Quantum Constraints on a Charged Particle Structure

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The crucial role of a Lorentz scalar Lagrangian density whose dimension is $[L^{-4}]$ ($\hbar = c = 1$) in a construction of a quantum theory is explained. It turns out that quantum functions used in this kind of Lagrangian density have a definite dimension. It is explained why quantum functions that have the dimension $[L^{-1}]$ cannot describe particles that carry electric charge. It is shown that the 4-current of a quantum particle should satisfy further requirements. It follows that the pion and the $W^{\pm}$ must be composite particles. This outcome is inconsistent with the electroweak theory. It is also argued that the 125 GeV particle found recently by two LHC collaborations is not a Higgs boson but a $t\bar{t}$ meson.

1 Introduction

The fundamental role of mathematics in the structure of theoretical physics is regarded as an indisputable element of the theory [1]. This principle is utilized here. The analysis relies on special relativity and derives constraints on the structure of equations of motion of quantum particles. The discussion examines the dimensions of wave functions and explains why spin-0 and spin-1 elementary quantum particles cannot carry an electric charge. This conclusion is relevant to the validity of the electroweak theory and to the meaning of recent results concerning the existence of a particle having a mass of 125 GeV [2, 3].

Units where $\hbar = c = 1$ are used in this work. Hence, only one dimension is required and it is the length, denoted by $[L]$. For example, mass, energy and momentum have the dimension $[L^{-1}]$, etc. Greek indices run from 0 to 3 and the diagonal metric used is $g_{\mu\nu} = (1, -1, -1, -1)$. The symbol $_{,\mu}$ denotes the partial differentiation with respect to x^{μ} and an upper dot denotes a differentiation with respect to time. The summation convention is used for Greek indices.

The second section shows that quantum functions have a definite dimension. This property is used in the third section where it is proved that Klein-Gordon (KG) fields and those of the $W^{\pm}$ particle have no self-consistent Hamiltonian. The final section contains a discussion of the significance of the results obtained in this work.

2 The dimensions of quantum fields

In this section some fundamental properties of quantum theory are used for deriving the dimensions of quantum fields. A massive quantum mechanical particle is described by a wave function $\psi(x^{\mu})$. The phase $\varphi(\alpha)$ is an important factor of $\psi(x^{\mu})$ because it determines the form of an interference pattern. For the present discussion it is enough to demand that the phase is an analytic function which can be expanded in a power series that contains more than one term. It means that in the following expansion of the phase,

$$\varphi(\alpha) = \sum_{i=0}^{\infty} a_i \alpha^i, \tag{1}$$

the inequality $a_i \neq 0$ holds for two or more values of the index i.

The requirement stating that all terms of a physical expression must have the same dimension and the form of the right hand side of (1) prove that α must be dimensionless. By the same token, in a relativistic quantum theory, α must also be a Lorentz scalar. (The possibility of using a pseudoscalar factor is not discussed here because this work aims to examine the parity conserving electromagnetic interactions of a quantum mechanical particle.) It is shown below how these two requirements impose dramatic constraints on acceptable quantum mechanical equations of motion of a charged particle.

Evidently, a pure number satisfies the two requirements. However, a pure number is inadequate for our purpose, because the phase varies with the particle's energy and momentum. The standard method of constructing a quantum theory is to use the Plank's constant $\hbar$ which has the dimension of the action, and to define the phase as the action divided by $\hbar$. In the units used here, $\hbar = 1$ and the action is dimensionless. Thus, a relativistic quantum theory satisfies the two requirements presented above if it is derived from a Lagrangian density $\mathcal{L}$ that is a Lorentz scalar having the dimension $[L^{-4}]$. Indeed, in this case, the action

$$S = \int \mathcal{L} d^4 x^{\mu} \tag{2}$$

is a dimensionless Lorentz scalar. It is shown below how the dimension $[L^{-4}]$ of $\mathcal{L}$ defines the dimension of quantum fields.

Being aware of these requirements, let us find the dimension of the quantum functions used for a description of three kinds of quantum particles. The Dirac Lagrangian density of a free spin-1/2 particle is [4, see p. 54]

$$\mathcal{L} = \bar{\psi}[\gamma^\mu i\partial_\mu - m]\psi. \tag{3}$$

Here the operator has the dimension $[L^{-1}]$ and the Dirac wave function ψ has the dimension $[L^{-3/2}]$.

The Klein-Gordon Lagrangian density of a free spin-0 particle is [4, see p. 38]

$$\mathcal{L} = \phi^*_{,\mu}\phi_{,\nu}g^{\mu\nu} - m^2\phi^*\phi. \tag{4}$$

Here the operator has the dimension $[L^{-2}]$ and the KG wave function ϕ has the dimension $[L^{-1}]$.

The electrically charged spin-1 $W^\pm$ particle is described by a 4-vector function W_μ. W_μ and the electromagnetic 4-potential A_μ are linear combinations of related quantities [5, see p. 518]. Evidently, they have the same dimension. Hence, like the KG field, the dimension of W_μ is $[L^{-1}]$.

The dimension of each of these fields is used in the discussions presented in the rest of this work.

3 Consequences of the dimensions of quantum fields

Before analyzing the consequences of the dimension of quantum fields and of the associated wave functions, it is required to realize the Hamiltonian's role in quantum theories. The following lines explain why the Hamiltonian is an indispensable element of Relativistic Quantum Mechanics (RQM) and of Quantum Field Theory (QFT). This status of the Hamiltonian is required for the analysis presented below.

The significance of hierarchical relationships that hold between physical theories is discussed in the literature [6, see pp. 1-6] and [7, see pp. 85, 86]. The foundation of the argument can be described as follows. Physical theories take the form of differential equations. These equations can be examined in appropriate limits. Now RQM is a limit of QFT. The former holds for cases where the number of particles can be regarded as a constant of the motion. Therefore, if examined in this limit, QFT must agree with RQM. By the same token, the classical limit of RQM must agree with classical physics. This matter has been recognized by the founders of quantum mechanics who have proven that the classical limit of quantum mechanics agrees with classical physics. The following example illustrates the importance of this issue. Let us examine an inelastic scattering event. The chronological order of this process is as follows:

a. First, two particles move in external electromagnetic fields. Relativistic classical mechanics and classical electrodynamics describe the motion.
b. The two particles are very close to each other. RQM describes the process.
c. The two particles collide and interact. New particles are created. The process is described by QFT.
d. Particle creation ends but particles are still very close to one another. RQM describes the state.
e. Finally, the outgoing particles depart. Relativistic classical mechanics and classical electrodynamics describe the motion.

Evidently, in this kind of experiment, energy and momentum of the initial and the final states are well defined quantities and their final state values abide by the law of energy-momentum conservation. It means that the specific values of the energy-momentum of the final state agree with the corresponding quantities of the initial state. Now, the initial and the final states are connected by processes that are described by RQM and QFT. In particular, the process of new particle creation is described only by QFT. Hence, RQM and QFT must "tell" the final state what are the precise initial values of the energy-momentum. It follows that RQM as well as QFT must use field functions that have a self-consistent Hamiltonian.

The Hamiltonian H and the de Broglie relations between a particle's energy-momentum and its wave properties yield the fundamental equation of quantum mechanics

$$i\frac{\partial\psi}{\partial t} = H\psi. \tag{5}$$

The Hamiltonian density $\mathcal{H}$ is derived from the Lagrangian density by the following well known Legendre transformation

$$\mathcal{H} = \sum_i \dot{\psi}_i \frac{\partial\mathcal{L}}{\partial\dot{\psi}_i} - \mathcal{L}, \tag{6}$$

where the index i runs on all functions.

The standard form of representing the interaction of an electric charge with external fields relies on the following transformation [8, see p. 10]

$$-i\frac{\partial}{\partial x^\mu} \to -i\frac{\partial}{\partial x^\mu} - eA_\mu(x^\nu). \tag{7}$$

Now let us examine the electromagnetic interaction of the three kinds of quantum mechanical particle described in the previous section. This is done by adding an interaction term $\mathcal{L}_{int}$ to the Lagrangian density. As explained above, this term must be a Lorentz scalar whose dimension is $[L^{-4}]$. The required form of the electromagnetic interaction term represents the interaction of charged particles with electromagnetic fields *and* the interaction of electromagnetic fields with charged particles. This term is written as follows [9, see p. 75]

$$\mathcal{L}_{int} = -j^\mu A_\mu. \tag{8}$$

Here j^μ is the 4-current of the quantum particle and A_μ is the electromagnetic 4-potential.

Charge conservation requires that j^μ satisfies the continuity equation

$$j^\mu_{,\mu} = 0. \tag{9}$$

The 0-component of the 4-vector j^μ represents density. It follows that its dimension is $[L^{-3}]$ and the electromagnetic interaction (8) is a term of the Lagrangian density. For this reason, it is a Lorentz scalar whose dimension is $[L^{-4}]$. Hence, a quantum particle can carry electric charge provided a self-consistent 4-current can be defined for it. Furthermore, a self-consistent definition of density is also required for a construction the Hilbert space where density is used for defining its inner product.

It is well known that a self-consistent 4-current can be defined for a Dirac particle [8, see pp. 8,9,23,24]

$$j^\mu = e\bar{\psi}\gamma^\mu\psi. \tag{10}$$

This expression has properties that are consistent with general requirements of a quantum theory. In particular, the 4-current is related to a construction of a Hilbert space. Here the density $\psi^\dagger\psi$ is the 0-component of the 4-current (10). As required, this quantity has the dimension $[L^{-3}]$. Thus, electromagnetic interactions of charged spin-1/2 Dirac particles are properly described by the Dirac equation.

Let us turn to the case of a charged KG or W^μ particle. Here the appropriate wave function has the dimension $[L^{-1}]$. This dimension proves that it cannot be used for constructing a self-consistent Hilbert space. Indeed, let ϕ denote a function of such a Hilbert space and let O be an operator operating on this space. Then, the expectation value of O is

$$< O >= \int \phi^* O \phi d^3x. \tag{11}$$

Now, $< O >$ and O have the same dimension. Therefore ϕ must have the dimension $[L^{-3/2}]$. This requirement is not satisfied by the function ϕ of a KG particle or by W^μ because here the dimension is $[L^{-1}]$. Hence, there is no Hilbert space for a KG or W^μ particle. For this reason, there is also no Hamiltonian for these functions, because a Hamiltonian is an operator operating on a Hilbert space. Analogous results are presented for the specific case of the KG equation [10].

The dimension $[L^{-1}]$ of the KG and the W^μ functions also yields another very serious mathematical problem. Indeed, in order to have a dimension $[L^{-4}]$, their Lagrangian density has terms that are *bilinear* in derivatives with respect to the space-time coordinates. Thus, the KG Lagrangian density is (4) and the W^μ Lagrangian density takes the following form [11, see p. 307]

$$\mathcal{L}_W = -\frac{1}{4}(\partial_\mu W_\nu - \partial_\nu W_\mu + g W_\mu \times W_\nu)^2. \tag{12}$$

As is well known, an operation of the Legendre transformation (6) on a Lagrangian density that is *linear* in time derivatives yields an expression that is *independent* of time derivatives. Thus, the Dirac Lagrangian density (3) yields a Hamiltonian that is free of time derivatives. On the other hand, the Hamiltonian density of the KG and W^μ particles depends on time derivatives. Indeed, using (5) , one infers that for these particles, the Hamiltonian density depends quadratically on the Hamiltonian. Hence, there is no explicit expression for the Hamiltonian of the KG and the W^μ particles.

Two results are directly obtained from the foregoing discussion. The Fock space, which denotes the occupation number of particles in appropriate states, is based on functions of the associated Hilbert space. Hence, in the case of KG or W^μ function there are very serious problems with the construction of a Fock space because these functions have no Hilbert space. Therefore, one also wonders what is the meaning of the creation and the annihilation operators of QFT.

Another result refers to the 4-current. Thus, both the KG equations and the W^μ function have a 4-current that satisfies (9) [11, see p. 12] and [12, see p. 199]. However, the contradictions derived above prove the following important principle: *The continuity relation* (9) *is just a necessary condition for an acceptable 4-current. This condition is not sufficient and one must also confirm that a theory that uses a 4-current candidate is contradiction free.*

The contradictions which are described above hold for the KG and the $W^\pm$ particles provided that these particles are elementary pointlike quantum mechanical objects which are described by a function of the form $\psi(x^\mu)$. Hence, *in order to avoid contradictions with the existence of charged pions and $W^\pm$, one must demand that the pions and the $W^\pm$ are composite particles.* Several aspects of this conclusion are discussed in the next section. It should also be noted that the results of this section are consistent with Dirac's lifelong objection to the KG equation [13].

4 Discussion

An examination of textbooks provides a simple argument supporting the main conclusion of this work. Indeed, quantum mechanics is known for more than 80 years. It turns out that the Hamiltonian problem of the hydrogen atom of a Dirac particle is discussed adequately in relevant textbooks [8, 14]. By contrast, in spite of the long duration of quantum mechanics as a valid theory, an appropriate discussion of the Hamiltonian solution of a hydrogen-like atom of a relativistic electrically charged integral spin particle is not presented in textbooks. Note that the operator on the left hand side of the KG equation [14, see p. 886]

$$(\partial_\mu + ieA_\mu)g^{\mu\nu}(\partial_\nu + ieA_\nu)\phi = -m^2\phi \tag{13}$$

is *not* related to a Hamiltonian because (13) is a Lorentz scalar whereas the Hamiltonian is a 0-component of a 4-vector.

An analogous situation holds for the Hilbert and the Fock spaces that are created from functions on which the Hamiltonian operates. Thus, in the case of a Dirac particle, the density $\psi^\dagger\psi$ is the 0-component of the conserved 4-current (10). This expression is suitable for a definition of the Hilbert space inner product of any pair of integrable functions

$$(\psi_i^\dagger, \psi_j) \equiv \int \psi_i^\dagger \psi_j \, d^3x. \tag{14}$$

Indeed, it is derivative free and this property enables the usage of the Heisenberg picture which is based on time-independent functions. Integration properties prove that (14) is linear in $\psi_i^\dagger$ and ψ_j. Thus,

$$(a\psi_i^\dagger + b\psi_k^\dagger, \psi_j) = a(\psi_i^\dagger, \psi_j) + b(\psi_k^\dagger, \psi_j).$$

Furthermore, $(\psi_i^\dagger, \psi_i)$ is a real non-negative number that vanishes if and only if $\psi_i \equiv 0$. These properties are required from a Hilbert space inner product. It turns out that the construction of a Hilbert space is the cornerstone used for calculating successful solutions of the Dirac equation and of its associated Pauli and Schroedinger equations as well.

By contrast, in the case of particles having an integral spin, one cannot find in the literature an explicit construction of a Hilbert space. Indeed, the $[L^{-1}]$ dimension of their functions proves that the simple definition of an inner product in the form $\int \phi_i^* \phi_j \, d^3x$ has the dimension $[L]$ which is unacceptable. An application of the 0-component of these particles 4-current [11, see p. 12] and [12, see p. 199] is not free of contradictions. Thus, the time derivative included in these expressions prevents the usage of the Heisenberg picture. Relation (7) proves that in the case of a charged particle the density depends on *external* quantities. These quantities may vary in time and for this reason it cannot be used in a definition of a Hilbert space inner product. In the case of the W^μ function, the expression is inconsistent with the linearity required from a Hilbert space inner product.

The results found in this work apply to particles described by a function of the form

$$\psi(x^\mu). \tag{15}$$

Their dependence on a single set of four space-time coordinates x^μ means that they describe an elementary pointlike particle. For example, this kind of function cannot adequately describe a pion because this particle is not an elementary particle but a quark-antiquark bound state. Thus, it consists of a quark-antiquark pair which are described by *two* functions of the form (15). For this reason, one function of the form (15) cannot describe a pion simply because a description of a pion should use a larger number of degrees of freedom. It follows that the existence of a π^+, which is a spin-0 charged particle, does not provide an experimental refutation of the theoretical results obtained above.

Some general aspects of this work are pointed out here. There are two kinds of objects in electrodynamics of Dirac particles: massive charged spin-1/2 particles and charge-free photons. The dimension of a Dirac function is $[L^{-3/2}]$ and the dimension of the electromagnetic 4-potential is $[L^{-1}]$. Now, the spin of any interaction carrying particle must take an integral value in order that the matrix element connecting initial and final states should not vanish. The dimension of an interaction carrying particle must be $[L^{-1}]$ so that the Lagrangian density interaction term have the dimension $[L^{-4}]$. These properties must be valid for particles that carry any kind of interaction between Dirac-like particles. Hence, the pions and the $W^\pm$ have integral spin and dimension $[L^{-1}]$. However, in order to have a self-consistent Hilbert and Fock spaces, a function describing an elementary massive particle must have the dimension $[L^{-3/2}]$. Neither a KG function nor the W^μ function satisfies this requirement.

The conclusion stating that the continuity equation (9) is only a *necessary condition* required from a physically acceptable 4-current and that further consistency tests must be carried out, looks like a new result of this work that has a general significance.

Before discussing the state of the $W^\pm$ charged particles, let us examine the strength of strong interactions. Each of the following arguments proves that strong interactions yield extremely relativistic bound states and that the interaction part of the Hamiltonian swallows a large portion of the quarks' mass.

A. Antiquarks have been measured directly in the proton [15, see p. 282]. This is a clear proof of the extremely relativistic state of hadrons. Indeed, for reducing the overall mass of the proton, it is energetically "profitable" to add the mass of two quarks because the increased interaction is very strong.

B. The mass of the ρ meson is about five times greater than the pion's mass. Now these mesons differ by the relative spin alignment of their quark constituents. Evidently, spin interaction is a relativistic effect and the significant π, ρ mass difference indicates that strong interactions are very strong indeed.

C. The pion is made of a u, d quark-antiquark pair and its mass is about 140 MeV. Measurements show that there are mesons made of the u, d flavors whose mass is greater than 2000 MeV [6]. Hence, strong interactions consume most of the original mass of quarks.

D. Let us examine the pion and find an estimate for the intensity of its interactions. The first objective is to find an estimate for the strength of the momentum of the pion's quarks. The calculation is done in units of fm, and 1 $fm^{-1} \simeq 200\, MeV$. The pion's spatial size is somewhat smaller than that of the proton [16]. Thus, let us assume that the pion's quark-antiquark pair are enclosed inside a box whose size is 2.2 fm and the pion's quark wave function vanishes on its boundary. For the x-component, one finds that the smallest absolute value of the momentum is obtained from a function of the form $\sin(\pi x/2.2)$. Hence, the absolute value of this component of the momentum is $\pi/2.2$. Thus, for the three spatial coordinates, one multiplies this number by $\sqrt{3}$ and another factor of 2 accounts for the quark-antiquark pair. It follows that the absolute value of the momentum enclosed inside a pion is

$$|\,\mathbf{p}| \simeq 1000\, MeV. \tag{16}$$

This value of the momentum is much greater than the pion's mass. It means that the system is extremely relativistic and (16) is regarded as the quarks' kinetic energy. Thus, the interaction consumes about 6/7 of the kinetic energy *and* the entire mass of the quark-antiquark pair. In other word, the pion's kinetic energy is about 7 times greater than its final mass. It is interesting to compare these values to the corresponding quantities of the positronium, which is an electron-positron system bound by the electromagnetic force. Here the ratio of the kinetic energy to the final mass is about 7/1000000. On the basis of this evidence one concludes that strong interactions must be much stronger than the experimental mass of the pion.

Relying on these arguments and on the theoretical conclusion stating that the $W^{\pm}$ must be composite objects, it is concluded that the $W^{\pm}$ particles contain one top quark. Thus, the W^{+} is a superposition of three meson families: $t\bar{d}$, $t\bar{s}$ and $t\bar{b}$. Here the top quark mass is 173 GeV and the mass of the W is 80 GeV [16]. The difference indicates the amount swallowed by strong interactions. This outcome also answers the question where are the mesons of the top quark? The fact that the $W^{\pm}$ is a composite particle which is a superposition of mesons is inconsistent with the electroweak theory and this fact indicates that the foundations of this theory should be examined.

Another result of this analysis pertains to recent reports concerning the existence of a new particle whose mass is about 125 GeV and its width is similar to that of the $W^{\pm}$ [2,3]. Thus, since the mass of the top quark is about 173 GeV and this quantity is by far greater than the mass of any other quark, it makes sense to regard the 125 GeV particle as a $t\bar{t}$ meson. For this reason, the $t\bar{t}$ meson is heavier than the 80 GeV $W^{\pm}$ which consists of one top quark and a lighter quark.

A $t\bar{t}$ mesonic structure of the 125 GeV particle explains naturally its quite sharp disintegration into two photons. Indeed, the disintegration of a bound system of charged spin-1/2 particle-antiparticle pair into two photons is a well known effect of the ground state of the positronium and of the π^0 meson. On the other hand, the results obtained in this work deny the $W^+ W^-$ disintegration channel of the 125 GeV particle, because the Ws are composite particles and a $W^+ W^-$ system is made of two quark-antiquark pairs. For this reason, their two photon disintegration should be accompanied by other particles. Hence, a W^+W^- two photon outcome should show a much wider energy distribution. This kind of $W^+ W^- \rightarrow \gamma\gamma$ disintegration is inconsistent with the quite narrow width of the 125 GeV data. It turns out that for a Higgs mass of 125 GeV, Standard Model Higgs decay calculations show that the $W^+W^- \rightarrow \gamma\gamma$ channel is dominant [17, see section 2.3.1]. However, it is proved in this work that the $W^+ W^-$ disintegration channel of the 125 GeV particle is incompatible with the data. Therefore, one denies the Higgs boson interpretation of the 125 GeV particle found at the LHC [2, 3]. This outcome is consistent with the Higgs boson inherent contradictions which are discussed elsewhere [10].

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