The Nuclear Shape Phase Transitions Studied within the Geometric Collective Model

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In the framework of the Geometric Collective Model (GCM), quantum phase transition between spherical and deformed shapes of doubly even nuclei are investigated. The validity of the model is examined for the case of lanthanide chains Nd/Sm and actinide chains Th/U. The parameters of the model were obtained by performing a computer simulated search program in order to obtain minimum root mean square deviations between the calculated and the experimental excitation energies. Calculated potential energy surfaces (PES's) describing all deformation effects of each nucleus are extracted. Our systematic studies on lanthanide and actinide chains have revealed a shape transition from spherical vibrator to axially deformed rotor when moving from the lighter to the heavier isotopes.

1 Introduction

The nuclear shape transitions were studied within the nuclear interacting boson model (IBM) [1–3]. The IBM-1 describes a system of a fixed number N of spin zero and two bosons (s and d bosons) subject to one- and two-body interactions. The IBM-1 reveals a transparent algebraic structure with U(6) as the dynamical group. Varying six free parameters of the model, one can reach three standard dynamical symmetries U(5), SU(3) and O(6) and two additional ones SU(3)* and O(6)* [2]. It turns out that these dynamical symmetries provide an appropriate framework for the description of low-energy collective motions of real nuclei with certain shape symmetries: The U(5) limit corresponds to spherical nuclei, the SU(3) and SU(3)* limits to axially symmetric nuclei with quadruple deformation (prolate and oblate shapes) and the O(6) and O(6)* limits to quadruply deformed nuclei that are unstable against the axial symmetry breaking. This is represented in the so called Casten triangle [2,4] with vertices corresponding to the standard dynamical symmetries and the other points to various transitional cases. Phase transitions between these shapes were studied, and it is known that the phase transition from U(5) to O(6) is second order, while any other transition within the Casten triangle from a spherical to a deformed shape is first order [5–15].

Alternative descriptions of nuclei at the critical point of phase transitions from spherical vibrator to deformed γ soft E(5) [16], and from spherical vibrator to deformed axially symmetric rotor X(5) [17], were proposed. These analytic solutions are obtained by introducing a square well potential in the Bohr Hamiltonian and yield parameter free predictions for both energies and electromagnetic transition probabilities. Empirical examples were suggested for both the proposed symmetries [18]. It was found [19, 20] that the X(5) predictions cannot be exactly reproduced by any point in the two parameter space of the IBM, whereas best agreement is obtained for parameters corresponding to a point close to, but outside, the shape phase transition region of the IBM. Since the IBM was formulated from the beginning in terms of creation and annihilation boson operators, its geometric interpretation in terms of shape variables is usually done by introducing a boson condensate with two shape parameters β and γ through the intrinsic state formalism (coherent state) [21]. The parameter β is related to the axial deformation of the system, while γ measures the deviation from axial symmetry. The equilibrium shape of the system is obtained by minimizing the intrinsic state. It is well know that the dynamical symmetry associated with U(5) corresponds to a spherical shape $\beta = 0$, the dynamical symmetry SU(3) is associated with an axially deformed shape $\beta \neq 0$ and $\gamma = 0, \pi/3$ and the dynamical symmetry O(6) is related to a γ-unstable deformed shape $\beta \neq 0$ and γ-independent.

A very flexible and powerful approach to describe nuclear collective excitations which is an extension of the Bohr-Mottelson vibrational Hamiltonian [22] is the GCM essentially based on the quadruple degrees of freedom [23,24]. The problem of nuclear collective motion is formulated by Bohr and Mottelson from the beginning in terms of the intrinsic parameters β, γ and the three Euler angels ω_i that characterize the orientation of a deformed nucleus.

The GCM is a macroscopic nuclear structure model in the sense that it considers the nucleus as a charged liquid drop with a definite surface, rather than a many-body system of constituent particles.

Neodymium isotopes are the members of the chain of nuclei which represent an ideal case for studying the influence of the shape transition from spherical to deformed nuclei. Therefore, in the chart of nuclei there is a very important lanthanide Nd/Sm transition region which exhibit a rapid structural change from spherical to well deformed when moving from the lighter to the heavier isotopes. Although this tran-

sitional region has been studied extensively in the framework of the IBM, the discussion of phase transitions has not always been treated in a proper way.

In the present paper, we have analyzed systematically the transitional region and phase transition in lanthanide and actinide chains of isotopes in the framework of GCM. For each isotope chain a fitting procedure is performed to get the model parameters. We have generated the PES to classify phase transitions and to decide if a nucleus is close to criticality. In these chains, nuclei evolve from spherical to deformed shapes.

2 The GCM Hamiltonian and the PES's

The Hamiltonian of the GCM [23] represents a concrete realization of the general Bohr Hamiltonian [22] describing the quadruple oscillations of the nuclear surface. The collective Hamiltonian restricted to quadruple deformations can be written in the notation of Rajah for tensor products of irreducible tensor operators. The α's are the well known collective coordinates, which are defined by the usual expansion of the nuclear radius in terms of spherical harmonics. The $\hat{\pi}$ is the covariant tensor of the canonically conjugate momenta. We start by writing the GCM Hamiltonian as:

$$\hat{H} = \hat{T} + \hat{V}. \tag{1}$$

The kinetic energy $\hat{T}$ up to second order is given by [2].

$$\hat{T} = \frac{1}{B_2}[\pi \times \pi]^0 + \frac{P_3}{3}\left[[\pi \times \alpha]^{(2)} \times \hat{\pi}\right]^{(0)} \tag{2}$$

where B_2 is the common mass parameter and P_3 is an enharmonic kinetic term which for simplicity, we set to zero here. A transformation to the intrinsic body fixed system leads to a formal separation of the rotational and vibrational variables expressed by the Euler angles and the shape parameters β and γ respectively. The potential energy V is given by

$$\begin{aligned} V &= C_2[\alpha \times \alpha]^{(2)} + C_3\left[[\alpha \times \alpha]^{(2)} \times \alpha\right]^{(0)} + \\ &+ C_4[\alpha \times \alpha]^{(0)}[\alpha \times \alpha]^{(0)} + \\ &+ C_5[\alpha \times \alpha]^{(0)}\left[[\alpha \times \alpha]^{(2)} \times \alpha\right]^{(0)} + \\ &+ C_6\left[[\alpha \times \alpha]^{(2)} \times \alpha\right]^{(0)}\left[[\alpha \times \alpha]^{(2)} \times \alpha\right]^{(0)} + \\ &+ D_6[\alpha \times \alpha]^{(0)}[\alpha \times \alpha]^{(0)}[\alpha \times \alpha]^{(0)}. \end{aligned} \tag{3}$$

The six stiffness parameters C_2, C_3, C_4, C_5, C_6 and D_6 occurring in the collective potential energy are constants for each nucleus. They are treated as adjustable parameters which have to be determined from the best fit to the experimental data, level energies, B(E2) transition strengths and quadruple moments. They depend however on the proton and neutron numbers due to shell structure. The potential energy, expressed in terms of the intrinsic variables β and γ, is

$$\begin{aligned} V(\beta,\gamma) &= C_2\tfrac{1}{\sqrt{5}}\beta^2 - C_3 rub\sqrt{\tfrac{2}{35}}\beta^3\cos(3\gamma) + \\ &+ C_4\tfrac{1}{5}\beta^4 - C_5\sqrt{\tfrac{2}{175}}\beta^3\cos(3\gamma) + \\ &+ C_6\tfrac{2}{35}\beta^6\cos^2(3\gamma) + D_6\tfrac{1}{5\sqrt{5}}\beta^6 \\ &= V_s(\beta) + V_{Po}(\beta,\gamma) + V_{na}(\beta,\gamma). \end{aligned} \tag{4}$$

Roughly speaking the C_2, C_4 and D_6 terms describe the γ-independent features of the PES. They form the contribution $V_s(\beta)$. The C_3 and C_5 terms are responsible for the prolate-oblate energy differences in the PES and are represented by $V_{po}(\beta,\gamma)$. The C_6 term is symmetric about the $\gamma = \pi/6$ axis and therefore can be used for the generation of non axial shape $V_{na}(\beta,\gamma)$. The selection of the eight parameters of the GCM Hamiltonian is impractical and difficult, because the available observation data are usually not sufficient to establish the qualitative nature of the GCM potential. It is therefore, often desirable to use a more tractable form of the model. In practice simplification for the GCM is to use a maximum of three parameters to describe all limits of nuclear structure: vibrator, rotor and γ-soft nuclei and transition regions in between. Then the potential energy up to the fourth power of β is simplified to be:

$$V(\beta,\gamma) = C_2\frac{1}{\sqrt{5}}\beta^2 - C_3\sqrt{\frac{2}{35}}\beta^3\cos(3\gamma) + C_4\frac{1}{5}\beta^4 \tag{5}$$

where $\beta \in [0,\infty]$ and $\gamma \in [0, 2\pi/3]$.

3 Critical Point Symmetries

The equilibrium shape associated with the GCM Hamiltonian can be obtained by determining the minimum of the energy surface with respect to the geometric variables β and γ, *i.e.* where the first derivative vanish.

Since the parameter C_3 controls the steepness of the potential, and therefore, the dynamical fluctuations in γ, it strongly affects the energies of excited intrinsic states. The parameter $C_3 = 0$ gives a γ-flat potential and an increase of C_3 introduces a γ-dependence in the potential with a minimum at $\gamma = 0$. Changing C_3 will indeed induce a γ-unstable to the symmetric rotor transition; it is best to simultaneously vary C_2 and C_4 as well.

The shape transition from vibrator to rotors is achieved by starting from the vibrator limit, lowering C_2 from positive to negative value, increasing C_4 to large positive value, with gradually increasing C_3 (lowering C_2 from positive to negative value, introducing a large positive C_4 and a positive C_3).

4 Numerical Results Applied to Lanthanide and Actinide chains

The first nucleus to be identified as exhibiting transition from spherical to axially deformed shapes was ^{152}Sm [18], followed by ^{150}Nd [24]. Further work on ^{152}Sm [25] and ^{150}Nd

[25, 26] reinforced this conclusion. In our calculation we will examine and systematically study the lanthanide $^{144-154}$Nd and $^{146-156}$Sm, isotopes and actinide $^{224-234}$Th and $^{230-238}$U isotopes because of the richness of available experimental data indicating a transition of nuclear shapes from spherical to deformed form. The optimized model parameters for each

Table 1: The GCM parameters by (MeV) as derived in fitting procedure used in the calculation.

Nucleus	C_2	C_3	C_4
^{144}Nd	12.46084	1.06407	−26.29034
^{146}Nd	7.98904	8.46249	−5.34827
^{148}Nd	−19.84450	41.41216	105.62500
^{150}Nd	−56.19267	83.37305	248.96600
^{152}Nd	−73.70551	104.57310	319.48270
^{154}Nd	−84.13947	118.02790	362.71460
^{146}Sm	14.49576	1.27688	−30.52593
^{148}Sm	8.89235	9.87290	−5.28215
^{150}Sm	−23.19850	47.32818	121.87500
^{152}Sm	−63.80397	93.79468	281.39990
^{154}Sm	−82.44842	116.19230	356.21830
^{156}Sm	−93.05583	129.83070	400.10950
^{224}Th	0.55766	4.96951	6.10300
^{226}Th	−0.11521	6.38937	9.70762
^{228}Th	−0.83906	7.98671	13.68875
^{230}Th	−1.63871	9.76153	18.10188
^{232}Th	−2.59264	11.71384	23.12250
^{230}U	−1.67560	9.76153	18.18437
^{232}U	−2.63289	11.71384	23.21250
^{234}U	−3.77666	13.84363	28.92012
^{236}U	−4.90299	16.15090	34.85125
^{238}U	−6.23928	18.63565	41.51437

nucleus was adjusted by fitting procedure using a computer simulated search program in order to describe the gradual change in the structure as neutron number varied and to reproduce the properties of the selected reliable state of positive parity excitation ($2_1^+, 4_1^+, 6_1^+, 8_1^+, 0_2^+, 2_3^+, 4_3^+, 2_2^+, 3_1^+$, and 4_2^+) and the two neutron separation energies of all isotopes in each isotopic chain. The resulting parameters are listed explicitly in Table 1. For the isotopic chains investigated here, the collective properties are illustrated by representing the calculated PES describing all deformation effects of the nucleus. We investigated the change of nuclear structure within these chains as illustrated in Figures 1-4. The PES's versus the deformation parameter β for lanthanide and actinide isotopic chains of nuclei evolving from spherical to axially symmetric well deformed nuclei. We remark that for all mentioned nuclei, the PES is not flat, exhibiting a deeper minimum in the prolate ($\beta > 0$) region and a shallower minimum in the oblate ($\beta < 0$) region. Relatively flat PES occur for the N = 86 nuclei ^{146}Nd and ^{148}Sm. A first order shape phase transition with change in number of neutrons when moving from the lighter to heavier isotopes, *i.e* U(5) - SU(3) transitional region are observed.

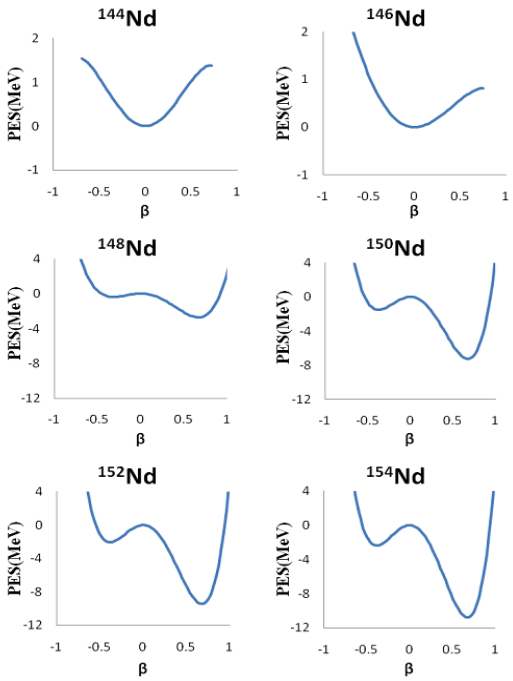

Fig. 1: PES calculated with GCM as a function of the shape parameter β for shape phase transition from spherical to prolate deformed for Neodymium isotope chain $^{144-154}$Nd.

The present results for $^{146-156}$Sm is in good agreement with Nilsson-Strutinsky (BCS)-calculations [26]. However, the existence of a bump in the PES is related to the success of the confined β-soft (BCS) rotor model, employing an infinite square well potential displaced from zero, as well as to the relevance of Davidson potentials [27, 28]. It also is related to the significant five-dimensional centrifugal effect [28, 29]. The actinide $^{228-234}$Th and $^{234-238}$U are all well-deformed rotors with energy ratio $E(4_1^+)/E(2_1^+)$ close to (3.3).

5 Conclusion

A simple approach of the GCM is discussed which reproduces the basic features of the three limits of the nuclear structure: spherical vibrator, axially symmetric rotor and γ-soft rotor, as well as the three phase shape transition regions linking them. The Hamiltonian is expressed as a series expansion in terms of surface deformation coordinates and a conjugate momentum. We considered only the lowest kinetic energy terms, so that the eigen problem for our Hamiltonian reduces to Schrodinger equation in five dimensional spaces. All calculations are performed for reference value of the common mass parameter, only a maximum of three parameters of the truncated form of GCM potential instead of the six are

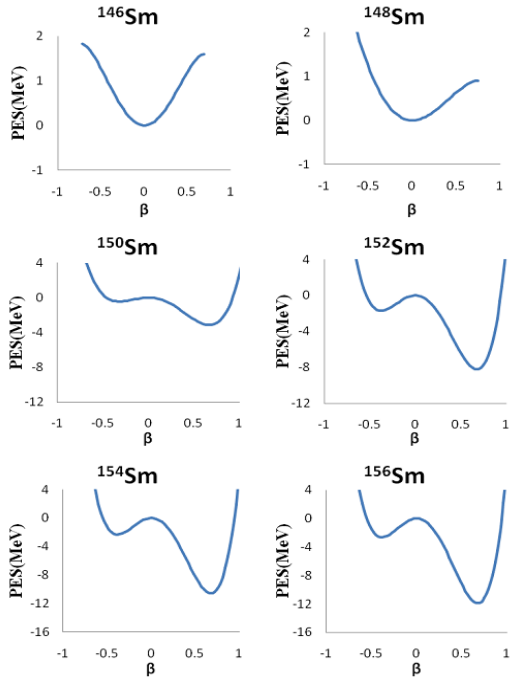

Fig. 2: PES calculated with GCM as a function of the shape parameter β for shape phase transition from spherical to prolate deformed for Samarium isotope chain $^{146-156}$Sm.

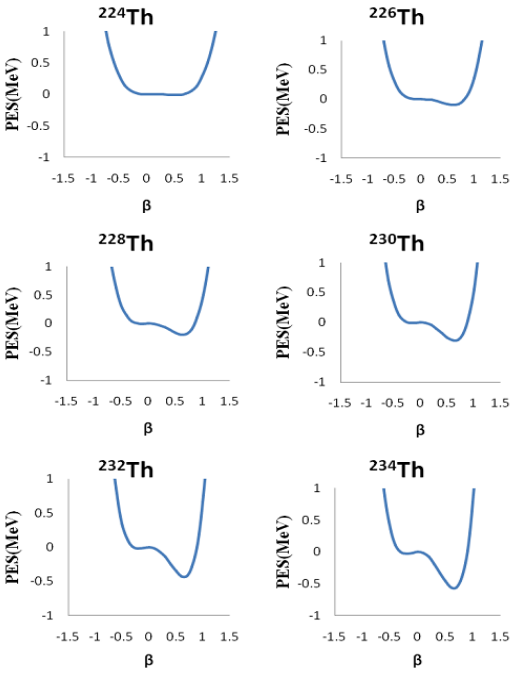

Fig. 3: PES calculated with GCM as a function of the shape parameter β for shape phase transition from spherical to prolate deformed for Thorium isotope chain $^{224-234}$Th.

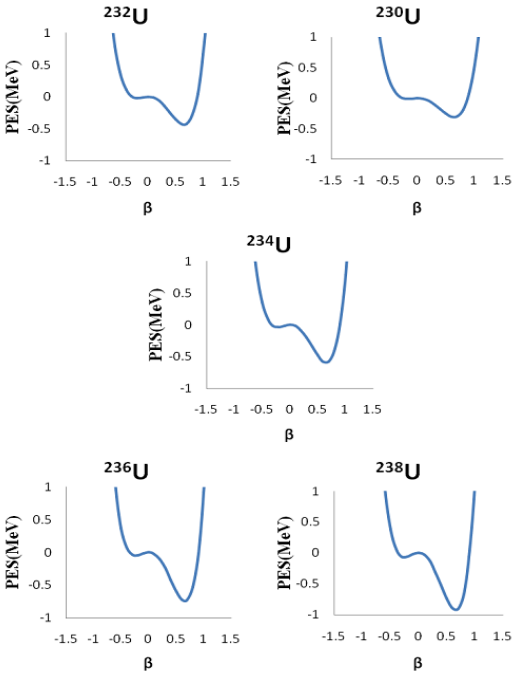

Fig. 4: PES calculated with GCM as a function of the shape parameter β for shape phase transition from spherical to prolate deformed for Uranium isotope chain $^{230-238}$U.

used. The parameter values for the description of a particular nucleus have been found through automated fitting of the nuclear energy levels.

The systematics of shape transitions versus neutron number is studied by the GCM. The capabilities of the model and the illustrative way of representing the collective properties by potential energy surfaces are demonstrated. For neutron number N = 90, the nucleus has a substantial static deformation, but for N = 80 the nucleus is soft or transitional and cannot be described as deformed.

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