# On the Luminosity Distance and the Hubble Constant

Yuri Heymann

3 rue Chandieu, 1202 Geneva, Switzerland. E-mail: y.heymann@yahoo.com

By differentiating luminosity distance with respect to time using its standard formula we find that the peculiar velocity is a time varying velocity of light. Therefore, a new definition of the luminosity distance is provided such that the peculiar velocity is equal to $c$. Using this definition a Hubble constant $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is obtained from supernovae data.

## 1 Introduction

The luminosity distance is an important concept in cosmology as this is the distance measure obtained from supernovae data using the distance modulus. The standard formula of the luminosity distance is $d_L = (1+z)\,d_M = d_M/a$, where $d_L$ is the luminosity distance and $d_M$ the comoving transverse distance [1, p. 421]. As shown below this definition implies that the peculiar velocity is a time varying velocity of light, and therefore a new definition is proposed where the speed of light is constant.

## 2 Definition of the luminosity distance and the peculiar velocity from light propagation

From there we will use the notation $r_L$ for the luminosity distance as it represents the radius of a sphere for light propagating from the center which is the point of emission of the light source. The standard formula of the luminosity distance for a flat Universe is as follows:

$$r_L = \frac{\chi}{a}\,, \tag{1}$$

and

$$\chi = c\int_0^{t_0}\frac{dt}{a}\,, \tag{2}$$

where $r_L$ is the luminosity distance, $\chi$ the comoving distance, $a$ the scale factor at the time of emission, $t$ the time which is equal to zero at the origin set at the center of the sphere from which light is emitted, and $t_0$ the time when light reaches the earth.

Let us apply the change of coordinates $T = t_0 - t$, where $T$ is the light travel time between the observer and the photon. Hence, $dt = -dT$, and (2) can be rewritten as follows:

$$\chi = -c\int_T^0\frac{dT}{a} = c\int_0^T\frac{dT}{a}\,. \tag{3}$$

By differentiating (1) with respect to $T$ we get:

$$\frac{dr_L}{dT} = \frac{\dot{\chi}}{a} - \frac{\dot{a}}{a^2}\chi\,. \tag{4}$$

As $I = \int_{t_1}^{t_2} f(t)\,dt$ leads to $\frac{dI}{dt} = \frac{dt_2}{dt}\,f(t_2) - \frac{dt_1}{dt}\,f(t_1)$, from (3) we get:

$$\dot{\chi} = \frac{c}{a}\,. \tag{5}$$

Using (1) we get:

$$\frac{\dot{a}}{a^2}\chi = \frac{\dot{a}}{a}\,r_L\,. \tag{6}$$

Because $H = \frac{1}{a}\frac{da}{dt} = -\frac{1}{a}\frac{da}{dT}$, equation (6) can be rewritten as follows:

$$\frac{\dot{a}}{a^2}\chi = -H\,r_L\,. \tag{7}$$

Combining (4), (5) and (7) we get:

$$\frac{dr_L}{dT} = \frac{c}{a^2} + H\,r_L\,. \tag{8}$$

The term $H\,r_L$ represents the expansion for the radius of our sphere, and $\frac{c}{a^2}$ is the peculiar velocity. From light propagation we see that the standard formula of luminosity distance implies a time varying velocity of light.

A new equation is proposed for the luminosity distance where the peculiar velocity is always equal to $c$. Considering a sphere of radius $r'_L$ for the propagation of light emitted from a point at the center, and that the sphere inflates over time due to the expansion of the Universe and the velocity of light, we obtain:

$$\frac{dr'_L}{dT} = c + H\,r'_L\,, \tag{9}$$

with boundary condition $r'_L = 0$ at $T = 0$. Where $r'_L$ is the luminosity distance, $T$ the light travel time between emission and reception of the light source, and $H$ the Hubble constant at time $T$.

## 3 Solving the equation of the luminosity distance

In this section we assume that the Hubble constant does not vary over time and is always equal to $H_o$.

By integrating (9) we get:

$$r'_L = \frac{c}{H_0}\left(\exp(H_0\,T) - 1\right)\,. \tag{10}$$

This equation can be rewritten as follows:

$$T = \frac{1}{H_0}\ln\left(1 + \frac{H_0}{c}\,r'_L\right)\,. \tag{11}$$

The expression of the light travel time versus redshift is as follows:

$$T = \int_{1/(1+z)}^{1}\frac{da}{H\,a} = \frac{1}{H_0}\ln(1+z)\,. \tag{12}$$

By combining (11) and (12) we get:

$$r'_L = \frac{c}{H_0} z\,. \tag{13}$$

## 4 Calculation of the Hubble constant from supernovae data

Let us compute the Hubble constant from supernovae using the relationship in (13) . In order to compute the luminosity distance we use the redshift adjusted distance modulus provided in [2] which is as follows:

$$m - M = -5 + 5\log r'_L + 2.5\log(1+z)\,. \tag{14}$$

The distance modulus $\mu = m - M$ is the difference between the apparent magnitude $m$ and the absolute magnitude $M$.

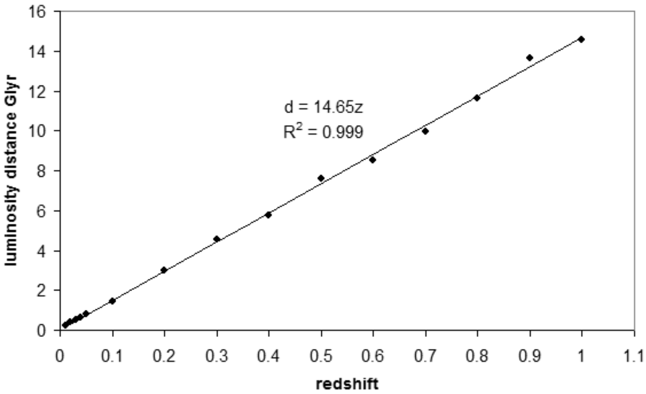


Fig. 1: Luminosity distance in Glyr versus redshift plot for supernovae. Data source: http://supernova.lbl.gov/Union/

In Fig. 1 we have a plot of the luminosity distance versus redshift that was obtained with (14) using supernovae data. This plot is rectilinear with a slope of 14.65 where the luminosity distance is expressed in *Glyr* (billion light years). The Hubble constant which is the inverse of the slope from (13) is equal to $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

## 5 Conclusion

In this study it has been shown that the standard formula of the luminosity distance implies that the peculiar velocity is a time varying velocity of light. Given our choice for the luminosity distance equation which is based on a peculiar velocity always equal to $c$, we find that the solution to this equation requires a Hubble constant that does not change over time in order to fit the supernovae data.

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### References

1. Weinberg S. Gravitation and Cosmology: Principles and applications of the general theory of relativity. John Wiley and Sons, 1972.
2. Heymann Y. Redshift Adjustment to the Distance Modulus. *Progress in Physics*, 2012, v. 1, 6–7.