Sampling the Hydrogen Atom

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A model is proposed for the hydrogen atom in which the electron is an objectively real particle orbiting at very near to light speed. The model is based on the postulate that certain velocity terms associated with orbiting bodies can be considered as being affected by relativity. This leads to a model for the atom in which the stable electron orbits are associated with orbital velocities where Gamma is n/α, leading to the idea that it is Gamma that is quantized and not angular momentum as in the Bohr and other models. The model provides a mechanism which leads to quantization of energy levels within the atom and also provides a simple mechanical explanation for the Fine Structure Constant. The mechanism is closely associated with the Sampling theorem and the related phenomenon of aliasing developed in the mid-20th century by engineers at Bell labs.

Since the emergence of quantum theory just over a century ago every model that has been developed for the hydrogen atom incorporates the same basic assumption. From Niels Bohr through de Broglie and Schrödinger up to and including the Standard Model all such theories are based on an assumption first put forward by John Nicholson.

Nicholson recognised that the units of Planck's constant are the same as those of angular momentum and so he reasoned that perhaps Planck's constant was a measure of the angular momentum of the orbiting electron. But Nicholson went one step further and argued that Planck's constant was the fundamental unit or quantum of angular momentum and therefore the angular momentum of the orbiting electron could only take on values which were an integer multiple of Planck's constant. This allowed Bohr to develop a model in which the energy levels of the hydrogen atom matched those of the empirically developed Rydberg formula [1]. When the Bohr model was superseded Nicholson's assumption was simply carried forward unchallenged into these later models.

Nicholson's assumption however lacks any mathematical rigour. It simply takes one variable, angular momentum, and asserts that if we allow it to have this characteristic quantization then we get energy levels which appear to be correct. In so doing it fails to provide any sort of explanation as to just why such a quantization should take place.

In the mid-20th century a branch of mathematics emerged which straddles the boundary between continuous functions and discrete solutions. It was developed by engineers at Bell Labs to address problems of capacity in the telephone network. While at first site there appears to be little to connect problems of network capacity with electrons orbiting atomic nuclei it is the application of these mathematical ideas which holds the key to explaining quantization inside the atom.

In the 1930's and 40's telecommunications engineers were concerned to increase the capacity of the telephone network. One of the ideas that surfaced was called Time Division Multiplexing. In this each of a number of incoming telephone lines is sampled by means of a switch, the resulting samples are sent over a trunk line and are decoded by a similar switch at the receiving end before being sent on their way. This allowed the trunk line to carry more telephone traffic without the expense of increasing the number of cables or individual lines. The question facing the engineers at the time was to determine the minimum frequency at which the incoming lines needed to be sampled in order that the telephone signal can be correctly reconstructed at the receiving end.

The solution to this problem was arrived at independently by a number of investigators, but is now largely credited to two engineers. The so called Nyquist-Shannon sampling theorem is named after Harry Nyquist [2] and Claude Shannon [3] who were both working at Bell Labs at the time. The theorem states that in order to reproduce a signal with no loss of information, then the sampling frequency must be at least twice the highest frequency of interest in the signal itself. The theorem forms the basis of modern information theory and its range of applications extends well beyond transmission of analogue telephone calls, it underpins much of the digital revolution that has taken place in recent years.

What concerned Shannon and Nyquist was to sample a signal and then to be able to reproduce that signal at some remote location without any distortion, but a corollary to their work is to ask what happens if the frequency of interest extends beyond this Shannon limit? In this condition, sometimes called under sampling, there are frequency components in the sampled signal that extend beyond the Shannon limit and maybe even beyond the sampling frequency itself.

A simple example can be used to illustrate the phenomenon. Suppose there is a cannon on top of a hill, some distance away is an observer equipped with a stopwatch. The job of the observer is to calculate the distance from his current location to the cannon. Sound travels in air at roughly 340 m/s. So it is simply a matter of the observer looking for the flash as the cannon fires and timing the interval until he

hears the bang. Multiplying the result by 340 will give the distance to the cannon in metres, let's call this distance D.

This is fine if the cannon just fires a single shot, but suppose the cannon is rigged to fire at regular intervals, say T seconds apart. For the sake of argument and to simplify things, let's make T equal to 1. If the observer knows he is less than 340 m from the cannon there is no problem. He just makes the measurement as before and calculates the distance D. If on the other hand he is free to move anywhere with no restriction placed on his distance to the cannon then there is a problem. There is no way that the observer knows which bang is associated with which flash, so he might be located at any one of a number of different discrete distances from the cannon. Not just any old distance will do however. The observer must be at a distance of D or $D + 340$ or $D + 680$ and so on, in general $D + 340n$. The distance calculated as a result of measuring the time interval between bang and flash is ambiguous. In fact there are an infinite number of discrete distances which could be the result of any particular measured value. This phenomenon is known as aliasing. The term comes about because each actual distance is an alias for the measured distance.

Restricting the observer to be within 340 m of the cannon is simply a way of imposing Shannon's sampling limit and by removing this restriction we open up the possibility of ambiguity in determining the position of the observer due to aliasing.

Let's turn the problem around a little. If instead of measuring the distance to the cannon the position of the observer is fixed. Once again to make things simpler, let's choose a distance of 340m. This time however we are able to adjust the rate of fire of the cannon until the observer hears the bang and sees the flash as occurring simultaneously. If the rate of fire is one shot per second then the time taken for the slower bang to reach the observer exactly matches the interval between shots and so the two events, the bang and the flash are seen as being synchronous. Notice that the bang relates, not to the current flash, but to the previous flash.

If the rate of fire is increased then at first, for a small increment, the bang and the flash are no longer in sync. They come back into sync however when the rate of fire is exactly two shots per second, and again when the rate is three shots per second. If we had a fast enough machine gun this sequence would extend to infinity for a rate of fire which is an integer number of shots per second. Notice that now the bang no longer relates to the previous flash, but to a previous flash. It is interesting to note also that if the rate of fire is reduced from once per second then the observer will never hear and see the bang and the flash in sync with one another and so once per second represents the minimum rate of fire which will lead to a synchronous bang and flash. In fact what we have here is a system that has as its solutions a base frequency and an infinite set of harmonic frequencies.

Suppose now that there is some mechanism which feeds back from the observer to the cannon to drive the rate of fire such that bang and flash are in sync, and suppose that this feedback mechanism is such as to always force the condition to apply to the nearest rate of fire which produces synchronisation.

We now have a system which can cause a variable, in this case the rate of fire of the gun, to take on a series of discrete values even though, in theory at least, the rate of fire can vary continuously. Equally important is that if the feedback mechanism is capable of syncing the system to the lowest such frequency then all the multiples of this frequency are also solutions, in other words if the base frequency is a solution then so are harmonics of the base frequency.

This idea that there are multiple discrete solutions which are harmonics of a base frequency is an interesting one since it couples the domains of the continuous and the discrete. Furthermore what the example of the cannon shows us is that any system which produces results which are a harmonic sequence must involve some sort of sampling process. This becomes clear if we consider the Fourier representation of a harmonic sequence. A harmonic sequence of the type described consists of a number of discrete frequencies, spreading up the spectrum and spaced equally in the frequency domain with each discrete frequency represented by a so called Dirac function. Taken together they form what is described as a Dirac comb, in this case in the frequency domain. The inverse Fourier transform of such a Dirac comb is itself another Dirac comb, only this time in the time domain, and a Dirac comb in the time domain is a sampling signal [4].

This link between a Dirac comb in the frequency domain and a corresponding Dirac comb in the time domain means that if ever we observe a set of harmonics in some natural process there must inevitably be some form of sampling process taking place in the time domain and vice versa.

One such example, in which this relationship has seemingly been overlooked, is found in the structure of the hydrogen atom.

By the beginning of the 20th century it was becoming evident that the universe was composed of elements which were not smooth and continuous but were somehow lumpy or granular in nature. Matter was made up of atoms, atoms themselves contained electrons and later it emerged that the atomic nucleus was itself composed of protons and neutrons.

Perhaps even more surprising was that atoms could only absorb or emit energy at certain discrete levels. These energy levels are characteristic of the atom species and form the basis of modern spectroscopy. The issue facing the scientists of the day was that this discrete behaviour is not associated with the discrete nature of the structure of the atom; that can easily be explained by asserting that any atom contains an integer number of constituent particles. Where energy levels are concerned, the quantization effects involve some sort of process that is taking place inside the atom.

The atom with the simplest structure is that of hydrogen,

comprising a single proton surrounded by an orbiting electron and work began to investigate its structure and to understand the mechanisms which gave it its characteristic properties.

The first such theoretical model was proposed by Niels Bohr [5]. Bohr used simple classical mechanics to balance the centrifugal force of the orbiting electron against the electrostatic force pulling it towards the atomic nucleus. He needed a second equation in order to solve for the radius and velocity of the orbiting electron and came upon the idea proposed by John Nicholson [6]. Nicholson reasoned that the units of Planck's constant matched those of angular momentum and so he proposed that the angular momentum of the orbiting electron could only take on values which were an integer multiple of was Planck's constant.

Bohr's equations worked, but they threw up a strange anomaly. In Bohr's model each energy level is represented by the orbiting electron having a specific orbit with its own particular orbital velocity and orbital radius. The really strange thing was that in order to fit with the conservation laws, transitions from one energy state to another had to take place instantly and in such a way that the electron moved from one orbit to another without ever occupying anywhere in between, a sort of discontinuity of position. This ability to jump instantaneously across space was quickly dubbed the Quantum Leap in the popular media, a phrase which still has resonance today.

Bohr reasoned that

$$l = mv_n r_n = n\hbar \tag{1}$$

$$\frac{Kq^2}{\hbar c} = \frac{mv_n^2}{r_n} \tag{2}$$

which means

$$v_n = \frac{Kq^2}{n\hbar} \tag{3}$$

$$r_n = \frac{n^2\hbar^2}{mKq^2}\,. \tag{4}$$

where m is the rest mass of the electron, q is the charge on the electron, r_n is the orbital radius for the nth energy level, v_n is the orbital velocity for the nth energy level, l is the angular momentum, K is the Coulomb force constant, $\hbar$ is Planck's constant.

Equation 1 represents Nicholson's assumption that angular momentum can only take on values which are integer multiples of Planck's constant.

Equation 2 balances the centrifugal force against the electrostatic force.

Equation 3 shows that the orbital velocity decreases with increasing energy level.

Equation 4 shows that the orbital radius increases as the square of the energy level and leads directly to the idea of the Quantum Leap.

It was widely accepted that the Bohr model contained substantial flaws. Not only did it throw up the quirky quantum leap, but it took no account of special relativity, it failed to explain why the electron orbit did not decay due to synchrotron radiation but most important of all it failed to explain the nature of the quantization of angular momentum*. The fact is that the assumption that angular momentum is quantized lacks any mathematical rigour, the assumption is arbitrary and expedient and fails to address the underlying question as to why and how such quantization occurs but merely asserts that if we make the assumption then the numbers seem to fit. Nevertheless, and despite this, the Bohr assumption has continued to be accepted and forms an integral part of every theory which has come along since.

In a paper published in 1905 Einstein had shown that light, which had hitherto been considered a wave, was in fact a particle [7]. In an effort to explain quantization the French mathematician Louis de Broglie turned this idea on its head and suggested that perhaps the electron was not a particle but should be considered as a wave instead. He calculated the wavelength of the electron, dividing Planck's constant by the electron's linear momentum and found that when he did so the orbital path of base energy state contained one wavelength; that of the second energy state contained two wavelengths and so on, in what appeared at first site to be a series of harmonics†.

On any other scale the wavelength of an object in orbit is associated with the orbital path length or circumference of the orbit and can be derived as a result of dividing the angular momentum of the orbiting object by its linear momentum. De Broglie instead chooses to associate the wavelength of the particle with the value of Planck's constant divided by the linear momentum, while at the same time assuming that the angular momentum of the particle was an integer multiple of Planck's constant. In choosing to substitute Planck's constant in this way instead of the angular momentum when calculating the wavelength, what de Broglie is doing is to coerce the wavelength of the electron to be an integer fraction of the orbital path length. Viewed in this light de Broglie's contribution can be seen as less of an insight and more of a contrivance.

If you were to observe an object in orbit, say a moon orbiting Jupiter or the proverbial conker‡ whirling on the end of a string, what you see is a sine wave. The orbiting object

*At first site it appears that the energy of the electron in the Bohr atom decreases with increasing energy level. However since the radius changes with energy level, the potential energy does also. When these two effects are combined, the energy levels increase with increasing energy level.

†In fact they are not harmonics of a single fundamental frequency, but instead each harmonic relates to a different base frequency and these two effects combine in such a way that they form a sub harmonic or inverse harmonic sequence

‡A conker is a horse chestnut on a string often used in a children's game

subtends a wave to an external observer of the form:

$$d = R\sin(\omega t) \tag{5}$$

or

$$d = R\sin(2\pi F t)\,. \tag{6}$$

where d is the displacement about some mean

For such a body we can easily calculate the orbital radius if we know the angular momentum and the linear momentum.

$$R = \frac{l}{p} = \frac{mvr}{mv}\,. \tag{7}$$

Furthermore we can identify the wavelength of such a wave with the orbital circumference which is simply.

$$\lambda = 2\pi R\,. \tag{8}$$

This is true for all orbiting objects no matter whether they are the size of a planet or the size of a conker.

By what rational then does de Broglie identify the wavelength of the orbiting electron, not with the angular momentum in this way, but with Planck's constant, which he believes, according to Bohr's assumption, to be an integer fraction of the angular momentum?

The alignment of wavelength with Planck's constant in this way cannot be justified either mathematically or mechanically. It is a contrivance which leads to the idea that there is some sort of wavelength which is an integer fraction of the orbital circumference. This is no miraculous discovery, not an insight into the workings of the atom, but an artificial device which reinforces and sustains the Bohr assumption without any basis in mechanics.

Other later models, such as that of Schrödinger, are based directly on the work of de Broglie and therefore inherently follow Bohr's assumption, up to and including the currently proposed Standard Model. Having been adopted by Bohr, later theorists simply continued with this working assumption and incorporated it into all subsequent models for the atom, without ever bothering to go back and justify it, until now it has become an item of received wisdom and an article of faith.

The trouble with all of these models is that the assumption proposed by Nicholson and adopted by Bohr is not based on finding any mechanism that leads to angular momentum being quantized in this way. The assumption was simply expedient — it just happens to give the values for the absorption and emission spectra of the hydrogen atom which match those of the Rydberg formula.

The year 1905 was an eventful one for Albert Einstein. In that year, he not only published his paper on the discrete nature of the photon but he also published two further seminal works as well as submitting his Ph.D. thesis. The most famous of his other papers concerned the dynamics of moving bodies [8]. This is the paper whose later editions contained the equation $e = mc^2$. The paper was based on a thought experiment and concerned the perception of time, distance and mass as experienced by two observers, one a stationary observer and one moving relative to the stationary observer at speeds approaching that of light.

What Einstein showed is that time elapses more slowly for a moving observer, that distances measured by a moving observer are foreshortened relative to those same distances measured by a stationary observer and that a stationary observer's perception of the mass of a moving object is that it has increased. All three effects occur to the same extent and are governed by a factor γ (Gamma). The time between two events observed by the stationary observer as time t is seen by the moving observer as time $T = t/\gamma$. Similarly the distance between two point measured by the stationary observer as distance d is seen by the moving observer as distance $D = d/\gamma$. As far as the stationary observer is concerned the mass of the moving object is seen to increase by this same factor γ.

Gamma is referred to as the Lorentz factor and is given by the formula:

$$\gamma = \frac{c}{\sqrt{c^2 - v^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\,. \tag{9}$$

Both observers agree on their relative velocity but go about calculating it in different ways. For the stationary observer the velocity of the moving observer is the distance travelled divided by the time taken as measured in his stationary domain. For the stationary observer the velocity is:-

$$v = \frac{d}{t}\,. \tag{10}$$

For the moving observer the distance as measured in his own domain is foreshortened by the factor Gamma, but the time taken to cover that distance reduced by the same factor Gamma.

$$v = \frac{D}{T} = \frac{\frac{d}{\gamma}}{\frac{t}{\gamma}} = \frac{d}{t}\,. \tag{11}$$

There is a great deal of experimental evidence to support Einstein's Special Theory. One of the more convincing experiments was carried out at CERN in 1977 and involved measuring the lifetimes of particles called muons in an apparatus called the muon storage ring [9]. The muon is an atomic particle which carries an electric charge, much like an electron, only more massive. It has a short lifetime of around 2.2 microseconds before it decays into an electron and two neutrinos.

In the experiment muons are injected into a 14m diameter ring at a speed close to that of light, in fact at 99.94% of the speed of light where Gamma has a value of around 29.33. The muons, which should normally live for 2.2 microseconds, were seen to have an average lifetime of 64.5 microseconds; that is the lifetime of the muon was increased

by a factor Gamma. This comes about because the processes which take place inside the muon and which eventually lead to its decay are taking place in an environment which is moving relative to us at 99.94% of the speed of light and in which time, relative to us, is running 29.33 times slower. Hence the muon, in its own domain, still has a lifetime of 2.2 microseconds, it's just that to us, who are not moving, this appears as 64.5 microseconds.

Travelling at almost the speed of light a muon would normally be expected to cover a distance of 660 metres or roughly 7.5 times around the CERN ring during its 2.2 microsecond lifetime, but in fact the muons travelled almost 20,000 metres or 220 times around the ring. This is because distance in the domain of the muon is compressed so what we stationary observers see as being 20,000 metres the muon sees as being just 660 metres.

Both parties agree that during its lifetime the muon completes some 220 turns around the ring. We stationary observers see this as having taken place in some 64.5 microseconds, corresponding to a frequency of 3.4 MHz, while the muon sees these 220 turns as having been completed in just 2.2 microseconds, corresponding to a frequency of 100 Mhz. Hence for the muon and indeed all objects orbiting at close to light speed orbital frequency is multiplied by a factor Gamma relative to that of a stationary observer and it is this multiplication of orbital frequency which holds the key to the discrete energy levels of the atom.

As well as this effect on orbital frequency the muon ring experiment serves to show that considerations of special relativity can be applied to objects in orbit, this despite the fact that object in orbit are subject to a constant acceleration towards the orbital centre. However where the orbital velocity is constant, it is reasonable and correct to apply considerations of special relativity around the orbital path. In effect what we are doing is to resolve the orbital velocity into two components, one tangential component which has a constant velocity and one radial where there is a constant acceleration.

We have seen that speed is invariant with respect to relativity. Both the moving object and the stationary observer agree on their relative speed. This invariance of speed is central to the derivation of special relativity and so is deemed to be axiomatic. There is however one circumstance where it is reasonable to suggest that this need not be the case. For a stationary observer we normally require the use of two clocks in order to measure velocity; one at the point of departure and one at the point of arrival (at least conceptually). An object which is in orbit however returns once per cycle to its point of departure and so we can measure the orbital period of such an object with a single clock provided we do so over a complete orbit.

Thus for an object in orbit it is possible to define two velocity terms relating to the tangential or orbital velocity*. The first of these I have called the Actual Velocity and is simply the distance around the orbit divided by the orbital period as measured by the stationary observer. The second velocity term is the distance around the orbit as measured by the moving observer divided by the orbital period as measured by the stationary observer. Such a velocity term straddles or couples the two domains, that of the orbiting object and that of the stationary observer and so could sensibly be called the "Coupling Velocity" or possibly the "Relativistic Velocity". A simple calculation shows that the Relativistic Velocity is related to the Actual Velocity by the same factor Gamma an hence:

$$v_R = \frac{D}{t} = \frac{d}{t\gamma} = \frac{v}{\gamma} . \tag{12}$$

Thus far Relativistic Velocity is only a definition. However there is one set of circumstances where such a velocity term can indeed be justified and that is when dealing with the equations of motion relating to objects in orbit. It is considered here to be meaningful to use this Relativistic Velocity term when dealing with orbital velocities such as occur when calculating angular momentum, centripetal and centrifugal force and acceleration.

Nicholson had suggested that because Planck's constant has the units of angular momentum that it was somehow associated with the angular momentum of the orbiting electron. Here we take up that idea and suggest that the angular momentum of the orbiting electron is equal to Planck's constant, but reject his other idea that angular momentum is quantized. Instead we assume that orbital velocity is affected by relativity and use this to derive the equations of motion of the orbiting electron.

Planck's constant is then seen, not as a fundamental quantum of angular momentum but instead as providing a limiting value for angular momentum. The effect would not be significant at low velocities, but if the electron orbiting the hydrogen atom were to do so at close to light speed then:

$$l = \hbar = (m\gamma)\, r \left(\frac{c'}{\gamma} \right) . \tag{13}$$

where l is the angular momentum, $\hbar$ is Planck's constant, m is the mass of the electron, r is the orbital radius of the electron, c' is the orbital velocity of the electron and is very close to c, the speed of light.

Both the mass term and the velocity term are affected by relativity. The mass term because mass increases by factor Gamma as the object's velocity approaches the speed of light and in this case the velocity term is affected because we are dealing with an object in orbit and it is therefore appropriate

*In fact it is possible to define a further two velocity terms, the relativistic distance divided by the relativistic time and the actual distance divided by the relativistic time. The first of these is the invariant velocity discussed earlier. As a stationary observer we do not have any direct access to the moving clock and so these velocities can only be described mathematically and appear to have no physical significance.

to use Relativistic Velocity which is the Actual Velocity divided by Gamma. However since we are concerned here with an orbital velocity very close to the speed of light, to a first approximation we can substitute c for c' in Equation 13.

$$l = \hbar = (m\gamma)\, r \left(\frac{c}{\gamma}\right). \tag{14}$$

The two Gamma terms will cancel. The terms for rest mass, Planck's constant and the speed of light are all constants, which must therefore mean that the orbital radius is also a constant

$$R = \frac{\hbar}{mc}. \tag{15}$$

This not unfamiliar term is known as the Reduced Compton Wavelength although here it takes on a new and special significance as the characteristic radius at which an electron will orbit at or near light speed. This serves to explain why the orbiting electron does not emit synchrotron radiation. It does not do so because it is not driven to orbit the atomic nucleus by virtue of being accelerated by forces towards the orbital centre in the normal way, instead it is constrained to orbit at this radius by the limiting effect of Planck's constant. It is as if the electron is orbiting on a very hard surface from which it cannot depart and which it cannot penetrate. Equation 15 also means that there is no need to introduce the idea of a quantum leap or later equivalents. If the electron is constrained to always orbit at a fixed radius, then changes in energy level have to take place as a result of changes in orbital velocity, with no accompanying change of radius. Indeed this idea that the electron orbits at constant radius is a necessary condition for the electron to be considered objectively real.

Substituting Relativistic Velocity into the force balance equation that Bohr himself used, but at an orbital velocity very close to that of light yields another interesting result*

$$\frac{Kq^2}{\hbar c} = \frac{(m\gamma)}{r}\left(\frac{c}{\gamma}\right)^2. \tag{16}$$

Which combines with Equation 15 and simplifies to give:

$$\frac{Kq^2}{\hbar c} = \frac{1}{\gamma}. \tag{17}$$

Readers may be familiar with the term on the left of this equation which is known as the Fine Structure Constant often written as α (Alpha). So for the base energy state of the atom

$$\gamma = \frac{1}{\alpha}. \tag{18}$$

α has a value of $7.2973525698 \times 10^{-3}$

From this and Equation 9 we can easily calculate the corresponding orbital velocity and frequency as measured by the stationary observer.

$$\frac{v}{c} = \sqrt{1-\alpha^2} = 0.999973371. \tag{19}$$

The orbital velocity turns out to be 99.9973% of the speed of light c, thus vindicating the first approximation made in Equation 14 and the frequency (in the domain of the stationary observer)

$$\omega_1 = \frac{v}{R} = 7.76324511 \times 10^{20}. \tag{20}$$

The physicist Richard Feynman [10] once said of Alpha that:

"It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a ***coupling***† *comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!"*

Equation 18 effectively solves the mystery, providing an explanation for the physical significance of the Fine Structure Constant. It is seen simply as the ratio of two velocities, the Relativistic Velocity and the Actual Velocity of the orbiting electron. Since these two velocities share the same orbital period, it can also be seen as the ratio of two orbital path lengths, the one traversed at non-relativistic speeds to that traversed by the orbiting electron at near light speed. The Fine Structure Constant is seen to be dynamic in nature. Its value relies on the fact that the electron is in motion, orbiting at near light speed; it does so at a speed that is necessary to maintain structural equilibrium within the hydrogen atom, since it is only by travelling at this speed that the structural integrity of the atom can be maintained. In the world of the atom, where there is no friction and in the absence of any sort of external input, the atom remains stable and, unless disturbed in some way, the electron will continue in this state indefinitely. In this sense it defines the speed at which the electron has to travel in order to achieve a stable orbit.

So far we have only considered the lowest or base energy state of the atom. We have seen that one of the effects of relativity is to multiply frequency in the domain of a moving object by Gamma. The frequency in the domain of the

*Once again since the orbital velocity is very close to the speed of light we can, to a first approximation, substitute c as the Actual Velocity

†My emphasis — the term Coupling Velocity resonates with the idea of Alpha as a coupling constant.

electron which corresponds to this stable state is simply calculated by multiplying by Gamma — equivalent to dividing by Alpha – to give.

$$\Omega = \frac{\omega_1}{\gamma} = 1.06378925 \times 10^{23} . \quad (21)$$

But just as was the case with the observer and the cannon if there is a frequency Ω at which the atom is stable then frequencies of $n\Omega$ must also be stable for all n = integer which in turn means that there are stable states for all

$$\gamma_n = \frac{n}{\alpha} \quad (22)$$

and so

$$r_n = R = \frac{\hbar}{mc} \quad (23)$$

and

$$\frac{v_n}{c} = \sqrt{\frac{n^2 - \alpha^2}{n^2}} . \quad (24)$$

Equation 23 shows that the orbital radius remains the same for all energy levels, while Equation 24 describes the orbital velocity for the nth energy state*. Table 1 shows the resulting orbital velocities for the first 13 energy states and the theoretically infinite state of the hydrogen atom and as you might expect they match the absorption and emission spectra of the hydrogen atom perfectly.

During the 1930's and 40's Einstein and Bohr disagreed over the nature of reality, with Bohr arguing that the laws of physics were different on the scale of the atom and that as a consequence reality becomes subjective in nature. Particles are not considered to discrete point particles in the classical sense, but instead are considered to be nebulous wave-particles which manifest themselves as either particles or as waves when subjected to some sort of observing process. Einstein on the other hand took the view that reality had to be objective and that particles must therefore be discrete point particles having deterministic position and velocity.

In the end the debate was largely resolved by default. Bohr simply outlived Einstein and so his ideas prevailed and form the basis of today's Standard Model. Einstein is nowadays often described as being an old man, set in his ways and unable to accept the new ways of thinking. But this is to misconstrue Einstein's position, which was one of principle.

Einstein had argued that the laws of physics are the same for all reference frames, while Bohr reasoned that the laws of physics are different on the scale of the atom. Einstein was concerned with reference frames of comparable scale that were in motion with respect to one another but it is logical to extend his idea to reference frames of differing scales. If we start from this position and pursue the idea that particles are objectively real and that the laws of physics are the same independent of scale then it is necessary to question our current understanding of the laws of physics. They must be deficient in some way and it is necessary to find a way in which the laws must be modified to describe the atom but which does not affect our understanding on all other scales.

The idea of relativistic velocity postulated here does just that. It provides a model for the structure and dynamics of the hydrogen atom which is consistent with particles which are objectively real. At the same time it does what all previous models have failed to do and provides a mechanism to explain exactly why the energy levels of the atom are quantized without the need of resorting to arbitrary assumptions. The idea of a Relativistic Velocity or Coupling Velocity, a velocity term which is affected by relativity, solves all of the problems that faced Niels Bohr with his model and produces a model for the hydrogen atom which matches the emission and absorption spectra of the atom.

Here quantization takes place with respect to the variable Gamma as the orbital velocity of the electron gets ever closer to the speed of light with increasing energy level, and not with respect to angular momentum as postulated by Bohr. Angular momentum for the orbiting electron remains substantially constant and equal to Planck's constant over all of its energy levels as the orbital velocity varies from 99.99733% of c for the base energy state upwards as energy levels increase, although never quite achieving the theoretical limit of 100%, while Gamma is constrained to take on values which are integer multiples of a base value, that value being the reciprocal of the Fine Structure Constant. Planck's constant takes on a new and special significance, not as the quantum of angular momentum of the existing models, but as a lower limit for angular momentum below which it cannot exist.

The orbital radius of the electron remains substantially constant irrespective of the energy level of the atom, a necessary condition for an objectively real electron, and so transitions from one energy state to another take place without the need to introduce the idea of discontinuity of position, inherent in the Bohr model, or its equivalent probability density functions and wave particle duality found in other more recent models. Such transitions are easily explained as simple changes in the orbital velocity of the electron over a dynamic range which lies very close to the speed of light. With no changes in orbital radius, changes in energy level involve no change in potential energy, only the kinetic energy of the orbiting electron changes between energy states.

Thus the morphology of the atom remains substantially unaltered for all energy levels. This is consistent with the atom having the same physical and chemical properties irrespective of energy level. The Bohr model, and indeed the standard model, would have us believe that the morphology of the atom changes substantially with energy level, with the orbital radius increasing as the square of the energy level with no theoretical upper limit. Such changes are difficult to rec-

*Notice that since the orbital radius remains substantially the same for all energy levels, there is no change in potential energy between the various different energy levels, only a change in kinetic energy.

n	v_n/c	$1/\gamma_n$	Energy eV	ΔEnergy	eV
1	0.999973371	0.007297559	7.76324511E+20	255485.925	13.607
2	0.999993343	0.003648853	7.76340016E+20	255496.130	3.402
3	0.999997041	0.002432577	7.76342887E+20	255498.020	1.512
4	0.999998336	0.001824435	7.76343892E+20	255498.682	0.850
5	0.999998935	0.001459549	7.76344357E+20	255498.988	0.544
6	0.999999260	0.001216291	7.76344610E+20	255499.154	0.378
7	0.999999457	0.001042536	7.76344762E+20	255499.255	0.278
8	0.999999584	0.000912219	7.76344861E+20	255499.320	0.213
9	0.999999671	0.000810861	7.76344929E+20	255499.364	0.168
10	0.999999734	0.000729775	7.76344977E+20	255499.396	0.136
11	0.999999780	0.000663432	7.76345013E+20	255499.420	0.112
12	0.999999815	0.000608146	7.76345040E+20	255499.438	0.094
13	0.999999842	0.000561366	7.76345061E+20	255499.452	0.081
∞	1.000000000	0.000000000	7.76345184E+20	255499.532	0.000

Table 1:

oncile with an atom who's physical and chemical properties remain the same for all energy levels.

The model explains all of the shortcomings found in the Bohr model, the absence of orbital decay due to synchrotron radiation and the need for a quantum leap. Bohr had ignored the effects of special relativity on the energy levels of the atom, even though they should have been small but significant at the velocities predicted by his model. Here they are fully integrated into the model.

The model sheds a new light on the nature of the wave particle duality. The electron is seen as a point particle in the classical sense, having deterministic position and velocity*. Electrons are thus objectively real. The electron has wavelike properties, but these derive from the orbital motion of an objectively real particle. The waves are seen as the projection of the circular orbit of the objectively real electron onto an external observer, in much the same way that we can describe the orbit of the moons of distant planets as having a wavelike nature. There is no need to invent the ether or what has more recently passed for the ether, the so called fabric of space time, as a medium in which these waves exist. In the final analysis where vacuum contains absolutely nothing, there is nothing to wave except the particle and that is precisely what the model provides.

The introduction of Relativistic Velocity has another major implication. It extends the laws of physics down to the scale of the atom and possibly beyond. With its introduction the same set of physical laws extends from a scale of approximately 10^{-20} m to 10^{20} m thus doing away with the notion that a different set of physical law applies on the scale of the atom. It is quite likely therefore that a single set of physical laws exists for all scales and throughout the universe.

Finally it provides a simple mechanical explanation for the existence and the value of the hitherto mysterious Fine Structure Constant.

Appendix 1 Derivation of Centripetal Acceleration under relativistic conditions

The idea that orbital velocity is affected by relativity is central to the theory presented here, so it is perhaps worthwhile examining this idea in a little more detail. Before doing so however it is necessary to restate that the use of Special Relativity in dealing with objects which have constant orbital velocity is entirely appropriate, this despite the fact that such objects are subject to acceleration. The velocity of an object which is in orbit can be considered as having two components, a tangential component and a radial component. For constant orbital velocity, the tangential component is itself constant and therefore can be dealt with using Special Relativity which affects the time and distance measured along the orbital path. Direct evidence to support this comes in the form of the Muon ring experiment described earlier.

Such an orbiting object is subject to constant acceleration towards the orbital centre and it is this acceleration which in effect maintains the circular path. Conventional wisdom has it that this centripetal acceleration is not affected by relativity, since it acts in a direction which is normal to the velocity of the object. Here it is argued that this cannot be the case since the distances involved in calculating centripetal acceleration derive directly from the distances travelled around the orbital path and that these distances are themselves affected by relativity. It can then be shown that this is equivalent to substituting Relativistic Velocity in place of Actual Velocity in the standard formula for calculating centripetal acceleration.

Einstein showed that objects which are travelling at close

*This is not to say that uncertainty does not exist, it does, but it is seen as a practical issue of measurement when the scale of the measurement tools is similar to that of the measured object and not as being an intrinsic property of the particle.

to light speed are affected in three ways, time in the domain of the moving observer advances at a slower rate than it does for a stationary observer, distance for the moving object is foreshortened in the direction of travel relative to that same distance as measured by the stationary observer. The mass of a moving object appears increased as far as the stationary observer is concerned. All three effects occur to the same extent by the factor Gamma (γ). Gamma is named after the Dutch physicist Hendrik Antoon Lorentz (1853 — 1928). Gamma is given by the formula

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} . \tag{25}$$

Examination of the effect of relativity on an object moving at close to the speed of light however reveals that both time and distance are scaled by a factor $1/\gamma$ and so from Equation 25

$$\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} . \tag{26}$$

It can be seen that this is the equation of a circle, more specifically a quadrant of a unit circle, since v is constrained to lie between 0 and c as shown in Figure 1.

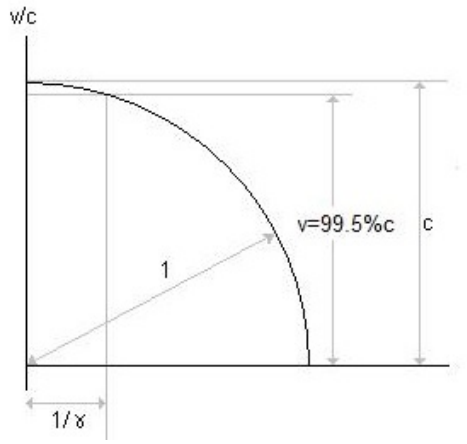

Fig. 1:

If the object under consideration is in circular orbit, then this quadrant can be superimposed on the orbital path to form a hemisphere. Objects orbiting at non-relativistic speeds see the path length around the orbit as being equal in length to the equator, while objects orbiting at higher speeds follow a path length described by a line of latitude on the hemisphere. An object orbiting at the theoretical maximum speed of light would then be pirouetting at the pole. We can consider the length of the orbital path as being represented by the line of latitude formed by a slicing plane which cuts through the hemisphere parallel to the equatorial plane. In Figure this is at approximately 15% of the speed of light c and so the orbital path length is just a little less than the equatorial path length, around 99%.

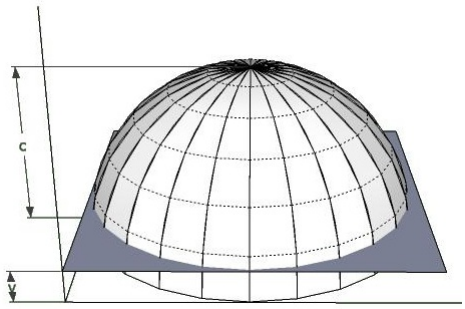

Fig. 2:

In Figure 3 the orbital velocity is approximately 80% of the speed of light and so the orbital path length as seen by the moving object is approximately 60% that for an object moving at non-relativistic speed

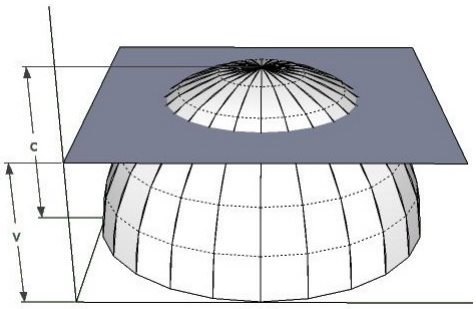

Fig. 3:

In Figure 4 the orbital velocity is around 98% of the speed of light and the corresponding orbital path length is approximately 20% of that for non-relativistic motion.

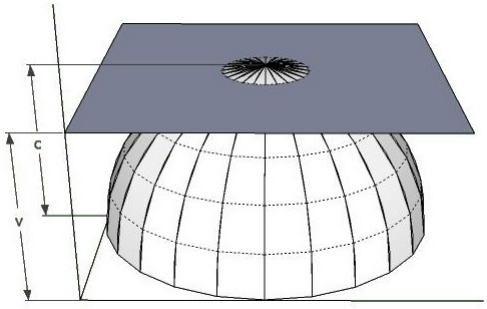

Fig. 4:

This hemispheric model of the motion of an orbiting object is useful because it allows us to visualise the orbital path

length as being foreshortened by relativity while at the same time the radius of the orbit is unaffected by relativity. The orbital geometry is non-Euclidean and in reality all takes place in just one plane. The introduction of this third dimension is just a device to allow us to visualise what is going on. The orbiting object sees the distance it travels around one orbit as being reduced by a factor Gamma, but nevertheless sees the orbital radius as being unaffected by relativity since this is at right angles to the direction of travel. Thus we can represent the radius of the orbit as being the distance from a point on the relativistic orbit to the centre of the hemisphere.

The term Actual Velocity has been adopted to describe the velocity of the orbiting object as seen by a stationary observer. This is easily calculated as the circumference of the orbital path, the equator of the hemisphere (d), divided by the orbital period (t), both measured by the stationary observer.

The theory postulates that there is a velocity term which is affected by Gamma. This is termed the Relativistic Velocity, but only becomes significant when the Actual Velocity is close to the speed of light. This velocity term can be calculated by taking the foreshortened distance around the line of latitude, which represents the orbital path as seen by the moving observer, divided by the orbital period as measured by a stationary observer. The foreshortened distance around the orbit is calculated as d/γ and the orbital period remains the same as for Actual Velocity (t) and hence this Relativistic Velocity is then easily calculated as $v_R = d/t\gamma$.

We can use this term directly in calculating the angular momentum of the orbiting object. This is simply a restatement of the argument used earlier. Angular momentum is the product of the mass, the velocity and the radius of an orbiting point object. However the mass of the object is affected by relativity, appearing to increase the mass by a factor Gamma (γ) and so:

$$l = (m\gamma)\, r \left(\frac{v_R}{\gamma} \right) . \tag{27}$$

However since for Gamma to take on a significant value v_R must be very close to c, the speed of light and so we can substitute c for v_R. Also since the angular momentum of an electron in orbit around an atomic nucleus is given by Planck's constant we can substitute this for l in Equation 27 to give:

$$l = \hbar = mcr . \tag{28}$$

In effect we are simply substituting Relativistic Velocity for Actual Velocity in the standard textbook formula for calculating angular momentum. This is recognising that the orbital velocity is the distance around the orbit as measured by the moving object divided by the orbital period as measured by a stationary observer.

We can of course use this same argument to substitute Relativistic Velocity for Actual Velocity in the formula for centripetal acceleration and hence derive expressions for centripetal and centrifugal forces. However in the case of centripetal acceleration it is also useful to derive an expression for the relativistic case from first principles.

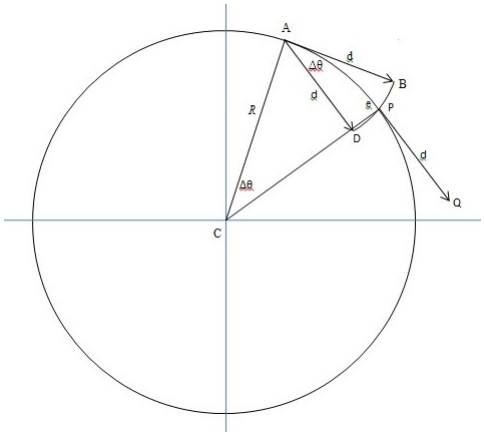

Fig. 5:

The formula for centripetal force was first derived by Christian Huygens in 1659 and describes a constant force acting on a body in circular motion towards the centre of the circle. When combined with Newton's second law this leads to the idea that a body in circular motion is subject to a constant acceleration towards the centre called centripetal acceleration.

It is customary when deriving the formula for centripetal acceleration to use velocity vectors directly. Here we take a slightly different approach and use the distance vectors instead. This is because in the proposed theory only the distance component of velocity is affected by relativity and not the time component. In other respects the derivation is the same as that found in many standard texts.

Consider an object in orbit around a point C at radius R. At a particular instant t the object is at point A and some short interval of time later Δt it is at point P, having moved through an angle subtended at the centre of the circle of $\Delta\theta$.

The vector representing the distance moved in time Δt is AB and has length d and is tangential to the circle, hence CAB is a right angle. At $t + \Delta t$ the object is at P and has a distance vector PQ, also of length d. We can translate the vector PQ to A forming AD. The vector BD then represents the distance moved towards the centre of the circle in time Δt. Note that for as $\Delta\theta$ tends to 0 the line BD tends to a straight line.

Then

$$d = R\Delta\theta . \tag{29}$$

Since APC and ABD are similar triangles (for small $\Delta\theta$)

$$e = d\Delta\theta \tag{30}$$

and the acceleration towards the centre of the circle is

$$a = \frac{e}{\Delta t^2} . \tag{31}$$

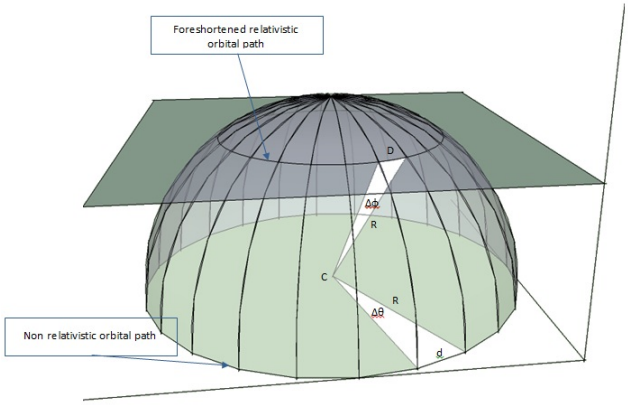

Fig. 6:

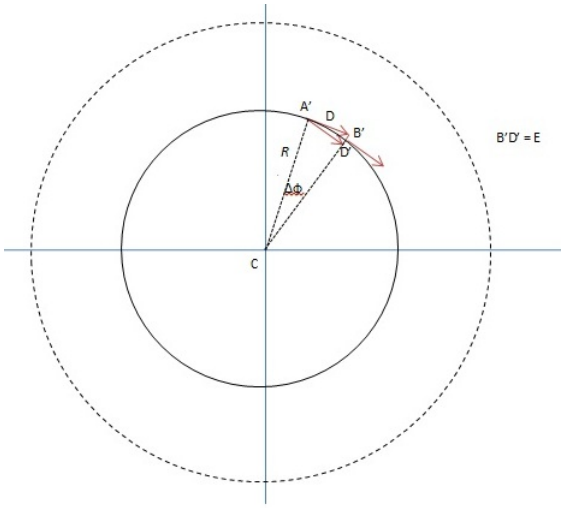

Fig. 7:

Therefore

$$a = \frac{R\Delta\theta^2}{\Delta t^2} . \tag{32}$$

Multiplying both top and bottom by R gives

$$a = \frac{R^2\Delta\theta^2}{R\Delta t^2} . \tag{33}$$

But since

$$v = \frac{d}{\Delta t} = \frac{R\Delta\theta}{\Delta t} . \tag{34}$$

Then

$$a = \frac{v^2}{R} . \tag{35}$$

When we take into consideration the effects of special relativity, the situation becomes a little more complicated. Although the orbital path is foreshortened, as represented by the line of latitude in Figure 6, and hence the circumference of this circle is reduced by a factor Gamma, the radius of the circle is not affected and remains the same as that for the equatorial orbital path.

Figure 6 attempts to show this by introducing a third dimension and using the hemispherical representation developed above. In reality however the radius and the orbital path are co-planar. It can be seen from Figure 6 that the angle subtended by a short segment of the circumference is less for the relativistic path than for the non-relativistic path. From Figure 6 it is evident that

$$\Delta\phi = \frac{\Delta\theta}{\gamma} \tag{36}$$

and

$$R\Delta\phi = \frac{R\Delta\theta}{\gamma} . \tag{37}$$

Figure 7 shows the foreshortened orbital path in plan view. The dashed circle represents the non-relativistic orbital path while the radii are shown dotted to indicate that they are not to scale in this representation.

The distance travelled during time Δt is foreshortened by relativity, instead of travelling a distance AB the object only travels a distance A'B'=D in Figure 7.

$$D = R\Delta\phi . \tag{38}$$

Once again the triangles CA'B' and A'B'D' are similar and so the distance travelled towards the centre of the orbit E is

$$E = D\Delta\phi . \tag{39}$$

Once again the triangles CA'B' and A'B'D' are similar and so the distance travelled towards the centre of the orbit E is

$$A = \frac{E}{\Delta t^2} . \tag{40}$$

Which is also

$$A = \frac{R\Delta\phi^2}{\Delta t^2} \tag{41}$$

Again we can multiply both denominator and numerator by R to give

$$A = \frac{R^2\Delta\phi^2}{R\Delta t^2} . \tag{42}$$

Which gives

$$A = \frac{R^2\Delta\theta^2}{R\Delta t^2\gamma^2} \tag{43}$$

and so

$$A = \frac{v^2}{R\gamma^2} . \tag{44}$$

Equation 44 represents a more general case for calculating centripetal acceleration. When the orbital velocity is low, under non-relativistic conditions, the value of Gamma is unity

and the formula can be simplified to the more familiar one shown in Equation 35. Effectively therefore the formula for centripetal acceleration under relativity substitutes Relativistic Velocity for Actual Velocity in the standard textbook formula.

It is the geometry of the triangle AB'D' which lies at the heart of the argument. Here it is argued that the length B'D' is affected by relativity even though it is measured in a direction at right angles to the direction of travel. This comes about because the lengths of the two sides AB' and AD' are both themselves affected by relativity and the triangle must have geometric integrity and so B'D' must also be scaled by relativity. If it was not then the triangle AB'D' would be a very strange triangle indeed. It would have to be an isosceles triangle in which the third side could be longer than the sum of the two other sides. The direction of the vectors AB' and AD' could not be preserved. Even in non-Euclidian geometry such a triangle would not be possible and so B'D' must be scaled by Gamma.

The measurement of time on the other hand can only take place in the domain of the observer, so the moving observer sees his time in his own domain and the stationary observer sees time in his domain. The two domains are related by a factor Gamma, but from the point of view of direct measurement this is a theoretical connection. In other words the stationary observer has no direct access to the moving clock and, vice versa, the moving observer has no direct access to the stationary clock.

Appendix 2 An Analytical Method for calculating Actual Velocity

A more analytical approach for calculating the value for c' can be found without the first approximation used above:

The equation for the value of gamma

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} . \tag{45}$$

From which

$$v = c\sqrt{\frac{\gamma^2 - 1}{\gamma^2}} . \tag{46}$$

Substituting this into the force balance equation gives

$$\frac{m_0 c^2(\gamma^2 - 1)}{R\gamma^3} = \frac{Kq^2}{R^2} . \tag{47}$$

Recognising that $\hbar = m_0 Rc$ and simplifying gives

$$\frac{\gamma^2 - 1}{\gamma^3} = \frac{Kq^2}{\hbar c} . \tag{48}$$

The term on the right hand side is the Fine Structure Constant which is denoted by α. Substituting and rearranging gives the following equation for γ.

$$\alpha\gamma^3 - \gamma^2 + 1 = 0 . \tag{49}$$

The numerical value for α^* is $7.2973525698 \times 10^{-3}$. Substituting this and calculating the three roots gives:

$\gamma = 137.028700944403$

$\gamma = -0.996384222264$

$\gamma = 1.0036823521665$

Only the first of these three values is significant. This cubic equation gives a more precise value for Gamma. By recognizing that v is very close to c in the force balance equation the value of Gamma can be calculated as:

Substituting in the equation for γ gives a value for v:

$$v = c\sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = 0.999973371c . \tag{50}$$

v is the Actual Velocity of the electron around its orbit and as can be seen it is very close to c, the velocity of light, being some 99.9973371% of c, which is in agreement with the method of first approximation to the first 8 significant figures.

Appendix 3 The Rydberg Formula

Joseph Jakob Balmer (1825–1898) was a Swiss mathematician and numerologist who, after his studies in Germany, took up a post teaching mathematics at a girls' school in Basel. A colleague in Basel suggested that he take a look at the spectral lines of hydrogen to see if he could find a mathematical relationship between them. Eventually Balmer did find a common factor† $h = 3.6456 \times 10^{-7}$ which led him to a formula for the wavelength of the various spectral lines.

$$\lambda = \frac{hm^2}{m^2 - 4} , \tag{51}$$

where m is an integer with value 3 or higher.

Balmer originally matched his formula for $m = 3, 4, 5, 6$ and based on this he predicted an absorption line for $m = 7$. Balmer's seventh line was subsequently found to match a new line in the hydrogen spectrum that had been discovered by Ångström.

Balmer's formula dealt with a particular set of spectral lines in the hydrogen atom and was later found to be a special case of a more general result which was formulated by the Swedish physicist Johannes Rydberg.

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) , \tag{52}$$

where λ is the wavelength of the spectral line, R_H is the Rydberg constant for hydrogen, n_1 and n_2 are integers and $n_1 < n_2$.

By setting n_1 to 1 and allowing n_2 to take on values of $2, 3, 4 \ldots \infty$ the lines take in a series of values known as the Lyman series. Balmer's series is obtained by setting $n_1 = 2$ and allowing n_2 to take on values of $3, 4, 5 \ldots \infty$. Similarly for other values of n_1 series of spectral lines have been named according to the person who first discovered them and so:

*CODATA - http://physics.nist.gov/cgi-bin/cuu/Value?alph

†h here is not to be confused with Planck's constant.

n_1	n_2	Series
1	$2 \ldots \infty$	Lyman series
2	$3 \ldots \infty$	Balmer series
3	$4 \ldots \infty$	Paschen series
4	$5 \ldots \infty$	Brackett series
5	$6 \ldots \infty$	Pfund series
6	$7 \ldots \infty$	Humfreys series

Other series beyond these do exist, but they are not named.

By substituting different values for R, it was found that Rydberg's formula worked for all so called *hydrogenic*[*] atoms.

The value of R_H can be found by considering the case where $n_1 = 1$ and $n_2 = \infty$, a condition which represents the maximum possible change in energy level within the hydrogen atom. R_H is then the wavelength of the absorption line associated with such an energy change and was calculated to have a value of 1.097×10^7

This was subsequently found to be given by the formula:

$$R_H = \frac{1}{4\pi} \frac{m_0 c \alpha^2}{\hbar} . \tag{53}$$

The highest possible energy level for the atom occurs when n, the energy level, equals the theoretical value of infinity. The corresponding value for the Actual Velocity would then be c, the speed of light.

The equation for the energy of an orbiting body of mass m with velocity v is easily obtained in any standard text and is given by:

$$e = \frac{1}{2} m v^2 . \tag{54}$$

If we assume that the electron is orbiting at near light speed then the maximum possible energy[†] of an electron orbiting the hydrogen nucleus where the orbital velocity has a theoretical value of c, the speed of light and the mass of the electron is m_0 is

$$e = \frac{1}{2} m_0 c^2 . \tag{55}$$

The energy potential for a hydrogen atom in any arbitrary energy state n is the difference between this maximum energy value and the energy of the nth state

$$e_n = \frac{1}{2} m_0 c^2 - \frac{1}{2} m_0 v_n^2 = \frac{1}{2} m_0 (c^2 - v_n^2) . \tag{56}$$

We saw earlier that gamma could be expressed in terms of c, the velocity of light and v, the Actual Velocity using Einstein's equation for special relativity and that $\gamma_n = n\gamma_0$

$$\gamma_n = \frac{c}{\sqrt{c^2 - v_n^2}} . \tag{57}$$

This is easily rearranged to give an expression for $c^2 - v^2$

$$c^2 - v_n^2 = \frac{c^2}{\gamma_n^2} \tag{58}$$

In the base energy state $n = 0$ and $\gamma_0 = 1/\alpha$

$$c^2 - v_0^2 = c^2 \alpha^2 \tag{59}$$

Hence the maximum energy potential for the atom is

$$e_p = \frac{1}{2} m_0 c^2 \alpha^2 . \tag{60}$$

Substituting numerical values for m_0, c and α gives the maximum energy potential of the atom as
$e_p = 2.18009839 \times 10^1 8$ Joules
or
$e_p = 13.6071$ eV.

The energy potential for any arbitrary energy level n is given by

$$e_{pn} = \frac{1}{2} \frac{m_0 c^2 \alpha^2}{n^2} . \tag{61}$$

Hence the difference between any two energy levels n and m is

$$e_{n,m} = \frac{1}{2} m_0 c^2 \alpha^2 \left(\frac{1}{n^2} - \frac{1}{m^2} \right) . \tag{62}$$

and the difference in orbital frequency is

$$\omega_{n,m} = \frac{1}{2} \frac{m_0 c \alpha^2}{\hbar} \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \tag{63}$$

This can be expressed in terms of wavelength, similar to the Rydberg formula, by dividing both sides by 2π to give

$$\frac{1}{\lambda_{n,m}} = \frac{1}{4\pi} \frac{m_0 c \alpha^2}{\hbar} \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \tag{64}$$

and

$$R_H = \frac{1}{4\pi} \frac{m_0 c \alpha^2}{\hbar} . \tag{65}$$

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[*]A hydrogenic atom is one which is ionized such that it has only one orbiting electron. In theory, at least, any atom can be ionized so as to become hydrogenic.

[†]Note that the electron is orbiting at the same radius for all energy levels, the potential energy of the electron therefore remains the same and all changes in energy level which are then associated with changes in kinetic energy and hence with the velocity of the electron.

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