(\pm i\varphi \delta x/\Delta x)$ with $\varphi = n/\sqrt{n''}$, the probability Π inferred here significantly differs from Π of eq. (6,5a) despite the same notation; the former is indeed a complex function, the latter coincides instead with eq. (6,1). Both are however definable in principle.Thus eq. (6,5b) still retains the essential concept of delocalization within an arbitrary uncertainty range, yet without concerning itself the ability of regarding the particle as a corpuscle in any specific point of Δx.

The following discussion concerns the case (iii). To accept both eqs. (6,5) together, we must acknowledge their different form, i.e. their different way to describe the particle delocalization inside Δx. This dual outcome reveals however the inadequacy of regarding the particle as mere corpuscle delocalized somewhere in its uncertainty range, as required by eqs. (1,1). Despite the particle must be anyway randomly moving in Δx, eq. (6,5b) is incompatible with the corpuscle-like behaviour of eq. (6,5a). A further difficulty to regard together eqs. (6,5a) and (6,5b) is that Π defined by this latter is not real, as instead $\Pi^* \Pi = |const|^2$ does. Yet just this property suggests a possible way out from this difficulty, i.e. supposing that eq. (6,5b) requires a wave-like propagation of the particle: so $\Pi^* \Pi$ could stand for particle wave amplitude whereas A', in fact regarded here as $A_0 A(t)$ without contradicting any previous step, could define frequency and phase of the particle wave. This idea is confirmed rewriting the exponential $x\Delta p$ of Π as $t\Delta\varepsilon$ dividing and multiplying by an arbitrary velocity v in order that $\pm ix\Delta p/\hbar\sqrt{n''}$ turns into $\pm it\Delta\varepsilon/\hbar\sqrt{n''}$. So $A(t)$ results defined just by this requirement, i.e.

$$\Pi = A_0 \exp[\pm i(c_x(x - x_o)\Delta p + c_t(t - t_o)\Delta\varepsilon)/\hbar\sqrt{n''}], \tag{6,6}$$

being c_x and c_t arbitrary coefficients of the linear combination expressing the most general way to unify the space and time functions. Calculate $\partial^2\Pi/\partial x^2 = -(c_x\Delta p)^2\Pi$ to extract the real quantity $c_x\Delta p$ from Π, and then by analogy $\partial^2\Pi/\partial t^2 = -(c_t\Delta\varepsilon)^2\Pi$ too; eliminating Π between these equations and noting that by dimensional reasons $(c_x\Delta p/c_t\Delta\varepsilon)^2 = v^{-2}$, the result $\partial^2\Pi/\partial x^2 - v^{-2}\partial^2\Pi/\partial t^2 = 0$ confirms, whatever v might be, the wave-like character of particle delocalization provided by eq. (6,5b). A similar wave equation could not be inferred from eq. (6,5a), according which the physical properties of the particle are related directly to the probability Π of eq. (6,1); instead, owing to the complex form of Π resulting from eq. (6,5b), the physical properties of the wave are related to $\Pi^*\Pi$. It is possible to eliminate this discrepancy introducing the complex function $\sqrt{\Pi}$ in place of Π and rewriting eq. (6,5b) as a function of the former instead of the latter; this idea agrees with that already exploited to find eqs.

(3,3). Dividing both sides by Π, eq. (6,5b) reads

$$\left(\pm\hbar\frac{\partial\sqrt{\Pi}}{\partial x}\right)^2 = -\left(p^{\S}\sqrt{\Pi}\right)^2, \qquad p^{\S} = \pm\frac{\Delta p}{2\sqrt{n''}}. \quad (6,7)$$

The notation emphasizes that $p^{\S}$ does not depend on x and is not a range; being defined as solution of the differential equation (6,7) only, its value is not longer related to Δp, i.e. it is an eigenvalue of $\sqrt{\Pi}$. This is possible because n'' is arbitrary like Δp, which allows that the ratio $\Delta p/2\sqrt{n''}$ behaves as a well determined quantity specified just by $p^{\S}$, whose value and signs correspond to either component of momentum along the x-axis where are defined positive δx and Δx. Thus eq. (6,7) reads

$$\pm\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial x} = p^{\S}\sqrt{\Pi}, \qquad \sqrt{\Pi} = \sqrt{A}\sqrt{\exp(\pm i\varphi\delta x/\Delta x)}. \quad (6,8)$$

So $\sqrt{\Pi}\sqrt{\Pi^*}$ expresses the probability to find the particle so-mewhere in Δx. Write thus

$$\sqrt{\Pi}\sqrt{\Pi^*} = \pm\frac{\hbar}{ip^{\S}}\frac{\sqrt{\Pi^*}\partial\sqrt{\Pi}}{\partial x}.$$

The right hand side is real and yields $\sqrt{\Pi}\sqrt{\Pi^*} = \delta x_0/\Delta x = A_0$, being $\delta x_0 = A_0\hbar\varphi/2p^{\S}$. As a proper value of A_0 certainly exists such that $\delta x_0 \le \Delta x$, then $\sqrt{\Pi}\sqrt{\Pi^*}$ agrees with a concept of probability similar to that of the initial definition $\delta x/\Delta x$ of eq. (6,1); yet this latter is replaced in the last equation by a constant value, which entails thus equal probability to find the particle in any sub-range δx_0 regardless of its size and position within Δx. The physical meaning of this result is emphasized integrating both sides of eq. (6,8) with respect to x in the sub-range $\delta x_0 = x_{02} - x_{01}$, which yields

$$p^{\S} = \pm\left(\int_{x_{01}}^{x_{02}}\sqrt{\Pi}\sqrt{\Pi^*}dx\right)^{-1}\int_{x_{01}}^{x_{02}}\left(\sqrt{\Pi^*}\frac{\hbar}{i}\frac{\partial}{\partial x}\sqrt{\Pi}\right)dx. \quad (6,9)$$

The average value of momentum is thus equal to the eigenvalue expected for the steady motion of a free particle (Ehre-nfest's theorem), which suggests regarding $\delta x_0/\Delta x$ as average probability that the particle is in the sub-range δx_0. It is clearly convenient therefore to define A_0 in order that $\delta x_0 = \Delta x$ through $\int\sqrt{\Pi}\sqrt{\Pi^*}dx = 1$, i.e. the momentum eigenvalue concerns the certainty that the particle is really delocalized in the total range Δx. Being this latter arbitrary, it allows considering in general the particle from $-\infty$ to ∞. The physical information provided by eq. (6,5b) is thus really different from that of eq. (6,5a), although being unquestionable the consistency of eqs. (6,8) and (6,9) with the initial eq. (6,1) despite their different formulation: both come indeed from the same uncertainty equations (1,1). So it is not surprising that the uncertainty is still inherent $\sqrt{\Pi}$ and consistent with the eigenvalue $p^{\S}$. It is evident at this point that the results hitherto inferred concern just the basic ideas through which has been formulated the early quantum mechanics; it is enough to regard in general the wave functions in analogous way, e.g. as it is shown below for the energy eigenfunction. So, write $\psi = const\sqrt{\Pi}$ and $\psi^* = const\sqrt{\Pi^*}$ to define the probability density of the particle within the volume $\Delta x\Delta y\Delta z$; this is just the volume to normalize $\psi\psi^*$. Being the uncertainty ranges arbitrary, this probability density concerns actually the whole space allowed to the particle. The normalization constant is inessential for the purposes of the present paper and not explicitly concerned hereafter. The result of interest is that, after having introduced the probability Π of eq. (1,1), one finds two distinct equations concurrently inferred from the respective eqs. (6,5)

$$\Delta p^{\S}\Delta x^{\S} = n^{\S}\hbar, \quad (6,10a)$$

$$\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial x} = \pm p^{\S}\sqrt{\Pi}. \quad (6,10b)$$

Two comments about eqs. (6,10):

(i) eq. (6,10a) is conceptually equal to the initial eq. (1,1), from which it trivially differs because of the size of the uncertainty ranges and related number of states; (ii) eq. (6,10b) defines a differential equation that calculates an eigenvalue of momentum through the probability that the particle be in a given point of its allowed range $\Delta x^{\S}$.

Eq. (6,10a) does not consider explicitly the particle, but only its delocalization inside $\Delta x^{\S}$ and thus its phase space; the same holds also for the momentum, whence the positions (2,1) and the indistinguishability of identical particles whose specific properties are disregarded "a priori". The unique information available concerns indeed the number of states $n^{\S}$ consistent with $\Delta x^{\S}$ and $\Delta p^{\S}$ for any delocalized particle; nothing requires considering the local dynamical variables themselves. The point of view of eq. (6,10b) is different: it considers explicitly the sub-range δx through $\sqrt{\Pi}$ and thus, even without any hypothesis about size and position of δx within $\Delta x^{\S}$, concerns directly the particle itself through its properties $\sqrt{\Pi}\sqrt{\Pi^*}$ and $p^{\S}$; both these latter are explicitly calculated solving the differential equation. Yet the common derivation of both eqs. (6,10) from the initial eq. (1,1) shows that actually the respective ways to describe the particle must be consistent and conceptually equivalent, as in effect it has been verified in section 2. This coincidence evidences the conceptual link between properties of the particles and phase space; it also clarifies why the quantum eigenvalues do not depend on the current values of the dynamical variables of the particles, even though calculated solving the differential equation (6,10b). Initially Π was introduced in eq. (6,1) as mere function of uncertainty ranges and sub-ranges of the phase space; thereafter, however, it has taken through the steps from eqs. (6,2) to (6,10) the physical meaning of wave function $\sqrt{\Pi}$ of the particle defining the momentum eigenvalue $p^{\S}$, which involves the mass of the particle. Eq.

(6,10b) introduces the operator formalism of wave mechanics. The approach starting directly from eqs. (1,1) has therefore more general character than the latter, which starts just postulating eq. (6,10b) here found instead as a corollary: the basic reason is that eq. (6,10a) contains less information than eq. (6,10b). These equations can be now regarded together once having acknowledged the kind of information inferred from eqs. (1,1). On the one side eqs. (6,10) introduce the wave/corpuscle dual nature of particles: eq. (6,10a) admits that the particle is somewhere in Δx, even though renouncing to know exactly where because of the delocalization; eq. (6,10b) instead regards the particle as a wave propagating within Δx thus still delocalized but excluding in principle the unknown position of a material corpuscle. On the other side eqs. (6,10) confirm that properties of particles and properties of phase space must not be regarded separately, rather they are intrinsically correlated: just for this reason the results of section 2 show that the numbers of quantum states (properties of the phase space) coincide with the quantum numbers that define the eigenvalues (properties of the wave function of the particle). Further properties of $\sqrt{\Pi} = \psi$ could be easily found, e.g. the concept of parity or the fact that the arbitrariness of the coefficients c_x and c_t previously introduced in the early expression $\Pi = A_0 \exp[\pm i(c_x x\Delta p + c_t t\Delta\varepsilon)/\hbar\sqrt{n''}]$ allows to write the more general form for this equation

$$\Pi = \sum_j A_{0j} \exp[\pm i(c_{xj} x\Delta p_j + c_{tj} t\Delta\varepsilon_j)/\hbar\sqrt{n''_j}].$$

All these assertions are well known since the early birth of the quantum theory and do not need further consideration here for sake of brevity; their evolution brings the theory up to today's formulation. It is more interesting to examine the same problem considering the time instead of the space coordinate. The steps to find the energy operator are conceptually identical to those so far reported; yet one regards the probability for the particle to be in δx at the time t, i.e. Π is defined as ratio between the time range $\delta t = t - t_o$ spent within a fixed δx and the total time range $\Delta t = t_2 - t_1$ spent elsewhere within Δx. Let us write then $\Pi = \delta t/\Delta t$ at fixed coordinate x; eqs. (6,2) and (6,4) read now $\Delta t^{-1} = \partial\Pi/\partial t$ and $(1-\Pi)\Pi\Delta\varepsilon^2 = n_- n_+ \hbar^2 (\partial\Pi/\partial t)^2$. Replacing position and momentum with time and energy in eq. (6,2), eqs. (6,7) read

$$\left(\pm\hbar\frac{\partial\sqrt{\Pi}}{\partial t}\right)^2 = -\left(\varepsilon^{\S}\sqrt{\Pi}\right)^2, \qquad \varepsilon^{\S} = \pm\frac{\Delta\varepsilon}{2\sqrt{n''}}. \quad (6,11)$$

The second eq. (6,8) reads now $\sqrt{A}\sqrt{\exp(\pm i\varphi\delta t/\Delta t)}$, which however is disregarded here because it appears included in eq. (6,6); the first eq. (6,8) becomes

$$-\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial t} = \pm\varepsilon^{\S}\sqrt{\Pi}. \quad (6,12)$$

With the upper sign at right hand side of eq. (6,12), the classical Hamiltonian written with the help of eq. (6,8) is consistent with the result $\varepsilon^{\S} = p^{\S 2}/2m$ in the particular case of a free particle having mass m and momentum $p^{\S}$. Yet the lower sign, also allowed as a consequence of eq. (6,11), shows the possibility of states with negative energy as well. The couple of equations (6,10) turns into

$$\Delta t^{\S}\Delta\varepsilon^{\S} = n^{\S}\hbar, \quad (6,13a)$$

$$-\frac{\hbar}{i}\frac{\partial\sqrt{\Pi}}{\partial t} = \pm\varepsilon^{\S}\sqrt{\Pi}. \quad (6,13b)$$

For this couple of equations hold the same considerations carried out for the corresponding eqs. (6,10). This section has shown that the operator formalism of wave mechanics is consequence itself of the concept of uncertainty. On the one side this result explains why the properties of quantum particles can be obtained as shown in section 2 even without solving any wave equation. On the other side it appears clearly that both chances of describing the quantum world are nothing else but mirror consequences of the dual wave/corpuscle behavior of particles. All considerations so far carried out do not require knowing anything about the concerned uncertainty ranges.

7 Heuristic aspects of quantum special relativity

Let us introduce now some comments about eqs. (5,9) and (5,11) before exploiting eqs. (5,12). The momentum and energy equations during the quantum fluctuation transient rewritten identically as follows

$$p(t) = \pm\frac{mv_{eff}/r_p}{\sqrt{1-(v_{eff}/c)^2}}, \quad \varepsilon(t) = \pm\frac{mc^2/r_\varepsilon}{\sqrt{1-(v_{eff}/c)^2}}, \quad (7,1)$$

$$v_{eff} = r_p v/r_\varepsilon, \qquad r_p = r_p(t), \qquad r_\varepsilon = r_\varepsilon(t),$$

evidence that the Einstein quantities of eqs. (5,11) turn into new constant expressions calculated with an effective velocity and multiplied by the respective functions of time; the previous velocity v does not longer appear explicitly into the equations. If v_{eff} is regarded as a constant, then v turns into a time variable without contradicting the Einstein equations, whose deterministic character does not admit any fluctuation and requires a steady value of v; the fluctuation has been instead introduced by admitting the quantum meaning of $\delta\varepsilon$, δp and δv. The notation of eqs. (7,1) emphasizes that energy and momentum are functions of time during the transient; regarding r_ε and r_p like time variables is reasonable, because according to eqs. (5,9) $\delta\varepsilon$ and δp are related to r_ε and r_p during the fluctuation. The physical meaning of r_ε and r_p is that of describing the cycle of values of energy and momentum, whereas r_p/r_ε controls the range of transient values allowed for the velocity. To be more specific, any energy fluctuation is characterized by an initial time t_{in} where $\varepsilon(t_{in}) = \varepsilon_{Ein}$ that successively increases to $\varepsilon(t) > \varepsilon_{Ein}$ at $t > t_{in}$ and then decreases down to the initial value ε_{Ein} at the time t_{end}. Note now that during the fluctuation transient must hold the inequality $r_p < r_\varepsilon$; otherwise, being v arbitrary e.g. very close

to c, the chance $r_p > r_\varepsilon$ could entail $\varepsilon(t)$ imaginary although being real ε_{Ein}. This would actually mean that the fluctuation is not allowed to occur. Thanks to the former inequality, instead, v can increase in principle even beyond c while still keeping $v_{eff} < c$; this can happen during the time range between t_{in} and t_{end} without divergent or imaginary quantities because under square root of the transient formulas appears v_{eff} only. This point is easily verified noting that $\varepsilon(t)/p(t) = c^2/v$, as already emphasized in section 5. Thus it must be also true that $\varepsilon(t)^2 = c^2 p(t)^2 + (mc^2)^2$ likewise eq. (5,11). Trivial manipulations yield $(v/c)^2 = (r_\varepsilon^2 - 1)/(r_p^2 - 1)$; so if $r_\varepsilon > r_p$ then is even allowed a value $v_* > c$ without contradicting neither eqs. (5,5) nor (5,11) that describe a steady behavior of the particle. According to eqs. (5,7), $r_p < r_\varepsilon$ requires

$$\delta\varepsilon(t)/\delta p(t) > \varepsilon_{Ein}/p_{Ein}. \qquad (7,2)$$

From an intuitive point of view, the transient proceeds for an observer in the lab frame according to the following steps: (i) $r_p = r_\varepsilon = 1$ at $t = t_{in}$, i.e. hold eqs. (5,11) with a value of $v_{eff} = v < c$ uniquely fixed by the initial motion of the particle; (ii) when r_p and r_ε start changing at $t > t_{in}$, the value of v_{eff} is still constrained by $v_{eff} < c$ but now $v > v_{eff}$ according to the inequality (7,2); (iii) at a later time $t_* < t_{end}$ it could even happen that $v_* > c$, although still being $v_{eff} < c$; (iv) subsequently r_p and r_ε tend again to 1 when the fluctuation cycle ends at $t \to t_{end}$ while $p(t) \to p_{Ein}$ and $\varepsilon(t) \to \varepsilon_{Ein}$, i.e. $v \to v_{eff} < c$. Thanks to the concept of quantum fluctuation, therefore, the increase of velocity $v_* > c$ in the step (iii) does not involve directly the value of v appearing in the steady formulas of ε_{Ein} and p_{Ein}, as indeed it results in eqs. (5,12); so the superluminal step (iii) is in principle possible. However, what about the chance of detecting it experimentally? Certainly the answer is not found via eqs. (7,1), which describe local quantities at the random and unspecified time t; on the other hand, since the particle travels, t is related to a corresponding x, random and unspecified as well. Throughout this paper it has been emphasized that information of physical interest is obtainable through uncertainty ranges only; thus the considerations just carried out, based on time and space local coordinates, have worth only to guess and assess the possible behavior of the particle at any $t_{in} \le t \le t_{end}$ and better understand the physical results inferred by consequence. Coherently with the approach so far followed, we discard once again the local dynamical variables and pay attention to the respective uncertainty ranges only. Exploit thus eqs. (5,12) to get information comparable with the experience, putting $\delta t = t_{end} - t_{in}$ and δl equal to the distance across which is measured the velocity. In this way we can calculate an *average* velocity $\delta l/\delta t$ whose value depends upon how the experiment is carried out. If δt is shorter than the time τ for the particle to travel the distance δl, then the superluminal effect it is not detectable, because the fluctuation starts and ends while the particle is still traveling within δl; this means that the fluctuation is an event entirely occurring within a space delocalization range. Yet nothing is known about what happens within this uncertainty range. In this case, when considering the average velocity of the particle, we can only acknowledge that this latter is anyway smaller than c, whereas any information about any possible event allowed to occur within δl remains in fact unaccessible; moreover eqs. (5,12) do not have themselves physical meaning, as they attempt to get physical insight within an uncertainty range. If however δt is longer than τ, then the superluminal effect is at least in principle detectable without contradicting the previous reasoning, because now the fluctuation extends throughout all the range δl and beyond; it is no longer a local event hidden by the uncertainty. So if the average velocity is measured in these experimental conditions, i.e. with δl sufficiently short or δt sufficiently long, the superluminal effect is in principle detectable. Note in this respect that a small value of m in the second eq. (5,12) corresponds to a longer time at right hand side, so the inequality (7,2) is more easily fulfilled for a particle not too heavy than for a heavy particle; indeed the former typically travels with values of v closer to c than the latter for energy reasons and also entails a longer δt, so it could effectively overcome the superluminal transition threshold fulfilling more likely the condition $\delta t > \tau$. Once fulfilling these conditions, a light particle appears traveling the space range $\delta l = v_* \delta t$ at speed $v_* > c$ in the laboratory reference system even during a moderate energy fluctuation and without violating any principle of quantum special relativity formulated in section 5; indeed $\delta l/\delta t$ does not calculate v_{eff} but the average transient of v. As a clarifying comparison recall that $\delta\varepsilon$ does not violate the energy conservation, it is simply a temporary derogation to this latter allowed by the uncertainty principle only; why not should something similar happen also for the velocity, if this latter does not cause divergent or imaginary results? Anyway, for the comparison with the experiment are enough just the two equations (5,12) that relate in the laboratory frame the distance δl traveled by the particle to the time δt during which the transient is still in progress; their ratio, assumed physically consistent with the time length of the fluctuation transient, reads

$$\frac{\delta l}{\delta t} = \frac{\frac{mc^2}{\sqrt{r_\varepsilon^2 - (r_p v/c)^2}} - \frac{mc^2}{\sqrt{1-(v/c)^2}}}{\frac{mv}{\sqrt{r_\varepsilon^2 - (r_p v/c)^2}} - \frac{mv}{\sqrt{1-(v/c)^2}}} = c\frac{c}{v}.$$

Since $v < c$, then $\delta l/\delta t > c$, which demonstrates a superluminal particle transfer during the quantum fluctuation cycle. If for instance $v = 0.99c$ then $\delta l/\delta t = 1.01c$. Note that instead the speed of the photon $v = c$ remains identically, universally and invariantly equal to c. Eqs. (5,5) have been written through time and space uncertainty ranges only. The Einstein relativity specifies the time range $\Delta t = t - t_o$ through a current time coordinate t and a lower boundary $t_o = x_o V/c^2$; both times have a deterministic physical meaning. This last result could be easily guessed also here, thinking that even t_o must depend on V/c and must be related to

the corresponding x_o. Thus a value $V > c$ would change the signs of Δt and $\Delta t'$ in eq. (5,5), i.e. the concept itself of sequence "before" and "after". Apart from the fact that such a conclusion would be illusory in the present theoretical frame because the uncertainty discards "a priori" the local coordinates, it is also essential in this respect a further remark. As shown before, the lack of physical information about t and t_o and $t - t_o$ does not prevent to infer the relativistic formulas of energy and momentum: yet, even specifying $t_o = x_o V/c^2$, the possible time-reversal during the quantum fluctuation cycle does not affect any result previously obtained. First of all because actually this cycle has not been specified, i.e. exchanging t_{end} with t_{in} does not change any step of the previous reasoning; moreover if the cycle starts with an initial energy ε_{Ein} and ends with the same final energy ε_{Ein}, any discrimination between beginning and ending of the cycle seems unphysical. Therefore, since the possible time reversal should be a local effect concerning the quantum fluctuation only, all the conclusions hitherto obtained still hold. Also note that $\delta l/\delta t = \varepsilon_{Ein}/p_{Ein} = c^2/v$; so the inequality (7,2) reads $\delta\varepsilon/\delta p > \delta l/\delta t$ as well, i.e. $\delta\varepsilon/\delta l > \delta p/\delta t$: the left hand side represents the force acting on the particle due to its fluctuation driven energy gap along its path, the right hand side represents the force due to the momentum change during the fluctuation time length. Saying that the former is greater than the latter means an excess force with respect to the mere momentum change having fully quantum origin, necessarily due to nothing else but the fluctuation in the case of a free particle. It seems reasonable to assume that just this excess force justifies the superluminal effect. As expected, neither δl nor δt enter explicitly into the calculation of the velocity; the ratio between two uncertainty ranges provides of course an average value during the transient, which is in effect allowed in the frame of the present approach. It is interesting to emphasize that a given $\delta\varepsilon/\delta l$, related to the energy growing along the path traveled by the particle, could be at increasing δl not greater than $\delta p/\delta t$, related to the given fluctuation time length; this is because δl and δt are two independent quantities, the former related to the experimental apparatus, the latter to a feature of the fluctuation. If δl increases up to a larger value Δl such that $\delta\varepsilon/\Delta l < \delta p/\delta t$ the superluminal effect is not observable. Indeed this is just in line with the previous considerations recalling that: (i) the effect is detectable if at the end of the path of the particle within δl the fluctuation is still in progress; (ii) if instead the fluctuation cycle ends while the particle is still traveling inside δl, then it becomes an event occurring within an uncertainty range and thus, as such, unobservable. If the model is correct, this is what to expect imagining to increase the size of δl up to Δl: the same kind of observation should yield a positive outcome if carried out in the experimental situation (i), but certainly a negative outcome if carried out in the experimental situation (ii). This also suggests a possible way to verify the considerations just carried out: to detect the same velocity fluctuation event of not-heavy particles with two detectors located in two different laboratories. Although the concept of their respective "distances" from the source is illusory for the reasons introduced in sections 3, it remains nevertheless still true that different locations, wherever they might be, provide different chances for the uncertainty of revealing or hiding experimentally the superluminal transition. Thus the random occurring/non-occurring of the superluminal effect should not be ascribed to human experimental errors but to a further probabilistic weirdness of the quantum world.

8 Discussion

The ordinary formulation of quantum mechanics contains the classical physics as a limit case but needs this latter to be formulated [17]. Regarding instead eqs. (1,1) as expressions of a fundamental principle of nature, and not as mere by-products of the commutation rules of operators, this ambiguous link between classical and quantum physics is bypassed. Section 6 has shown that eqs. (1,1) entail as a corollary the operator formalism of wave mechanics; yet the present approach appears more general than that based on this latter. As shown in sections 4 and 5, it automatically introduces since the beginning the non-locality and non-reality into the description of quantum systems. In principle the quantum uncertainty does not prevent knowing exactly one dynamical variable only; being the size of all ranges arbitrary by definition, one must admit even the chance $\Delta x \to 0$ that means local position of a particle exactly known. The same reasoning holds separately for the momentum as well. Independent ranges however do not provide physical information on the observable properties of the quantum world. These observables require abandoning separate certainties independently allowed; the physical meaning of the ranges changes when considering together two conjugate dynamical variables, which also means discarding the classical realism and localism as well but gaining the eigenvalues. Does the moon exist regardless of whether one observes it? According to the approach sketched in section 2 this question should be better reformulated, for instance as follows: do the properties of the moon we know exist regardless of a possible observer? Yet if nobody observes the moon, nobody could define the properties "we know"; these latter are the outcomes of some kind of measurement, i.e. they are triggered themselves by a previous measurement interaction. Repeating this reasoning back in the time the conclusion is that before the first recording of light beam escaping from the moon nobody would even know the existence of the moon; in which case would become physically irrelevant the prospective physical properties of an object still to be discovered. In this sense it appears understandable that the properties we know exist when observations are carried out. Hence what we call moon is just the result of an interaction between an observer and an object sufficiently close to the Earth to be observable. As concerns the localism it is appro-

priate to think about an action at a spooky distance, since the local coordinates defining the distance are actually an arbitrary extrapolation to the quantum world of a classical way of thinking. This idea appeared since the early times of birth of quantum mechanics, when the deterministic concept of trajectory was irreversibly abandoned. The operator formalism requires a wave function of time and space coordinates; these latter identify in turn a region of space where however has physical meaning the mere probability density to find the particle only. Thus the wave function denies the classical meaning of the local coordinates, e.g. position and momentum or energy and time, as a function of which is however itself calculated. In this respect the present approach formulates an even more indeterministic and drastic view of the reality: to discard the local values since the beginning. In this sense, eqs. (1,1) seem a step ahead with respect to the operator formalism; even though seemingly more agnostic, they avoid handling the local variables to define and solve the appropriate wave equations from which are extracted the eigenvalues, i.e. the observables, in a probabilistic conceptual context. Here indeed we refuse "a priori" the physical usefulness of introducing time and space local coordinates and, in general, local quantities that do no longer appear in the eigenvalues; yet, even so the results are identical. This suggests that actually is the uncertainty the fundamental concept behind the results, a sort of essential information directly related to the knowledge we can afford; for instance, the arbitrariness of the quantum numbers of wave mechanics, due to the mathematical features of the solutions of differential equations, is replaced by that of the number of states; indeed the results show that the latter have a physical meaning identical to the former. Eqs. (1,1) provide these numbers since the beginning. This is the reason of the straightforward character of the present approach, which indeed does not require solving any differential equation but proceeds through trivial algebraic manipulations of the formulae. The arbitrariness seems a concept with negative valence, especially in science; yet it played an essential role in deriving eqs. (3,5) from eq. (3,2); on this step are based eqs. (1,1). The section 2 shows that these equations plug the classical definition of angular momentum into the quantum world thanks to two concepts: introducing the number of states and eliminating local information. The section 6 has shown why the indistinguishability of identical particles is a natural consequence of these premises; in the operator formalism instead it must be purposely introduced as a postulate and appropriately handled from a mathematical point of view, recall for instance the early Slater determinants. Moreover the section 4 has shown why the present approach entails inherently even the non-locality and the non-reality of the quantum world: while evidencing their link with the quantization of the physical observables, these weird features are automatically required by eqs. (1,1) through n. Eventually, let us emphasize that the present way of regarding the quantum world is compatible with the special relativity. The paper [15] has inferred its basic principles as corollaries, in section 7 some results particularly significant have been obtained: the invariant interval, the Lorentz transformations of time and length, the energy and momentum equations of a free particle, the rest energy of particle, the existence of antimatter and the concept of mass itself. The key idea underlying these results is the way to regard the relativistic intervals: to discard their deterministic definition, early introduced by Einstein, and regard them as uncertainty ranges. As shown before, this simple conceptual step is enough to plug into the quantum world even the special relativity. Moreover, the quantum way to infer the relativistic equations has opened the way to admit a typical quantum phenomenon, the energy fluctuation, able to account for unexpected effects otherwise precluded by the early deterministic basis of special relativity formulated by Einstein.

9 Conclusion

The approach based uniquely on eqs. (1,1) contains inherently the requirements of non-locality and non-reality that characterize the quantum world. This kind of approach is also consistent with the special relativity, whose basic statements were found as corollaries in previous paper.

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