Genesis of the "Critical-Acceleration of MOND" and Its Role in "Formation of Structures"

Hasmukh K. Tank

Indian Space Research Organization, 22/693, Krishna Dham-2, Vejalpur, Ahmedabad-380015, India
E-mail: tank.hasmukh@rediffmail.com, hasmukh.tank1@gmail.com

As an attempt to explain the "flattening of galaxies rotation-curves", Milgrom proposed a Modification of Newtonian Dynamics MOND, in which he needed a new constant of nature a_0 , termed as "critical-acceleration-of MOND", in his best-fit empirical formula. But so far it has been an ad-hoc introduction of a new constant. Whereas this article proposes: (i) a genesis of this constant; (ii) explains its recurrences in various physical situations; and (iii) its role in determining the size and radii of various structures, like: the electron, the proton, the nucleus-of-atom, the globular-clusters, the spiral-galaxies, the galactic-clusters and the whole universe. In this process we get a new interpretation of "the cosmological-red-shift", that the linear part of the cosmological-red-shift may not be due to "metric-expansion-of-space"; and even the currently-believed "accelerated-expansion" may be slowing down with time.

1 Introduction

The observations of "flattening of galaxies rotation curves" are generally explained by assuming the presence of "dark-matter", but there is no way to directly detect it other than its presumed gravitational effect. M. Milgrom [1] proposed an alternative explanation for the "galaxies rotation curves", by modifying Newton's law of gravitation, for which he needed an ad-hoc introduction of a new constant of nature a_0, termed as "critical-acceleration of MOND", of the order of magnitude: 1.2×10^{-10} meter per seconds squared. But so far it has been an ad-hoc introduction of a new constant; and there has been no explanation for why its value is this much. Sivaram noticed its recurrences in various physical situations. This author has been of the opinion that the matching of values of the "anomalous decelerations of the four space-probes": Pioneer-10, Pioneer-11, Galileo and Ulysses and the "deceleration of cosmologically-red-shifting-photons" can not be an accidental coincidence. Now, this article presents a genesis of this "critical-acceleration of MOND". And based on this genesis, the formation of various structures, like the electron, the proton, the nucleus-of-atom, the globular-clusters, the spiral-galaxies, the galactic-clusters and the whole universe, are explained here.

2 Genesis of the "critical acceleration of MOND"

R.K. Adair, in his book "Concepts in Physics" [2] has given a derivation, that the sum of "gravitational-potential-energy" and "energy-of-mass' of the whole universe is, strikingly, zero! i.e.

$$M_0 c^2 - \frac{G M_0 M_0}{R_0} = 0 \qquad (1)$$

where M_0 and R_0 are total-mass and radius of the universe respectively, and G is Newton's gravitational constant; i.e.

$$\frac{G M_0 m}{R_0} = m c^2. \qquad (2)$$

Where m is mass of any piece of matter. That is, the relativistic-energy of any piece of matter of mass m is equal to its "cosmic-gravitational-potential-energy". So the "cosmic-gravitational-force" experienced by every piece of matter is:

$$\frac{G M_0 m}{R_0^2} = m \frac{c^2}{R_0}. \qquad (3)$$

We know that $R_0 H_0 = c$, so, $R_0 = c/H_0$. Here H_0 is Hubble's constant; i.e.

$$\frac{G M_0 m}{R_0^2} = m H_0 c \qquad (4)$$

where m is mass of any object; and $H_0 c$ is a "cosmic-constant-of-acceleration". $H_0 c = 6.87 \times 10^{-10}$ meter/second^2. In the next section we will see the recurrences of this "cosmic-constant-of-acceleration" in various physical situations.

3 Observable recurrences of "the cosmic-constant-of-acceleration"

Inter-galactic-photons experience the "cosmological red-shift". We can express the cosmological red-shift z_c in terms of de-acceleration experienced by the photon [3, 4], as follows:

$$z_c = \frac{f_0 - f}{f} = \frac{H_0 D}{c}$$

i.e.

$$\frac{h \Delta f}{h f} = \frac{H_0 D}{c}$$

i.e.

$$h \Delta f = \frac{h f}{c^2} (H_0 c) D. \qquad (5)$$

Here: h is Plancks constant, f_0 is frequency of photon at the time of its emission, f is the red-shifted frequency measured on earth, H_0 is Hubble's constant, and D the luminosity-distance.

That is, the loss in energy of the photon is equal to its mass (hf/c^2) times the acceleration $a = H_0c$, times the distance D travelled by it. Where: H_0 is Hubble-parameter. And the value of constant acceleration a is: $a = H_0c$, $a = 6.87 \times 10^{-10}$ meter/sec^2.

Now, we will verify that the accelerations experienced by the Pioneer-10, Pioneer-11, Galileo and Ulysses space-probes do match significantly with the "cosmic-constant-of-acceleration". Slightly higher value of decelerations of the space-probes is then explained.

Carefully observed values of de-accelerations [5]:
For Pioneer-10:

$$a = (8.09 \pm 0.2) \times 10^{-10} \ m/s^2 = H_0c. \qquad (6)$$

For Pioneer-11:

$$a = (8.56 \pm 0.15) \times 10^{-10} \ m/s^2 = H_0c. \qquad (7)$$

For Ulysses:

$$a = (12 \pm 3) \times 10^{-10} \ m/s^2 = H_0c. \qquad (8)$$

For Galileo:

$$a = (8 \pm 3) \times 10^{-10} \ m/s^2 = H_0c. \qquad (9)$$

For Cosmologically-red-shifted-photon,

$$a = 6.87 \times 10^{-10} \ m/s^2 = H_0c. \qquad (10)$$

This value of acceleration is also the "critical-acceleration" of modified Newtonian dynamics MOND,

$$a_0 = H_0c \qquad (11)$$

and the rate of "accelerated-expansion of the universe"

$$a_{exp} = H_0c. \qquad (12)$$

According to Weinberg, mass of a fundamental-particle can be obtained from the "fundamental-constants" as follows:
Mass of a fundamental-particle,

$$m = \left(\frac{h^2 H_0}{cG}\right)^{1/3}$$

i.e.

$$\frac{Gm}{(h/mc)^2} = H_0c. \qquad (13)$$

That is, the self-gravitational-acceleration of Weinberg's [7] "fundamental-particle" is also equal to the "cosmic-constant-of-acceleration".

Reason why the apparent value of deceleration of the cosmic-photon is slightly small:

When the extra-galactic-photon enters our own milky-way-galaxy, the photon also experiences the gravitational-blue-shift, because of the gravitational-pull of our galaxy. The photon of a given frequency, if it has come from a near-by-galaxy, then it gets more blue-shifted, compared to the photon which has come from very-very far-distant-galaxy; so the galaxy which is at closer distance, appears at more closer distance, than the galaxies at far-away-distances. That is, the cosmic photon decelerated during its long inter-galactic-journey, and then accelerated because of the gravitational-pull of our milky-way galaxy; so we measure slightly lesser value of H_0; $H_0c = 6.87 \times 10^{-10}$ meter per seconds squared. But if we could send the Hubble-like Space-Telescope out-side our milky-way-galaxy, then the value of H_0c will match perfectly with the value of deceleration of all the four space-probes; $= 8.5 \times 10^{-10}$ meters per seconds squared.

Currently, the whole values of "anomalous accelerations of the space-probes" are "explained" in terms of radiation-pressure, gas-leakage... etc. So here we can explain the slight differences in their values of decelerations in terms of radiation-pressure, gas-leakage etc! Thus, the matching of values of decelerations of all the four space-probes is itself an interesting observation; and its matching with the deceleration of cosmologically-red-shifting-photons can not be ignored by a scientific mind as a coincidence. There is one more interesting thing about the value of this deceleration as first noticed by Milgrom, that: with this value of deceleration, an object moving with the speed of light would come to rest exactly after the time T_0 which is the age of the universe.

4 Formation of structures

Sivaram [6] has noticed that:

$$\begin{aligned} \frac{G\,M_0}{R_0^2} &= \frac{G\,m_p}{r_p^2} = \frac{G\,m_e}{r_e^2} = \frac{G\,m_n}{r_n^2} \\ &= \frac{G\,M_{gc}}{R_{gc}^2} = \frac{G\,M_{gal}}{R_{gal}^2} = \frac{G\,M_{cg}}{R_{cg}^2} \end{aligned} \qquad (14)$$

$$= \text{the "critical-acceleration" of MOND}$$
$$= H_0c.$$

(Here: M_0 and R_0 are mass and radius of the universe respectively, m_p and r_p are mass and radius of the proton, m_e and r_e are mass and radius of the electron, m_n and r_n are mass and radius of the nucleus of an atom, M_{gc} and R_{gc} are mass and radius of the globular-clusters, M_{gal} and R_{gal} are mass and radius of the spiral-galaxies, and M_{cg} and R_{cg} are mass and radius of the galactic-clusters respectively).

That is, the self-gravitational-pulling-force experienced by all the above bodies will be: Self-gravitational-force F = (mass of the body, say a galaxy) times (a constant value of deceleration H_0c).

For the formation of a stable structure, the "self-gravitational-acceleration" of a body of mass m should be equal to

the value of "cosmic-constant-of-acceleration" H_0c. In the expressions of eq. 14 above we found that: at the "surface" of the electron, the proton, the nucleus-of-atom, the globular-clusters, the spiral-galaxies, and the galactic-clusters this condition is beautifully satisfied. That is:

$$\frac{GM}{R^2} = H_0c. \qquad (15)$$

Where M and R represent mass and radius of the above objects. And the size and radius of the above structures get decided as follows: i.e.

$$\frac{GM}{R^2} = H_0c = \frac{c^2}{R_0}$$

i.e.

$$R^2 = \frac{GM}{c^2}R_0$$

i.e.

$$R = (r_G R_0)^{1/2} \qquad (16)$$

where r_G is "gravitational-radius" of the above objects. This is how all the structures get formed, beginning from the electron to the galactic-clusters.

5 Explanation for the "flattening of galaxies rotation-curves"

As seen in the expression-15, the condition for the formation of a stable structure is: $GM/R^2 = H_0c$ where M and R are mass and radius of a galaxy. That is, the centripetal acceleration at the surface of a structure is:

$$\frac{v^2}{R} = \frac{GM}{R^2} = H_0c \qquad (17)$$

i.e.

$$v^2 = RH_0c. \qquad (18)$$

Now, by dividing both the sides of the above expression by a distance r greater than R, the acceleration towards the center of spiral-galaxy experienced by a star at a distance r form the center is:

$$\frac{v^2}{r} = \frac{R}{r}H_0c. \qquad (19)$$

Where $r > R$.

So, the velocity of the stars at the out-skirts of spiral galaxies is:

$$v = \left[\left(\frac{GM}{c^2}\frac{GM_0}{c^2}\right)^{1/2} a_0\right]^{1/2} \qquad (20)$$

i.e.

$$v = \left[\left(\frac{GM}{c^2}R_0\right)^{1/2}\frac{c^2}{R_0}\right]^{1/2} \qquad (21)$$

i.e.

$$v = \left[\frac{M}{M_0}\right]^{1/4} c, \qquad (22)$$

a constant velocity. The above expression-22 is equal to Milgrom's expression: $(v^2/r) = [(GM/r^2)a_0]^{1/2}$ because $a_0 = GM_0/R_0^2$. This is how we can explain the "flattening of galaxies rotation-curves".

6 Conclusion

We presented here the genesis or root of the "critical-acceleration of MOND", that it follows from the equality of "gravitational potential-energy" and "energy-of-mass" of the universe; and showed that there are as many as fifteen physical situations where we find recurrences of this "cosmic-constant-of-acceleration". The sizes of various structures like the electron, the proton, the nucleus-of-atom, the globular-clusters, the spiral-galaxies, the galactic-clusters and the whole universe get decided based on the condition that: the "self-gravitational-acceleration" of them all should be equal to the "cosmic-constant-of-acceleration" H_0c. The flattening of galaxies rotation curves at the out skirts of spiral galaxies also emerge from the above-mentioned equality.

We are sure that the space-probes Pioneer-10 et al. did show decelerations of the order H_0c. Now, similar to the space-probes, if the cosmologically red-shifting photons also decelerate due to the "cosmic-gravitational-force" then the linear part of the cosmological-red-shift may not be due to the "metric-expansion-of-space"; only the recently-discovered accelerated-expansion may be due to the "metric-expansion-of-space"; and its rate H_0c suggests that even the receding galaxies may be getting decelerated like the space-probes! Thus we may be able to explain even the "accelerated-expansion of the universe" without any need for dark-energy.

References

1. Milgrom M. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophysical Journal*, 1983, v. 270, 365.

2. Adair R.K. Concepts in Physics. Academic Press, New York, (1969), p. 775.

3. Tank H.K. A new law emerging from the recurrences of the "critical-acceleration" of MOND, suggesting a clue to unification of fundamental forces. *Astrophysics and Space Science*, 2010, v. 330, 203–205.

4. Tank H.K. Some clues to understand MOND and the accelerated expansion of the universe. *Astrophysics and Space Science*, 2011, v. 336, no. 2, 341–343.

5. Anderson J.D., Laing P.A., Lau E.L., Liu A.S., Nieto M.M., and Turyshev, S.G. Indication, from Pioneer 10, 11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration. *Physical Review Letters*, 1998, v. 81, 2858–2861 [(Comment by Katz J.I.: *Physical Review Letters*, 1999, v. 83, 1892; Reply: *Physical Review Letters*, 1999, v. 83, 1893].

6. Sivaram C. Some aspects of MOND and its consequences for cosmology. *Astrophysics and Space Science*, 1994, v. 215, 185–189.

7. Weinberg S. Gravitation and Cosmology. John Wiley & Sons, (1972), p. 620.