# Quantum Uncertainty and Fundamental Interactions

Sebastiano Tosto
Italy. Email: stosto@inwind.it, stosto44@gmail.com

The paper proposes a simplified theoretical approach to infer some essential concepts on the fundamental interactions between charged particles and their relative strengths at comparable energies by exploiting the quantum uncertainty only. The worth of the present approach relies on the way of obtaining the results, rather than on the results themselves: concepts today acknowledged as fingerprints of the electroweak and strong interactions appear indeed rooted in the same theoretical frame including also the basic principles of special and general relativity along with the gravity force.

## 1 Introduction

The state of a classical particle is specified by its coordinates and momentum; the dynamical variables $x, p_x, y, p_y, z, p_z$, assumed known at any time, define the 6-dimensional space usually called "phase space". Knowing the state of a particle means determining these six quantities that describe its motion and energy. Since the state of a classical system is identified by the distribution of corresponding points in the phase space, any finite volume $V_{ps} = (\delta x\delta y\delta z)(\delta p_x\delta p_y\delta p_z)$ should seemingly contain an infinite number of states. Because of the uncertainty principle, however, these six quantities are not simultaneously known; the impossibility of defining the corresponding points in the phase space compels instead introducing a lower limit to the volume of phase space physically significant. Since such an elementary volume has size $V^o_{ps} = (dxdydz)(dp_xdp_ydp_z) = \hbar^3$, any finite volume $V_{ps}$ enclosing measurable combinations of coordinates and conjugate momenta consists of a finite number $V_{ps}/V^o_{ps}$ of elementary volumes. The quantum uncertainty was inferred by W. Heisenberg as a consequence of the operator formalism of wave mechanics, on which relies the quantum theory: the wave function $\psi = \psi(x, t)$ replaces the lack of definable quantum values of $x$ concurrently associable to the conjugate $p_x$. However most physicists believe unsatisfactory a theory based on the wave function $\psi$ without direct physical meaning [1]; indeed $\psi\psi^*$ only has the statistical meaning of probability density and contains the maximum information obtainable about a physical system. The wave function characterizes a pure state, represented by a single "ket" vector to which corresponds a well defined eigenvalue, whereas in general a particle is found in a mixture of states; so the result of a measurement on a quantum state represents a probability distribution of finding the particle in a given volume of phase space. The density matrix is the mathematical tool to describe mixed quantum states by means of a distribution function of coordinates and momenta. Owing to the statistical character of the knowledge we can afford in the quantum world, the Wigner function $W(x, p)$ [2] aims to represent a quantum state in terms of a joint probability distribution involving both coordinates and momenta, in formal analogy with the classical statistics; the former is therefore a correction to the latter. The quantum $x$ and $p$ distributions are appropriately described by the respective marginal distributions $\int_{-\infty}^{+\infty} W(x, p)dp$ and $\int_{-\infty}^{+\infty} W(x, p)dx$ under the normalization condition $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} W(x, p)dpdx = 1$, whereas the expectation value for any operator function is weighed by $W(x, p)$ as $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} W(x, p) f(x, p)dpdx$. Other relevant features of $W(x, p)$, well known [3], are omitted here for brevity. Also the Wigner function, however, although providing significant information about the quantum states, presents conceptual difficulties: it is not a real probability distribution in the classical sense, it is a quasi-probability that can even take negative values; moreover it can represent the average value of an observable but not, in general, also its higher power moments.

To bypass both these difficulties inevitably inherent the wave formalism, the present theoretical model implements an approach conceptually different: it exploits directly the statistical formulation of quantum uncertainty, which therefore becomes itself a fundamental assumption of the model and reads in one space dimension

$$\Delta x\Delta p_x = n\hbar = \Delta t\Delta\varepsilon. \qquad (1,1)$$

This set of $2n$ equation disregards since the beginning the local dynamical variables of the particles forming the quantum system and simply counts its number $n$ of allowed states. Are therefore required the following positions

$$x_i \to \Delta x_i, \qquad t \to \Delta t, \qquad i = 1..3. \qquad (1,2)$$

No hypotheses are made about the uncertainty ranges, which are by definition unknown, unknowable and arbitrary. In quantum mechanics the square complex wave function of space and time variables contains the maximum information about a quantum system, which has therefore probabilistic character. The present model intends instead starting from a *minimal* information about any quantum system, still based on the failure of the physical concept of points definable in the quantum phase space but trusting on the idea that a minimum information is consistent with the maximum generality: despite the knowledge of one dynamical variable only is in principle allowed even in the quantum world, the present

model disregards "a priori" the local values of both conjugate dynamical variables. This means renouncing even to the concept of probability density provided by the wave function of a particle, while also disregarding the related concept of wave packet to describe its propagation; in the present model it is only possible to say that if the particle moves during a time range $\Delta t$ throughout its uncertainty range $\Delta x$, then its average velocity component is $v_x = \Delta x/\Delta t$ regardless of any local feature of its actual delocalization motion. So eqs (1,1) require by definition $\Delta\varepsilon = v_x \Delta p_x$. In fact the positions (1,2) ignore both local dynamical variables, not as a sort of approximation to simplify some calculation but conceptually and since the early formulation of any quantum problem; accordingly, the delocalization of a quantum particle in its uncertainty range is conceived in its most agnostic form, i.e. waiving any kind of information about its position and motion. Thus, regarded in this way, eqs (1,1) exclude the concept itself of probability density and contextually also the definition of Wigner function linking the Schrodinger equation to the marginal distributions in the phase space; both equations are bypassed along with the concept of wave equation itself. Eqs (1,1) merely list the eigenvalues of pure states, indeed they are a set of equations corresponding to the respective values of $n$; so they also skip the probability with which in a mixed state each eigenvalue could be measured. Despite waiving themselves the concept of probability density through the positions (1,2), eqs (1,1) enable however also this kind of probabilistic information; it is essential indeed to mention that the wave formalism is obtainable as a corollary of eqs (1,1) [4], which means that all considerations previously introduced are in fact comprised also in the present theoretical model: one infers first from eqs (1,1) the operator formalism and then proceeds as usual. In this way the wave formalism, with its conceptual weakness, loses its rank of fundamental root of our knowledge about the quantum world, becoming indeed a mere by-product of eqs (1,1); yet, even so it still represents an added value to the physical information by introducing the concept of probability density that partially overcomes the total agnosticism of eqs (1,1).

What however about the chance of formulating any physical problem exploiting directly the eqs (1,1) only? Is legitimate the belief that the equations enclosing conceptually the wave formalism as a corollary also enclose the inherent physical information. The question that arises at this point concerns just the real chance of obtaining physical information once abandoning the typical ideas and mathematical tools of wave mechanics: is really redundant the concept of probability density? Several papers have demonstrated the effectiveness of this alternative approach, e.g. [5,6]; moreover, without the need of hypotheses on $n$ and on the uncertainty ranges defined by eqs (1,1), the paper [7] has shown the possibility of extending the mere quantum horizon of these equations, initially concerned, also to the special and general relativity. The positions (1,2) compel focusing the attention on the uncertainty ranges and related numbers of states, i.e. on the phase space, rather than on the specific coordinates of the particles concerned by the particular physical problem. In fact, the local dynamical variables are conceptually disregarded since the beginning in the present model. Put for instance $\Delta x = x - x_o$: if either boundary coordinate, say $x_o$, is defined by the origin of the coordinate system $R$, then it determines the position of $\Delta x$ in $R$; the other boundary coordinate $x$ determines its size. The crucial point is that both $x_o$ and $x$ are arbitrary, unknown and unknowable by fundamental assumption; the reference system $R$ is therefore "a priori" arbitrary, unspecified and unspecifiable as well, whence the equivalence of all reference systems whenever implementing the positions (1,2) to describe the quantum world. Otherwise stated, eqs (1,1) do not specify any particular reference system because analogous considerations hold for all uncertainty ranges they introduce. Moreover $n$ is itself arbitrary as well; it merely symbolizes a sequence of numbers of allowed states, not some specific value in particular. Let therefore eqs (1,1) be defined in any $R$ and rewrite them as $\Delta x' \Delta p'_x = n' = \Delta\varepsilon' \Delta t'$ in any $R'$: it is self-evident that actually these equations are indistinguishable because $n$ and $n'$ do so as well. Whatever a specific value of $n$ might be in $R$, any change to $n'$ e.g. because of the Lorentz transformations of the ranges is physically irrelevant: it means replacing an arbitrary integer in the former set with another integer of the latter set. In effect, two examples of calculation reported below highlight that modifying the range sizes from primed to unprimed values does not affect any result, in agreement with their postulated arbitrariness: no range size is expected to appear in the quantum eigenvalues. Hence the eqs (1,1) have general character, regardless of any particular reference system to be appropriately specified; this holds also if $R$ and $R'$ are inertial and non-inertial, since no hypothesis has been assumed about them [7]. On the one hand this entails obtaining the indistinguishability of identical particles as a corollary, regardless of which particle in a set could be that actually delocalized in a given uncertainty range; indeed no particle is specifically concerned "a priori". On the other hand it also entails that the properties of motion of the particle, and thus the marginal distributions of its dynamical variables, are disregarded by assumption and skipped by consequence when formulating any physical problem. To better understand the following of the paper, these remarks are now exemplified examining shortly the non-relativistic quantum angular momentum $\mathbf{M}$, on the one side to highlight how to exploit the positions (1,2) and on the other side to show why the minimal information accessible through eqs (1,1) is in fact just that available through the usual operator formalism of wave mechanics.

Consider the classical component $M_w = \mathbf{r} \times \mathbf{p} \cdot \mathbf{w}$ of $\mathbf{M}$ along an arbitrary direction defined by the unit vector $\mathbf{w}$, being $\mathbf{r}$ the radial distance of any particle from the origin of an arbitrary reference system and its momentum. The positions (1,2) compel $\mathbf{r} \to \Delta\mathbf{r}$ and $\mathbf{p} \to \Delta\mathbf{p}$ and enable the

number $l$ of states to be calculated only considering the total ranges $\Delta\mathbf{r}$ and $\Delta\mathbf{p}$ of distances and momenta physically allowed to the particle, about which no hypothesis is necessary; let us show that the random local values $\mathbf{r}$ and $\mathbf{p}$ themselves have instead no physical interest. So $M_w = (\Delta\mathbf{r} \times \Delta\mathbf{p}) \cdot \mathbf{w} = (\mathbf{w} \times \Delta\mathbf{r}) \cdot \Delta\mathbf{p}$, i.e. $M_w = \Delta\mathbf{W} \cdot \Delta\mathbf{p}$, where $\Delta\mathbf{W} = \mathbf{w} \times \Delta\mathbf{r}$. If and $\Delta\mathbf{W}$ are orthogonal, then $M_w = 0$; else, rewriting $\Delta\mathbf{W} \cdot \Delta\mathbf{p}$ as $(\Delta\mathbf{p} \cdot \Delta\mathbf{W}/\Delta W)\,\Delta W$ with $\Delta W = |\Delta\mathbf{W}|$, the component $\pm\Delta p_W = \Delta\mathbf{p} \cdot \Delta\mathbf{W}/\Delta W$ of $\Delta\mathbf{p}$ along $\Delta\mathbf{W}$ yields $M_w = \pm\Delta W \Delta p_W$.

Thus, according to eqs (1,1), $M_w = \pm l\hbar$, being $l$ the usual notation for the number of states of the angular momentum. As expected, $M_w$ is a multi-valued function because of the uncertainties initially postulated for $\mathbf{r}$ and $\mathbf{p}$. One component of $\mathbf{M}$ only, e.g. along the $z$-axis, is knowable; repeating the same approach for the $y$ and $x$ components would trivially mean changing $\mathbf{w}$. Just this conclusion suggests that the average values $< M_x^2 >$, $< M_y^2 >$ and $< M_z^2 >$ should be equal; so the quantity of physical interest to describe the properties of quantum angular momentum is $l$, as a function of which $M^2$ is indeed inferred as well. The components averaged over the possible states summing $(l\hbar)^2$ from $-L$ to $+L$, where $L$ is an arbitrary maximum value of $l$, yield $< M_i^2 >= \sum_{l_i=-L}^{l_i=L} (\hbar l)^2/(2L{+}1)$ and thus $M^2 = \sum_{i=1}^{3} < M_i^2 >= L(L{+}1)\hbar^2$.

The physical definition of angular momentum is enough to find quantum results completely analogous to that of the wave mechanics even disregarding any local detail about the angular motion. This result has been reminded here as it introduces several significant considerations useful in the following: (i) eqs (1,1) and the positions (1,2) plug the classical physics into the quantum world; (ii) no hypothesis is necessary about the motion of the particle nor about its wave/matter nature to infer the quantum result; (iii) trivial algebraic manipulations replace the solution of the pertinent wave equation; (iv) the result inferred through eqs (1,1) only is consistent with that of the wave mechanics; (v) the local distance between the particles concerned in the angular motion does not play any role in determining $l$; (vi) the number of allowed states plays actually the role of angular quantum number of the operator formalism of wave mechanics; (vii) the amount of information accessible for the angular momentum is not complete like that of the classical physics, but identical to that of the wave formalism; (viii) eqs (1,1) rule out "a priori" any chance of hidden variables hypothetically encodable in the wave function, i.e. local values of any kind that could in principle enhance our knowledge about $M_w$ and $M^2$ to obtain a more complete description of the angular quantum system; (ix) the eigenvalues, i.e. the physical observables, are actually properties of the phase space rather than properties of specific particles, whence the indistinguishability of identical particles here inferred as a corollary of eqs (1,1); (x) the numbers of states are here simply counted; (xi) the positions (1,2) are consistent with the concept of classical coordinate in the limit case $\Delta x \to 0$, which means that the random local variable $x_o \le x \le x_1$ tends to a classical local value uniquely and exactly defined; (xii) the total arbitrariness of the boundary values of the ranges is necessary to ensure that any local value is allowed for the corresponding classical variables; (xiii) the range sizes do not play any role in determining the eigenvalues of angular momentum, their conceptual reality, i.e. the total uncertainty about both conjugate dynamical variables of a quantum particle, is the unique hypothesis of the present model. The same holds of course for any other uncertainty range.

These ideas have been extended and checked in the papers [5,6] also for more complex quantum systems like hydrogenlike and many electron atoms/ions and diatomic molecules; also these papers allowed concluding that eqs (1,1) efficiently replace the standard approach of wave mechanics, without requiring the concept of probability density and thus without need of calculating marginal distributions in the phase space through the Wigner functions. In these papers the interaction is described via the Coulomb potential energy between charged particles; in other words, one assumes already known the Coulomb law to calculate for instance the energy levels of hydrogenlike atoms. This point is easily highlighted considering for simplicity the non-relativistic hydrogenlike energy levels; also this topic, already introduced in [5], is reported here for completeness.

Assuming the origin $O$ of an arbitrary reference system $R$ on the nucleus, the classical energy is $\varepsilon = p^2/2m - Ze^2/r$ being $m$ the electron mass. Since $p^2 = p_r^2 + M^2/r^2$, the positions (1,2) $p_r \to \Delta p_r$ and $r \to \Delta r$ yield $\varepsilon = \Delta p_r^2/2m + M^2/2m\Delta r^2 - Ze^2/\Delta r$. Two numbers of states, i.e. two quantum numbers, are expected because of the radial and angular uncertainties. Eqs (1,1) and the previous result yield $\varepsilon = n^2\hbar^2/2m\Delta r^2 + l(l+1)\hbar^2/2m\Delta r^2 - Ze^2/\Delta r$ that reads $\varepsilon = \varepsilon_o + l(l+1)\hbar^2/2m\Delta r^2 - E_o/n^2$ with $E_o = Z^2e^4m/2\hbar^2$ and $\varepsilon_o = (n\hbar/\Delta r - Ze^2m/n\hbar)^2/2m$. Minimize $\varepsilon$ putting $\varepsilon_o = 0$, which yields $\Delta r = n^2\hbar^2/Ze^2m$ and $\varepsilon = [l(l+1)/n^2 - 1]E_o/n^2$; so $l \le n-1$ in order to get $\varepsilon < 0$, i.e. a bound state. Putting thus $n = n_o + l + 1$ one finds the electron energy levels $\varepsilon_{el} = -E_o/(n_o + l + 1)^2$ and the rotational energy $\varepsilon_{rot} = l(l+1)E_o/n^4$ of the atom as a whole around $O$. Hold also here all considerations introduced for the angular momentum, in particular it appears that the range sizes do not play any role in determining the energy levels. The physical meaning of $\Delta r$, related to the early Bohr radius, appears noting that

$$\varepsilon_{el} = -\frac{E_o}{n^2} = -\frac{Ze^2}{2\Delta r}, \quad \Delta r = \frac{n^2\hbar^2}{Ze^2m}, \quad E_o = \frac{Z^2e^4m}{2\hbar^2}, \quad (1,3)$$

i.e. $\varepsilon_{el}$ is due to charges of opposite sign delocalized within a diametric distance $2\Delta r$ apart. As previously stated, nucleus and electron share a unique uncertainty radial range: in general, the greater $m$, the closer its delocalization extent around the nucleus. Also note that $n$ and $l$ are still properties of the phase space, but now they describe the whole quantum sys-

tem "nucleus + electron" rather than the nucleus and the electron separately. Since the first eq (1,3) does not depend explicitly on the kind of particles forming the concerned hydrogenlike atom, $m$ or the reduced mass are actually hidden into $\Delta r$; it is possible to link $\varepsilon_{el}$ to the known condition $n\lambda = 2\pi\Delta r$, according which an integer number of steady electron wavelengths is defined along a circumference of radius $\Delta r$. For such electron waves one finds

$$\varepsilon_{el} = -\frac{\pi Z e^2}{n\lambda} = -\frac{\alpha}{n}\frac{Z p_\lambda c}{2}, \quad p_\lambda = \frac{h}{\lambda}, \quad \alpha = \frac{e^2}{\hbar c}. \qquad (1,4)$$

Note that introducing $\alpha$ to express the quantum energy levels compels defining the De Broglie momentum. Even in this form $\varepsilon_{el}$ is still related to the reduced mass of the system, which can be introduced via the momentum $p_\lambda$; thus eq (1,4) holds in general for any system of charges. Moreover, the factor $Z/2$ apart, appears interesting that the energy levels of the system $\varepsilon_{el}$ are linked to the kinetic energy $p_\lambda c$ of the running electron wave circulating along the circumference of radius $\Delta r$ via the coefficient $\alpha/n$. On the one hand, this result emphasizes the electromagnetic character of the interaction between electron and nucleus; on the other hand, the key role of the quantum uncertainty in determining the allowed energy levels of eqs (1,3) also evidences the kind of interaction itself. The more general question that arises at this point is therefore: do eqs (1,1) provide themselves any hint also about the physical essence of the fundamental interactions? The standard model [8-11] provides a satisfactory description of the fundamental forces of nature. So the present paper does not aim to replicate the electro-weak model or the chromodynamics, which indeed would be useless and unexciting; nevertheless seems useful to propose a simplified approach aimed to show (i) that the fundamental interactions are inferable from eqs (1,1) only and (ii) that exists a unique conceptual root common to all fundamental interactions. This task is in effect particularly valuable because the present model has already accounted for the gravity force [7] and for the basic principles of special and general relativity.

The purpose of the paper is to examine the ability of eqs (1,1) to describe also other kinds of possible interactions and their relative strengths at comparable energies; it will be also shown that further information is obtained about the vector bosons associated with the respective kinds of interactions. Therefore the worth of the present paper rests mostly on the chance of finding concepts today known as fingerprints of the electroweak and strong interactions in the frame of a unique logical scheme based on the quantum uncertainty and including the relativity. The paper [7] has somewhat concerned the electromagnetic interactions, while also showing that all concepts of quantum wave formalism are indeed obtained through the present approach. Here we concern in particular the weak and strong interactions between nuclear and subnuclear particles. The next sections will describe the possible features of these interactions.

## 2 Physical background of the interactions

Let us show that the concept of interaction relies in the frame of the present model entirely on eqs (1,1). Consider first an isolated particle of mass $m$ and momentum component $p_x^\infty$ free to move in an ideal infinite range. When confined in a time-space uncertainty range $\Delta x$, however, its energy changes by an amount $\Delta\varepsilon$ given by

$$\Delta p_x^2/2m = (n\hbar)^2/2m\Delta x^2, \qquad \Delta p_x = p_x^{conf} - p_x^\infty;$$

i.e. $\Delta p_x$ is by definition the range including any change of local momentum component $p_x$ occurring when the free particle turns from a non-confined to a confined state within $\Delta x$.

Since no process occurs instantaneously in nature, let $\Delta t$ be the confinement time range corresponding to $\Delta p_x$: to the confinement process corresponds thus the arising of a force field whose component $\Delta F_x = \Delta p_x/\Delta t = F_x^{conf} - F_x^\infty$ is related to $\Delta\varepsilon$, being clearly $\Delta F_x = \Delta\varepsilon/\Delta x = \Delta p_x^2/2m\Delta x^3$. By definition $\Delta F_x$ includes any random $F_x^\infty \le F_x \le F_x^{conf}$: in the present model the local dynamical variables are replaced by corresponding ranges of values, so the classical force $F_x$ at the local coordinate $x$ is replaced by a range of possible forces active within $\Delta x$. Actually the result $\Delta p_x/\Delta t = \Delta\varepsilon/\Delta x$ could have been inferred directly from eqs (1,1) without need of any remark; yet these considerations highlight that a force field in a space time uncertainty range is the only information available on the particle once accepting the eqs (1,1) as the unique assumption of the model.

Clearly, once concerning one particle only, energy and force component cannot be related to any form of interaction; rather both have mere quantum origin. Also, $\Delta\varepsilon$ and $\Delta F_x$ tend obviously to zero for $\Delta x \to \infty$; hence if $p_x^\infty$ changes to $p_x^{conf}$ concurrently with the arising of a force component acting on the particle, then $p_x^\infty$ must be constant by definition as it represents the momentum of the particle before its confinement driven perturbation. This again appears from the standpoint of eqs (1,1): $\Delta x \to \infty$ requires $\Delta p_x \to 0$ for any finite number of states regardless of $\Delta t$. Since an uncertainty range infinitely small tends to a unique classical value of its corresponding quantum random variable and since this holds regardless of $\Delta t$, then the limit value must be a constant: so $p_x^\infty = const$ corresponds by necessity to $F_x^\infty = 0$.

Despite the present model allows reasoning on $\Delta F_x$ only, a first corollary is the inertia principle that holds for a lonely particle in an infinite space time delocalization range. Other interesting consequences follow for any finite $\Delta x = x_2 - x_1$: the notation emphasizes that instead of considering the particle initially in an infinite unconfined range, we are now interested to describe its behavior in a confined state, e.g. in the presence of two infinite potential walls $\Delta x$ apart. Clearly this means introducing the corresponding $\Delta p_x = p_2^{conf} - p_1^{conf}$: again the eqs (1,1) compel writing $\Delta\varepsilon/\Delta x = \Delta p_x^2/2m\Delta x^3$ when $p_x^\infty$ has turned into a local $p_1^{conf} \le p_x \le p_2^{conf}$, which entails once more $\Delta F_x = \Delta p_x/\Delta t$ within $\Delta x$. These ideas are

now extended to the interaction forces. Rewrite first the force field component $\Delta\varepsilon/\Delta x = \Delta p_x^2/2m\Delta x^3$ of a particle confined within $\Delta x$ as follows

$$\Delta F_x = \frac{\hbar^2}{2}\frac{n}{m}\frac{n}{V}, \qquad V = \Delta x^3. \tag{2,1}$$

Even the one-dimensional case defines the delocalization volume $V$ because, being $\Delta x$, $\Delta y$ and $\Delta z$ arbitrary, any value allowed to $\Delta x\Delta y\Delta z$ is also allowed to $\Delta x^3$. Is crucial the fact that the range of each force component is proportional to $n/m$, number of allowed states per unit mass, times $n/V$, number of allowed states per unit delocalization volume. Consider now two free particles $a$ and $b$ in their own uncertainty ranges $\Delta x_a$ and $\Delta x_b$; hold separately for them the relationships $\Delta\varepsilon_a = (n_a\hbar)^2/2m_a\Delta x_{xa}^2$ and $\Delta\varepsilon_b = (n_b\hbar)^2/2m_b\Delta x_{xb}^2$. These particles are non-interacting, as their $n_a$ and $n_b$ are assumed independent each other like $\Delta x_a$ and $\Delta x_b$ themselves; nothing in these equations accounts for the most typical and obvious consequence of any kind of interaction, i.e. some relationship between their allowed states or between their delocalization ranges. Two free particles do not share by definition any kind of link, any possible coincidence of allowed states would be accidental and transient only. Consider now their possible interaction; a reasonable chance of linking their allowed states is to assume, for instance, that the particles share the same uncertainty range. If $\Delta x$ is unique for both particles, then their allowed states must be somehow linked because of eqs (1,1); in other words, even being still $n_a \neq n_b$, the random values of local momentum components $p_{xa}$ and $p_{xb}$ are subjected to the constrain $n_a/\Delta p_{xa} = n_b/\Delta p_{xb} = \Delta x/\hbar$. Note for instance that $\Delta r$ of eq (1,3) includes by definition all possible distances between electron and nucleus, which implicitly means that both particles share the same uncertainty range where the interaction occurs; so $n$ and $l$ characterizing the electron energy levels of the hydrogenlike system result from the change of the early quantum numbers, e.g. $n^{free}$ and $l^{free} = 0$, owned by each particle independently of the other before interaction. In this respect two relevant points are: (i) the interaction driven change $\delta n$ of the number $n$ of states and (ii) the physical meaning of the related $\delta[(n/m)(n/V)]$.

As concerns the point (i), consider $\Delta\varepsilon\Delta t = n\hbar$ in an arbitrary reference system $R$ and let $n$ be allowed to change from any initial value $n_1$ to any successive value $n_2$ during a fixed time range $\Delta t$; whatever $n_1$ and $n_2$ might be, this is admissible because $\Delta t$ is arbitrary. The notation emphasizes that a given value of $\delta n = n_2 - n_1$ is obtainable regardless of the initial value $n_1$ because $n_2$ is arbitrary; so $\delta n = 1, 2, ..$ anyway, regardless of the specific value of $n_1$. Calculate next the change $\delta\Delta\varepsilon$ of $\Delta\varepsilon$ as a function of $\delta n$ during $\Delta t$, which reads now $(\Delta\varepsilon_{n_2} - \Delta\varepsilon_{n_1})/\Delta\varepsilon_{n_1} = \delta n/n_1$ with obvious meaning of symbols. Note that in general the series expansion of $\log(\Delta\varepsilon)$ around $\log(\Delta\varepsilon_{n_1})$ reads

$$\log(\Delta\varepsilon_{n_2}) = \log(\Delta\varepsilon_{n_1}) + \frac{\Delta\varepsilon_{n_2} - \Delta\varepsilon_{n_1}}{\Delta\varepsilon_{n_1}} - \frac{1}{2}\left(\frac{\Delta\varepsilon_{n_2} - \Delta\varepsilon_{n_1}}{\Delta\varepsilon_{n_1}}\right)^2 + ..$$

so that

$$\log\left(\frac{\Delta\varepsilon_{n_1+\delta n}}{\Delta\varepsilon_{n_1}}\right) = \frac{\delta n}{n_1} - \frac{1}{2}\left(\frac{\delta n}{n_1}\right)^2 + \frac{1}{3}\left(\frac{\delta n}{n_1}\right)^3 - ..$$

$$\Delta\varepsilon_{n_1} = \frac{n_1\hbar}{\Delta t}, \quad \delta n = 1, 2, \ldots \tag{2,2}$$

This equation describes the size change of the energy range $\Delta\varepsilon_{n_1}$ as long as the number of allowed states increases with respect to the initial value $n_1$: so $\Delta\varepsilon_{n_1+\delta n}$ with $\delta n = 1$ describes the first increment of energy range size with respect to $\Delta\varepsilon_{n_1}$, then $\delta n = 2$ the next size increment and so on; in short, eq (2,2) describes how are modified the random local values $\varepsilon_{n_1+\delta n}$ included in $\Delta\varepsilon_{n_1+\delta n}$ at $\delta n$ progressively increasing. Instead $\Delta\varepsilon_{n_1}$ plays here the role of a fixed reference range with respect to which is calculated $\Delta\varepsilon_{n_1+\delta n}$. For reasons that will be clear in the next section 5, it is mostly interesting to examine the particular case of $n_1$ such that

$$\Delta\varepsilon_{n_2} - \Delta\varepsilon_{n_1} << \Delta\varepsilon_{n_1}, \qquad \delta n/n_1 << 1. \tag{2,3}$$

Let us truncate thus the series expansion (2,2) at the first order of approximation under the assumption (2,3) and simplify the notation putting $i = \delta n$; one finds (i=1,2,...)

$$n_1 \log\left(\frac{\Lambda_i}{\Lambda}\right) = i, \qquad \Lambda_i = \Delta\varepsilon_{n_1+\delta n}, \qquad \Lambda = \Delta\varepsilon_{n_1}. \tag{2,4}$$

Despite the generality of eqs (2,2), is particularly significant for the purposes of the present paper the case of a quantum system consisting of an arbitrary number of particles, each one delocalized in its own uncertainty range: if these latter are non-interacting, then let the energy of the system be included within the range $\Delta\varepsilon_{n_1}$ and be $n_1$ its total number of states; if instead all particles are delocalized in the same space-time range, then their interaction changes the energy range of the system to $\Delta\varepsilon_{n_1+\delta n}$ characterized of course by a new number of states $n_2 = n_1 + \delta n$.

As concerns the point (ii), we expect according to eq (2,1) that from $\Delta\varepsilon_a$ and $\Delta\varepsilon_b$ of the two free particles follow because of the interaction the changes $\delta\Delta\varepsilon_a = (\hbar^2/2)\delta(n_a^2/m_a\Delta x_a^2)$ and $\delta\Delta\varepsilon_b = (\hbar^2/2)\delta(n_b^2/m_b\Delta x_b^2)$. The expressions of the corresponding changes of the initial confinement force components $\Delta F_{xa} = \Delta\varepsilon_a/\Delta x_a$ and $\Delta F_{xb} = \Delta\varepsilon_b/\Delta x_b$ from the non-interacting to the interacting state read thus

$$\delta\Delta F_{xa} = (\hbar^2/2)\delta\left[(n_a/m_a)(n_a/V_a)\right]$$

$$\delta\Delta F_{xb} = (\hbar^2/2)\delta\left[(n_b/m_b)(n_b/V_b)\right].$$

These equations agree with the previous idea, i.e. the forces are related to changes of the allowed numbers of states per unit mass and delocalization volumes of the particles $a$ and $b$: in effect the interaction between two particles consists of forces acting on both of them and requires that the respective numbers of states are affected as well. More precisely

$\delta[(n/m)(n/V)]$ means that are modified during the interaction not only the states allowed to the particles themselves, but also that of the delocalization space surrounding them. Clearly the former are consequences of the latter. In other words, the fact that $\delta(n/m)$ requires explicitly also the concurrent $\delta(n/V)$ compels thinking: (i) that a particle interacts with another particle because it generates a field that propagates outwards through the space volume $V$ and (ii) that just in doing so this field changes the number of states allowed to the other particle; i.e. the changes of number of states of each particle are somehow correlated, as previously stated. Since no event occurs instantaneously in nature, $\delta(n/V)$ requires an appropriate time range to be realized, i.e. the propagation rate is finite in agreement with the existence of an upper limit obliged by eqs (1,1) [7]; in this way the interaction exchanges information about physical features and strength of the related force between particles. The most natural way to acknowledge this way of regarding two interacting particles is to admit that they exchange intermediate virtual particles that propagate, whence $\delta(n/V)$, and carry the necessary information that affects in turn the real particles themselves, whence $\delta(n/m)$; indeed $n$ defining $n/V$ is the same as that defining $n/m$, i.e. the change $\delta(n/m)$ of states allowed to the particle is actually just that $\delta(n/V)$ of the space around it. Strictly speaking, however, one should say more appropriately space-time, and not simply space: indeed $\Delta x$ defining $V$ in eq (2,1) is actually $\Delta x = \Delta x(\Delta t)$ because of eqs (1,1) themselves. So the finite time range required for $\delta(n/m)$ to occur is nothing else but the finite time range required to propagate $\delta(n/V)$ and to come back, i.e. to allow exchanging the interaction carriers. Interaction force and propagation of force carriers through $V$ are therefore according to eq (2,1) two basic aspects of the interaction. In principle these carriers could be massive or massless, in which case one expects $(\hbar^2c^2)\delta[(n/\varepsilon)(n/V)]$, but they must have anyway boson character in order that the aforesaid forces affect the allowed states of the interaction partners while minimizing their exchange energy. It has been already demonstrated in [7] that as a consequence of eqs (1,1) integer or half-integer spin particles have a different link to the respective numbers of allowed states: an arbitrary number of the former can be found in a given quantum state, instead one particle only of the latter kind can be found in a given quantum state. Consider a multi-body interaction, where an arbitrary number of force carriers is to be expected: fermion carriers would require a corresponding number of quantum states with energy progressively increasing, whereas a unique ground state allows any number of boson carriers; as it will be shown below, the former case would be incompatible with a unique amount of energy to be transferred between all interacting particles and thus with at a minimum transfer energy. The corpuscles that mediate the fundamental forces of nature are indeed well known in literature as vector bosons, which also suggests the existence of a pertinent boson energy field. An interesting consequence of eq (2,1) comes from the chance of rewriting it as $(m/n\hbar)\Delta F_x = (\hbar/2)(n/V)$. Note that at left hand side appears the ratio $\hbar/m$ having physical dimensions of diffusion coefficient; write therefore $\Delta F_x = D^* n\hbar/2V$ with $D^* = n\hbar/m$. Moreover the fact that the physical dimensions of $F/D^*$ are $mass/(length \times time)$ suggests the position

$$\frac{\Delta F_x}{D^*} = \frac{\hbar}{2}\frac{n}{V} = \frac{du_\omega}{d\omega}, \quad D^* = \frac{n\hbar}{m}, \tag{2,5}$$

having at the moment mere formal meaning: if $\omega$ represents a frequency and $u_\omega$ an energy density, the physical dimensions of both sides are $energy \times time/volume$. So $\Delta F_x = D^* du_\omega/d\omega$ agrees with the idea that the force field is due to a diffusion-like flux of particles. This appears properly handling $du_\omega/d\omega$: indeed it is possible to write $du_\omega/d\omega = \omega V d\mathrm{C}/dx$ once more via dimensional requirement, being $\mathrm{C} = m/V$ or $\mathrm{C} = \varepsilon/c^2V$ the concentration of massive or massless carriers. Hence $\Delta F_x = \omega V D^* d\mathrm{C}/dx$ i.e. $\Delta F_x = -\omega V J_x$; the minus sign means of course an incoming flux of messenger particles if $J_x > 0$, yet both signs possible for $d\mathrm{C}$ reveal a complex fluctuation driven space distribution of interaction carriers randomly moving forwards and backwards between the real particles. This result is easily understood: in a volume $V$ where are delocalized interacting particles, boson carriers with density C are exchanged at frequency $\omega$ according to a Fick-like law that generates the force field $\Delta F_x$; the flow $J_x$ of vector bosons crosses an ideal plane perpendicular to the flow moving at rate $\omega\Delta x$ consistently with an energy $\Delta F_x \Delta x/V$ per unit volume. The diffusion coefficient of the bosons is quantized. In [12] has been demonstrated the quantum nature of the diffusion process and also the link between particle flow and concentration gradient driven Fick's law, as a consequence of which the statistical nature of the entropy also follows; this latter result is further inferred in the next section 7 in an independent way, see eqs (7,7). Eq (2,5) is immediately verifiable considering the cubic volume $V = \Delta x^3$ of space of eq (2,1) filled with photons. Let $\Delta x = \lambda$ be the longest wavelength allowed in $V$ to a steady electromagnetic wave with nodes at the opposite surfaces of the cube, whose side is therefore $\lambda/2$; thus $V = (\lambda/2)^3$, whereas $u_\omega = (\hbar\omega/2)/V$ is the corresponding zero point energy density of the oscillating electromagnetic field. So, with $\lambda = c/\nu$ one finds $du_\omega = 4n(\nu/c)^3\hbar d\omega$; since by definition $\hbar d\omega = h d\nu$, and thus $du_\omega = (2\pi)^{-1} du_\nu$, this result reads $du_\nu = (8\pi(\nu/c)^3 h d\nu)n$. In section 7 it will be shown that the number of states $n$ allowed to the photons trapped within the cube is given by $(\exp(h\nu/kT) - 1)^{-1}$, whence the well known result

$$\frac{du_\nu}{d\nu} = \frac{8\pi h\nu^3}{c^3}n, \qquad n = \frac{1}{\exp(h\nu/kT) - 1}. \tag{2,6}$$

It is interesting the fact that the black body law comes immediately from the same idea that shows the existence of messenger bosons mediating the interaction between particles. Clearly $\Delta x^3$ represents the black body volume.

Recall now that, in agreement with the arbitrariness of $n$, the ranges of eqs (1,1) can be regarded as arbitrary functions of time through $\Delta t$; read for instance $\Delta x = x - x_o$ with $x = x(\Delta t)$ and $x_o = x_o(\Delta t)$, being in general $x(\Delta t)$ and $x_o(\Delta t)$ different time functions. Of course no hypothesis is necessary about these functions, which are undefined and undefinable. Hence the size of $\Delta x$ is in general an arbitrary function of time itself, whereas the concept of derivative relies in the frame of eqs (1,1) only as mere ratio of uncertainty ranges. This idea generalizes the previous definition of force field $\Delta F_x = F_x^{conf} - F_x^{\infty}$. For instance $\Delta p_x/\Delta t$ takes the physical meaning of force field component $\Delta\varepsilon/\Delta x$ generated within $\Delta x$ by the change rate of all $p_x$ compatible with $\Delta p_x$ during $\Delta t$, whatever the physical reason affecting $p_x$ might be. Moreover, being the range sizes arbitrary, these ratios can even take the local physical meaning elucidated by the familiar notations $\Delta\varepsilon \to d\varepsilon$, $\Delta t \to dt$ and $\Delta p_x \to dp_x$. In other words, the local concept of derivative is here a particular case of that of ratio of arbitrarily sized uncertainty ranges. There is no contradiction between $\Delta\varepsilon/\Delta t$ and $d\varepsilon/dt$, which have both mere conceptual meaning and in fact are both indeterminable: the former because of the arbitrariness of the range boundaries, the latter because the local variables $p_x$ and $t$ around which shrink the respective ranges are arbitrary as well. The consistency of this position with the concept of covariancy has been concerned in [7]; in this paper and in [4] has been also shown that just the evanescent concept of distance required by the agnostic positions (1,2) in fact determines the non-locality of the quantum world. Exploit now eqs (1,1) to calculate in any reference system $R$ an arbitrary size change $d\Delta p_x$ of $\Delta p_x = p_x - p_{ox}$ as a function of that, $d\Delta t$, of the time uncertainty range $\Delta t$, assuming that $n$ remains constant during $d\Delta t$; hence during $d\Delta t$ the size of $\Delta x$ necessarily changes by an amount $d\Delta x$ as well. Of course this reasoning can be reversed: a force field arises within the space-time range $\Delta x$ because of its deformation $d\Delta x$ that in turn, because of eqs (1,1), requires the momentum range $\Delta p_x$ deformation as well [7]. Is evident the link of these ideas with the foundations of relativity. Differentiating eqs (1,1) and dividing by $d\Delta t$, one finds $d\Delta p_x/d\Delta t = -(n_x\hbar/\Delta x^2)(d\Delta x/d\Delta t)$. Of course, in $R'$ one would obtain $d\Delta p'_x/d\Delta t' = -(n'_x\hbar/\Delta x'^2)(d\Delta x'/d\Delta t')$; yet any consideration carried out about the unprimed equation can be identically carried out on the primed equation. In the present model there is no local value defined in $R$ that changes into a new value in $R'$, while any uncertainty range undefined in $R$ remains undefined in $R'$ too; so considering primed and unprimed range sizes means actually renaming a unique undefined range. The same holds of course for the ratios of any two ranges. If in particular $\Delta t = t - t_o$ is defined with constant $t_o$, since actually even this latter could be itself a function of $t$ without changing anything so far introduced, then one finds in any $R$

$$\frac{d\Delta p_x}{dt} = -\frac{n_x\hbar}{\Delta x^2}v'_x = F_x - F_{ox}, \qquad (2,7)$$

$$F_x = \dot{p}_x, \qquad F_{ox} = \dot{p}_{ox}, \qquad v'_x = \frac{d\Delta x}{d\Delta t}.$$

Having replaced any local distance $x$ with the uncertainty range $\Delta x$ including it, the local force $F_x$ is replaced by a corresponding range $\Delta F_x$ including local values of force. The notation $n_x$ emphasizes that the arbitrary number $n$ of states refers here to the $x$ components of $\Delta\mathbf{p}$, $\mathbf{v}'$, $\mathbf{F}$ and $\mathbf{F}_o$; of course are likewise definable $n_y$ and $n_z$ too. Moreover note that $v'_x$ is conceptually different from $v_x$ introduced in section 1: despite both have formally physical dimensions of velocity, the latter only is the actual average velocity of any real particle traveling through its delocalization range $\Delta x$ during $\Delta t$, the former is the deformation extent $d\Delta x$ of $\Delta x$ during the time increment $d\Delta t$. So $v_x$ is self-defined without need of further considerations, the physical meaning of $v'_x$ is instead strictly related to that of $F_x$ concurrently inferred. This distinction is inherent the character of the present theoretical model that, as previously remarked, concerns the uncertainty ranges of the phase space where any particle could be found rather than the particle itself; however the examples of the angular momentum and hydrogenlike energy levels have shown that working on the uncertainty ranges that define a physical property allows to gain information on the related behavior of the particle and on the given law itself. Eqs (2,7), reported here for clarity, have been early introduced in [7] and therein exploited to infer as a corollary in the particular case of constant $p_{ox}$ (i) the equivalence principle of general relativity, (ii) the coincidence of gravitational and inertial mass and then (iii) the Newton gravity law as a particular case; actually this law results to be the first order approximation of a more general equation allowing to calculate some interesting results of general relativity, for instance the perihelion precession of planets.

Also in the present model, therefore, the deformation of the space time quantum delocalization range entails the arising of a force as a corollary of eqs (1,1). In this paper we propose a further way of handling eq (2,7): in agreement with the purpose of this paper, i.e. to infer various forms of interaction between particles from a common principle, it is enough to rewrite eqs (2,7) in different ways and examine the respective consequences. The fine structure constant $\alpha$ enables $\hbar$ to be eliminated from eqs (2,7), which read in c.g.s. units for simplicity

$$F_x - F_{ox} = \pm\frac{e'e}{\Delta x^2}, \qquad e' = \pm\frac{n_x v'_x}{\alpha c}e. \qquad (2,8)$$

Here $\Delta F_x = F_x - F_{ox}$ is the force field between two charges $e$ and $e'$ interacting through their linear charge densities $e/\Delta x$ and $e'/\Delta x$: i.e. even the electric interaction force relies on a physical basis similar to that of the gravity force. The double sign accounts for both chances that $\Delta x$ expands or shrinks at deformation rate $\pm v'_x$, which is a decisive parameter to express the respective states of charge. If $v'_x = 0$ then $e' = 0$, i.e. it corresponds to a chargeless particle; of course the related electric force is null, i.e. $F_x = F_{ox}$ accounts for

other forces possibly acting on the particle, for instance the gravity; this case, concerned in [7] to emphasize the link between quantum theory and relativity, is skipped here. Moreover holds an obvious boundary condition on $n_x$, i.e. a value of $n_x$ must necessarily exist such that $e' = \pm e$. Be $n'$ this value such that by definition $n'v'_x = \alpha c$; being $n'_x$ arbitrary integer and $v'_x$ arbitrary as well, this position is certainly possible. Then

$$e' = \pm(n_x/n')e. \qquad (2,9)$$

Here the double sign agrees with the chances allowed for $e$ depending on the expansion or contraction of $\Delta x$. It is reasonable to assume that $n' = 3$; considering also the deformation rates $\pm v'_y$ and $\pm v'_z$ of $\Delta y$ and $\Delta z$ defined likewise to $v'_x$, the number of states is actually counted as $n' = n_x + n_y + n_z$ with ground values $n_x = n_y = n_z = 1$, while being $1 \leq n_{xi} \leq n'$ depending on the number of respective force components $F_{xi} - F_{oxi}$ actively contributing to $n'$. Consider first the $x$-component, eq (2,7), only. If $n_x = n' = 3$, then $e'_{(3)} = \pm e$ corresponds to electron and proton charges; $F_x - F_{ox}$ of eq (2,8) is the related Coulomb force component. The case $n_x = 2$ yields $e'_{(2)} = \pm(2/3)e$, whereas $n_x = 1$ yields $e'_{(1)} = \pm(1/3)e$; accordingly $F_x - F_{ox}$ must have a characteristic physical meaning that will be concerned in section 5. The same result would be obtained considering the $y$ or $z$ components corresponding to eq (2,7). Hence fractional charges are in principle to be expected in nature. It is easy guess how many particles with fractional charges, the well known quarks, are to be expected. Consider the four chances corresponding to the double signs of $e'_{(1)}$ and $e'_{(2)}$ and the three deformation rates $v'_x$, $v'_y$ and $v'_z$; the previous discussion has exemplified the link of $e'$ with $v'_x$ only, yet an analogous reasoning holds of course also for $v'_y$ and $v'_z$. Instead three different situations are in general compatible with $e'_{(1)}$ and $e'_{(2)}$ when (i) $v'_x \neq 0$ only, (ii) $v'_x \neq 0$ and $v'_y \neq 0$ only, (iii) $v'_x \neq 0$ and $v'_y \neq 0$ along with $v'_z \neq 0$ too. Since $n_x$, $n_y$, $n_z$ are independent and arbitrary, one could replace the second eq (2,8) for instance with $\pm n_x v'_x/\alpha c \pm n_y v'_y/\alpha c$, obtaining thus $\pm(n_x \pm n_y)/n'$ as done to infer eq (2,9); then one could combine $n_x$ and $n_y$ in order to obtain again ratios having the same values $\pm 1/3$ and $\pm 2/3$ previously found, but involving now both $v'_x$ and $v'_y$ instead of $v'_x$ only. Analogous considerations hold for the case (iii) that involves also $v'_z$. In (i) the vector $\mathbf{F} - \mathbf{F}_o$ is oriented along one of the axes, here the $x$-axis, in (ii) it lies on one coordinate plane, here the $x-y$ plane; the components of $\mathbf{F} - \mathbf{F}_o$ arbitrarily oriented correspond in general to (iii), whereas a null vector is instead related to $\mathbf{v}' = 0$ i.e. $e' = 0$. Anyway, whatever the linear combination of $v'_x$, $v'_y$ and $v'_z$ might be, it is reasonable to think that these ways of inferring $e'_{(1)}$ and $e'_{(2)}$ are physically different from that involving $v'_x$ only; otherwise stated, to the various ways of finding a given kind of charge correspond different particles. With the aforesaid 3 chances for each sign of $e'_{(1)}$ and $e'_{(2)}$ we expect therefore a variety of 12 particles in total. Since this number is reasonably expected to include particles and antiparticles, a sensible conclusion is that we should have 6 quarks and 6 antiquarks: for instance, to the $(n_x - n_y)e/n'$ quark charge corresponds the $(n_y - n_x)e/n'$ antiquark charge. Now the first problem is how to sort the charge signs between particles and antiparticles; in principle one could think the former as the ones having $e'_{(1)} = +e/3$ and $e'_{(2)} = +2e/3$, the latter as the ones with both negative signs. In this way, however, considering all values of charges compatible with $n$ from 1 to $n'$, one should conclude that in nature the mere charge signs discriminate particles and antiparticles. Since this is not the case, it is more sensible to expect that $e'_{(1)} = -e/3$ and $e'_{(2)} = +2e/3$, for instance, identify quarks whereas the inverted signs identify the corresponding antiquarks: likewise exist as a particular case particles with either integer charge whose antiparticles have either opposite charge.

Moreover if two charge states $-e/3$ and $+2e/3$ are consistent with six particles physically distinguishable, then each quark requires three chances of a new property, which is indeed well known and usually called color charge: each quark can exist in three quantum states, i.e. it can take three different color states. Being the quarks characterized by several quantum numbers, this way of justifying their number does not mean a specific color uniquely assigned to each one of them; rather it means introducing a number of internal freedom degrees of color that make two fractional charges consistent with six distinguishable particles. Anyway, since also anti-quarks exist for which hold the same considerations, three anti-colors must exist too.

Eventually, let us calculate how many kinds of bosons are necessary to describe the interactions between quarks via boson exchanges able to modify their initial color states. Consider for instance a charmed meson identically symbolized as $\{c\bar{c}\}$ or $\{\bar{c}c\}$ and assume that each boson mediating the quark interaction is specifically entrusted with changing one couple color-anticolor only: let for instance the exchange of one boson turn $r$ into $\bar{r}$ and vice-versa. The mesons $\{c\bar{c}\}$ and $\{\bar{c}c\}$, formally obtained by quark-antiquark and antiquark-quark exchanges, are clearly identical and indistinguishable. Imagine therefore of turning all colors of $c$, whatever they might be, into the corresponding anticolors of $\bar{c}$, whose anticolors are at once turned into the respective colors. How many exchanges of color states into the respective anticolor states are consistent with the identity of $c\bar{c}$ and $\bar{c}c$? Given two objects, $c$ and $\bar{c}$, each one of which can be found in three quantum states, the three colors, the trivial answer is $2^3$; eight exchanges are not only enough to turn all color states of $c$ into the respective anticolor states, which means by definition obtaining $\bar{c}$ from $c$, but also purposely necessary, as each single exchange generates a new quantum configuration of states physically distinguishable from that previously existing. Since a total of eight color-anticolor exchanges are required to account for as many different configurations, eight is also the number of different bosons required to make the aforesaid couple of identical mesons effectively indistinguishable. These different chances

of interaction, each one characterized by its own specific energy, should be someway correlated to and described by the existence of as many such particles representing the possible exchanges, i.e. just eight vector bosons. Also these particles are well known and usually called gluons. Is this reasoning extensible also to three-quark particles like neutron or proton? The quark-gluon plasma of these latter is necessarily more complex than that of the mesons, so the question arises whether the 8 gluons previously introduced are enough to describe also such three quark systems. Consider the proton $uud$ and the antiproton $\overline{uu}\overline{d}$. The conversion $uu \to \overline{uu}$ has been already described. As concerns $d \to \bar{d}$, still holds an analogous reasoning: a specific kind of gluon undertakes to change one color into the anticolor, another kind of gluon does the same with another color and so on. However the kind of gluon exchanges that turns red into antired of the quark $u$ cannot differ from that acting similarly on the quark $d$: it would mean that each gluon "recognizes" its own quark on which to act, i.e. we should admit that different $\delta(n/m)$ require different $\delta(n/V)$ depending on the respective $m$. But nothing in the previous eq (2,1) allows this conclusion, rather it seems true exactly the contrary because $\Delta x$ defining $V$ has nothing to do with $m$ therein delocalized: indeed, as above stated, the indistinguishability of identical particles is just due to the possibility that any particle could be found in a given range. So it is more reasonable to think that each kind of gluon exchange affects a specific color, not the color of specific quark only; otherwise stated, the total number of gluons in a nucleon is greater than that in a meson without necessarily compelling a new kind of gluons, i.e. any gluon in the tree-quark system turns one specific color regardless of whether that color is of a quark $d$ or $u$. This way of thinking allows that the gluons transmit the interaction between different quarks modifying their $\delta(n/m)$, i.e. their color quantum states, regardless of $m$. So, when counting the number of different gluons that allow the three-quark particle/ antiparticle exchanges the result is the same as that previously computed.

These short remarks are enough for the purposes of the present paper; further considerations on other properties like strangeness, isospin and so on, whose conservation rules are necessary for instance to describe the decay of complex particles consisting of two or three quarks, are well known and thus omitted here for brevity. The remainder of the paper aims to describe the fundamental interactions by implementing the ideas hitherto exposed.

## 3 The quantum interactions

Divide all sides of eqs (1,1) by $e^2\Delta x$ and recall that in general $\Delta p_x = (v_x/c^2)\Delta\varepsilon$. An intuitive hint to this equation, already concerned in [7] and important also for the present purposes, is quickly reported here for completeness. Let in an arbitrary reference system $R$ a photon travel at speed $c$ through an arbitrary delocalization range $\Delta x^{(c)}$, so that eqs (1,1) read $\Delta x^{(c)}\Delta p_x^{(c)} = n^{(c)}\hbar = \Delta t^{(c)}\Delta\varepsilon^{(c)}$; the superscripts emphasize that the ranges are sized in order to fulfill this delocalization condition during an appropriate time range $\Delta t^{(c)}$. Then $c\Delta p_x^{(c)} = \Delta\varepsilon^{(c)}$. To find how scale the sizes of the momentum and energy ranges with respect to $\Delta p_x^{(c)}$ and $\Delta\varepsilon^{(c)}$ in the case of a massive particle traveling at slower rate $v_x < c$ through $\Delta x^{(c)}$, write $\Delta x^{(c)}\Delta p_x^{(v)} = n^{(v)}\hbar = \Delta t^{(c)}\Delta\varepsilon^{(v)}$. Since neither $v_x$ nor $c$ appear explicitly in this equation, it is also possible to write $n^{(v)}\hbar = \Delta t^{(c)}\Delta\varepsilon^{(c)} = \Delta t^{(v)}\Delta\varepsilon^{(v)}$; this is indeed true if $\Delta t^{(c)}$ and $\Delta\varepsilon^{(c)}$ scale like $\Delta t^{(v)} = (c/v_x)\Delta t^{(c)}$, as it is reasonable, and $\Delta\varepsilon^{(v)} = (v_x/c)\Delta\varepsilon^{(c)}$. Replacing these positions in the former equation yields $\Delta x^{(c)}\Delta p_x^{(v)} = \Delta t^{(c)}(v_x/c)\Delta\varepsilon^{(c)}$ whence $c\Delta p_x^{(v)} = (v_x/c)\Delta\varepsilon^{(c)}$. Actually the superscripts can be omitted because they do not identify particular range sizes; both $\Delta p_x^{(v)}$ and $\Delta\varepsilon^{(c)}$ are indeed arbitrary like $v_x$ itself. The superscripts are also irrelevant as concerns the functional relationship between the local values of the respective variables, which reads $p_x = (v_x/c^2)\varepsilon$ regardless of how the respective uncertainty ranges are defined. Note that $p_x$ and $\varepsilon$, exactly determined in classical physics and in relativity, are instead here random values within the respective uncertainty ranges. Also note that an identical reasoning in $R'$ solidal with the particle would yield $p'_x = (v'_x/c^2)\varepsilon'$: this is therefore a quantum expression relativistically invariant. This kind of reasoning has been carried out in [7] to show the connection between quantum mechanics and relativity. Now instead consider for the next discussion the following equations directly inferred from eqs (1,1)

$$\frac{n\hbar v_x}{\Delta x} = \Delta\varepsilon, \qquad v_x = \frac{\Delta x}{\Delta t}, \qquad v_x \le c. \tag{3,1}$$

The last position does not merely emphasize a feature in principle expected for any velocity, it takes a special relevance in the present context. Being $\Delta\varepsilon$ and $\Delta x$ arbitrary, one could write $\Delta p_x = \Delta\varepsilon^o v_x^o/c^2$ too, with $v_x^o$ and $\Delta\varepsilon^o$ still fulfilling the given $\Delta p_x$. The total arbitrariness of the range sizes plays a key role in the following reasoning based on $v_x\Delta\varepsilon = v_x^o\Delta\varepsilon^o$: if $v_x = c$, then necessarily $v_x^o < c$ and $\Delta\varepsilon^o > \Delta\varepsilon$. Examine step by step this point writing identically eq (3,1) as follows

$$\frac{e^2}{\Delta x} = \frac{\alpha}{n}\frac{v_x^o}{c}\Delta\varepsilon^o, \qquad \frac{v_x v_x^o}{c^2} = \frac{\Delta\varepsilon}{\Delta\varepsilon^o}, \qquad \Delta\varepsilon \le \Delta\varepsilon^o. \tag{3,2}$$

The last position emphasizes that both chances $\Delta\varepsilon^o = \Delta\varepsilon$ and $\Delta\varepsilon^o \neq \Delta\varepsilon$ are equally possible. If $\Delta\varepsilon = \Delta\varepsilon^o$, then $v_x = v_x^o$ compels concluding $v_x = v_x^o = c$ only; so eqs (2,7) and (3,2) yield $e^2/\Delta x = \chi\Delta\varepsilon$, being $\chi = \alpha/n$ a proportionality factor. This means correlating the potential energy $e^2/\Delta x$ of two electric charges to $\Delta\varepsilon$, introduced through $\Delta p_x$ and thus having the meaning of kinetic energy range. On the one hand $\Delta\varepsilon^o \neq \Delta\varepsilon$ requires different $v_x^o$ and $v_x$, thus both velocities or at least either of them smaller than $c$, whence the inequality; on the other hand, relating the physical meaning of the velocities hitherto introduced to that of the boson carriers that

mediate the interaction force between particles: $v_x^o = c$ requires massless bosons, $v_x^o < c$ massive bosons. Therefore the arbitrariness of $\Delta\varepsilon$ and $\Delta\varepsilon^o$ justifies the conclusion that either chance of range sizes prospects different results for eqs (3,2) and (3,1), despite their common origin from eqs (1,1). Two questions arise at this point: (i) whether these equations describe two different interactions or two different appearances of a unique interaction, (ii) whether or not it is possible to infer from both equations a relationship like $e^2/\Delta x = \chi\Delta\varepsilon$ despite their formal difference. The answers rely on the fact that in eq (3,2) appears explicitly the Coulomb charge $e$ inherent the definition of $\alpha$, in eq (3,1) it does not necessarily hold; nothing compels assuming that even the energy $n\hbar v_x/\Delta x$ is by necessity referable to a Coulomb energy.

If $n\hbar v_x/\Delta x$ does, then the common origin of these equations from eqs (1,1) is a good reason to expect that the chances of massive or massless vector bosons are merely two different ways of manifesting a unique kind of interaction; rewriting the inequality as $\Delta\varepsilon^o = \Delta\varepsilon + \delta\varepsilon$, with $\delta\varepsilon \geq 0$ of course arbitrary like $\Delta\varepsilon$ and $\Delta\varepsilon^o$, both chances are in principle acceptable depending on the amount of energy at which the interaction occurs. In other words $\delta\varepsilon > 0$ is an additional energy range motivated by the arbitrariness of $\Delta\varepsilon$, which indeed admits introducing also $\Delta\varepsilon^o$ too, and justifying the presence of massive vector bosons. By consequence the chance of finding a unique link like $e^2/\Delta x = \chi\Delta\varepsilon$ between potential and kinetic energies is to be reasonably expected; so, fixing an arbitrary $\Delta\varepsilon$ allows assessing via $\chi$ the relative strengths of both interactions at comparable values of $\Delta\varepsilon$ and respective characteristic lengths $\Delta x$. The physical consequences of this reasoning are exposed in section 4.

If instead $n\hbar v_x/\Delta x$ is an energy not referable to that between integer charges, in fact nothing hinders thinking that it is directly related to the aforesaid fractional charges; according to eq (2,8), $v_x = \Delta x/\Delta t$ is physically different from $v'_x = d\Delta x/d\Delta t$. Then eq (3,1) describes an interaction prospectively different from that of eq (3,2); so the former equation must be considered regardless of the latter to check what kind of physical information follows from the considerations of section 2. Also the consequences inferred from these equations are expectedly different; in particular the link $\chi$ between potential and kinetic energies should be reasonably different in either case just mentioned. In other words, $\chi$ can be compared for similar $e^2/\Delta x$ and $\Delta\varepsilon$ to characterize the relative strengths of the various kinds of interactions. The physical consequences of this reasoning are exposed in section 5.

These are the key ideas to be further highlighted below. The dual way of elaborating a unique principle, the statistical formulation of quantum uncertainty, has an intrinsic physical meaning coherent with the purposes of the present paper, i.e. to demonstrate that kinds of interaction apparently different are in fact consequences of a unique principle. In other words, eqs (3,2) and (3,1) are the starting point to distinguish two cases, which will be discussed separately under the only conceptual constraint of being mutually self-consistent. The following sections 4 and 5 aim to outline the respective ways to link the potential and kinetic energies.

## 4 The interaction according to eqs (3,1) and (3,2)

The following discussion concerns the ways to reduce the eqs (3,1) and (3,2), regarded together, to the form $e^2/\Delta x = \chi\Delta\varepsilon$ in both cases $\delta\varepsilon = 0$ and $\delta\varepsilon > 0$. Consider first $\delta\varepsilon = 0$, which requires $v_x^o = v_x = c$ and thus massless boson carriers. So the unique result possible is

$$\frac{e^2}{\Delta x} = \chi_{em}\Delta\varepsilon, \qquad \chi_{em} = \frac{\alpha}{n}. \tag{4,1}$$

Here $\alpha/n$ emphasizes the electromagnetic interaction in analogy with eq (1,4).

The further chance $\delta\varepsilon > 0$ requiring the condition $v_x^o < c$ prospects instead the presence of massive boson carriers; thus $\delta\varepsilon > 0$, related to the formation of massive carriers, represents reasonably the energy gap with respect to the former case of eq (4,1) involving massless carriers only. While heavy vector bosons are the physical consequence of the concurring inequalities $v_x^o < c$ and $\delta\varepsilon > 0$, the arbitrariness of $v_x^o$ prevents the possibility of deciding a priori either chance for $\delta\varepsilon$ and compels the conclusion that a unique kind of interaction is actually compatible with both chances. It will be shown that the interaction energy related to the possible size of $\Delta x$ discriminates either chance. Despite both chances are incorporated into a unique conceptual frame, further considerations are necessary in this case. Write the first eq (3,2) as follows

$$\frac{e^2}{\Delta x} = \frac{\alpha^2}{n^2}\frac{\Delta\varepsilon^o}{q^o}, \qquad q^o = \frac{e^2}{n\hbar v_x^o}, \qquad v_x < c. \tag{4,2}$$

Since eqs (3,2) require $\Delta\varepsilon^o/q^o = (c/v_x)(n/\alpha)\Delta\varepsilon$, the obvious inequality

$$(n/\alpha)^2 > v_x/c \tag{4,3}$$

yields $\Delta\varepsilon^o/q^o > (\alpha/n)\Delta\varepsilon$. Hence a value $q_w > q^o$ certainly exists such that

$$\Delta\varepsilon^o/q_w = (\alpha/n)\Delta\varepsilon. \tag{4,4}$$

Replacing this result into the first eq (4,2), one finds

$$\frac{e^2}{\Delta x_w} = \chi_w\Delta\varepsilon, \qquad \chi_w = \left(\frac{\alpha}{n}\right)^3, \qquad \Delta x_w = \frac{q_w}{q^o}\Delta x. \tag{4,5}$$

The first equation is formally analogous to eq (4,1) a scale factor $q_w/q^o$ for $\Delta x$ apart, while $\alpha/n$ is replaced by the much smaller quantity $(\alpha/n)^3$; hold however for $\chi_w$ considerations analogous to that previously carried out for $\chi_{em}$, i.e. it links kinetics and potential energies. The explicit form of the inequality (4,3) reads $(n\hbar c)^2 > e^4(v_x/c)$, so that $(n\hbar c/\Delta x)^2 > (e^2/\Delta x)^2(v_x/c)$ and thus $(c\Delta p_x)^2 > (e^2/\Delta x)^2(c\Delta\varepsilon/v_x^o\Delta\varepsilon^o)$; as $c\Delta p_x = \Delta\varepsilon^o v_x^o/c$, i.e. $c\Delta p_x = (q_w v_x^o\alpha/nc)\Delta\varepsilon$ according to eq (4,4), the inequality (4,3) reads

$$(\zeta\Delta\varepsilon)^3 > (e^2/\Delta x)^2\Delta\varepsilon, \qquad \zeta = \frac{q_w v_x^o\alpha}{nc} = w\left(\frac{n}{\alpha}\right)^2. \tag{4,6}$$

Hence an energy $\varepsilon_0 > 0$ certainly exists such that

$$\zeta^3\Delta\varepsilon^3 - (e^2/\Delta x)^2\Delta\varepsilon - \varepsilon_0^3 = 0. \quad (4,7)$$

Regarding $\zeta$ as a constant through an appropriate choice of $q_w$, not yet specified and here accordingly defined, let us solve the eq (4,7) in order to introduce three real sizes $\Delta\varepsilon_j$, $j = 1, 2, 3$. Note that this does not mean assigning definite values to the size of $\Delta\varepsilon$, which remains indeed arbitrary and unknown like any uncertainty range because of $\Delta x$; solving eq (4,7) means examining the physical information consistent with some particular range sizes that fulfil the inequality (4,6). One finds

$$\varepsilon_0 = \left(\frac{2\sqrt{3}}{9}\right)^{1/3}\zeta^{-1/2}\frac{e^2}{\Delta x}, \qquad \Delta\varepsilon_1 = \frac{2}{\zeta^{3/2}\sqrt{3}}\frac{e^2}{\Delta x}, \quad (4,8)$$

$$\Delta\varepsilon_{2,3} = \Delta\varepsilon_2 = \Delta\varepsilon_3 = -\frac{1}{\zeta^{3/2}\sqrt{3}}\frac{e^2}{\Delta x}.$$

The former equation is the condition to make null the imaginary parts of the roots $\Delta\varepsilon_2$ and $\Delta\varepsilon_3$ that, as emphasized by the last equation, result by consequence coincident. As expected, all quantities expressed here as a function of $\Delta x$ are in fact arbitrary like this latter. The constant $\zeta$ can be eliminated from the equations; so

$$\frac{\Delta\varepsilon_1}{\varepsilon_0} = \frac{2}{\zeta\sqrt{3}}\left(\frac{9}{2\sqrt{3}}\right)^{1/3}, \qquad \frac{\Delta\varepsilon_1}{\varepsilon_0^3} = \frac{3}{(e^2/\Delta x)^2}, \quad (4,9)$$

$$\frac{e^2}{\Delta x} = \varepsilon_0\sqrt{3\frac{\varepsilon_0}{\Delta\varepsilon_1}}.$$

It is interesting to rewrite eq (4,7) as $(\zeta^3\Delta\varepsilon^2 - (e^2/\Delta x)^2)\Delta\varepsilon = \varepsilon_0^3$, which yields

$$\Delta t = n\hbar\zeta^3\frac{(e^2/\Delta x)^2}{\varepsilon_0^3}\left[\left(\frac{\Delta\varepsilon}{e^2/\Delta x}\right)^2 - \zeta^{-3}\right].$$

In this way $\Delta\varepsilon^3$ splits into a multiplicative factor $\Delta\varepsilon$, related to $\Delta t$ through eqs (1,1), times a factor merging together $\Delta\varepsilon^2$ and $(e^2/\Delta x)^2$. Let us specify in particular $\Delta x$ as $\Delta x_w$ of eq (4,5); owing to the last eq (4,9), one finds then

$$\Delta t_w = \frac{3n\hbar\zeta^3}{\Delta\varepsilon_{1w}}\left[\left(\frac{n^3}{\alpha^3}\right)^2 - \zeta^{-3}\right], \quad \Delta\varepsilon_{1w} = \frac{2}{\zeta^{3/2}\sqrt{3}}\frac{e^2}{\Delta x_w}. \quad (4,10)$$

Despite $\Delta x$ is unknown and arbitrary by definition, when it is specified as the range $\Delta x_w$ purposely pertinent to eq (4,5) the former equation takes the form $\Delta t \propto (n/\alpha)^6$ plus a term $\tau = 3n\hbar/\Delta\varepsilon_{1w}$. If $\Delta\varepsilon_{1w}$ and $n$ are large enough so that $\tau << (n/\alpha)^6$, then $\Delta t_w$ and the factor $\chi_w$ linking $e^2/\Delta x_w$ and $\Delta\varepsilon$ of eq (4,5) fulfill the well known condition

$$\Delta t_w \propto \chi_w^{-2}.$$

Note now that

$$\Delta\varepsilon_1 + \Delta\varepsilon_2 + \Delta\varepsilon_3 = 0 \quad (4,11)$$

and that eq (4,7) is directly related to $v_x^o/c < 1$ because it comes from the inequalities (4,2) and (4,3). Moreover each energy range by definition introduces its own random value of energy; this suggests that are related to eq (4,5) three characteristic energies, i.e. three corresponding massive particles, whose energies are by definition included within the uncertainty ranges of eqs (4,11).

Consider in general three energy ranges $\Delta\varepsilon_j = \varepsilon_j' - \varepsilon_j''$, being $j = 1..3$, of course with both $\varepsilon_j'$ and $\varepsilon_j''$ arbitrary and unknown; define then the energies $\eta_j$ included within them as $\eta_j = (\varepsilon_j' + \varepsilon_j'')/2$, i.e. as average values of the respective boundary values. It is immediate to realize that the condition $\sum\Delta\varepsilon_j = 0$ is compatible with $\sum\eta_j \neq 0$; indeed $\sum(\varepsilon_j' - \varepsilon_j'')/2 = 0$ reads identically $\sum(\varepsilon_j' + \varepsilon_j'')/2 - \sum\varepsilon_j'' = 0$, whence in general $\sum\eta_j = \sum\varepsilon_j'' \neq 0$. Repeat this reasoning regarding $\eta_j$ as the average values of the specific energy ranges of eq (4,11). The fact that $\eta_{tot} = \eta_1 + \eta_2 + \eta_3 \neq 0$ agrees with the idea of interaction energy; indeed no constrain could be definable for three independent free particles. On the one hand the chance of replacing any quantum range with its average, as done here for $\Delta\varepsilon_j$ and $\eta_j$, has a general valence because the range sizes are arbitrary, undefined and undefinable like the average value inferred from their boundaries. Since any value allowed to the former is also allowed to the latter, considering $\eta_j$ instead of $\Delta\varepsilon_j$ does not exclude the point of view of eqs (1,1): replacing an arbitrary value with another arbitrary value corresponds to replace $n$ with $n'$, which is however immaterial because both symbolize sets of integer values and not specific values. On the other hand the ranges (4,11), regarded all together, fulfill globally the energy conservation regardless of whether $\Delta\varepsilon_j \neq 0$ or $\Delta\varepsilon_j = 0$; as just shown, however, the same does not necessarily hold for $\eta_{tot}$. To make also this latter compliant with the eq (4,11), let us assume therefore that $\eta_{tot}$ has a finite lifetime of the order of $\hbar/\eta_{tot}$. Let $\Delta t_w$ be this lifetime. In agreement with eq (4,10), during $\Delta t_w$ the sum $\sum\Delta\varepsilon_j$ is still globally null likewise as before and after their actual transient appearance; in this way the massive particles concerned by the respective energy ranges are jointly involved as concurrent physical properties inherent eq (4,5) and thus the present kind of interaction. The physics of the weak interactions is well known. Here, as a significant check of these ideas, we propose a simple energy balance to infer the energies $\eta_j$ and thus $\eta_{tot}$ exploiting just the requirement that the $\eta_j$ must be regarded all together.

A possible interpretation of the equal sizes and negative signs of $\Delta\varepsilon_2$ and $\Delta\varepsilon_3$, despite in the present model the ranges are always introduced positive by definition, is that their sum with $\Delta\varepsilon_1$ equal to zero requires interacting particles; as explained in section 2, no relationship would be possible by definition for free particles. Let two of them, say $\eta_2$ and $\eta_3$,

interact in order to release the energy necessary to form also $\eta_1$. The fact that $\eta_2 = \eta_3$ because of $\Delta\varepsilon_2 = \Delta\varepsilon_3$ means that their interaction occurs regarding identically either of them in the field of the other one; together, therefore, these particles provide the energy necessary to allow the kind of interaction here concerned. The simplest hypothesis is that the particles $\eta_2$ and $\eta_3$ have charges of opposite signs whereas $\eta_1$ is neutral, thus fulfilling the global charge conservation before, during and after their lifetime; if so, the energy gain of Coulomb energy at an appropriate interaction distance justifies also the neutral particle $\eta_1$. In this way the model allows the existence of three range sizes whose finite lifetimes agree with the finite values of the respective heavy bosons $\eta_j$. This conclusion is summarized as follows

$$\eta_{tot} = \eta_1 + (\eta_2 + \eta_3), \qquad \eta_2 = \eta_3.$$

The second equation emphasizes that actually $\eta_2$ and $\eta_3$ form a Coulomb system of charges, whose energy transient uniquely defined likewise $\varepsilon_{el}$ of eq (1,4) characterizes the present kind of interaction. This idea suggests to estimate $\eta_{tot}$ just computing the energy levels of the system of charges $\eta_2$ and $\eta_3$ by analogy with that of a hydrogenlike atom. Exploit for simplicity the previous non-relativistic equations (1,3) and (1,4); owing to the generality of these equations, there is no reason to exclude that analogous considerations hold at least approximately also here putting of course the charge $Z = 1$ and describing the system of charged bosons $\eta_2$ and $\eta_3$ as due to $-\varepsilon_{el} = \pi e^2/n\lambda$. It is necessary to take into account however that now also the neutral particle $\eta_1$ contributes to $\eta_{tot}$, in agreement with the idea of regarding the particles all together. Guess first according to eqs (4,8) that the mass of $\eta_1$ should have the same order of magnitude of $\eta_2$ and $\eta_3$, so that $\eta_{tot} \approx 3\eta_2$; the chance of identifying $\eta_{tot}$ with $-\varepsilon_{el}$ is consistent with this idea simply putting

$$\eta_{tot} = \pi\eta_2, \quad \eta_2 = \eta_3 = e^2/n\lambda, \quad \eta_1 = (\pi - 2)e^2/n\lambda. \quad (4,12)$$

In other words, eq (1,4) suggests that the expected coefficient $\approx 3$ must be actually regarded as $\pi$. Despite the non-relativistic reasoning, these conclusions are correct because confirmed by the experience. The experimental masses of the $W^{\pm}$ and $Z^0$ vector bosons are $m_{W^{\pm}} = 80.39$ GeV and $m_{Z^0} = 91.19$ GeV respectively, for a total mass of $m_{tot} = 251.97$ GeV; in effect

$$m_{tot} = 3.134\, m_{W^{\pm}} \qquad m_{Z^0} = 1.134\, m_{W^{\pm}}$$

are compatible with the values expected for $\pi$ and $\pi - 2$. Trivial considerations show that the reduced Compton lengths $\bar{\lambda}$ of the vector bosons consistent with $e^2/n\lambda$ are $\bar{\lambda}_{\eta_1} = n\lambda/((\pi - 2)\alpha)$ and $\bar{\lambda}_{\eta_2} = \bar{\lambda}_{\eta_3} = n\lambda/\alpha$, having introduced explicitly the masses $m_j = \eta_j/c^2$. These results are confirmed considering the zero point energy $\Delta p_j^2/2m_j$ of the vector bosons $\eta_j$, where $\Delta p_j = p_2 - p_1$ is the gap between its momentum $p_2$ after confinement within a given delocalization range $\Delta x_w$ and its initial momentum $p_1$ in an ideal unconfined state; hence the corresponding energy gap after confinement within $\Delta x_w$ resulting from the $x$, $y$ and $z$ components is $\Delta p_j^2/2m_j = 3(n^2\hbar^2/2m_j\Delta x_w^2)$. Assume now that the confinement energy $\Delta p_j^2/2m_j$ is just the energy $\eta_j = m_j c^2$ itself that determines the space-time scale of this kind of interaction, i.e.

$$\eta_j = \frac{3}{2}\frac{c^2\Delta p_j^2}{\eta_j}; \quad (4,13)$$

then $\Delta x_w = (3/2)^{1/2} n\hbar c/\eta_j$, i.e. for $\eta_2$ and $\eta_3$

$$\Delta x_w = (3/2)^{1/2} n^2 \lambda(\hbar c/e^2). \quad (4,14)$$

For $n = 1$ therefore $\Delta x_w$ coincides with $\bar{\lambda}_{\eta_2} = \bar{\lambda}_{\eta_3}$ a trivial numerical factor $\sqrt{3/2}$ apart; an identical conclusion holds of course for $\eta_1$ too, the numerical factor $(\pi - 2)\sqrt{3/2}$ apart. This confirms the assumed link between delocalization extent and energy of the force carriers, which allows identifying $\eta_{tot} = -\varepsilon_{el}$ in agreement with eq (1,3).

Put first $n = 1$ in eqs (4,12). The value of $\lambda$ corresponding to the energies of the particles $\eta_2$ and $\eta_3$ is $\lambda = 1.79\times10^{-20}$ m, so that $\bar{\lambda}_{\eta_2} = \bar{\lambda}_{\eta_3} = 2.45 \times 10^{-18}$ m and $\bar{\lambda}_{\eta_1} = 2.15 \times 10^{-18}$ m; the characteristic range $\Delta x_w$ of interaction is thus of the order of $10^{-18}$ m. Since the classical proton radius $r_p = e^2/m_p c^2$ is about 0.8768 fm according to recent measurements [13], the above energies concern a sub-nuclear scale interaction; vice-versa, one could estimate the correct scale of energy of the vector bosons requiring an interaction that occurs at the sub-nuclear extent at which one calculates $\chi_w = \alpha^3 = 3.9 \times 10^{-7}$.

So far we have considered $n = 1$. What however about $n > 1$? First of all, $\Delta x_w$ becomes $n$ times larger than the aforesaid Compton lengths of $\eta_j$; this deviation means a longer range allowed to the interaction. Moreover, according to eqs (4,12) $\eta_{tot} \to 0$ for $n \to \infty$; at this limit the aforesaid space scale of interaction is inconsistent with the corresponding energies of massive boson carriers, which therefore should expectedly require an appropriate threshold energy to be activated. For $n \to \infty$ is thus allowed the less energy expensive and longer range interaction with $\delta\varepsilon = 0$ only, in agreement with the initial idea that $\delta\varepsilon \neq 0$ is related to the boson masses. This conclusion is intuitively clear, but what about the energy threshold? According to the eqs (4,12) the energies $\eta_1$, $\eta_2$ and $\eta_3$ downscale with $n$, whereas according to eq (4,14) $\Delta x_w$ upscales with $n^2$; so the lower threshold for the existence of massive bosons, i.e. for the validity of these equations themselves, concerns $n$ of $\eta_{tot}^{(n)} = -\varepsilon_{el}(Z = 1, n) = (\pi/n)e^2/\lambda$: it is required that the interaction distance of the hydrogenlike system of charges enable the energy to create vector bosons. The inequality $\eta_{tot}^{(n)} > e^2/\lambda$, which holds for $n \leq 3$, ensures that, whatever the masses $\eta_2^{(n)}$ and $\eta_3^{(n)}$ might be, the energy gain due to their Coulomb interaction accounts not only for the energy $e^2/\lambda$ of the system of charged particles themselves but also for the surplus required by the neutral particle $\eta_1^{(n)}$. Clearly the threshold corresponds to the

value $\eta^{(3)}_{tot} = (\pi/3)e^2/\lambda$, i.e. about 81 GeV; the corresponding Compton lengths of the bosons are $\lambda^{(3)}_{\eta_2} = \lambda^{(3)}_{\eta_3} = 3\lambda/\alpha$ and $\lambda^{(3)}_{\eta_1} = 3\lambda/((\pi-2)\alpha)$. In fact even for $n = 3$ these lengths are of the order of $10^{-17}$ m, i.e. still consistent with a subnuclear range. At energy below this threshold, i.e. $n \geq 4$, eq (4,1) only describes the interaction. Of course the most favorable condition for this interaction to occur is that with $n = 1$, which ensures the maximum binding energy given by eq (4,12) and corresponds to the shortest interaction distance and maximum values of the three boson masses in fact experimentally detected. The model admits however even the possible existence of lighter bosons. In conclusion, the different energy scales characterize the features of eqs (4,1) or (4,5) because of different values of $n$; both equations describe however the same kind of interaction.

## 5 The interaction according to eq (3,1)

The starting point of this section is the eq (3,1) that reads

$$\frac{n\hbar v_x}{\Delta x} = \chi_s \Delta\varepsilon, \qquad \chi_s = 1. \tag{5,1}$$

The lack of coefficient at right hand side of eq (3,1) is tentatively interpreted here as the presence of coefficient $\chi_s = 1$. Being $v_x$ and $n$ arbitrary, it is certainly possible to introduce a proportionality constant $\xi$ defined as $n\hbar v_x = \xi e^2$; so eq (3,1) reads $(\xi e^2/\Delta x)/\Delta\varepsilon = 1$. Usually a proportionality constant linking two quantities that fulfill a given condition or a given physical law is of the order of the unity, unless some specific reason compels an appropriate hypothesis about its actual order of magnitude. Since here even $\Delta x$ and $\Delta\varepsilon$ are arbitrary, however, it is difficult to guess a valid reason to compel $\xi$ very different from the unity. So, in terms of order of magnitude, the position $\xi \approx 1$ seems reasonable although not thoroughly demonstrated, whence the tentative conclusion quoted in eq (5,1). On the other hand, once having reduced this equation to the form $(e^2/\Delta x)/\Delta\varepsilon = \chi_s$, one can compare $\chi_s = \xi^{-1} \approx 1$ with $\chi_{em} \approx \alpha$ and $\chi_w \approx \alpha^3$ defined by the equations (4,1) and (4,5) formally similar, of course under the assumption that the ranges at left hand sides defining these values are comparable as well. Even without a specific reason to exclude the plain idea $\chi_s \approx 1$, a better assessment of this conclusion appears however necessary: the lack of $e^2$ at left hand side, replaced by $n\hbar v_x$, allows handling eq (5,1) in order to introduce the interaction between the fractional charges concerned in section 2; but this chance, suggested by eqs (2,8) and (2,9) that anyway do no exclude themselves $\xi \approx 1$, is justified only revising the term $e^2/\Delta x$.

Consider again the eq (2,7) $F_x = -a'/\Delta x^2 + F_{ox}$ with $a' = n\hbar v'_x$ in the simplest case where both $a'$ and $\dot{p}_{ox} = F_{ox}$ are constants. Actually these constants could likely be first order approximations only of series developments whose higher order terms are neglected; yet, even this approximate meaning of the eq (2,7) is enough for the present discussion. Assuming $F_{ox} < 0$ likewise as the first addend in order to describe an attractive force, $F_x$ is compatible with a potential energy $U_i$ of the $i$-th quark having the form

$$U_i = -\frac{a}{\Delta x} + b\Delta x + U_0 \tag{5,2}$$

being $U_0$, $a$ and $b$ appropriate integration constants; the latter is clearly related to $F_{ox}$. Considering $\Delta U_i = U_i - U_0$ one recognizes a well know formula, the so called "asymptotic freedom", describing the interaction between quarks; of course in the present model where any local distance $x$ randomly included by its quantum uncertainty range is replaced by a range of distances $\Delta x$, the local value of potential energy $U_i$ turns into a range $\Delta U_i$ of allowed values. Let us examine the eq (5,2) in two particular cases where (i) $a/\Delta x \approx b\Delta x$ and (ii) $a/\Delta x \approx U_0$; the arbitrary size of $\Delta x$ justifies in principle both chances. The former case holds when $\Delta x^{(\mathrm{i})} \approx \sqrt{a/b}$ and yields $U^{(\mathrm{i})}_i \approx U_0$; according to the chance (ii) $\Delta x^{(\mathrm{ii})} \approx a/U_0$ yields instead $U^{(\mathrm{ii})}_i \approx b\Delta x^{(\mathrm{ii})} = ba/U_0$. This means that a delocalization extent of the system quark + gluons around $\Delta x^{(\mathrm{i})}$ the potential energy is approximately of the order of $U_0$, around a range $\Delta x^{(\mathrm{ii})}$ the potential energy increases linearly with $\Delta x$. Define $a$ and $b$ in agreement with eqs (2,8) and (2,9) in order that eq (5,2) takes a reasonable form. Put $a$ proportional to the electric charge $c_i^2 = (\pm(n_i/n')e)^2$, i.e. $a = a_o c_i^2$ via the proportionality constant $a_o$; also, let analogously be $b$ proportional to the color quantum number $C_j$, i.e. $b = b_o C_j^2$ with $j = 1 \cdot \cdot 3$. The subscripts symbolize the $i$-th quark in the $j$-th color quantum state; in this way $b = 0$ for a colorless Coulomb particle with $n_i = n'$, in which case the eq (5,2) turns, according to eq (2,9), into the classical potential energy $-e^2/\Delta x' + U_0$ of two Coulomb charges attracting each other. This reasoning suggests that the color quantum number should have the form $C_j = f_{j1}(n' - n_i)^2 + f_{j2}(n' - n_i)^4 + \cdot\cdot$, where $f_{j1}$ and $f_{j2}$ are appropriate coefficients of series expansion fulfilling the actual value of $C_j$ whatever it might be; it is interesting the fact that the electric charge depends on $n_i/n'$, the color charge on $n' - n_i$. As concerns $\Delta x' = \Delta x/a_o$, note that multiplying the size of $\Delta x$ by any factor yields a new range still arbitrary and thus still compliant with eqs (1,1); for the same reasons introduced in the previous section, i.e. because any size possible for $\Delta x$ is allowed to $\Delta x'$ as well, the notation $\Delta x'$ means in fact nothing else but renaming $\Delta x$. In summary, the Coulomb potential appears to be a particular case of eq (5,2), whose local features are described by the aforesaid chances; the expressions of $U^{(\mathrm{i})}_i$ and $U^{(\mathrm{ii})}_i$ are

$$\begin{gathered} \Delta x^{(\mathrm{i})} = \sqrt{\frac{a}{b}}, \qquad U^{(\mathrm{i})}_i = U_0, \\ a = l_a\varepsilon_a\left(\frac{c_i}{e}\right)^2, \qquad b = \frac{\varepsilon_b}{l_b}C_j^2, \\ \Delta x^{(\mathrm{ii})} = \frac{a}{U_0}, \\ U^{(\mathrm{ii})}_i = \frac{ab}{U_0} = \frac{\varepsilon_a\varepsilon_b}{U_0}\frac{l_a}{l_b}\left(\frac{c_iC_j}{e}\right)^2 = b\Delta x^{(\mathrm{ii})}. \end{gathered} \tag{5,3}$$

The constant energies $\varepsilon_a$ and $\varepsilon_b$ together with the constant lengths $l_a$ and $l_b$ describe the physical dimensions of $a$ and $b$ without need of proportionality factors. Note that $l_b \to \infty$, compels $\Delta x^{(\mathrm{i})} \to \infty$ and $b \to 0$; as the color is introduced by $b$, this agrees with a constant Coulomb potential $U_i^{(\mathrm{i})} = U_0$ of a colorless particle. By definition therefore $l_a\varepsilon_a = e^2$ for $n_i/n' = 1$, whereas it is expected to take a different value for $n_i/n' < 1$: the new value of $l_a\varepsilon_a/e^2$ when $e^2$ is replaced by $(n_i/n')e^2$ is known in the literature as $\alpha_s \approx 1$. In summary, eqs (5,3) yield

$$U_i^{(\mathrm{i})} = U_0, \quad U_i^{(\mathrm{ii})} = U_0'\left(\frac{c_i}{e}\right)^2, \quad U_0' = \frac{\alpha_s\varepsilon_b e^2 C_j^2}{l_b U_0}. \qquad (5,4)$$

Appears here once more the importance of the delocalization range $\Delta x$: in eq (4,14) $\Delta x_w$ controlled either appearance of the electroweak interaction, in eqs (5,3) two different range sizes $\Delta x \approx \Delta x^{(\mathrm{i})}$ or $\Delta x \approx \Delta x^{(\mathrm{ii})}$ emphasize either feature of $U_i$: in (ii) it depends upon the fractional charge, in (i) it does not because $-a/\Delta x$ is balanced by $b\Delta x$ despite both terms describe attractive force.

Let us concern now eq (5,2) in a more general way. The features of $U_i$ as a function of $\Delta x$ are related to $\delta[(n/m)(n/V)]$ because $\Delta x$ defines $V$, eq (2,1), and also because the eq (5,2) comes directly from $\Delta F_x$ of eq (2,7). What is distinctive here with respect to the gravitational or Coulomb interaction is the mere fact of having put $F_{ox} \neq 0$; so the consequent form of $U_i$ with $b \neq 0$ describes a peculiar kind of attractive force that increases with $\Delta x$. Another remarkable point is that $\Delta F_x$ is not necessarily that between different quarks only, because eq (2,7) concerns a mere effect of confinement that holds even for an isolated quark; rather it seems more appropriate to think that the interaction between different quarks strictly replicates an intrinsic feature of the potential energy due to the confinement effect even of a single particle, which also involves its messenger bosons. In fact, in the present model $\Delta x$ is by definition the delocalization range of one particle; the arising of any form of interaction is due to the presence of a further particle that possibly shares the same delocalization range. In general the number of states within a system of interacting particles is related to their energy, to their masses and to the whole space volume in which they are delocalized: eq (2,2) shows indeed that if $n_1$ is the number of states of the system with its particles supposed non-interacting, then $\delta n$ is the change consequent to their interaction, while $\Delta\varepsilon_{n_1+\delta n}$ is the concurrent energy change from the initial $\Delta\varepsilon_{n_1}$. According to the considerations of section 2, in the present case $V$ is the time space delocalization volume of one quark and its interaction messengers, the gluons. If a further quark could share this $V$, then the quarks interact. If the delocalization volume $V$ is filled with gluons of both quarks mediating their interaction, then the change $\delta(n/V)$ stimulates a question: are the particles that mediate the interaction interacting themselves? Clearly, from the standpoint of eqs (2,7) and (5,2) this question holds even for one quark only within $V$. A positive answer would explain why $\Delta F_x$ increases when pulling apart the interacting quarks, e.g. of a nucleon or meson, or even a lonely quark and its gluon system; in the latter case a greater delocalization range describes indeed the chance of mowing away the gluons from their own quark, which however increases the energy of the system. To emphasize how the position $F_{ox} \neq 0$ answers the question, suppose that the quark-gluon and gluon-gluon interactions does not allow distinguishing the interaction between a quark and "its own" gluons from that of these latter with another identical quark; this would mean distinguishing identical particles, which is however forbidden by eqs (1,1) [7]. If the gluons are not mere interaction messengers but rather self-interacting messengers, then eq (5,2) describes the asymptotic freedom simply as a feature of one quark and its own system of gluons, i.e. even without necessarily requiring a further quark; otherwise stated, a net splitting of gluons from a quark interferes even with their propensity to follow another quark. The concept of asymptotic freedom is linked to the energy constrain that explains why do not exist bare quarks without gluons and bare gluons without quarks. Calculate the change of $U_i$ as a function of $\Delta x$ as $\Delta U_i = (\partial U_i/\partial\Delta x)\Delta x$ at the first order; the force field $\Delta F_x = -\partial U_i/\partial\Delta x$ acting on quark and its gluon system delocalized in $\Delta x$ can be calculated in particular at the delocalization extents $\Delta x^{(\mathrm{i})}$ or $\Delta x^{(\mathrm{ii})}$. Replacing here the previous results, one finds $\Delta F_x^{(\mathrm{i})} = -2b$ and $\Delta F_x^{(\mathrm{ii})} = -b(1 + U_0/U^{(\mathrm{ii})})$.

It will be shown in the next section that $U_0' \approx 2U_0 \approx 1\,\mathrm{MeV}$; so, being $U_i$ a monotonic function of $\Delta x$, results $\Delta x^{(\mathrm{ii})} \lesssim \Delta x^{(\mathrm{i})}$ because $U_i^{(\mathrm{ii})} \lesssim U_i^{(\mathrm{i})}$ according to eq (5,4). If $\Delta x^{(\mathrm{ii})}$ is of the order of the proton radius, i.e. $10^{-15}$ m, then according to eq (5,3) $b$ results of the order of 1 GeV/fm, as it is well known. Then, inside a proton the force field at (i) is about twice than that at (ii); of course $b\Delta x$ further increases for $\Delta x > \Delta x^{(\mathrm{i})}$, i.e. outside the actual radius of the proton. This means that extending delocalization range of the quark/gluon system from $\Delta x^{(\mathrm{ii})}$ to $\Delta x^{(\mathrm{i})}$ and then to any $\Delta x > \Delta x^{(\mathrm{i})}$, i.e. allowing quark and gluons to have more space to move apart each other, corresponds to a greater energy; this is not surprising once having found that $U_i^{(\mathrm{ii})}$ is already in the region of linear increase of $U_i$ as a function of $\Delta x$. The dependence of $U_i$ on $\Delta x$ is trivially self-evident; the reasoning about $\Delta x^{(\mathrm{ii})}$ and $\Delta x^{(\mathrm{i})}$ allows to quantify this evidence with specific reference to the sub-nuclear length scale.

The behavior of $U_i$ and the concept of asymptotic freedom equation are straightforward consequences of eq (2,7) and thus of eqs (1,1); this feature of the strong interaction is indeed characterized by the concept of uncertainty, which in particular prevents specifying the actual size of $\Delta x$. From the present standpoint only, therefore, no kind of correlation appears in principle between quark generations and chances (i) and (ii) inherent the eq (5,2). Yet, it seems intuitive that either chance for $\Delta x$ and thus either behavior of potential energy should be selectively related to the energies characteristic of

the three generations of quarks. This supposition will be confirmed in the next section, at the moment one must only admit that both chances are allowed to occur.

Now let us revert to the opening question of this section, i.e. how to regard the energy term $n\hbar v_x/\Delta x$ and $\chi_s$ of eq (5,1). The conceptual analogy of $\chi_s$ with $\chi_{em}$ and $\chi_w$ of eqs (4,1) and (4,5) was in principle legitimated by the arbitrariness of $v_x$ in defining $(e^2/\Delta x)/\Delta\varepsilon = \chi_s$ with $\chi_s$ expectedly of the order of the unity. Exploit now eq (2,2), for simplicity regarded again at the first order only

$$n_1 = \frac{\delta n}{\delta \log(\Delta\eta')}, \quad \delta \log(\Delta\eta') = \log(\Delta\varepsilon_{n_1+\delta n}) - \log(\Delta\varepsilon_{n_1}),$$

introducing two further energy uncertainty ranges $\Delta\varepsilon_o$ and $\Delta\varepsilon$ whose sizes are by definition intermediate between that of $\Delta\varepsilon_{n_1}$ and that of $\Delta\varepsilon_{n_1+\delta n}$, i.e. $\Delta\varepsilon_{n_1} \le \Delta\varepsilon_o < \Delta\varepsilon \le \Delta\varepsilon_{n_1+\delta n}$. Hence eq (2,2) rewritten as a function of these new ranges takes the form

$$\begin{gathered} \gamma n_1 = \frac{\zeta\delta n}{\delta \log(\Delta\eta)}, \quad \gamma = \gamma(\Delta\eta), \quad \zeta = \zeta(\Delta\eta), \\ \delta \log(\Delta\eta) = \log(\Delta\varepsilon) - \log(\Delta\varepsilon_o). \end{gathered} \tag{5,5}$$

Now $\Delta\varepsilon_o$ plays the role of fixed reference energy range, likewise as the early $\Delta\varepsilon_{n_1}$ did. The correction coefficients $\gamma$ and $\zeta$ account for the fact that $n_1$ and $\delta n = n_2 - n_1$ were early defined for $\Delta\varepsilon_o \equiv \Delta\varepsilon_{n_1}$ and $\Delta\varepsilon \equiv \Delta\varepsilon_{n_1+\delta n}$, being therefore $\gamma = 1$ and $\zeta = 1$; having changed the ranges at right hand sides, clearly $\gamma$ and $\zeta$ must be replaced here by $\gamma n_1$ and $\zeta\delta n$ with $\gamma \neq 1$ and $\zeta \neq 1$, whence their definitions of functions of $\Delta\varepsilon$ once having fixed $\Delta\varepsilon_o$. So the previous eq (2,2) becomes a particular case of the present result (5,5), which reads now

$$\begin{gathered} \beta(\Delta\eta) = \frac{\delta g}{\delta \log(\Delta\eta)}, \quad \beta(\Delta\eta) = \gamma n_1, \\ \delta g = \zeta n_2 - \zeta n_1 = \delta(\zeta n). \end{gathered} \tag{5,6}$$

The third equation is interesting as it defines the new range $\delta g$. Let the function $\zeta$ be someway proportional to $\Delta\eta$, i.e. let $\zeta$ decrease with $\Delta\eta$; also, consider the particular case where $\Delta\eta$ is so small that the notation $\delta g$ can replaced by the familiar differential symbol $dg$ whatever the actual $\delta n$ might be. Being the range sizes arbitrary, this position about $\delta g$ is not a hypothesis; it focuses the attention on a particular chance of $\Delta\eta$ that must be taken into account simply because it is allowed and thus to be actually expected. Since a smaller and smaller uncertainty range identifies better and better a local value of the random variable included by its boundaries, $\delta \log(\Delta\eta)$ tends to $d \log(\eta)$; hence the former equation (5,6) tends to the known beta function $\beta(\eta) = dg/d \log(\eta)$ defining the coupling constant $g$ at the energy scale defined by $\eta$. This particular limit case helps thus to understand the physical meaning of the ratio in the first eq (5,6), merely written as a function of ranges instead of local values. It is clear the interest to take now $\Delta\eta$ comparable with $\Delta\varepsilon$ of eq (4,1) and (4,5) in order to infer from $\beta(\Delta\eta)$ the function $g(\Delta\eta) \equiv \chi_s$ to be compared with the respective $\chi_{em}$ and $\chi_w$. The next task is to calculate the first eq (5,6) in order to confirm that $\chi_s$ is of the order of the unity. To this purpose let us expand $\beta$ in series of powers of $\delta g$, i.e. $\beta = \beta_o + \beta_1\delta g + \beta_2\delta g^2 + \cdots$: the coefficient $\beta_o$ must be equal to zero because of eqs (5,6), whereas $\beta_1 = 0$ as well to fulfill the reasonable condition $\partial\beta/\partial(\delta g) = 0$ of minimum $\beta$ for $\delta g = 0$. Hence $\beta = \beta_2\delta g^2$, neglecting the higher order terms, requires $\delta g = (\beta_2\delta \log(\Delta\eta))^{-1}$; this appears replacing $1/(\delta \log(\Delta\eta))$ in eq (5,6), which indeed turns into $\beta(\Delta\eta) = \beta_2(\delta g)^2$. According to the fourth eq (5,5), $\delta g = \beta_2/(\log(\Delta\varepsilon/\Delta\varepsilon_o))$ is reducible to the well known form

$$\delta g = \frac{\xi}{\zeta \log(\Delta\eta^2/\Delta\varepsilon_o^2)}, \quad \frac{2\zeta}{\xi} = \beta_2, \quad \Delta\varepsilon_o \approx 0.2 \text{ GeV}. \tag{5,7}$$

The order of magnitude of $\Delta\varepsilon_o$ is easily justified recalling the eq (2,5) of section 2 and the conclusions thereafter inferred: $\Delta\varepsilon_o$ implies that to $\Delta t \approx \hbar/\Delta\varepsilon_o$ corresponds the path $\delta x \approx \hbar c/\Delta\varepsilon_o$ of gluons moving at the light speed to carry the interaction between quarks. The given value of $\Delta\varepsilon_o$ is therefore consistent with the order of magnitude $\delta x \approx 10^{-15}$ m previously quoted for the strong interaction. The result (5,7) and the value of $\Delta\varepsilon_o$ are well known outcomes of quantum chromodynamics; further considerations, in particular about the constants $\xi$ and $\zeta$, are omitted for brevity. This paper aims indeed to show the consistency of the present model based uniquely on eqs (1,1) with the standard features of the strong interactions, not to repeat known concepts.

## 6 The quark and lepton masses

This section consists of two parts, the first of which concerns the ability of eq (2,4) to describe the ideal masses of isolated quarks. Correlating these masses to the energy ranges $\Lambda_i \equiv \Delta\varepsilon_{n_1+\delta n}$ is in principle sensible first of all regarding the various quarks as a unique class of particles: there would be no reason to expect that different kinds of particles of dissimilar nature are all described by a unique law simply changing a unique distinctive index, here represented by $i \equiv \delta n$. Moreover must hold for the energies of the various quarks a common sort of functional dependence upon $\delta n$ like that of $\Delta\varepsilon_{n_1+\delta n}$. Eventually, this dependence must still hold even replacing these ranges with the respective average energies $< \varepsilon_{n_1+\delta n} >$ calculated as described in section 4. This last requirement suggests correlating the quark masses with these averages in agreement with the eq (2,4), tanks to the fact that both $< \varepsilon_{n_1+\delta n} >$ and $\Delta\varepsilon_{n_1+\delta n}$ are consistent with their own $\delta n$. Indeed an incremental index $\delta n$ representing the quark energies is defined replacing in eq (2,2) $\log(\Delta\varepsilon_{n_2})$ and $\log(\Delta\varepsilon_{n_1})$ with $\log(< \varepsilon_{n_1} >)$ and $\log(< \varepsilon_{n_2} >)$; a procedure completely analogous yields an equation of the average quantities fully corresponding to eq (2,4). The second point has been explained: the self-interaction of quarks justifies in principle $\delta n$

simply admitting that the various quarks are characterized by different self-interaction strengths and thus by distinctively different values of $\delta n$. So the critical step is the first one, i.e. whether or not $\Delta F_x$ of eq (2,5) really governs the self-interactions of all quarks in order that all of them are related to a unique law (2,4) of $\delta n$. This means in practice: (i) regarding one quark delocalized in its own uncertainty range; (ii) thinking that various quarks are characterized by different $\delta(n/m)$ because of their own kind of self-interaction; (iii) assuming that in fact the eq (2,4) accounts for the different numbers of states that characterize uniquely the various quarks. If the functional dependence described by the eq (2,4) is consistent with the three points just mentioned, then $\Lambda_i$ describes the ideal masses of the quarks as a function of $i$; also, the point (ii) shows that the energies of this class of particles are really related to their number of allowed states through the self-interaction between quark and gluons.

The estimated masses $Q_i$ of the quarks quoted in literature [14] are reported here:

$$\begin{array}{l} Q_u = 1.7 \leftrightarrow 3.3 \text{ MeV} \\ Q_d = 4.1 \leftrightarrow 5.8 \text{ MeV} \\ Q_s = 80 \leftrightarrow 130 \text{ MeV} \\ Q_c = 1.18 \leftrightarrow 1.34 \text{ GeV} \\ Q_b = 4.13 \leftrightarrow 4.85 \text{ GeV} \\ Q_t = 170.7 \leftrightarrow 173.3 \text{ GeV} \end{array} \qquad (6,1)$$

The mass interval of the "b" quark actually merges two intervals, that reported for the $\overline{MS}$ "mass-independent subtraction scheme" and that of the "1S mass" scheme [14]; the respective mass intervals are $4.19^{+0.18}_{-0.06}$ GeV and $4.67^{+0.18}_{-0.06}$ GeV [15].

It is known that these literature data represent estimates instead of experimental values, as actually isolate quarks do not exist; because of their confinement, the masses are indirectly inferred from scattering experiments. In fact the masses depend on their different combinations in various hadrons and mesons. So the values quoted above must be regarded with carefulness when compared with the results of theoretical calculations. Nevertheless the intervals of values (6,1) do not overlap, which suggests that their order of magnitude is somehow related to and thus at least indicative of the ideal masses of isolated quarks; by consequence it seems also sensible to expect that the sought values of quark masses should fall within these intervals. In lack of further information, therefore, exploit the intervals (6,1) to calculate the average values $Q_i$:

$$\begin{array}{l} {Q_u}^{(2/3)} = 2.50 \text{ MeV} \\ {Q_d}^{(-1/3)} = 4.95 \text{ MeV} \\ {Q_s}^{(-1/3)} = 105 \text{ MeV} \\ {Q_c}^{(2/3)} = 1.26 \text{ GeV} \\ {Q_b}^{(-1/3)} = 4.49 \text{ GeV} \\ {Q_t}^{(2/3)} = 172 \text{ GeV} \end{array} \qquad (6,2)$$

The superscripts indicate the charges of the respective quarks. These averages have neither specific physical meaning nor come from some particular assumption, they merely represent preliminary starting points defined within realistic intervals; thus their worth is that of reasonable inputs to carry out calculations. The validity of the results inferred in this way relies mostly on their self-consistency; the only initial information is that any sensible output calculated starting from the values (6,2) should expectedly fall within the intervals (6,1). Regard therefore the available data as mere reference values to clarify with the help of eq (2,4) what do $Q_i$ vs $i$ might actually mean in the present context. According to the reasoning carried out in the previous section let us try preliminarily to correlate $Q_i$ with $\Lambda_i$ putting $\Lambda_i/\Lambda = ((Q_i/U_i)/q)^{1/b}$, where $q$ is a proportionality constant and $b$ a coefficient to be determined by successive calculations; this coefficient fulfills the chance that if $< \Delta\varepsilon_{n_2} > \approx < \Delta\varepsilon_{n_1} >$, i.e. $< \varepsilon_{n_2} > \approx < \varepsilon_{n_1} >$, then the corresponding ratio $(Q_i/U_i q)^{1/b}$ with increasing $b$ anyway matches the limit behavior of $\Lambda_i/\Lambda$ whatever $q$ and $U_i$ might be. Initially $U_i$ is justified as mere dimensional factor to be determined; the next results will show that actually it results to be just the potential energy of eq (5,2). Let us sort now the various $Q_i$ by increasing value to check if really the estimated quark masses fulfill the logarithmic dependence of eq (2,4) upon the incremental number of states $i$, which therefore takes from now on values from 1 to 6. In this way each mass is progressively related to its own increasing $i$. This expectation is indeed reasonable because $i \equiv \delta n$ defines $\Lambda_i \equiv < \varepsilon_{n_1+\delta n} >$ with respect to a ground reference state number, to which corresponds the reference energy range $\Lambda \equiv < \varepsilon_{n_1} >$. Being by definition $\Lambda_i \equiv \Lambda$ for $\delta n = 0$, one also expects that holds for the eq (2,4) the boundary condition

$$Q_0/U_0 \equiv q \qquad i = 0 \qquad (6,3)$$

whatever $b$ might be; this fact justifies the proposed notation. When handling sets of data, regression calculations are in general needed; the outcomes of these calculations are usually expressed as power series development of an appropriate parameter. Implementing the linear eq (2,4) with the values (6,2) as a function of $i$, means therefore calculating the best fit coefficients $a$ and $b$ of the form $\log(Q_i/U_i) = a + ib$; clearly $n_1$ has been included in the regression coefficients. This is easily done regarding $\Lambda_i$ and $\Lambda$ of eq (2,4) as follows

$$\log(Q_i/U_i) = a + bi, \qquad a = \log(q), \qquad 1 \le i \le 6. \qquad (6,4)$$

The factor $q$ linking $U_i$ to the reference energy $\Lambda$ is determined by the boundary condition (6,3); this holds of course even in the presence of higher order terms. The plain first order approximation decided for $i$ agrees with the intent of the present paper: to describe the quarks through an approach as simple as possible and compatible with the minimum amount of input data needed for an unambiguous assessment of re-

sults. So, owing to eqs (5,3) and (5,4), one expects

$$a + bi = \begin{cases} \log(Q_i/U_0) \\ \log\left(Q_i/U_0'(c_i/e)^2\right) \end{cases} \qquad U_0' = \frac{\alpha_s \varepsilon_b e^2 C_j^2}{l_b U_0} \qquad (6,5)$$

Now the $\Delta x$-dependent behavior of $U_i$ can be checked: if these equations of $U_i$ and the position $\Lambda_i/\Lambda \propto (Q_i/U_i)^{1/b}$ are correct, then both chances (5,3) should somehow appear when exploiting the logarithmic law. A series of plots shows this point step by step starting from the raw data (6,2).

The various $Q_i$ are preliminarily plotted vs $i$ taking all $U_i$ equal to a constant; this first result is reported in fig 1. The boxes represent the input data, the letters between {} identify the quarks, the dot lines describe tentatively their possible connection; the best fit dashed line has a mere indicative meaning of preliminary reference trend. The various points are not completely random, rather they roughly follow an identifiable increase with $i$. It appears that couples of the various $Q_i$ lie along three lines reasonably parallel each other; so, according to eq (6,4), these lines should be characterized by a unique best fit coefficient $b$ and differ by the coefficient $a$ only. Yet, since each line must be handled in order to fulfill the condition (6,3), the different $a$ are irrelevant: indeed the three regression lines $\log(Q_i) = a_k + bi$, with $k = 1..3$, must be actually plotted as $\log(Q_i/q_k) = bi$ putting $a_k = \log(q_k)$. In effect the fig 2 shows that once having forced the three dotted connections to cross the origin, all quark masses are perfectly aligned along a unique best fit line, whose regression coefficients are: $a_k = 4.7,\ 5.1,\ 5.4$; the respective values of $b$ range between 0.967 and 0.985, i.e. it is reasonably unchanged. Clearly are here concerned the masses of isolated quarks, since the raw data (6,2) have been plotted one by one independently each other. The relevant conclusion is that of having confirmed the validity of eq (2,4) and (2,1): $\Delta x$ has physical meaning of delocalization range of a unique quark. Considering that the masses spread over 5 orders of magnitude, the result is certainly interesting. If one would calculate the masses of quarks through this plot, however, four constants must be known: three $a_k$ and $b$: too many, to consider physically meaningful this way of exploiting eq (2,4). The worth of fig 2 is merely heuristic. It must be noted, however, that significant information about $b$ can be obtained through very simple considerations. In the linear regression (6,4), the best fit coefficient $b$ weights the increase of $\log(Q_i)$ as a function of the incremental number of states $i$. Consider in particular the highest mass $Q_6$ of the top quark, corresponding to $i = 6$: the greater $b$, the greater the calculated value of $Q_6$. So $b$ is expected to be proportional to $Q_6$. Moreover for the same reason $b$ controls also the masses of lighter quarks for $i < 6$; the link of $Q_6$ with the masses of all quarks, inherent the plot of fig 2, suggests that the proportionality constant should reasonably have form and physical dimensions somehow related to all quark masses. Put therefore $b = (\sum_{i=1}^{6} Q_i)^{-1} Q_6$,

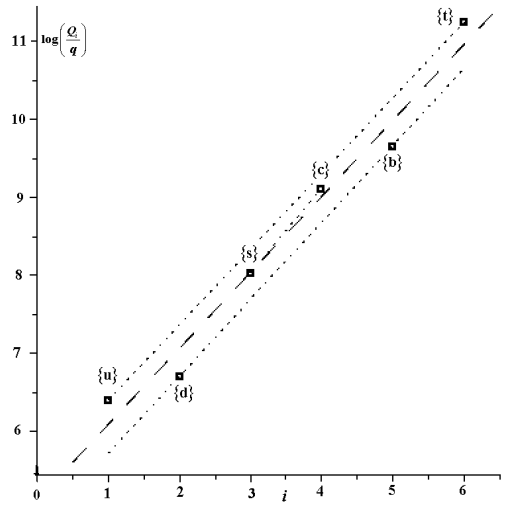


Fig. 2: Plot of $\log(Q_i/q_k)$ vs $i$; three values of $q_k$ calculated via the boundary condition (6,3) enable a unique trend line of the quark masses with a unique constant $U_0$.

in which case $Q_6$ is normalized with respect to the total energy of all possible states allowed between $\Delta\varepsilon_{n_1}$ and $\Delta\varepsilon_{n_1+\delta n}$. Hence the estimates (6,2) yield

$$\frac{Q_6}{\sum_{i=1}^{6} Q_i} = 0.967.$$

In effect, the value of $b$ calculated in this way is very close to that determined in (6,6) via best fit regression.

Yet even three input data to calculate the quarks masses

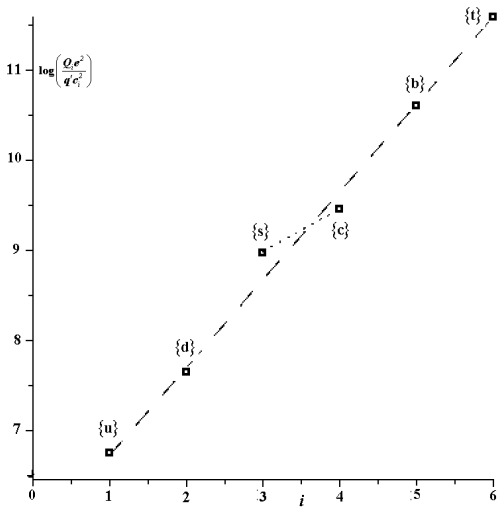


Fig. 4: Plot of $\log(Q_i^*/q')$ vs $i$ with $Q_i^* = Q_i/x_i^2$: here $x_i = const$ for the quarks {c} and {s} and $x_i = c_i/e$ for the other quarks.

Fig. 5: Plot of $\log(Q_i^*/q_k')$ vs $i$; $x_i$ are defined in fig 4, $q_k'$, with $k = 1, 2$, are calculated in order to fulfil the condition (6,3).

are still too many; certainly there is something else not yet evidenced by the plot of fig 2. Moreover this result, while showing that the idea of concerning the masses of isolated quarks is basically correct, does not highlight anything about the potential energies $U_i$ of eqs (6,5), at the most it could account for $U_0$ only. Since the idea of considering $Q_i/q_k$ is theoretically too naive, let us regard the various $Q_i$ all together. If so however, despite the previous warnings, the plot of fig 1 is unsatisfactory; owing to the logarithmic ordinate scale, the deviations of the various $Q_i$ from the best fit line are markedly large. Seems however decipherable an unambiguous configuration of these points; this plot prospects the chance of better results. An improved connection between quark masses and $i$ must have exclusively physical valence: here the problem does not concern a random dispersion of experimental measurement errors, but the relationship between masses of isolated quarks and bound quarks on the basis of data extrapolated from the experience; the challenge is to extract the former from the latter trusting to their initial order of magnitude only. The fig 3 reports a new plot where the ratios $(Q_i/U_0)/q$ are replaced by the respective $Q_i e^2/q'c_i^2$, being $c_i$ the electric charges of the various quarks; $e$ is clearly introduced for dimensional reasons. The chance $Q_i e/qc_i$ is not mentioned because found of scarce interest after preliminary checks. From a numerical point of view, therefore, the plain $Q_i$ are now corrected by fractional charge factors $(-1/3)^2$ and $(2/3)^2$. In this way the logarithmic terms are handled exactly as before, which allows the comparison with the former plot: the figure 3 reports again a new best fit line. Now the linear trend of $\log(Q_i e^2/q'c_i^2)$ as a function of $i$ is significantly better than that of fig 1; the {s} and {c} quarks only, both second generation quarks, deviate appreciably from the best fit line; their calculated values consistent with the linear best fit trend are respectively 51 MeV and 1.9 GeV, well outside the literature intervals (6,1). Considering that the orders of magnitude calculated are however globally correct, two chances are in principle admissible: either the literature estimates of the masses of these quarks must be replaced by the values calculated here or some further physical reason, not yet taken into account, enables to modify just these values and align them with the others. The former option is in principle acceptable according to the previous warnings on the literature quark masses, but would conflict with the plot of fig 1: both masses of these quarks were correctly aligned on a similar best fit line before introducing the correction due to their electric charges. So the

latter option seems more stimulating.

Replace therefore $Q_i e^2/q' c_i^2$ of the quarks {c} and {s} only with $Q_i/const$. This idea works well defining *const* appropriately, i.e. in order to fit $Q_i$ of these two quarks to the main best fit line of the other quarks. The fig 4 reports the same data of fig 3, yet replacing $e^2/c_i^2$ of the quarks {s} and {c} only with a unique value not dependent on $c_i$; now $Q_i^*/q'$ with $Q_i^* = Q_i/x_i^2$ includes both chances through $x_i$. The ideal line joining these quark masses is reasonably parallel to the four quark best fit line, i.e. the plot of these two quarks differs trivially from that of the other quarks by the value of the constant $a$ only. As before, in fact this means admitting two values of $a$: one for the main best fit line, another one for the second generation quark best fit line; of course both values must make the best fit lines compliant with the condition (6,3) via a unique $b$. The result is shown in fig 5: despite replacing $c_i/e$ with a unique constant is certainly an approximation, nevertheless all quark masses are reasonably represented by a unique eq (2,4). In conclusion, the path from fig 1 to fig 5 was aimed to verify that effectively the logarithmic law (2,4) is expressed via the ratio $Q_i/U_i$ vs the incremental number $i$ of states. The plot of the quark masses $Q_i$ is described by the following equation

$$\begin{array}{lll} \log\left(\dfrac{Q_i^*}{Q_0}\right) = bi & Q_i^* = \dfrac{Q_i}{x_i^2} & b = 0.9723 \\ x_i = c_i/e & Q_0 = 0.556 \text{ MeV} & 1^{st}, 3^{rd} \text{ gen.} \\ x_i = 0.3644 & Q_0 = 1.118 \text{ MeV} & 2^{nd} \text{ generation} \end{array} \qquad (6,6)$$

So $b$ is very similar to that of fig 2. The double value of $U_0$ corresponds to the two regression constants $a$ allowing to merge the best fit lines of fig 4 according to the condition (6,3); $Q_i^*$ plays the role of an "effective mass" of quarks. The reliability of the results inferred from the plots is assessed recalculating via eqs (6,6) the quark masses and comparing them to the starting values (6,2); one finds $2.32,\ 5.44,\ 1.22 \times 10^2,\ 1.14 \times 10^3,\ 4.50 \times 10^3,\ 1.69 \times 10^5$ MeV that agree reasonably with the literature intervals (6,1). As mentioned at the beginning of this section, this is the basic requirement to be fulfilled. To assess this result also note however that the values (6,2) do not have the rank of experimental data, to be necessarily matched as exactly as possible; as stated before, they have a mere indicative meaning of reference values. Hence the conclusion is that the eqs (6,6) yield a sensible result, while having also the merit of verifying the positions (6,5) strictly related to eqs (5,3). But the most interesting remark concerns $U_i$, which depends explicitly on the charges $c_i$ in the first and third generation of quarks only; in the second generation it does not, which brings to mind the respective limit cases introduced in eqs (5,3) and further emphasized in eqs (6,5). The generations of quarks are indeed described by $\log(Q_i/U_i) = bi$ with $U_i$ defined by the following equations

$$\begin{array}{ll} U_i^{(\mathrm{ii})} = 0.556(c_i/e)^2 \text{ MeV} & 1^{st}, 3^{rd} \text{ generation,} \\ U_i^{(\mathrm{i})} = 0.148 \text{ MeV} & 2^{nd} \text{ generation.} \end{array}$$

The superscripts are assigned to the generations of quarks by comparison with eqs (5,4) and (6,5); so $U_0 = 0.148$ MeV and $U_0' = 0.556$ MeV.

Some further remarks on this result are also useful. The first concerns the plots of figs 2 and 5: despite the former has been obtained from $\log(Q_i/q_k)$ and the latter from $\log(Q_i/U_i)$ that involves the potential energy, both plots look like and fit surprisingly well the logarithmic law (2,4) despite the quark masses spread over 5 orders of magnitude. These plots are not trivial duplicates: it is interesting the fact that $Q_i/U_i$ takes both forms $Q_i(e/c_i)^2$ and $Q_i/const$, while are determined $U_0$ and $U_0'$. On the one hand is remarkable the fact of having identified the mass range as the reason that discriminates the chances (i) and (ii) of eqs (6,5): indeed the mass range of the second generation of quarks is well defined with respect to that of the first and third generations. On the other hand, the fact that both chances are merged in the same plot is itself a further fingerprint of the quantum uncertainty, early introduced because of the mere arbitrariness of $\Delta x$. The third remark confirms the fact that $\Delta x$ is not necessarily the distance between two quarks, it can also be the delocalization range of one quark only; the fact that the plot of fig 5 overlaps very well that of fig 2 shows that even isolated quarks must be regarded as self-interacting and that the interaction potential energy between quarks, the well known eq (5,2) is a replica of the self-interaction potential energy. This conclusion, also supported by the fact that the plot of fig 5 is better than that of fig 1 by introducing $Q_i/c_i^2$ and not $Q_i/c_i$, explains why eq (5,2) describing the interaction between different quarks holds also for isolated quarks. The fourth remark concerns the values of the constants $U_0$ and $U_0'$ reported in eqs (6,6), which describe the asymptotic freedom introduced in the previous section.

Note eventually that the considerations hitherto carried out have assumed already known the quark masses; also, in eqs (6,6) appear several constants to be known "a priori" to carry out the calculations. Moreover, the literature estimates (6,1) appear now as values well configured in the frame of eq (2,4) but not directly supported by experimental measurements. In this respect, a sound proof of their meaning would be to calculate them contextually to other well known and well determined particle mass. The merit of this first part of the section is to have checked the eqs (5,2) and (5,3) via the logarithmic law of eq (2,4). Yet it is also possible to extend further this idea considering together both lepton and quark masses. Indeed a simple question arises at this point: does the eq (6,4) hold also for the leptons? The fact that quarks and leptons are both fundamental bricks of matter suggests the idea that the eq (6,4) could hold for both classes of particles. Moreover note an interesting coincidence: the number of leptons is 6, like that of the quarks. Is this a mere accident or is there some correlation between each quark and each lepton? The next part of the section will show that considering together both kinds of particles allows obtaining all of their

masses as a consequence of a unique principle.

The literature data on the masses $L_i$ of the 6 leptons are summarized here:

$$\begin{array}{l} e \to 0.51 \text{ MeV}, \quad \mu \to 105.66 \text{ MeV}, \\ \tau \to 1776.84 \text{ MeV}, \quad \nu_e \to \ < 2.2 \text{ eV}, \\ \nu_\mu \to < 170 \text{ KeV}, \quad \nu_\tau \to \ < 15.5 \text{ MeV}. \end{array} \tag{6,7}$$

The difficulty of comparing calculated and experimental masses concerns now the neutrinos, because of their very scarce interaction with matter and because the neutrino flavor eigenstates are not the same as the mass eigenstates due to the neutrino oscillations [17]. However, being the masses of electron, muon and tau well known, the strategy to carry out the next calculations is: (i) to assume preliminarily the eq (6,4) for the masses of the leptons; (ii) to fit the masses of the neutrinos to the profile required by the logarithmic law via an appropriate correction factor downscaling their upper limit values (6,7); (iii) to look for a unique best fit calculation including both leptons and quarks; (iv) to infer some conclusion about the physical meaning of such a result.

Since the most important task of this section is to find a correlation between the lepton and quark masses previously determined and to confirm the validity of the previous results, the approach proposed here does not concern directly eq (2,4) rewritten in the form (6,4) $\log(L_i) = a' + b'i$ involving the lepton masses only; rather we start looking since the beginning for a connection between $L_i$ and $Q_i$. Let us show first of all that such a link actually exists, i.e. that are physically sensible logarithmic laws having the forms $\log\left(Q_i^*\right) \pm \log(L_i)$ with $Q_i^*$ defined in eqs (6,6). From $\log(Q_i^*) = a_Q + bi + ci^2 + \cdot\cdot$ and $\log(L_i) = a_L + b'i + c'i^2 + \cdot\cdot$, with $a_Q = \log(Q_0)$ and $a_L = \log(L_0)$ regression constants, one finds first $\log(Q_i^*) \pm \log(L_i) = a_Q \pm a_L + (b \pm b')i + (c \pm c')i^2 + \cdot\cdot$; the higher powers of $i$ have been skipped for brevity, whereas the dimensional factors $Q_0$ and $L_0$ are included in the constants $a_Q$ and $a_L$ as in eq (6,4). The fig 6 evidences that the idea of plotting $\log(Q_i^*) + \log(L_i)$ and $\log(Q_i^*) - \log(L_i)$ vs $i$ is sensible: in fact both curves are reasonably definable through appropriate best fit coefficients. To obtain these plots, the neutrino masses, quoted in literature through the respective upper limits only, have been downscaled to the following values

$$\nu_e = 1.802 \text{ eV}, \nu_\mu = 3481.6 \text{ eV}, \nu_\tau = 1.549 \times 10^7 \text{ eV}. \tag{6,8}$$

Moreover the various $L_i$ have been sorted by increasing mass like the respective $Q_i^*$. This sorting criterion establishes a one-to one correspondence between leptons and quarks that reads

$$\begin{array}{lcccccc} \textit{leptons} & \nu_e & \nu_\mu & e & \nu_\tau & \mu & \tau \\ & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \textit{quarks} & u & d & s & c & b & t \end{array} \tag{6,9}$$

Before commenting this correspondence and confirming the validity of eq (2,4) also for the leptons, let us repeat here preliminarily the reasoning previously carried out for the quarks. Calculate $(\sum_{i=1}^6 L_i)^{-1} L_6$ exploiting the values (6,7) and (6,8); one finds

$$\frac{L_6}{\sum_{i=1}^6 L_i} = 0.935 \qquad \left(\frac{Q_6}{\sum_{i=1}^6 Q_i}\right)^2 = 0.936$$

which shows that the lepton equation is related to that of the quarks. To explain this result assume that the normalized values of $L_6$ and $Q_6$ are correlated, i.e. $L_6/\sum_j L_j = b' Q_6/\sum_j Q_j$, being $b'$ a constant; imposing then $b' = b$, in order that also $L_6/\sum_j L_j$ be proportional to $b$ of eq (6,5) for the same aforesaid reasons, one finds the given result. These considerations put a constrain on the best fit coefficients of $Q_i$ and $L_i$ vs $i$. The fig 6 suggests the reasonable chance of introducing a further arbitrary constant $b_o$ that defines the more general linear combinations $\log\left(Q_i^*\right) \pm b_o \log(L_i) = a''_Q \pm b_o a''_L + b''_\pm i + \cdot\cdot$. Hence, multiplying side by side these equations and collecting the constants at right hand side, it must be also true that

$$(\log(Q_i^*))^2 - b_o^2 (\log(L_i))^2 = a''^2_Q - b_o^2 a''^2_L + \cdot\cdot$$

skipping even the first power of $i$. In effect the advantage of having introduced the arbitrary coefficient $b_o$ is that it can be defined in order to make even the first order term negligible with respect to the constant term, whence the notation reported here; so, neglecting all powers of $i$, the right hand side reduces to a constant. The last equation reads thus

$$(\log(Q_i^*))^2 = a + (\log(L_i))^2 b, \quad a = a''^2_Q - b_o^2 a''^2_L, \quad b = b_o^2.$$

Now implement again the input data listed in (6,7), (6,8) and (6,2) to check if this last equation correlates sensibly the sets of leptons and quark masses via two constants $a$ and $b$

Fig. 6: Plot of $\log(Q_i^*/Q_0) \pm \log(L_i/L_0)$ vs $i$; $Q_0$ and $L_0$ are dimensional best fit constants. $Q_0$ is defined in eqs (6,6).

only; these constants are clearly best fit coefficients that describe the correspondence (6,9). If the zero order approximation just introduced is correct, then trivial regression calculations should yield a sensible statistical correlation of all masses. The best fit coefficients consistent with the zero order approximation of the last equation are

$$a = 45.49178521, \qquad b = 1.039628847. \tag{6,10}$$

So the best fit equation is

$$\log\left(Q_i^{bf}/x_i^2\right) = \pm\sqrt{a + b(\log(L_i))^2};$$

the notation stresses that $Q_i^*$ of eqs (6,6) are replaced by values $Q_i^{bf}$ of $Q_i$ determined by the regression, while the various $x_i$ are of course still that defined in eq (6,6). This result is readily checked calculating

$$Q_i^{bf} = x_i^2 \times 10^{\pm\sqrt{a+b\left(\log(L_i)\right)^2}} \tag{6,11}$$

via the respective lepton masses $L_i$ listed in (6,7), (6,8) and comparing with $Q_i$ reported in (6,1). Note that, because of the exponentials, the decimal places of the regression coefficients are important to reproduce the results of the following calculations. All of the values calculated with the positive sign in eq (6,11)

$$\begin{array}{ll} Q_u^{bf} = 2.50\times10^6 \text{ eV} & Q_d^{bf} = 4.97\times10^6 \text{ eV} \\ Q_s^{bf} = 1.08\times10^8 \text{ eV} & Q_c^{bf} = 1.22\times10^8 \text{ eV} \\ Q_b^{bf} = 4.45\times10^9 \text{ eV} & Q_t^{bf} = 1.75\times10^{11} \text{ eV} \end{array} \tag{6,12}$$

fit surprisingly well the values (6,2) and, mostly important, fall within the estimated intervals (6,1); it is worth noticing that the agreement is much better than that obtained through eqs (6,6). A further remark in this respect is the following. When carrying out the regression calculations with random input data, have been traced the percent deviations of the resulting values of quark and lepton masses with respect to the respective input values; the best self-consistency was found with the true data; the conclusion is that the regression is not mere calculation procedure, but rather a real physical representation of the masses. This also supports the idea that the average values (6,2) of the estimated intervals (6,1) could have an actual physical meaning. Yet are also allowed the following results calculated with the minus sign

$$\begin{array}{ll} q_u^{bf} = 7.91\times10^{-8} \text{ eV} & q_d^{bf} = 2.48\times10^{-9} \text{ eV} \\ q_s^{bf} = 1.64\times10^{-10} \text{ eV} & q_c^{bf} = 1.45\times10^{-11} \text{ eV} \\ q_b^{bf} = 2.77\times10^{-12} \text{ eV} & q_t^{bf} = 1.13\times10^{-12} \text{ eV} \end{array} \tag{6,13}$$

The former set of energies has a literature check through the estimates (6,1), the latter set does not; yet there is no reason to exclude the values (6,13), whose physical meaning will appear shortly. In the latter case the subscripts have a formal physical meaning only, merely reminiscent of the respective quark masses (6,12); nevertheless, it is possible to show the key role of these further energies for the physics of quarks and leptons.

Any statistical regression concerns by definition whole sets of values; here eq (6,11) correlates all masses of leptons and that of all quarks reported in (6,2) and (6,7), (6,8) according to their representation (6,9). The best fit coefficients (6,10) are therefore the fingerprint of *all* masses. Various simulations have been indeed carried out (i) altering deliberately some selected input values of either set of masses, (ii) altering either whole set of masses and (iii) altering both whole sets of masses by means of arbitrary multiplicative factors to find out how the corresponding results are affected; the results, compared with that of eq (6,11) obtained from true values, confirm of course that anyway the new regression coefficients differ from (6,10). The obvious conclusion is that, for some specific reason, just the quoted coefficients (6,10) identify uniquely the fundamental masses of our universe: $a$ is related to their measure units, as previously explained, $b$ controls instead the link between quarks and lepton masses at increasing values of $i$. Actually one coefficient only is enough to identify all masses; the other is merely associated to it, being concurrently calculated. Otherwise stated, one could assume as a fundamental assumption one of these coefficients only, the other one results consequently determined by the unique set of quark and lepton masses consistent with the former one. Is clear the importance of understanding the specific physical meaning of the particular couple of coefficients (6,10) able to account for the fundamental masses of our universe as a function of *one* predetermined input. Besides the numerical calculation of these masses, however, it seems reasonable to expect that some physical idea is still hidden in eq (6,11).

To investigate this point consider the following equation

$$q_i^o = x_i^2 \times 10^{\pm\sqrt{a+\left(\log(L_i)\right)^2}} \tag{6,14}$$

inferred from (6,11) leaving unchanged $a$ while replacing instead $b$ with the unity. This equation results formally from $\left(\log(q_i^o)\right)^2 = (\log(L_i))^2 + a$, which is interesting because $q_i^o$ and $L_i$ can be interchanged simply changing the sign of $a$ but not its absolute value. Of course the various $q_i^o$ so defined are no longer quark masses; being still related to the respective true lepton masses $L_i$, however, also $q_i^o$ are somehow related to $Q_i$.

It is very significant to regard eqs (6,14) thinking $Q_i$ correlated to $L_i$, which in turn are correlated to $q_i^o$ via one additive constant $a$ only.

So far the experimental masses of quarks and leptons have been introduced as a matter of fact, thus finding that a unique equation, (6,11), accounts for all of them simply postulating a well defined and unique couple of regression constants. Eq (6,14) adds to this standpoint a new perspective: the existence

of a field whose quanta are related to the $q_i^o$, as a function of which are first calculated $L_i$ via eq (6,14) and then $Q_i$ via eq (6,11). The number of input data confirms that highlighted before, i.e. the quoted value of $a$ only; the masses of both quarks and leptons appear then as consequences of a unique kind of particles, just the $q_i^o$, since the only possible regression of $L_i$ with $Q_i$ consistent with the given $a$ is that with the concurrent value $b$. This explains why $q_i^o$ have been defined keeping $a$ and changing $b$ only; even without appearing explicitly appearing in eq (6,14), we know that the latter is required to be just that consistent with the former.

Note now that also eq (6,14) allows two sets of of values, $q_i^{o+}$ and $q_i^{o-}$, defined by either possible sign of the exponential; it is easy to realize that, likewise as the values (6,12) and (6,13), also now from a numerical point of view $q_i^{o+} >> q_i^{o-}$. This appears regarding all $q_i^o$ together: the resulting total energies corresponding to the positive and negative signs are $\sum_{i=1}^{6} q_i^{o+} = 1.29 \times 10^{11}$ eV and $\sum_{i=1}^{6} q_i^{o-} = 8.189 \times 10^{-8}$ eV. Define therefore the linear combination $q_i^{o+} - q_i^{o-}$ and sum together all $i$-th terms; one obtains again a total energy

$$\varepsilon_H = 129 \text{ GeV}.$$

Regardless of the numerical values, however, the physical meaning of each term $q_i^{o+} - q_i^{o-}$ is profoundly different from that of the terms $q_i^{o+}$ and $q_i^{o-}$ regarded separately: the masses $m_i$, charges $c_i$, spins $s_i$, colors $C_i$ and so on of these virtual particles, expectedly the same for $q_i^{o+}$ and $q_i^{o-}$ whatever they might be as a consequence of eq (6,14), subtract each other and thus do no longer appear in $q_i^{o+} - q_i^{o-}$. This point is easily highlighted and explained. Actually the eq (6,14) establishes the numerical values of the new energies $q_i^{o+}$ and $q_i^{o-}$, not their specific forms about which nothing has been hypothesized or is known. The most natural way to regard these quantities, in full line with the basic ideas of the present model, is to relate the various $q_i^o$ to appropriate energy uncertainty ranges as done in eq (2,4); this means assuming for instance

$$q_i^{o+} = \varepsilon_i^+(m_i, c_i, s_i, C_i, ..) - \varepsilon_i^+(0, 0, 0, 0, ..)$$
$$q_i^{o-} = \varepsilon_i^-(m_i, c_i, s_i, C_i, ..) - \varepsilon_i^-(0, 0, 0, 0, ..)$$

with

$$\varepsilon_i^-(m_i, c_i, s_i, C_i, ..) \approx \varepsilon_i^-(0, 0, 0, 0, ..)$$

as well. As repeatedly stressed, both boundaries of any uncertainty ranges are arbitrary. Here we are interested to consider in particular ranges fulfilling the following condition about the upper boundaries:

$$\varepsilon_i^+(m_i, c_i, s_i, C_i, ..) = \varepsilon_i^-(m_i, c_i, s_i, C_i, ..).$$

These positions agree with $q_i^{o+} >> q_i^{o-}$ and also yield

$$q_i^{o+} - q_i^{o-} = \varepsilon_i^-(0, 0, 0, 0, ..) - \varepsilon_i^+(0, 0, 0, 0, ..)$$

that defines $q_i^{o+} - q_i^{o-}$ as the energy uncertainty range of a massless, spinless, chargeless, colorless,.. virtual particle, having in particular boson character. So, when summing up all these terms one finds a total boson energy having the value just quoted. This peculiar energy that accounts for the lepton and quark masses corresponds to a *composite* particle consisting of the sum of 6 terms $q_i^{o+} - q_i^{o-}$ rather than to a truly elementary particle. This conclusion is supported by the fact that the lifetime $\Delta t_H$ of such a particle should reasonably result from that of its longest life constituent term with $i = 1$, i.e. $\Delta t_H = \hbar/(q_1^{o+} - q_1^{o-})$; one calculates in this way via eq (6,14)

$$q_1^{o+} - q_1^{o-} = 2.50 \text{ MeV}, \qquad \Delta t_H = 2.63 \times 10^{-22} \text{ s}.$$

These last results are reasonable and fully agree with the outcomes of recent experimental measurements.

## 7 The quantum statistical distributions

This section investigates further consequences of eq (2,2). This part of the paper is thus significant because just this equation leads to eq (2,4), which has been heavily involved to infer the asymptotic freedom equation (5,7) of quarks and the masses of quarks and leptons; confirming once more eq (2,2) means therefore to correlate these results to another fundamental topic of quantum physics concerned in the present section, i.e. the statistical distributions of quantum particles. Eqs (1,1) link the energy range $\Delta\varepsilon$ including the possible energies of a quantum system to its number $n$ of allowed states: the change of energy range size $\delta\Delta\varepsilon = (\hbar/\Delta t)\delta n$ during a given time rage $\Delta t$ has been concerned in section 2 to calculate the related change $\delta n = n_2 - n_1$ of $n$, thus obtaining eq (2,4). In that case $n_1$ was regarded as a fixed quantity, i.e. as a reference number of states as a function of which to define $\delta n$. Now we generalize these ideas: both $n_1$ and $n_2$ are allowed to change in a quantum system characterized by an initial number of states $n_o$. If so $\hbar/\Delta t$ can be identically rewritten as $\hbar/\Delta t = \Delta\varepsilon_{n_1}/n_1$ or $\hbar/\Delta t = \Delta\varepsilon_{n_2}/n_2$, because both right hand sides are equivalent reference states in defining $\delta n$. So, being both chances alike as well, it is reasonable to expect that $\hbar/\Delta t \propto K_t/(n_1 n_2)$ with $K_t = K_t(\Delta t)$ proportionality factor having physical dimensions of an energy. This position is possible in principle because $\Delta t$ is arbitrary; so, whatever $n_1$ and $n_2$ might be, certainly exists a time length $\Delta t = \Delta t(n_1, n_2)$ that fulfills the proposed correlation. From a formal point of view, assume that $\Delta\varepsilon/n$ of the system is described during $\Delta t$ by the linear combination $a_1\Delta\varepsilon_{n_1}/n_1 + a_2\Delta\varepsilon_{n_2}/n_2$, being $a_1$ and $a_2$ appropriate time dependent coefficients; if so, then $K_t = a_1 n_2 \Delta\varepsilon_{n_1} + a_2 n_1 \Delta\varepsilon_{n_2}$ is defined just by the equation $\delta\Delta\varepsilon/\delta n = \hbar/\Delta t = K_t/(n_1 n_2)$. Since all quantities at right hand side are arbitrary, for simplicity let us approach the problem in the particular case where $K_t$ is regarded as a constant in the following. This chance is obviously also obtainable defining appropriately $a_1$ or $a_2$ or both during $\Delta t$. The following discussion will show that even this particular case is far reaching and deserves attention.

Write $n_2 = n_o \pm j$ and $n_1 = \pm j$, being $n_o$ a reference fixed number of states and $j$ a variable integer accounting for the change of $n_1$ and $n_2$; of course both $n_o$ and $j$ are arbitrary and independent each other, which yields indeed $n_2 - n_1 = n_o$ or $n_2 - n_1 = n_o \pm 2\delta j$ depending on the signs of $j$. In this way it is possible to describe a steady system with its $n_o$ initial states or an evolving system where is allowed a new number $n' \neq n_o$ of states; since now both $n_1$ and $n_2$ are allowed to change, $\delta n = \pm 2\delta j$. Simplifying the notations, the equation inferred from $\delta\Delta\varepsilon/\delta n = K_t/(n_1 n_2)$ of interest for the following discussion reads

$$\frac{\delta\Delta\varepsilon_j}{\delta j} = \frac{2K}{j(n_o \pm j)}, \qquad \delta j = 1,\ 2,.. \tag{7,1}$$

where $K$ must be intended as the constant replacing $K_t$ previously introduced; it is allowed to take both signs, which is avoids writing explicitly $\pm\delta j$. The notation $\Delta\varepsilon_j$ emphasizes the variable number of states appearing at right hand side. To proceed on, consider the case where both $j$ and $n_o$ are large enough to regard approximately the former as a continuous variable, so that $\delta j << j$; so the left hand side can be handled, for mere computational purposes only, as $d\Delta\varepsilon_j/dj$; hence $\Delta\varepsilon_j$ calculated solving the differential equation, results to be

$$\Delta\varepsilon_j = (K'\varepsilon_o/n_o)\log(n_o/j \pm 1) + const, \quad 2K = -K'\varepsilon_o, \tag{7,2}$$

being *const* the integration constant; $K'$ is an arbitrary dimensionless constant and $\varepsilon_o$ an arbitrary constant energy. Consider now two boundary conditions of eq (7,2) concerning the respective limit cases (i) $n_o << j$ and (ii) $n_o >> j$. From a mathematical point of view, note that eq (7,2) is obtained by integration of eq (7,1) with respect to $j$ regardless of $n_o$; hence one could think the cases (i) and (ii) as due to fixed integration limits on $dj$ for two different values of $n_o$ consistent with either inequality, of course without modifying the result of the integration and the subsequent considerations.

In the case (i) holds $n_o/j + 1$ only; putting $const = 0$ and expanding in series the logarithmic term, the right hand side of eq (7,2) reads

$$\begin{gathered} \Delta\varepsilon_j = \frac{w_j K'\varepsilon_o}{j}, \\ w_j = 1 - \frac{n_o}{2j} + \frac{n_o^2}{3j^2} - \ldots, \quad 0 < w_j < 1. \end{gathered} \tag{7,3}$$

Let $j$ be defined between two arbitrary numbers of states $j_1$ and $j_2 > j_1$; moreover define now $K'$ in order that the sum of all terms $K'w_j$ introduced in the last equation over all values of $j$ fulfills the following condition

$$j_1 \leq j \leq j_2, \qquad \pi_j = K'w_j, \qquad K'\sum_{j_1}^{j_2} w_j = \sum_{j_1}^{j_2} \pi_j = 1;$$

then the result is

$$\pi_j = \frac{j\Delta\varepsilon_j}{\sum_{j=j_1}^{j_2} j\Delta\varepsilon_j}, \quad \varepsilon_o = \sum_{j=j_1}^{j_2} j\Delta\varepsilon_j, \quad \frac{n_o}{j_1} << 1. \tag{7,4}$$

The inequality ensures that is fulfilled the initial condition of the case (i) concerned here, whereas the first eq (7,4) shows the probabilistic character of $\pi_j$ resulting from the previous positions.

Consider now the limit case (ii). Despite the second eq (7,3) requires in principle a very large number of series terms to express $n_o/j >> 1$, even tending to infinity, there is no reason to exclude that the second equation (7,4) defining $j\Delta\varepsilon_j$ still holds: being $K'$ arbitrary, it can be still defined in order to fulfill the inequality $K'\Sigma_j(1 - n_o/2j + n_o^2/3j^2 + ..) < 1$ whatever the ratio $n_o/j$ might be. On the one hand this inequality can be accepted in principle even though the series consists of an infinite number of terms; in fact the series does not need to be explicitly computed, which makes plausible also the position $\pi_j = K'w_j$. On the other hand, however, in this way the result $j\Delta\varepsilon_j = K'\varepsilon_o w_j$ is not explicitly inferred: the left hand side of the last inequality is indeed undefined. Otherwise stated, without the straightforward hint coming from the case (i) the eqs (7,4) could have been hypothesized only and then still introduced in the case (ii) as plausible inputs but without explanation. Actually, the assessment of the limit case (i) and the subsequent considerations on $w_j K'$ are the points really significant of the present reasoning: while extending the physical meaning of $\pi_j$ and $j\Delta\varepsilon_j$ also to the case (ii), they ensure the compatibility of the limit cases (i) and (ii). Once again, the arbitrariness of the numbers of states plays a key role to carry out the reasoning.

Looking back to eq (7,2) and multiplying by $j$ both sides, let us write

$$j\Delta\varepsilon_j = K'\varepsilon_o(j/n_o)\log(n_o/j \pm 1) + const\, j. \tag{7,5}$$

According to eqs (7,4) $j\Delta\varepsilon_j/K'\varepsilon_o = w_j$; so, neglecting 1 with respect to $n_o/j$ in agreement with the present limit case (ii) and summing all terms $w_j$, eq (7,5) yields

$$W = -\sum_{j=j_1}^{j_2}\left(\frac{j}{n_o}\right)\log\left(\frac{j}{n_o}\right) - \sigma\frac{const}{K'\varepsilon_o}, \qquad \sigma = \sum_{j=j_1}^{j_2} j. \tag{7,6}$$

It is useful now to rewrite eq (7,6) as a function of a new variable $\xi_j$

$$W = -q\sum_{j=j_1}^{j_2}\xi_j\log(\xi_j), \quad const = -\frac{K'\varepsilon_o}{n_o}\log(q), \quad \xi_j = \frac{j}{n_o q},$$

where $q$ is a proportionality factor not dependent on $j$; it has been defined according to the second equation to eliminate the second constant addend of eq (7,6). The next step is to define $j$, so far simply introduced as an arbitrary integer without any hypothesis on its actual values, in order that $W$ has specific physical meaning with reference to a thermodynamic system characterized by a number $s$ of freedom degrees. To this purpose assume that $j$ can take selected values $n^s$ only, with $n$ arbitrary integer. This is certainly possible:

nothing hinders calculating the eq (7,2) as a function of $n_o/n^s$ instead of any $j$ progressively increasing; in this way also the eq (7,6) accordingly calculated takes a specific physical meaning consistent with that of the ratios $n^s/n_o$. Clearly this does not mean trivially renaming $j$: now $n^s$ reads $\Delta x\Delta p/\hbar^s$, where $\Delta x = \Delta x_1 \cdot \cdot \Delta x_s$ and $\Delta p = \Delta p_1 \cdot \cdot \Delta p_s$. Since therefore $\Delta x\Delta p$ symbolizes a volume in a $s$-dimensional phase space, $\Delta x\Delta p/\hbar^s$ represents the number of states allowed in this volume. It is known that this ratio introduces the statistical formulation of the entropy [16]; so putting $const/K'\varepsilon_o$ proportional to a new quantity $S_0$, one finds

$$S = -q\sum_{n=n_1}^{n_2}\xi_n\log(\xi_n), \qquad S_0 = -q\log(\Omega),$$
$$\frac{const}{K'\varepsilon_o} = \frac{1}{\zeta}\frac{S_0}{q}, \qquad \Omega = q^{\zeta/n_o}. \tag{7,7}$$

The notation of the first sum emphasizes that now $j$ takes values corresponding to the possible $n^s$. The constant of eq (7,6) has been therefore related in the last equation to $S_0$. The second equation can be regarded as a particular case of the former when the thermodynamic probabilities $\xi_j$ are all equal; while in eq (7,2) $j$ was an arbitrary number progressively increasing fro $j_1$ to $j_2$, in eq (7,7) its relationship to $n^s$ does not exclude the chance of coincident values for equal volumes of phase space. It is well known that the results so far exposed introduce the statistical definition of entropy a trivial proportionality factor apart. Note that this result has been obtained in a very different context [12], i.e. to show the quantum character of the Fick diffusion laws as a consequence of eqs (1,1) only; despite the different topic, the theoretical frame is however exactly the same as that hitherto concerned.

Let us return now to the early eq (7,2). Define as usual the energy range as $\Delta\varepsilon_j = \varepsilon'' - \varepsilon'$, so that the eq (7,2) reads $n_o(const + \varepsilon' - \varepsilon'')/K = \log(n_o/j \pm 1)$. Exploit once again the fact that in general the boundary values of the uncertainty ranges are arbitrary; hence, whatever the sign and values of $K$ and $const$ might be, the left hand side can be rewritten as $(\varepsilon_j - \varepsilon_o)/K$, being of course both $\varepsilon_j$ and $\varepsilon_o$ still arbitrary. So the number of states $j$ of the eq (7,2) reads

$$j = \frac{n_o}{\exp((\varepsilon_j - \varepsilon_o)/K) \mp 1}, \quad \Delta\varepsilon = \varepsilon_j - \varepsilon_o = n_o(const + \varepsilon' - \varepsilon'').$$

The second equation reports again the starting point from which is inferred the former equation to emphasize that, despite the arbitrariness of the boundary values that define the size of the energy uncertainty range, the specific problem determines the values of physical interest. For instance in eq (2,6) has been inferred the Planck law identifying $\Delta\varepsilon_j$ with $h\Delta\nu_j$; clearly the number of states therein appearing is to be identified here with $j$, whereas $n_o$ can be taken equal to 1 because the photons are bosons. Here the upper sign requires signs of $K$ and $\varepsilon_j - \varepsilon_o$ such that $(\varepsilon_j - \varepsilon_o)/K > 0$ because the number of states $j$ must be obviously positive; instead the lower sign allows in principle both $\varepsilon_o < \varepsilon_j$ and $\varepsilon_o > \varepsilon_j$, as in effect it is well known. To understand these conclusions, let us exploit the reasonable idea that the number $j$ of states allowed for a quantum system is related to the number $N$ of particles of the system. Recall another result previously obtained exploiting eqs (1,1) [7]: half-integer spin particles can occupy one quantum state only, whereas one quantum state can be occupied by an arbitrary number of integer spin particles. In the former case therefore $j$ is directly related to $N$, i.e. $j = N$ and $n_o = 1$, in the latter case instead in general $N >> j$ without a specific link between $j$ and $N$. Yet the arbitrariness of $n_o$ makes $j$ suitable to represent any $N$ also in this case as $N = \sum j = n_o\sum(\exp(\Delta\varepsilon_j/K) - 1)^{-1}$. In the classical case where $\Delta\varepsilon_j >> K$, this equation is the well known partition function.

## 8 Discussion

After the early papers concerning non-relativistic quantum physics [5,6], the perspective of the eqs (1,1) was extended to the special and general relativity; the gravitational interaction was indeed inferred as a corollary just in the present theoretical frame. The problem of examining more in general also other possible forms of quantum interaction appeared next as a natural extension of these results. This paper aimed indeed to infer some basic concepts on the fundamental interactions possible in nature. Even without ambition of completeness and exhaustiveness, the chance of finding some outstanding features unambiguously typical of the electromagnetic, weak and strong interactions has the heuristic value of confirming the fundamental character of eqs (1,1): seems indeed significant that the weird peculiarities of the quantum world are directly related not only to the physical properties of the elementary particles but also to that of their fundamental interactions, which are described in a unique conceptual frame including also the gravity and the Maxwell equations [7]. Now also the gravitational coupling constant, so far not explicitly concerned, is inferred within the proposed conceptual frame. The starting point is again the eq (2,7) rewritten as follows

$$v'_x = -\frac{\Delta F_x\Delta x^2}{n\hbar}, \qquad v'_x = \frac{d\Delta x}{d\Delta t}, \qquad \Delta F_x = F_x - F_{ox}. \tag{8,1}$$

By means of this equation the paper [7] has emphasized the quantum nature of the gravity force, approximately found equal to $\Delta F_x = Gm_am_b/\Delta x^2$ for two particles of mass $m_a$ and $m_b$; also, the time dependence of $p_x$ or $p_{ox}$ of $\Delta p_x = p_x - p_{ox}$ was alternatively introduced to infer the equivalence principle of relativity as a corollary. In the present paper, instead, both boundary values of the momentum component range have been concurrently regarded as time dependent to infer the expected potential energy (5,2) of the strong interactions: the reasoning is in principle identical, although merely carried out in a more general way; the form of eq (5,2) comes putting in eq (2,7) both $\dot{p}_x \neq 0$ and $\dot{p}_{ox} \neq 0$, which is the

generalization of the relativistic reasoning carried out in [7]. In fact the eq (2,7), straightforward consequence of eqs (1,1) and thus valid in general, has been reported also in the present paper to better understand these results through its underlying reasoning: what changes is the way it can be exploited to describe specific physical problems, as it has been also emphasized about the physical meaning of $v'_x$. Now we are interested to implement a particular case of eq (2,7), i.e. the Coulomb law quoted in eq (2,8). The procedure followed below does not need any additional hypothesis with respect to these considerations: it is enough to specify appropriately $\Delta F_x$ in eq (8,1).

Consider first the eq (2,8): in the particular case $e' = e$ it yields the Coulomb law $F_x - F_{ox} = \Delta F_x = \pm e^2/\Delta x^2$. Replace this expression into eq (8,1), which reads then

$$v'_x = \pm e^2/n\hbar. \tag{8,2}$$

The ± sign is a trivial feature of the velocity component $v'_x$ along the arbitrary $x$-axis, it is in fact of scarce interest for the purposes of the present discussion. More interesting is the fact that putting $v'_x = (\alpha/n)c$, as done to infer eq (2,9), one obtains the identity $\alpha/n = e^2/n\hbar c$. This result supports the idea that $v'_x/c$ of eq (8,2) effectively represents a coupling constant: it reads $\alpha/n$, just the electromagnetic coupling constant found in eq (1,4).

Consider now the gravity force $\Delta F_x = Gm_a m_b/\Delta x^2$ and replace this expression into eq (8,1): so $v'_x = Gm_a m_b/n\hbar$. Comparing this result with the case of the electric force propagating between charged masses, one finds

$$\alpha_G = v'_x/c = Gm_a m_b/n\hbar c. \tag{8,3}$$

Is obvious the reason why the gravitational coupling constant, recognizable at the right hand side, has been formally obtained through elementary considerations identical to that of eq (8,2): the unique eq (8,1) turns into either result simply depending on whether one replaces $\Delta F_x$ with $e^2/\Delta x^2$ or $Gm_a m_b/\Delta x^2$. Eqs (8,2) and (8,3) suggest that the gravitational and electromagnetic field propagate at the same rate $c$: as emphasized when discussing the physical meaning of $v_x$ and $v'_x$ in section 2, the latter is the deformation rate of the space-time range $\Delta x$ that determines $\Delta F_x$, whereas is instead $v_x$ the real propagation rate of the respective messenger particles in the interaction space-time range $\Delta x$; in both cases $\Delta x/\Delta t = c$.

These results are not end points, they have heuristic character. Let us start from eq (8,3) considering for simplicity $m_a = m_b = m$, so that $m = m_P\sqrt{n\alpha_G}$; i.e. any $m$ is proportional to the Planck mass, the proportionality factor being just $\sqrt{n\alpha_G}$. Owing to the small values of $\alpha_G$, one expects that large values of $n$ are required to fit even small masses. Although $\alpha_G$ depends in general on the specific values of the masses, it is interesting to examine its minimum value corresponding to the particular case where both $m_a$ and $m_b$ represent the lightest elementary particle, the electron neutrino. As concerns the ratio $m_{\nu e}/m_P$ note that $m_{\nu e}$ is a real particle, $m_P$ is a mere definition; so for the former only holds the idea that any particle confined in an arbitrary uncertainty range $\Delta x$ is characterized in principle by a momentum component gap $\Delta p_x = p_x^{conf} - p_x^{\infty}$ with respect to an ideal unconfined state, see eq (2,1). For the reasoning is irrelevant how an electron neutrino could be confined in practice, because $\Delta x$ is arbitrary; it could even be the full diameter of the whole universe. It is instead significant in principle that, as already shown in section 4 about the weak interaction boson vectors, it is possible to write for the electron neutrino a delocalization energy $\Delta\varepsilon_{\nu e} = \Delta p_x^2/2m_{\nu e}$ valid for any real object; this reasoning has been in effect exploited in eq (4,13). These considerations aim to conclude that, whatever $\Delta p_x$ might be, the equation

$$m_{\nu e} = \Delta p_x^2/2\Delta\varepsilon_{\nu e} \qquad \Delta\varepsilon_{\nu e} = m_{\nu e}c^2 \tag{8,4}$$

suggests $m_{\nu e}$ proportional to a reciprocal energy range $\Delta\varepsilon_{\nu e}$ that in turn should be proportional to $c^2$. If this reasoning is physically sensible, then $m_{\nu e}/m_P \propto c^{-2}$ suggests by consequence $m_{\nu e}/m_P \propto \alpha^2$; since the fine structure constant is proportional itself to $c^{-1}$, this position simply means including $e^2/\hbar$ into the proportionality constant. Write therefore

$$m_{\nu e}/m_P = \alpha^2/N$$

having called $1/N$ the proportionality constant. The ratio at left hand side is immediately calculated with the help of the first value (6,8), it results equal to $1.5\times10^{-28}$; the factor $\alpha^2 \approx 5.3\times10^{-5}$ calculates $N$ equal to $3.5\times10^{23}$, a value surprisingly similar to well known $N = 6.02\times10^{23}$ for the ratio at right hand side. The agreement between these values is really unexpected: while the position $m_{\nu e}/m_P \propto \alpha^2$ could be acceptable at least in principle, is really difficult to understand what the Avogadro number has to do with the present problem. A reasonable idea is to regard $\alpha^2/N$, perhaps a mere numerical accident, as a whole factor between ordinary mass units and Planck mass units. To support this statement replace in eq (8,4) $\Delta\varepsilon_{\nu e}$ with $m_{\nu e}c^2$, regarded as the average of the boundary values of $\Delta\varepsilon_{\nu e}$; for the following order of magnitude estimate this replacement is acceptable. So, recalling that $\Delta p_x^2 = (n\hbar/\Delta x)^2$ and that actually to calculate $\Delta\varepsilon_{\nu e}$ one should consider $\Delta p_x^2 + \Delta p_y^2 + \Delta p_z^2$, eq (8,4) reads $\Delta x = n\hbar c\sqrt{3/2}/m_{\nu e}c^2$; putting $n = 1$, one finds $\Delta x = 1.3\times10^{-7}$m. Replace now $\Delta\varepsilon_{\nu e}$ with $(N/\alpha^2)\Delta\varepsilon_{\nu e}$: the factor previously found to convert $m_{\nu_e}$ into Planck mass units should now convert the energy $\varepsilon_{\nu e}$ from the ordinary units into Plank energy units. Indeed $\Delta x = n\hbar c\sqrt{3/2}\alpha^2/Nm_{\nu e}c^2$ calculated again with $n = 1$ results equal to $1.1\times10^{-35}$m, which is reasonably comparable with the Planck length $l_P = 1.6\times10^{-35}$m. Actually this result could be expected, because it is based on regarding the energy $\Delta\varepsilon_{\nu e} = \Delta p_x^2/2m_{\nu e}$ as $\Delta\varepsilon_{\nu e} = \Delta p_x^2c^2/2\Delta\varepsilon_{\nu e}$, as already done in section 4; accordingly, this means identifying $\Delta\varepsilon_{\nu e}$ calculated from the confinement uncertainty equation with the mass $m_{\nu e}$

of the particle itself via the factor $c^2$. This idea was found reasonable to calculate the characteristic length of the weak interaction, eq (4,14), and appears adequate also here because it shows that the conversion factor of $m_{ve}$ into $m_P$ also converts $\varepsilon_{ve}$ into $E_P$.

The main reason for having proposed this result is to stimulate (i) further considerations on the link between $\alpha$ and $\alpha_G^{(ve)}$ and (ii) a greater attention to $N$ when searching fundamental relationships between the constants of nature. Another numerical accident, which is worth noticing here because perhaps of possible interest, concerns the key coefficients (6,10); indeed $\pi a/b = 137.469$, which differs from 137.036 by about 0.3% only. It has been remarked the obvious fact that even small deviations of any lepton or quark mass from the input values (6,2) and (6,7), (6,8) affect the regression coefficients (6,10). So, at least from a numerical point of view, it is sensible to suppose that a very fine-tuning of some among these input values could match exactly the fine structure constant. This optimization is certainly justified: indeed the electron, muon and tau masses only are experimentally known with a degree of accuracy such to exclude any minimum revision; instead, for the reasons previously remarked, there are ample margins of small adjustment for the neutrino and isolate quark masses implemented in the present calculations. On the one hand, such an effort is physically sensible only guessing a good physical reason to expect that the regression coefficients should be actually related to $\alpha$; on the other hand is evident the interest to provide such an explanation, wholly physical and not merely numerical, of the coefficients that determine the fundamental masses of our universe.

Some further points are still to be better clarified; they pose several questions, some of which are still unanswered. One of them concerns the correspondence (6,9) between leptons and quarks: is it really mere consequence of the increasing order of their masses, thus a mere definition to exploit eq (2,4), or is it actually due to something else still hidden in the correspondence (6,9) and not yet evidenced? But perhaps the most amazing point is that also the leptons fulfill the eq (2,4) just thanks to this correspondence. In the case of quarks, the dependence of their masses on $i \equiv \delta n$ was tentatively explained through the self-interaction of bare quarks with their own clouds of gluons and the self-interaction between these latter: with reference to eq (2,1), a different interaction strength is related both to a dissimilar $n/m$ and to a dissimilar $n/V$, thus explaining not only the different $m$ of the various quarks but also the equations (5,3) and (6,5). Yet thereafter also the leptons have been handled through the eq (2,4) simply guessing an analogy of behavior for both kinds of fundamental particles of our universe. But, strictly speaking from a physical point of view, why should the lepton masses depend on $\delta n$? On the one side the extension of the eq (2,4) certainly works well, because the well known masses of electron, muon and tau particles fit the proposed scheme; the fact of having included these masses among the results calculated through eq (6,10) supports also the values of the masses not experimentally available. On the other side, however, in lack of a self-interaction mechanism characteristic of the quarks only, the question arises: is justified a similar mechanism for the vacuum polarization around the real charges with formation of virtual particle-antiparticle pairs? Does the interaction between these couples of virtual particles/antiparticles surrogate the self-interaction of the quark-gluon plasma? Work is in advanced progress on these points.

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