E(5) Characters to ^{100}Ru Isotope

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The positive and negative parity states of ^{100}Ru isotope are studied within the frame work of the interacting boson approximation model ($IBA-1$). The calculated levels energy, potential energy surfaces, $V(\beta,\gamma)$, and the electromagnetic transition probabilities, $B(E1)$ and $B(E2)$, show that ^{100}Ru isotope has $E(5)$ Characters. Staggering effect, $\Delta I = 1$, has been observed between the positive and negative parity states. The electric monopole strength, $X(E0/E2)$, has been calculated. All calculated values are compared to the available experimental, theoretical data and reasonable agreement has been obtained.

1 Introduction

The mass region $A = 100$ has been of considerable interst for nuclear structure studies as it shows many interesting features. These nuclei show back bending at high spin and shape transitions from vibrational to γ-soft and rotational characters. Many attempts have made to explore these structural changes which is due mainly to the n-p interactions.

Experimentally, the nuclear reaction ^{100}Mo (α,xn) [1] has been used in studying levels energy of ^{100}Ru. Angular distribution, γ-γ coincidences were measured, half-life time has calculated and changes to the level scheme were proposed. Also, double beta decay rate of ^{100}Mo to the first excited 0^+ state of ^{100}Ru has been measured experimentally [2] using γ-γ coincidence technique.

Doppler-shift attenuation measurements following the ^{100}Ru (n, $n'\gamma$) reaction [3] has used to measure the life times of the excited states in ^{100}Ru. Absolute transition rates were extracted and compared with the interacting boson model description. The 2^+(2240.8 keV) state which decays dominantly to the 2^+ via 1701 keV transition which is almost pure $M1$ in nature considered as a mixed-symmetry state. Again ^{100}Ru has been studied [4] experimently and several levels were seen where some new ones are detected below 3.2 MeV.

Theoretically many models have been applied to ruthenium isotopes. Yukawa folded mean field [5] has applied to ^{100}Ru nucleus while the microscopic vibrational model has applied to ^{104}Ru and some other nuclei with their daughters [6]. The latter model was successful in describing the yrast 0^+ and 2^+ states of most of these nuclei and also some of their half-lives.

The very high-spin states of nuclei near A$\approx$100 are investigated by the Cranked Strutinsky method [7] and many very extended shape minima are found in this region. Interacting boson model has been used in studying *Ru* isotopes using a $U(5)$–$O(6)$ transitional Hamiltonian with fixed parameters [8-10] except for the boson number N. Hartree-Fock Bogoliubov [11] wave functions have been tested by comparing the theoretically calculated results for ^{100}Mo and ^{100}Ru nuclei with the available experimental data. The yrast spectra, reduced $B(E2, 0^+ \rightarrow 2^+)$ transition probabilities, quadrupole moments $Q(2^+)$ and g factors, g(2^+) are computed. A reasonable agreement between the calculated and observed values has been obtained.

The microscopic anharmonic vibrator approach (MAVA) [12] has been used in investigating the low-lying collective states in $^{98-108}$Ru. Analysis for the level energies and electric quadrupole decays of the two-phonon type of states indicated that ^{100}Ru can interpreted as being a transitional nucleus between the spherical anharmonic vibrator ^{98}Ru and the quasirotational $^{102-106}$Ru isotopes.

A new emprical approach has been proposed [13] which is based on the connection between transition energies and spin. It allows one to distinguish vibrational from rotational characters in atomic nuclei. The cranked interacting boson model [14] has been used in estimating critical frequencies for the rotation-induced spherical-to-deformed shape transition in $A = 100$ nuclei. The predictions show an agreement with the back bending frequencies deduced from experimental yrast sequences in these nuclei.

The aim of the present work is to use the $IBA-1$ [15, 16] for the following tasks:

1. Calculating the potential energy surfaces, $V(\beta,\gamma)$, to know the type of deformation existing for ^{100}Ru;
2. Calculating levels energy, electromagnetic transition rates $B(E1)$ and $B(E2)$;
3. Studying the relation between the angular momentum I and the rotational angular frequency $\hbar\omega$ for bending in ^{100}Ru;
4. Calculating staggering effect to see beat patterns and detect any interactions between the (+ve) and (−ve) parity states, and
5. Calculating the electric monopole strength $X(E0/E2)$.

2 (IBA-1) model

2.1 Level energies

IBA-1 model was applied to the positive and negative parity states of ^{100}Ru isotope. The Hamiltonian employed in the

nucleus	EPS	$PAIR$	ELL	QQ	OCT	HEX	$E2SD(eb)$	$E2DD(eb)$
^{100}Ru	0.5950	0.000	0.0085	−0.0200	0.0000	0.0000	0.1160	−0.3431

Table 1: Parameters used in IBA-1 Hamiltonian (all in MeV).

present calculation is:

$$\begin{aligned} H = \; & EPS \cdot n_d + PAIR \cdot (P \cdot P) \\ & + \tfrac{1}{2}\, ELL \cdot (L \cdot L) + \tfrac{1}{2}\, QQ \cdot (Q \cdot Q) \\ & + 5\, OCT \cdot (T_3 \cdot T_3) + 5 HEX \cdot (T_4 \cdot T_4) \end{aligned} \tag{1}$$

where

$$P \cdot p = \frac{1}{2} \left[\begin{array}{l} \left\{ (s^\dagger s^\dagger)^{(0)}_0 - \sqrt{5} (d^\dagger d^\dagger)^{(0)}_0 \right\} x \\ \left\{ (ss)^{(0)}_0 - \sqrt{5} (\tilde{d}\tilde{d})^{(0)}_0 \right\} \end{array} \right]^{(0)}_0 , \tag{2}$$

$$L \cdot L = -10 \sqrt{3} \left[(d^\dagger \tilde{d})^{(1)} x (d^\dagger \tilde{d})^{(1)} \right]^{(0)}_0 , \tag{3}$$

$$Q \cdot Q = \sqrt{5} \left[\begin{array}{l} \left\{ (S^\dagger \tilde{d} + d^\dagger s)^{(2)} - \dfrac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} x \\ \left\{ (s^\dagger \tilde{d} + + \tilde{d} s)^{(2)} - \dfrac{\sqrt{7}}{2} (d^\dagger \tilde{d})^{(2)} \right\} \end{array} \right]^{(0)}_0 , \tag{4}$$

$$T_3 \cdot T_3 = - \sqrt{7} \left[(d^\dagger \tilde{d})^{(2)} x (d^\dagger \tilde{d})^{(2)} \right]^{(0)}_0 , \tag{5}$$

$$T_4 \cdot T_4 = 3 \left[(d^\dagger \tilde{d})^{(4)} x (d^\dagger \tilde{d})^{(4)} \right]^{(0)}_0 . \tag{6}$$

In the previous formulas, n_d is the number of boson; $P \cdot P$, $L \cdot L$, $Q \cdot Q$, $T_3 \cdot T_3$ and $T_4 \cdot T_4$ represent pairing, angular momentum, quadrupole, octupole and hexadecupole interactions between the bosons; EPS is the boson energy; and $PAIR$, ELL, QQ, OCT, HEX is the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions.

2.2 Transition rates

The electric quadrupole transition operator employed in this study is:

$$\begin{aligned} T^{(E2)} = \; & E2SD \cdot (s^\dagger \tilde{d} + d^\dagger s)^{(2)} + \\ & + \tfrac{1}{\sqrt{5}}\, E2DD \cdot (d^\dagger \tilde{d})^{(2)} . \end{aligned} \tag{7}$$

$E2SD$ and $E2DD$ are adjustable parameters.

The reduced electric quadrupole transition rates between $I_i \to I_f$ states are given by:

$$B(E_2, I_i - I_f) = \frac{[< I_f \parallel T^{(E_2)} \parallel I_i >]^2}{2 I_i + 1} . \tag{8}$$

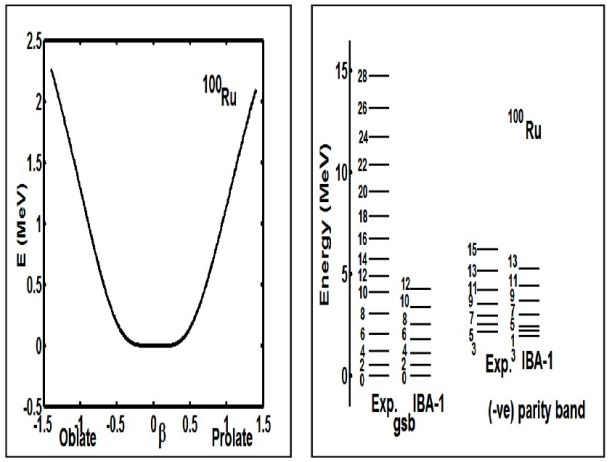

Fig. 1: A: Potential energy surfaces for ^{100}Ru. B: Comparison between exp. [19] and theoretical IBA-1 energy levels.

3 Results and discussion

3.1 The potential energy surfaces

The potential energy surfaces [17], $V(\beta, \gamma)$, as a function of the deformation parameters β and γ are calculated using:

$$\begin{aligned} & E_{N_\Pi N_\nu}(\beta, \gamma) = \; < N_\pi N_\nu; \beta\gamma \,|H_{\pi\nu}| N_\pi N_\nu; \beta\gamma > \; = \\ & = \zeta_d (N_\nu N_\pi) \beta^2 (1 + \beta^2) + \beta^2 (1 + \beta^2)^{-2} \times \\ & \times \left\{ k N_\nu N_\pi [4 - (\bar{X}_\pi \bar{X}_\nu) \beta \cos 3\gamma] \right\} + \\ & + \left\{ [\bar{X}_\pi \bar{X}_\nu \beta^2] + N_\nu (N_\nu - 1) \left(\frac{1}{10}\, c_0 + \frac{1}{7}\, c_2 \right) \beta^2 \right\} , \end{aligned} \tag{9}$$

where

$$\bar{X}_\rho = \left(\frac{2}{7} \right)^{0.5} X_\rho \qquad \rho = \pi \;\; \text{or} \;\; \upsilon . \tag{10}$$

The calculated potential energy surfaces, $V(\beta, \gamma)$, are presented in Fig. 1A. The flat potential in the critical symmetry point has supported quite well the $E(5)$ characters to ^{100}Ru nucleus. Also, the energy ratios presented in Table 4 as well as the existance of ^{100}Ru isotope between the spherical anharmonic vibrator ^{98}Ru and γ - soft ^{102}Ru nuclei [9] supported the $E(5)$ characters.

3.2 Energy spectra

The energy of the positive and negative parity states of ^{100}Ru isotope are calculated using computer code PHINT [18]. A

$I_i^+ I_f^+$	B(E2)	$I_i^+ I_f^+$	B(E1)
0_1 Exp*. 2_1	0.490(5)	$1_1\ 0_1$	0.0030
0_1 Theor. 2_1	0.4853	$1_1\ 0_2$	0.1280
$2_1\ 0_1$	0.0970	$3_1\ 2_1$	0.1211
$2_2\ 0_1$	0.0006	$3_1\ 2_2$	0.0415
$2_2\ 0_2$	0.0405	$3_1\ 2_3$	0.0002
$2_3\ 0_1$	0.0000	$3_2\ 2_1$	0.0024
$2_3\ 0_2$	0.0759	$3_2\ 2_2$	0.0197
$2_3\ 0_3$	0.0087	$3_2\ 2_3$	0.2126
$2_4\ 0_3$	0.0066	$5_1\ 4_1$	0.2533
$2_4\ 0_4$	0.0588	$5_1\ 4_2$	0.0480
$4_1\ 2_1$	0.1683	$5_1\ 4_3$	0.0006
$4_1\ 2_2$	0.0142	$7_1\ 6_1$	0.3950
$4_1\ 2_3$	0.0319	$7_1\ 6_2$	0.0446
$6_1\ 4_1$	0.2039	$9_1\ 8_1$	0.5439
$6_1\ 4_2$	0.0179	$9_1\ 8_2$	0.0342
$6_1\ 4_3$	0.0242	$11_1\ 10_1$	0.6983
$8_1\ 6_1$	0.2032		
$8_1\ 6_2$	0.0183		
$8_1\ 6_3$	0.0157		
$10_1\ 8_1$	0.1678		
$10_1\ 8_2$	0.0175		

Table 2: Calculated $B(E2)$ and $B(E1)$ in ^{100}Ru.

I_i^+	I_f^+	$I_{f'}^+$	$X_{if'f}\,(E0/E2)^{100}$Ru
0_2	0_1	2_1	0.027
0_3	0_1	2_1	0.347
0_3	0_1	2_2	0.009
0_3	0_1	2_3	0.042
0_3	0_2	2_1	0.086
0_3	0_2	2_2	0.002
0_3	0_2	2_3	0.010
0_4	0_1	2_2	0.010
0_4	0_1	2_3	0.010
0_4	0_1	2_4	0.113
0_4	0_2	2_2	0.030
0_4	0_2	2_3	0.034
0_4	0_2	2_4	0.340
0_4	0_3	2_1	0.454
0_4	0_3	2_2	0.010
0_4	0_3	2_3	0.011
0_4	0_3	2_4	0.113

Table 3: Calculated $X_{if'f}\,(E0/E2)$.

Energy Ratios	$E_{4_1^+}/E_{2_1^+}$	$E_{2_2^+}/E_{2_1^+}$
$E(5)$	2.19	2.20
Exp. [19]	2.27	2.52
$IBA-1$	2.12	2.11

Table 4: Energy ratios of $E(5)$ characters to ^{100}Ru.

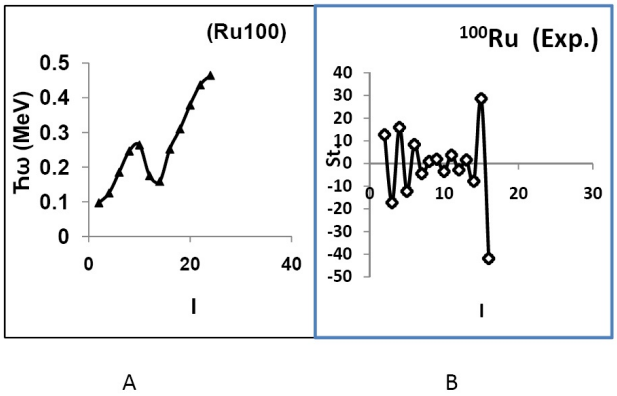

Fig. 2: A: Angular momentum I as a function of $(\hbar\omega)$. B: $(\Delta I = 1)$, staggering pattern for ^{100}Ru isotope.

comparison between the experimental spectra [19] and our calculations, using values of the model parameters given in Table 1 for the ground state band are illustrated in Fig. 1B. The agreement between the calculated levels energy and their correspondence experimental values are slightly higher especially for the higher excited states. We believe this is due to the change of the projection of the angular momentum which is due mainly to band crossing.

Unfortunately there is no enough measurements of electromagnetic transition rates $B(E1)$ or $B(E2)$ for ^{100}Ru nucleus. The only measured $B(E2, 0_1^+ \to 2_1^+)$ is presented, in Table 2 for comparison with the calculated values[20]. The parameters $E2SD$ and $E2DD$ displayed in Table 1 are used in the computer code NPBEM [18] for calculating the electromagnetic transition rates after normalized to the available experimental values. No new parameters are introduced for calculating electromagnetic transition rates $B(E1)$ and $B(E2)$ of intraband and interband.

The moment of inertia I and angular frequency $\hbar\omega$ are calculated using equations (11, 12):

$$\frac{2I}{\hbar^2} = \frac{4I-2}{\Delta E(I \to I-2)}\,, \tag{11}$$

$$(\hbar\omega)^2 = (I^2 - I + 1)\left[\frac{\Delta E(I \to I-2)}{(2I-1)}\right]^2\,. \tag{12}$$

The plot in Fig. 2A show back bending at angular momentum $I^+ = 10$. It means, there is a crossing between the $(+ve)$ and $(-ve)$ parity states in the ground state band which

was confirmed by calculating the staggering effect where a beat pattern has been observed, Fig. 2B.

3.3 Electric monopole transitions

The electric monopole transitions, $E0$, are normally occurring between two states of the same spin and parity by transferring energy and zero unit of angular momentum. The strength of the electric monopole transition, $X_{if'f}(E0/E2)$, [21] can be calculated using equations (13, 14); results are presented in Table 3

$$X_{if'f}(E0/E2) = \frac{B(E0, I_i - I_f)}{B(E2, I_i - I_{'f})}\,, \tag{13}$$

$$X_{if'f}(E0/E2) = (2.54\times 10^9)\,A^{3/4}\times \frac{E_\gamma^5(\mathrm{MeV})}{\Omega_{KL}}\,\alpha(E2)\,\frac{T_e(E0, I_i - I_f)}{T_e(E_2, I_i - I_{'f})}\,. \tag{14}$$

Here: A is mass number; I_i is spin of the initial state where E0 and E2 transitions are depopulating it; I_f is spin of the final state of E0 transition; $I_{'f}$ is spin of the final state of E2 transition; E_γ is gamma ray energy; Ω_{KL} is electronic factor for K,L shells [22]; $\alpha(E2)$ is conversion coefficient of the E2 transition; $T_e(E0, I_i - I_f)$ is absolute transition probability of the $E0$ transition between I_i and I_f states, and $T_e(E_2, I_i - I_{'f})$ is absolute transition probability of the $E2$ transition between I_i and $I_{'f}$ states.

3.4 The staggering

The presence of $(+ve)$ and $(-ve)$ parity states has encouraged us to study staggering effect [23–25] for ^{100}Ru isotope using staggering function equations (15, 16) with the help of the available experimental data [19].

$$St(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2)\,, \tag{15}$$

with

$$\Delta E(I) = E(I+1) - E(I)\,. \tag{16}$$

The calculated staggering patterns are illustrated in Fig. 2B and show an interaction between the $(+ve)$ and $(-ve)$ parity states for the ground state band of ^{100}Ru.

3.5 Conclusions

IBA-1 model has been applied successfully to ^{100}Ru isotope and:

1. The levels energy are successfully reproduced;
2. The potential energy surfaces are calculated and show $E(5)$ Characters to ^{100}Ru;
3. Electromagnetic transition rates $B(E1)$ and $B(E2)$ are calculated;
4. Bending for ^{100}Ru has been observed at angular momentum $I^+ = 10$;
5. Strength of the electric monopole transitions $X_{if'f}(E0/E2)$ are calculated; and
6. Staggering effect has been calculated and beat pattern has been obtained, showing an interaction between the $(-ve)$ and $(+ve)$ parity states.

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