Proper Space Kinematics

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It is desirable to understand the movement of both matter and energy in the universe based upon fundamental principles of space and time. Time dilation and length contraction are features of Special Relativity derived from the observed constancy of the speed of light. Quantum Mechanics asserts that motion in the universe is probabilistic and not deterministic. While the practicality of these dissimilar theories is well established through widespread application inconsistencies in their marriage persist, marring their utility, and preventing their full expression. After identifying an error in perspective the current theories are tested by modifying logical assumptions to eliminate paradoxical contradictions. Analysis of simultaneous frames of reference leads to a new formulation of space and time that predicts the motion of both kinds of particles. *Proper Space* is a real, three-dimensional space clocked by proper time that is undergoing a densification at the rate of c. Coordinate transformations to a familiar *object space* and a mathematical *stationary space* clarify the counterintuitive aspects of Special Relativity. These symmetries demonstrate that within the local universe stationary observers are a forbidden frame of reference; all is in motion. In lieu of Quantum Mechanics and Uncertainty the use of the imaginary number i is restricted for application to the labeling of mass as either material or immaterial. This material phase difference accounts for both the perceived constant velocity of light and its apparent statistical nature. The application of Proper Space Kinematics will advance more accurate representations of microscopic, macroscopic, and cosmological processes and serve as a foundation for further study and reflection thereafter leading to greater insight.

1 Introduction

The planets dancing in the heavens, an apple falling to earth each kindle curiosity about the dynamical universe. The mysteries of the unseen world and its apparent influences on daily life inspire wonder and imagination. Such observations drive the search for hidden constraints that govern the actions of atomic particles and molecules, ballistic objects, and celestial bodies. Guided by tools of logic, intuition, and creativity philosophers, scientists, and mathematicians strive to model laws that describe movement in each realm. Many years of disparate effort and the resulting accumulation of knowledge demonstrate that there are underlying commonalities that apply across all physical scales. This connectedness prompts the realization that searching for unifying first principles based upon fundamental aspects of space and time is an attainable goal. Understanding the foundation that the universe is built upon enables the continuing pursuit of deeper and more profound truths and further illuminates the miracle of human existence.

In 1905 Albert Einstein published his landmark work *Zur Elektrodynamik bewegter Körper* [1] (translated as *On the Electrodynamics of Moving Bodies* [2]). He stated that it was well known that under transformation to a moving reference frame Max-well's equations acquired asymmetries that were not present in nature. Einstein resolved these inconsistencies by introducing two fundamental principles [2]:

1. *The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion.*
2. *Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body.*

The first postulate identified inertial frames of reference. The second postulate emphasized the constancy of the speed of light. From these followed the development of Special Relativity as a basis for motion.

Although the efficacy of Special Relativity cannot be denied it is a mathematical physics derived from the observations of light approaching any observer at the same speed regardless of the specific frame of reference. Any element of a theory that behaves identically under all applications must itself lie outside this theory and for this reason the action of discrete quanta requires a separate and distinctly different explanation.

This leads to the hard-fought and hard-won triumph of the Copenhagen interpretation of Quantum Mechanics culminating in its emergence as the preeminent theory of modern physics [3]. Owing to their experimental origins the composition of each theory contains mathematical elements that are not immediately obvious and consequently can act as obstacles to understanding and usage. If the basic realities of space and time are known then it is possible to properly explain the curious details of motion of all objects in the native environment and show that they proceed in a logical and intuitive way from this physical foundation.

This research is motivated by a personal failure of understanding attributable to the lack of a fundamental mechanics capable of explaining all rudimentary motion in the universe as derived from the basic condition of spacetime. Guided by instinct and curiosity the contemporary scientific theories and the corresponding philosophies are explored through a careful analysis of perspective; long-held premises are tested and discarded by virtue of the necessity to eliminate contradiction. The result of the methodology described in this paper addresses a kinematics which describes free motion without forces and interactions and with no regard for collisions and the associated quantities of energy, momentum and mass. A first principles theory is significant in that it can immeasurably improve physics on every level by serving as a foundation for the advancement of larger fields of research. The sluggish pace of grand unification, the overwrought complexity of string theory, the extremes of quantum gravity, the perplexity of dark matter, and the simplistic seeming three body problem are currently unresolved issues in physics [4]. These problems along with technological improvements to solar cell efficiency and medical scanning devices are among those that can potentially benefit from the application of Proper Space Kinematics.

2 Methodology

As a part of natural skepticism and scientific inquiry it is often useful to be able to replicate the research process both as a test of results and as a guide to understanding. In theoretical work much of the effort is introspective and it is impossible to retrace the labyrinthine mental pathways that lead to these results. In light of this difficulty it is practical to detail the initial impetus that motivated the author and to provide an overview of the techniques employed in the striving for enlightenment.

It is always more difficult to understand the fundamental principles that govern a system when the only perspectives available lie within the system itself. For this reason it is desirable to find a vantage point or frame of reference that lies outside the system so as not to be influenced by or subject to whatever constraints are imposed upon its occupants. In reviewing the basic elements of Special Relativity it is troubling that there are inconsistencies in the currently used theory between the common explanations and the mathematical model. While the equations purport to explain motion from an exterior viewpoint it is a theory of *relative* motion that performs as if a massive object occupies the choice of origin. This fallacy compounds the suspicion that an accurate picture of reality may not be known and necessitates the need for further exploration of this phenomena the source of which must thereafter be inferred from these confused aspects. In a similar mien the self-circular reasoning involved in using light itself as a mediator to measure lightspeed is also an obstacle to understanding and conceals basic mechanisms that are vital to accurately model the system mathematically.

Other concerns arise from a thoughtful analysis of the present philosophy. If the lightspeed barrier is a limiting condition then this implies that the velocity of an object is a more important kinematical consideration than position or acceleration. A cursory examination of the invariant interval suggests that its spatial and temporal components act in opposition to each other across varied reference frames although the use of hyperbolic functions would conversely imply a conjunction of underlying influences. The question of balance imparts an impression of rotation along a spectrum instead of a deviation from zero which is compounded by the inability to rotate a vector of zero length and might lead to the conclusion that nothing is static. The Quantum Mechanical proposition that the universe is unknowable at its most basic level and the ensuing enigma of wave-particle duality raise further reservations. Intuitively the structure of the universe should be based on the least number and simplest of principles although wisdom dictates that allowances be made for the possibility of deliberate design.

Logic is a weak tool for dissecting a system that is known to have defects in its application and for this reason a trial-by-solution is likely to be ineffective. Therefore the course of action must include an exploration using physical intuition and not only a mathematical manipulation of equations. This is accomplished through repeated testing of both implicit and explicit assumptions to find the origins of paradoxical situations and then to remove these faults. The movements of both energy and matter in spacetime are studied with careful consideration of perspective in an attempt to unravel the knot of relativity and to imagine an extrauniversal viewpoint. Producing an accurate answer to the dilemmas detailed here requires substantial time for trial calculations, for searching through potentially applicable literature, and for reevaluating conventional concepts through quiet reflection.

3 Results

The natural universe is undergoing a process of *densification* and is described here as being composed of three real spatial coordinates and one real monodirectional temporal counter. Densification is defined for this demesne as an increase in the density of space that occurs in the measure of distance between any two disparate points clocked by proper time and progressing at the rate of c. Previously referred to as lightspeed the particular value of the *characteristic velocity* as it has been measured serves as a label for the universe as well as all residents. It is further assumed that the inhabited universe is infinite though possibly bounded, is fixed relative to any preternatural background, if one exists, and is not undergoing additional physical alteration. The kinematics of finitesimal objects is derived for the movement of noninteracting rigid bodies traveling at constant speed. The premise of constant speed translates across all *spaces*. Initially this derivation is done without the qualification of particles as either matter or

energy. For the purposes of this paper it is practical and sufficient for understanding to consider equations of motion of only one dimension since any path traversed at constant speed can be parameterized as such and densifies at the same rate; extrapolation to all three dimensions is a straightforward task.

Length and time are measured with a ruler and a clock [5]. *Proper Space* is denoted by the variable z and experiences densification dependent on proper time which is denoted by the independent variable tau τ. In this case the clock is also embedded within the ruler and is not considered an additional physical dimension. In *object space* space and time are treated on equal footing as independent dimensions and are denoted by x and t, respectively. These variables have local values that manifest densification as contraction and dilation in mimicry of many of the details of Special Relativity and continue to suffer from dependence on frame-specific relative velocity.

Measurements of physical observables are made in object space and converted to values in proper space where the action originates. The coordinate transformation for length or displacement involves the scale change

$$dz = f\,dx. \tag{1}$$

The unitless scale factor f is defined for densification as a *density of points* which is represented by a ratio of infinities increasing from unity as

$$f = \left(\frac{dx + cdt}{dx}\right) = 1 + \left(\frac{cdt}{dx}\right). \tag{2}$$

Simple substitution of (2) into (1) yields the coordinate transformation between spaces

$$dz = dx + cdt. \tag{3}$$

This is the conversion for points in space with an explicit dependence on elapsed time. Contrary to expectation with densification a scale transformation from object space to proper space takes a form that is reminiscent of a Galilean boost [6].

The burgeoning density of proper space requires the use of additional notation for the proper *waxing velocity*, denoted by w, while in object space the concept of velocity is retained as it is traditionally used and remains denoted by v. The relationship between the two quantities is

$$w \triangleq \frac{dz}{d\tau} = \alpha\,(v + c)\,. \tag{4}$$

Values for the velocity in object space persist within the range of $(-c, c)$ while values for the waxing velocity are always positive within the range of $[c, 0)$. Open endpoints of each interval are forbidden for the same reason; denizens of the universe must always experience the advancement of proper time in some nonzero fraction. Accordingly values for the *temporal dilation coefficient*, marked by alpha α, vary as $[1, 0)$. Infinite dilation is taboo and is expressed by the avoidance of an asymptotic value of zero for α.

In a break from prior theories of motion it is important that velocities in all spaces are measured from a special class of perspectives hereinafter referred to as *proper frames*. The choice of axes may be made without particular regard for position but must be boosted to the specific velocity whereby t reaches the maximum expression of τ and experiences densification at its fullest flowering. Proper frames can be thought of as critical points and specific values associated with these perspectives are $w = c$, $v = 0$ and $\alpha = 1$.

For the sake of completeness it is worthwhile to also define a *stationary space*, denoted by y, which advances with the preceding variable of proper time τ. This nonphysical construct may be mathematically advantageous as it allows for the use of global variables that forgo dependence on relative perspective but carries the caveat that the space is not demonstrative of physical reality. The scale-densification — to — boost technique above is repeated to provide the transformation to proper space as

$$dz = dy + cd\tau. \tag{5}$$

Measurements of length or distance are converted from object space to corresponding values in stationary space through the transitive property with application of (3) and (5) to yield

$$dy + cd\tau = dx + cdt. \tag{6}$$

For stationary space a *pseudovelocity* is defined as u and takes on the values $(-c, c)$. Values of u are somewhat analogous to velocities v in object space e.g., adopting the value of zero in a proper frame where $dt = d\tau$. The relation for the two quantities is

$$u \triangleq \frac{dy}{d\tau} = \alpha\,(v + c) - c. \tag{7}$$

As proper space and stationary space both share the variable τ as proper time the relationship between velocities is more simple as

$$w = u + c. \tag{8}$$

The choice of alphabetically proximate variables is a mnemonic convenience that is intended to be familiar and resemble current definitions but not to imply any other mathematical relationship including equivalence with commonly used spatial unit vectors. The invariant variable s is reserved for possible future use.

4 Discussion

Change is the true nature of the universe and the densification of proper space depicts the most authentic representation of space and time. A static ruler of fixed length is a forbidden item; an absolutely stationary observer is a nonsensical frame of reference that does not exist. Although this picture of reality is not mathematically convenient it is the correct philosophy to accurately model basic kinematics. Object(-*ive*) space is the milieu where action is perceived and measurements are made. The coordinate transformation to proper

space takes the form of a boost centered on c which arises, not surprisingly, from the defining feature of the universe. This conversion yields the advantage to the waxing velocity which can always be rotated since it is never zero as objects must experience some positive slice of proper time. Objects moving at the same rate as densification do not experience the passage of proper time and therefore cannot inhabit this universe. It should not be overlooked that the transformation is originally a scale change whereby the size of massive objects is growing relative to the coordinate system with the densification. It is the growth of the span between the center of mass of an object and any other contained point within that same object that is seemingly retarded in entities not occupying a proper frame of motion. Consideration of the action of only infinitesimal points does not reveal this detail. It is helpful if the time-dependent metric tensor is visualized as the ruler growing shorter and shorter thus creating an illusion of inflation. The author supposes that the idea of densification within fixed boundaries is an option that Einstein either discarded or failed to consider and is the source of his self-critical vacillation regarding the Cosmological Constant [7].

In a brief departure from kinematics an examination of multiple perspectives clarifies the necessity for a preferred frame of interaction. Collisions cannot have different outcomes in different frames otherwise every incident can be transformed into a destructive event. Synchronization to a proper frame is a sufficient condition to preserve the integrity of any physical interaction; the regimentation also reemphasizes the significance of velocity. This interpretation of simultaneity provides the means to intellectually resolve the well-known gedanken paradox [8]: what are the ages of the traveling twins? There currently exists an abundance of experimental and observational data which can be used to determine the validity of proper frames. The incongruity of superluminal travel can be rectified by application of the results discussed here and the presence of tachyons is discarded.

Terminology relating to motion must be used cautiously since the concepts involved vary among the different spaces despite a similarity in formulation. Calculations done in stationary space remove some of the difficulties of perspective that are inherent to the other spaces but readers are warned to remember that this is not a physical reality. In object space it is time that slows and space that contracts as a function of speed to the detriment of the occupying objects. A sequence of snapshots in proper space shows that movement in any direction produces an apparent spatial *and* temporal dilation based upon the movement of a mass impinging on the budding densification. Part of the virtue of proper space is that the object itself is not actually altered and the perception of dilation occurs only in the direction of motion while densification continues unabated along all other axes. Along with the increase in movement this retardation of proper space and proper time is demonstrated as a decrease in the waxing velocity although the moving particle still perceives densification continuing at c. A reasonable choice for a functional definition of w is the hyperbolic secant as a function of the *angle of dilation*, represented by phi φ, and demonstrated in $w = \operatorname{csech}(\varphi)$ making it more akin to a speed than a velocity. The positive-definite, even function is a rotation of phi through the real interval $(-\infty, \infty)$ as measured from a proper frame and this run equates with the previously detailed bounds for w of $[c, 0)$. The choice of hyperbolic functions is preferred over the circular transcendental equations as the hyperbolics are independent of the imaginary number i.

Consideration of the relative velocity between bulk objects with determinate length requires the use of a proper frame. A measurement of relative velocity is inadequate to completely determine the true states of objects in the system; two measurements are required to establish the correct scalings for space and time. Take the example in object space of two masses *at rest* to a specific proper frame as well as to each other; the waxing velocity of each frame in proper space is c. While the relative velocity in object space between the centers of mass remains at zero in proper space the relative velocity is characteristic and not zero as might be anticipated. This discrepancy can be partly reconciled by acknowledging the supplemental velocity acquired in proper space which is imparted by the densification of the gap between the two masses. Accordingly the correct velocities between the center of mass frames are emphasized by primed coordinates and subscripts enumerate the frames of reference for separate and distinct objects as

$$u' \triangleq u_2 - u_1, \tag{9.1}$$

$$w' \triangleq u' + c, \tag{9.2}$$

$$\text{and } v' \triangleq \left(\frac{w'}{\alpha'}\right) - c. \tag{9.3}$$

The sense of relative motion is preserved by these transformations; the distinction of an *alias* versus an *alibi* transformation is highlighted [9]. To determine the relative velocity in object space measurements are made there first, converted to pseudovelocities and the relative velocity calculated then reverted to object space. All direct measurements are relative with v' equal to v from a proper frame. Although this computation avoids direct expression of quantities in proper space the kernel of the action lies there.

The primed alpha coefficient α' serves as both the relative temporal dilation between objects as well as the transformation between frames in proper space. It is defined as a ratio in the range of real positive numbers $(0, \infty)$ and is most easily understand as an exponential with argument given as the difference between two angles and shown here

$$\alpha' \triangleq \left(\frac{\alpha_2}{\alpha_1}\right) = e^{-(\varphi_2 - \varphi_1)}. \tag{10}$$

These definitions in combination with some computation restore the hyperbolic tangent in a composition of velocities in

object space and yield a result that is in correspondence with rapidity [10]. The assertion that values of α' can exceed 1 is a specific deficiency in the conventional measurement of relative velocity. Attend to these calculations with care as variables of the traditional theory are ill-defined by the muddled use of mixed perspective due to a misconception in the choice of laboratory frame.

The derivation is accomplished to this point without the need for i; further descriptions of the manifest complexity of nature require the use of imaginary numbers. The kinematics is extended to distinguish between the movements of the two types of mass by applying a label of *material* or *immaterial* (i-material) to all particles whether they are matter or energy; the two types are interchangeable provided the exchange is done en masse. Real and imaginary objects occupy overlapping worlds within the same universe because the phase dichotomy causes a perception of near invisibility between the two categories of mass in which the contrary object collapses to a dimensionless point. As seen before with dilation the flattening is perceptual and not actual. The alternately phased object appears to ignore densification and to therefore exist in a forbidden state. In that the object doesn't seem to experience scaling it performs as with a waxing velocity of zero and erroneously claims relative velocities as $w' = c \pm c$ and $v' = 0 \pm c$. The relative motion of the oppositely phased objects either approaches or recedes depending on the relative angle of dilation. The tipping point occurs when $\varphi_1 = \varphi_2$ and $\alpha' = 1$ and can serve as a test provided it is possible to produce a series of identical immaterial objects. The author defers the specific method for this production to the expertise of experimentalists.

The expression $E = mc^2$ acquires a new complexion after revisiting the outmoded concepts of the rest mass of matter and the mass equivalence of energy. The characteristic velocity measured between real and imaginary particles is superficial and acts as a screening value whereby information is hidden from the casual observer but still preserved. Relying only on light as a mediator to comprehend motion introduces inaccuracies that must be corrected. A single physical measurement of an immaterial object is underdetermined and wrongly constrains the associated parameters of velocity and imaginary mass. Consequently the sources of wave nature are found to originate from the complex quality of mass and not directly from the tableau of spacetime. The seeming lack of determinate states which is the hallmark of Quantum Mechanics illustrates its subservience to statistical models and elucidates its failure of completeness and its misappropriation of fundamental reality.

5 Conclusions

Maintaining an open-minded attitude of skepticism lies at the heart of the scientific method; challenging established ideas is not necessarily an effort towards rebellion and anarchy. Persistent testing is an important undertaking in the quest to further humanity's understanding of life, the universe and everything. The author is awakened to the fact that the peculiar consequences of Albert Einstein's Special Relativity and subsequent geometric interpretation of space and time originate from observation and the theory does not proceed directly from a foundational source. Relying on relative viewpoints to predict motion has an inherent handicap and in combination with the confused measurement of lightspeed initially serve as motivation for study. The approach to creating a kinematics involves keeping a critical eye on perspective and attempting to dispel paradoxes in order to see through to the metaphysical center. It is a mistake to rely totally on mathematical models of nature as they are ultimately flawed and physicists must constantly endeavor to look beyond constructed images of reality. If the basic realities of space and time are known then it is possible to properly explain the curious details of motion of all objects in the native environment and show that they proceed in a logical and intuitive way from this physical foundation. The success of such a hypothesis would be the pedestal on which the future of physics could be built and would have a far-reaching influence on science and greatly impact its application to technology in addition to answering important philosophical questions.

The elegance of Proper Space Kinematics is that it proceeds directly from the fundamental concept that the fabric of the universe densifies at the unique quantity and quality of the characteristic velocity c maturing with an inescapable duration of proper time. This insight into the inner workings of space and time solidifies realizations regarding the arrow of time and the spectre of irreversible entropy. It is not surprising that in a study of motion appearances are deceiving and this deception necessitates a transformation to positions in other spaces which are difficult to visualize since the use of a time-dependent metric is not a well-developed field of study with much pertinent literature. Spatial densification is understood by a study of the steadily mounting density of points (Mind the infinities!) whereby a scale change converts the growing size of objects to the form of a boost. Care at the beginning: reconceptions of velocity and movement lead to new definitions such as proper space's waxing velocity and the interrelated temporal dilation coefficient. Additionally boosting perspective to any proper frame provides the linkage that shows these points of view can be logically related and provides for surety over the use of four-vectors and four-velocities. Scrutiny of these results discerns that stationary space is a fictitious point of view that proves to be a useful tool.

Densification clarifies the observed nuances of motion more clearly than Special Relativity by eschewing stationary states and shedding new light on the evolution of the aging universe. Scale expansion of objects is found to be a new source of motion where movement hinders the passage of time and limits experience. Thought problems are revisited

and explained by the introduction of new concepts such as proper frames providing ample opportunity for testing the validity of these new ideas; experimental and mathematical verification have many available avenues to explore. This kinematics shows that the movement of objects does not cause a physical change but merely alters appearances. As particles always experience their own perspective as characteristic the presented composition of velocities accurately details the difference between spectators and participants. The duality of mass shows that the landscape of space is a perpendicular reality for matter and energy which can be tested by manufacturing an experimental watershed. The screening between imaginary material phase shifts creates a Quantum confusion due to underdetermined measurements that the author feels does finally vindicate Einstein's intuition. (No Dice!)

As a first principles theory which meets the onus of the stated hypothesis Proper Space Kinematics claims jurisdiction over all motion in the universe. Proper motion supplants the golden relics of relative and absolute motion; the dubious lessons of Quantum nature must be extracted and distilled for their essential truths. As seen with Isaac Newton in his 1687 *Philosophiae Naturalis Principia Mathematica* [11] in the continuing quest for deeper insight new ideas are a harbinger for chaos as fundamental changes in understanding prompt the reevaluation of physics on every level and in every niche. The potential impact on science and its application expands from the theoretical to the technological to hopefully improve the quality of human life and reinvigorate the search for profundity. The author proposes that the next step in this study is to complete a mechanics in full generality with metric-tensor formalism to include a derivation of canonical coordinates with energy and momentum and an examination of accelerating objects with interactions via both collisions and forces-at-a-distance. Delving further raises a rich multitude of questions: Is densification in the universe constant? What does this mean for cosmology and the birth and death of the universe? Are there other characteristic parallel universes that are unseen? Is there a greater realm? How do these results apply to the standard model? Was the creation of life and homo sapiens sapiens an accident? Why are we here? Physicists have always searched the universe for bedrock on which to stand but to live in harmony with our world we must instead navigate the rising tide of space and time and learn to walk on water.

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