New Constraints on Quantum Theories

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Hierarchical relationships between physical theories are discussed. It is explained how a lower rank theory imposes constraints on an acceptable structure of its higher rank theory. This principle is applied to the case of quantum mechanics and quantum field theory of massive particles. It is proved that the Dirac equation is consistent with these constraints whereas the Klein-Gordon equation, as well as all other second order quantum equations are inconsistent with the Schrödinger equation. This series of arguments undermines the theoretical structure of the Standard Model.

1 Introduction

The equations of motion are regarded as the basis of a physical theory. A mathematical analysis of these equations yields the complete form of a given theory and of its details. The validity of a mathematically correct physical theory should be consistent with two kinds of tests. Thus, it must agree with relevant experimental data and it must also be consistent with well established physical principles. (Evidently, the latter represent many experimental data in a concise form.) The following simple example illustrates the latter point. A new theory is unacceptable if its final results are inconsistent with the law of energy conservation. This point shows the significance of physical constraints that restrict the number of acceptable physical theories and guide theoretical and experimental efforts to take promising directions.

The definition of a domain of validity is an important element of a theory. For example, mechanics is the science used for predicting the motion of bodies. It is very successful in the case of the motion of planets moving around the sun. On the other hand, it cannot predict the motion of an eagle flying in the sky. This example does not mean that mechanics is incorrect. It means that mechanics is a very satisfactory science for a set of experiments. For example, Newtonian mechanics is acceptable for cases where the following conditions hold: the velocity is much smaller than the speed of light, the classical limit of quantum mechanics holds, and the force can be calculated in terms of position, time and velocity. The set of experiments where a given theory is successful is called the theory's domain of validity. This issue is used in the rest of this work.

The definition of the domain of validity illustrates an important aspect of the correctness of a physical theory. Indeed, this notion should be regarded in a relative sense. Thus, many measurements are given together with experimental error. For this reason, even if we know that a given theory is not perfect, it still can be regarded as a correct theory for cases where the theory's errors are smaller than the experimental errors.

In this work units where $\hbar = c = 1$ are used. In this system of units one kind of dimension applies and here it is the length $[L]$. Thus, the dimension of every physical quantity takes an appropriate power of $[L]$. For example, mass, energy and momentum take the dimension $[L^{-1}]$. The metric is diagonal and its entries are $(1, -1, -1, -1)$. Greek indices run from 0 to 3. The subscript symbol $_{,\mu}$ denotes the partial differentiation with respect to x^μ.

2 The dimensions of quantum fields

Consider the two sets of experiments S_A and S_B defining the domains of validity of the physical theories A and B, respectively.

Fig. 1 illustrates the hierarchical relationships between theories A and B. Here the sets S_A and S_B consist of all experiments that are described correctly by theory A and B, respectively. The set S_A is a subset of S_B. This relationship means that all experiments that are described successfully by theory A are also described successfully by theory B, but not vice versa. For this reason it can be stated that theory B has a more profound meaning because it is also valid for cases where theory A is useless. However, this fact does not mean that theory A is wrong, simply because this theory can be used successfully for all cases that belong to its domain of validity S_A.

This kind of relationships between theories has been recognized a long time ago. For example, A. Einstein mentions special relativity and general relativity and explains why special relativity should not be regarded as a wrong theory. The reason is that special relativity holds in cases where a flat space-time can be regarded as a good description of the

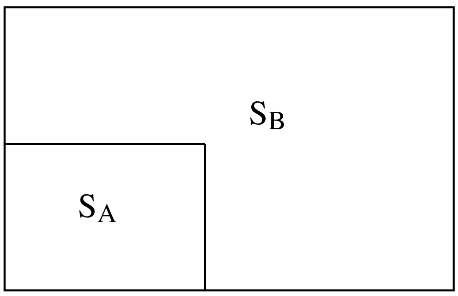

Fig. 1: Domains of validity of two theories (see text).

physical conditions. Similarly, considering electrostatics and Maxwellian electrodynamics, he explains why electrostatics is a good theory for cases where the charge carriers can be regarded as motionless objects (see [1], pp. 85, 86).

The issue of hierarchical relationships between theories is also discussed in Rohrlich's book (see [2], pp. 1–6). Here one can find explanation showing the hierarchical relationships between several pairs of theories. This discussion provides the reader with a broader overview of the structure of existing physical theories and of their hierarchical relationships.

As pointed out above, a physical theory that takes a higher hierarchical position has a more profound meaning. The rest of this work relies on another result obtained from these relationships. Thus, *a well established physical theory imposes constraints on appropriate limits of a higher rank theory.* For example, this requirement is satisfied by relativistic mechanics, whose low velocity limit agrees with Newtonian mechanics (see [3], pp. 26–30). Similarly, the classical limit of quantum mechanics agrees with classical physics (see [4], pp. 19–21 and [5], pp. 133–141). Below, this principle is called *constraints imposed by a lower rank theory.* It is shown in this work that this principle provides powerful constraints on the acceptability of physical theories.

3 Hierarchical Relationships Between Quantum Theories

Let us discuss the hierarchical relationships between three quantum theories of massive particles: non-relativistic quantum mechanics (QM), relativistic quantum mechanics (RQM) and quantum field theory (QFT) (see fig. 2). Thus, QM takes the lowest hierarchical rank because it is valid for cases where the absolute value of the momentum's expectation value is much smaller than the particle's self-mass. RQM is valid for cases where the number of particles can be regarded as a constant of the motion. QFT is a more general theory and RQM is its appropriate limit. The inherent relationships between these theories are well documented in the literature. Thus, S. Weinberg makes the following statement. "First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schrödinger, Heisenberg, Pauli, Born, and others in 1925-1926, and has been used ever since in atomic, molecular, nuclear and condense matter physics" (see [6], p. 49).

The Schrödinger equation takes the following form

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\Delta\psi + U\psi. \tag{1}$$

An analysis of this equation yields an expression for a conserved current whose density is (see e.g. [4], pp. 53–55)

$$\rho = \psi^*\psi. \tag{2}$$

Relation (2) proves that the dimension of the Schrödinger function is

$$[\psi] = [L^{-3/2}]. \tag{3}$$

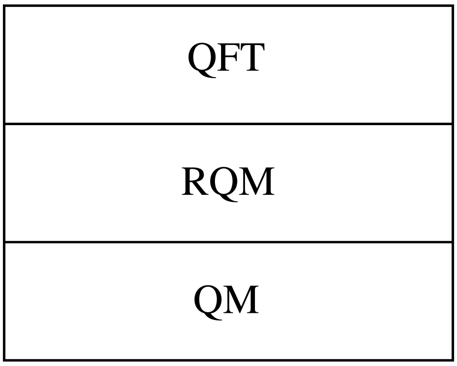

Fig. 2: Hierarchical relationships between three quantum theories (see text).

Here the expression for density depends only on the wave function and contains no derivatives. The form of the density (2) is an important element of the theory because it enables a construction of a Hilbert space of the time-independent functions which belong to the Heisenberg picture.

Let us examine the structure of QFT. The vital role of the Lagrangian density in QFT can be briefly described as follows. The phase is an indispensable element of quantum theories. Being an argument of an exponent which can be expanded in a power series, the phase must be a dimensionless Lorentz scalar. Thus, the phase is defined as a Lorentz scalar action (divided by $\hbar$). The following expression shows how the action is obtained from a given Lagrangian density $\mathcal{L}$

$$S = \int \mathcal{L}\, d^4x. \tag{4}$$

This expression proves that a dimensionless Lorentz scalar action is obtained from a Lagrangian density that is a Lorentz scalar whose dimension is $[L^{-4}]$.

This property of the Lagrangian density is used in an examination of two kinds of QFT theories. Let us begin with the first order Lagrangian density of a free Dirac field ψ_D (see [7], p. 54)

$$\mathcal{L}_D = \bar\psi_D[\gamma^\mu i\partial_\mu - m]\psi_D. \tag{5}$$

Now, the dimension $[L^{-4}]$ of the Lagrangian density and the dimension $[L^{-1}]$ of the operators ∂_μ and m prove that the dimension of the Dirac field ψ_D is $[L^{-3/2}]$. This value agrees with that of the Schrödinger function (3). It means that the Dirac field theory satisfies the dimension constraints imposed by the lower rank theory of QM.

A different result is obtained from the second order complex Klein-Gordon (KG) equation. The Lagrangian density of this equation is (see [7], p. 38)

$$\mathcal{L}_{KG} = g^{\mu\nu}\phi^*_{,\mu}\phi_{,\nu} - m^2\phi^*\phi. \tag{6}$$

Here the dimension of the operators is $[L^{-2}]$. Using the dimension $[L^{-4}]$ of the Lagrangian density, one infers that the dimension of the KG function ϕ is $[L^{-1}]$. On the other hand, it is shown in (3) that the dimension of the Schrödinger wave function is $[L^{-3/2}]$. This outcome means that the complex KG function ϕ violates a constraint imposed by a lower rank theory.

It turns out that this inconsistency holds for other quantum equations where the dimension of their field function is $[L^{-1}]$. Thus, a dimension $[L^{-1}]$ is a property of the following field: the Yukawa particle (see [8], p. 211), the electroweak $W^{\pm}, Z$ bosons (see [9], p. 307) and the Higgs boson (see [10], p. 715). For this reason, quantum theories of all these particles are inconsistent with the dimensional constraint imposed by the Schrödinger equation.

One can also see immediately that the Yukawa and the Z fields introduce to the Lagrangian density an interaction term with a fermion ψ which takes the form

$$\mathcal{L}_{Int} = g\bar{\psi}\phi\psi. \tag{7}$$

This kind of interaction means that the field ϕ of each of these particles is a *real* field (in a mathematical sense). This conclusion stems from the facts that the action and the integration factor d^4x are real. These properties mean that all terms of a Lagrangian density must be real. Now, since g and the product $\bar{\psi}\psi$ are real, one finds that ϕ is real. Evidently, a theory of a real field is inconsistent with another constraint of QM. Indeed, QM uses a *complex* wave function and for this reason the non-relativistic limits of the *real* field of Yukawa and of Z particles also violate a second kind of constraint.

4 Concluding Remarks

It is explained in this work how hierarchical relationships between physical theories can be used for deriving necessary conditions that an acceptable higher rank theory must satisfy. This issue is applied to QFT theories and the non-relativistic limit of their field function is compared with properties of non-relativistic quantum mechanics. It is explained how such a comparison provides a powerful criterion for the acceptability of physical theories. The discussion examines the dimension of quantum functions of several specific theories and compares the dimension of QFT theories with that of the lower rank non-relativistic Schrödinger theory. It turns out that the Dirac field satisfies this criterion whereas the Klein-Gordon and the Yukawa theories as well as those of the $W^{\pm}, Z$ and the Higgs boson fail to satisfy this criterion.

An important evaluation of a theoretical idea is a comparison of its outcome with experimental results. Referring to this issue, one should note that a field function $\psi(x^{\mu})$ which is used in QM, RQM and QFT depends on a *single* set of four space-time coordinates x^{μ}. For this reason, $\psi(x^{\mu})$ describes an elementary point-like particle. The following example illustrates this matter. A pion consists of a quark-antiquark pair of the u, d flavor and each quark is described by a function that depends on its own 4-coordinates x^{μ}. Hence, a pion cannot be described by a function $\psi(x^{\mu})$, simply because this function has a smaller number of independent coordinates. It turns out that experimental data of all spin-1/2 Dirac particles, namely, leptons and quarks, are consistent with their pointlike attribute. On the other hand the pion, which was the original KG candidate is not pointlike and the $\pi^{\pm}$ mesons have a charge radius which is not much smaller than that of the proton [11]. There is still no experimental data concerning pointlike properties of the $W^{\pm}, Z$ and the Higgs boson.

As is well known, the $W^{\pm}, Z$ and the Higgs bosons are cornerstones of the Standard Model. It means that the series of arguments presented in this work undermines the theoretical structure of the Standard Model. Evidently, a physical theory that has an inconsistent structure is unacceptable. Hence, people who still adhere to the Standard Model must show why the arguments presented above are incorrect. It is also interesting to note that the results of this work are consistent with Dirac's lifelong objection to the second order KG equation of a spin-0 boson (see [12], pp. 3, 4).

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