LETTERS TO PROGRESS IN PHYSICS

A More Elegant Argument that $P \neq NP$

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In April 2011, I published a paper in *Progress in Physics* entitled "An elegant argument that $P \neq NP$" [1]. Since then, I have discovered how to make that argument much simpler. In this letter, I present this argument.

Consider the following problem: Let $\{s_1, \ldots, s_n\}$ be a set of n integers and t be another integer. We want to determine whether there exists a subset of $\{s_1, \ldots, s_n\}$ for which the sum of its elements equals t. We shall consider the sum of the elements of the empty set to be zero. This problem is called the SUBSET-SUM problem [2].

Let $k \in \{1, \ldots, n\}$. Then the SUBSET-SUM problem is equivalent to determining whether there exist sets $I^+ \subseteq \{1, \ldots, k\}$ and $I^- \subseteq \{k+1, \ldots, n\}$ such that

$$\sum_{i \in I^+} s_i = t - \sum_{i \in I^-} s_i \,.$$

There is nothing that can be done to make this equation simpler. Then since there are 2^k possible expressions on the left-hand side of this equation and 2^{n-k} possible expressions on the right-hand side of this equation, we can find a lower-bound for the worst-case running-time of an algorithm that solves the SUBSET-SUM problem by minimizing $2^k + 2^{n-k}$ subject to $k \in \{1, \ldots, n\}$.

When we do this, we find that $2^k + 2^{n-k} = 2^{\lfloor n/2 \rfloor} + 2^{n - \lfloor n/2 \rfloor} = \Theta(\sqrt{2^n})$ is the solution, so it is impossible to solve the SUBSET-SUM problem in $o(\sqrt{2^n})$ time with a deterministic and exact algorithm. This lower-bound is tight [1]. And this conclusion implies that $P \neq NP$ [2].

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References

1. Feinstein C.A. An elegant argument that $P \neq NP$. *Progress in Physics*, 2011, v. 2, 30–31.
2. Cormen T.H., Leiserson C.E., and Rivest R.L. Introduction to Algorithms. McGraw-Hill, 1990.