The Heisenberg Uncertainty Principle and the Nyquist-Shannon Sampling Theorem

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The derivation of the Heisenberg Uncertainty Principle (HUP) from the Uncertainty Theorem of Fourier Transform theory demonstrates that the HUP arises from the dependency of momentum on wave number that exists at the quantum level. It also establishes that the HUP is purely a relationship between the effective widths of Fourier transform pairs of variables (i.e. conjugate variables). We note that the HUP is not a quantum mechanical measurement principle *per se*. We introduce the Quantum Mechanical equivalent of the Nyquist-Shannon Sampling Theorem of Fourier Transform theory, and show that it is a better principle to describe the measurement limitations of Quantum Mechanics. We show that Brillouin zones in Solid State Physics are a manifestation of the Nyquist-Shannon Sampling Theorem at the quantum level. By comparison with other fields where Fourier Transform theory is used, we propose that we need to discern between measurement limitations and inherent limitations when interpreting the impact of the HUP on the nature of the quantum level. We further propose that while measurement limitations result in our perception of indeterminism at the quantum level, there is no evidence that there are any inherent limitations at the quantum level, based on the Nyquist-Shannon Sampling Theorem.

1 Introduction

The Heisenberg Uncertainty Principle is a cornerstone of quantum mechanics. As noted by Hughes [1, see pp. 265–266], the interpretation of the Principle varies

- from expressing a limitation on measurement as originally derived by Heisenberg [2] (Heisenberg's microscope),
- to being the variance of a measurement carried out on an ensemble of particles [3] [4],
- to being inherent to a microsystem [5], meaning essentially that there is an indeterminism to the natural world which is a basic characteristic of the quantum level.

Greenstein retains only the first and last alternatives [6, see p. 51].

However, the Heisenberg Uncertainty Principle can be derived from considerations which clearly demonstate that these interpretations of the principle are not required by its mathematical formulation. This derivation, based on the application of Fourier methods, is given in various mathematical and engineering textbooks, for example [7, see p. 141].

2 Consistent derivation of the Heisenberg Uncertainty Principle

In the Fourier transform literature, the Heisenberg Uncertainty Principle is derived from a general theorem of Fourier theory called the Uncertainty Theorem [7]. This theorem states that the effective width of a function times the effective width of its transform cannot be less than a minimum value given by

$$W(f)\, W(\tilde{f}) \geqslant 1/2 \tag{1}$$

where f is the function of interest and $\tilde{f}$ is its Fourier transform. $W(f)$ is the effective width of function f, defined by

$$|W(f)|^2 = \frac{\int_{-\infty}^{\infty} |f(u)|^2 [u - M(f)]^2 du}{\int_{-\infty}^{\infty} |f(u)|^2 du} \tag{2}$$

and $M(f)$ is the mean ordinate defined by

$$M(f) = \frac{\int_{-\infty}^{\infty} |f(u)|^2 u du}{\int_{-\infty}^{\infty} |f(u)|^2 du}. \tag{3}$$

There are several points that must be noted with respect to this derivation:

Eq.(1) applies to a Fourier transform pair of variables. Taking the simple case of time t and frequency ν to illustrate the point: If we consider the function f to be the function that describes a time function t, then the width of the function, $W(f)$, can be denoted as $W(f) = \Delta t$. The Fourier transform of function t is the frequency function ν and the width of this function can be denoted as $W(\tilde{t}) = W(\nu) = \Delta\nu$. Substituting in (1), the Uncertainty Theorem then yields

$$\Delta t\, \Delta\nu \geqslant 1/2. \tag{4}$$

However, if one wishes to use the circular frequency $\omega = 2\pi\nu$ instead, (4) becomes

$$\Delta t\, \Delta\omega \geqslant \pi. \tag{5}$$

It is thus necessary to take special care to clearly identify the Fourier transform variable used as it impacts the R.H.S. of the resulting Uncertainty relation (see for example [8] and [9, pp. 21–22]).

Equations (4) and (5) above correspond to the following definitions of the Fourier transform respectively [8]:

Equation (4):

$$f(t) = \int_{-\infty}^{\infty} \tilde{f}(\nu) \exp(2\pi i \nu t) d\nu \tag{6}$$

$$\tilde{f}(\nu) = \int_{-\infty}^{\infty} f(t) \exp(-2\pi i \nu t) dt \tag{7}$$

Equation (5):

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega) \exp(i\omega t) d\omega \tag{8}$$

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-i\omega t) dt \tag{9}$$

Sometimes the factor $1/2\pi$ is distributed between the two integrals (the Fourier and the Inverse Fourier Transform Integrals) as $1/\sqrt{2\pi}$. In Physics, (8) and (9) are preferred, as this eliminates the cumbersome factor of 2π in the exponential (see for example [10, p. 12]), but care must then be taken to ensure the resulting factor of $1/2\pi$ in (8) is propagated forward in derivations using that definition.

Using the relation $E = h\nu$, where h is Planck's constant, in (4) above, or the relation $E = \hbar\omega$, where $\hbar = h/2\pi$, in (5) above, one obtains the same statement of the Heisenberg Uncertainty Principle namely

$$\Delta E \, \Delta t \geqslant h/2 \tag{10}$$

in both cases.

Similarly for the position x, if we consider the function f to be the function that describes the position x of a particle, then the width of the function, $W(f)$, can be denoted as $W(f) = \Delta x$. The Fourier transform of function x is the function $\tilde{x} = \lambda^{-1}$ and the width of this function can be denoted as $W(\tilde{x}) = W(\lambda^{-1}) = \Delta(\lambda^{-1})$ which we write as $\Delta\lambda^{-1}$ for brevity. You will note that we have not used the wavenumber function k, as this is usually defined as $k = 2\pi/\lambda$ (see for example [11] and references). Substituting in (1), we obtain the relation

$$\Delta x \, \Delta\lambda^{-1} \geqslant 1/2. \tag{11}$$

In terms of the wavenumber k, (11) becomes

$$\Delta x \, \Delta k \geqslant \pi. \tag{12}$$

Given that the momentum of a quantum particle is given by $p = h/\lambda$ or by $p = \hbar k$, both (11) and (12) can be expressed as

$$\Delta x \, \Delta p \geqslant h/2. \tag{13}$$

Equations (10) and (13) are both different statements of the Heisenberg Uncertainty Principle.

The R.H.S. of these equations is different from the usual statement of the Heisenberg Uncertainty Principle where the value $\hbar/2$ is used instead of the value $h/2$ obtained in this analysis. The application of (4) to circular variables (i.e. using ω in (4) instead of (5)) would result in the (incorrect) expression

$$\Delta t \, \Delta\omega \geqslant 1/2 \tag{14}$$

and the more commonly encountered (incorrect) expression

$$\Delta E \, \Delta t \geqslant \hbar/2. \tag{15}$$

However, Heisenberg's original derivation [2] had the R. H.S. of (13) approximately equal to h, and Greenstein's rederivation [6, see p. 47] of Heisenberg's principle results in the value $h/2$. Kennard's formal derivation [12] using standard deviations established the value of $\hbar/2$ used today. This would thus seem to be the reason for the use of the value $\hbar/2$ in the formulation of the Heisenberg Uncertainty Principle.

Recently, Schürmann et al [13] have shown that in the case of a single slit diffraction experiment, the standard deviation of the momentum typically does not exist. They derive the conditions under which the standard deviation of the momentum is finite, and show that the R.H.S. of the resulting inequality satisfies (13). It thus seems that (13) is the more general formulation of the Heisenberg Uncertainty Principle, while the expression with the value $\hbar/2$ derived using standard deviations is a more specific case.

Whether one uses $\hbar/2$ or $h/2$ has little impact on the Heisenberg Uncertainty Principle as the R.H.S. is used to provide an order of magnitude estimate of the effect considered. However, the difference becomes evident when we apply our results to the Brillouin zone formulation of Solid State Physics (as will be seen in Section 5) since this now impacts calculations resulting from models that can be compared with experimental values.

3 Interpretation of the Heisenberg Uncertainty Principle

This derivation demonstrates that the Heisenberg Uncertainty Principle arises because x and p form a Fourier transform pair of variables. It is a characteristic of Quantum Mechanics that conjugate variables are Fourier transform pairs of variables. Thus the Heisenberg Uncertainty Principle arises because the momentum p of a quantum particle is proportional to the de Broglie wave number k of the particle. If momentum was not proportional to wave number, the Heisenberg Uncertainty Principle would not exist for those variables.

This argument elucidates why the Heisenberg Uncertainty Principle exists. Can it shed light on the meaning of the Heisenberg Uncertainty Principle in relation to the basic nature of the quantum level? First, we note that the Uncertainty Principle, according to Fourier transform theory, relates the effective width of Fourier transform pairs of functions or variables. It is not a measurement theorem *per se*. It does not describe what happens when Fourier transform variables are measured, only that their effective widths must satisfy the Uncertainty Principle.

Indeed, as pointed out by Omnès [14, see p. 57], ”it is quite legitimate to write down an eigenstate of energy at a well-defined time”. Omnès ascribes this seeming violation of the Heisenberg Uncertainty Principle to the fact that time is not an observable obtained from an operator like momentum, but rather a parameter. Greenstein [6, see p. 65] makes the same argument. However, time t multiplied by the speed of light c is a component of the 4-vector x^μ and energy E divided by c is a component of the energy-momentum 4-vector P^μ. The time component of these 4-vectors should not be treated differently than the space component. The operator versus parameter argument is weak.

What Omnès' example shows is that the impact of the effective widths Δt and ΔE of the Heisenberg Uncertainty Principle depends on the observation of the time function t and of the energy function E that is performed. A time interval Δt can be associated with the time function t during which is measured the energy eigenstate function E which itself has a certain width ΔE, with both widths (Δ) satisfying (10). This example demonstrates that the Heisenberg Uncertainty Principle is not a measurement theorem as often used. Rather, it is a relationship between the effective widths of Fourier transform pairs of variables that can have an impact on the observation of those variables.

A more stringent scenario for the impact of the energy-time Heisenberg Uncertainty Principle is one where the time and energy functions are small quantities. For example, we consider the impact of Δt on the observation of τ_n, the lifetime of an atom in energy eigenstate n, and the impact of ΔE on the transition energy E_{mn}, for a transition between states n and m during spectral line emission. The conditions to be able to observe τ_n and E_{mn} are:

$$\tau_n \geqslant \Delta t \tag{16}$$

$$E_{mn} \geqslant \Delta E. \tag{17}$$

Using (10) in (16),

$$\tau_n \geqslant \Delta t \geqslant h/(2\Delta E). \tag{18}$$

Hence

$$\Delta E \geqslant \frac{h}{2}\frac{1}{\tau_n}. \tag{19}$$

As state n increases, the lifetime τ_n decreases. Eq.(19) is thus more constrained in the limit of large n. Using the following hydrogenic asymptotic expression for τ_n from Millette et al [15]

$$\tau_n \sim \frac{n^5}{\ln(n)} \tag{20}$$

into (19), (17) becomes

$$E_{mn} \geqslant \Delta E \gtrsim \frac{h}{2}\, k\, \frac{\ln(n)}{n^5} \tag{21}$$

where $1/k$ is the constant of proportionality of (20) given by

$$k = \frac{2^6}{3}\sqrt{\frac{\pi}{3}}\, Z^2\alpha^3 c R_H \tag{22}$$

where Z is the nuclear charge of the hydrogenic ion, α is the fine-structure constant, and R_H is the hydrogen Rydberg constant. Eliminating the middle term, (21) becomes

$$E_{mn} \gtrsim \frac{h}{2}\, k\, \frac{\ln(n)}{n^5}. \tag{23}$$

Applying L'Hôpital's rule, the R.H.S. of the above equation is of order

$$\text{R.H.S.} \sim O\left(\frac{1}{n^5}\right) \text{ as } n \to \infty \tag{24}$$

while the L.H.S. is of order [16, see p. 9]

$$\text{L.H.S.} \sim O\left(\frac{1}{n^2}\right) \text{ as } n \to \infty. \tag{25}$$

Given that (24) tends to zero faster than (25), (23) is satisfied. Both τ_n, the lifetime of the atom in energy eigenstate n, and the transition energy E_{mn} for the transition between states n and m satisfy the conditions for observation of the spectral line emission. Thus for the time interval Δt, given by (16), associated with the time function τ_n for the transition energy function E_{mn} which itself has a certain width ΔE, given by (17), both Δ's satisfy (10) as expected, given the observation of spectral line emission.

4 Quantum measurements and the Nyquist-Shannon Sampling Theorem

At the quantum level, one must interact to some degree with a quantum system to perform a measurement. When describing the action of measurements of Fourier transform variables, one can consider two limiting measurement cases: 1) truncation of the variable time series as a result of a fully interacting measurement or 2) sampling of the variable time series at intervals which we consider to be regular in this analysis, in the case of minimally interacting measurements. As we will see, the action of sampling allows for measurements that otherwise would not be possible in the case of a single minimal interaction.

It should be noted that the intermediate case of a partial measurement interaction resulting for example in a transfer of energy or momentum to a particle can be considered as the truncation of the original time series and the initiation of a new time series after the interaction. The advantage of decomposing measurement actions in this fashion is that their impact on Fourier transform variables can be described by the Nyquist-Shannon Sampling Theorem of Fourier transform theory. This theorem is a measurement theorem for Fourier transform variables based on sampling and truncation operations.

The Nyquist-Shannon Sampling Theorem is fundamental to the field of information theory, and is well known in digital signal processing and remote sensing [17]. In its most basic form, the theorem states that the rate of sampling of a signal (or variable) f_s must be greater than or equal to the Nyquist sampling rate f_S to avoid loss of information in the sampled signal, where the Nyquist sampling rate is equal to twice that of the highest frequency component, f_{max}, present in the signal:

$$f_s \geqslant f_S = 2 f_{max}. \tag{26}$$

If the sampling rate is less than that of (26), aliasing occurs, which results in a loss of information.

In general, natural signals are not infinite in duration and, during measurement, sampling is also accompanied by truncation of the signal. There is thus loss of information during a typical measurement process. The Nyquist-Shannon Sampling theorem elucidates the relationship between the process of sampling and truncating a variable and the effect this action has on its Fourier transform [18, see p. 83]. In effect, it explains what happens to the information content of a variable when its conjugate is measured.

Sampling a variable x at a rate δx will result in the measurement of its conjugate variable $\tilde{x}$ being limited to its maximum Nyquist range value $\tilde{x}_N$ as given by the Nyquist-Shannon Sampling theorem:

$$\tilde{x} \leqslant \tilde{x}_N \tag{27}$$

where

$$\tilde{x}_N = 1/(2\delta x). \tag{28}$$

Combining these two equations, we get the relation

$$\tilde{x}\,\delta x \leqslant 1/2, \quad \text{for}\, \tilde{x} \leqslant \tilde{x}_N. \tag{29}$$

Conversely, truncating a variable x at a maximum value x_N ($x \leqslant x_N$) will result in its conjugate variable $\tilde{x}$ being sampled at a rate $\delta\tilde{x}$ given by the Nyquist-Shannon Sampling theorem $\delta\tilde{x} = 1/(2x_N)$ resulting in the relation

$$\delta\tilde{x}\, x \leqslant 1/2, \quad \text{for}\, x \leqslant x_N. \tag{30}$$

The impact of the Nyquist-Shannon Sampling theorem is now considered for a particle's position x and momentum p. Applying the theorem to the case where a particle's trajectory is truncated to x_N, we can write from (30), for $x \leqslant x_N$,

$$x\,\delta\lambda^{-1} \leqslant 1/2, \quad \text{for}\, x \leqslant x_N \tag{31}$$

or

$$x\,\delta k \leqslant \pi, \quad \text{for}\, x \leqslant x_N \tag{32}$$

which becomes

$$x\,\delta p \leqslant h/2, \quad \text{for}\, x \leqslant x_N \tag{33}$$

where δp is the p-domain sampling rate and the x values can be measured up to x_N (corresponding to the equality in the equations above).

Conversely, applying the theorem to the case where a particle's trajectory is sampled at a rate δx, one can also write from (29), for $\tilde{x} \leqslant \tilde{x}_N$, where $\tilde{x}$ stands for either of λ^{-1}, k, or p,

$$\delta x\,\lambda^{-1} \leqslant 1/2, \quad \text{for}\, \lambda^{-1} \leqslant \lambda_N^{-1} \tag{34}$$

or

$$\delta x\,k \leqslant \pi, \quad \text{for}\, k \leqslant k_N \tag{35}$$

which becomes

$$\delta x\,p \leqslant h/2, \quad \text{for}\, p \leqslant p_N \tag{36}$$

where δx is the x-domain sampling rate and k_N is the wave number range that can be measured. For the case where the equality holds, we have $k_N = \pi/\delta x$ where k_N is the Nyquist wave number, the maximum wave number that can be measured with a δx sampling interval.

Sampling in one domain leads to truncation in the other. Sampling (δx) and truncation (x_N) in one domain leads to truncation (k_N) and sampling (δk) respectively in the other. As x and k form a Fourier transform pair in quantum mechanics, the Nyquist-Shannon Sampling theorem must also apply to this pair of conjugate variables. Similar relations can be derived for the E and ν pair of conjugate variables.

5 Implications of the Nyquist-Shannon Sampling Theorem at the quantum level

Equations (32) and (35) lead to the following measurement behaviors at the quantum level:

Lower-bound limit: If the position of a particle is measured over an interval x_N, its wave number cannot be resolved with a resolution better than sampling rate δk as given by (32) with $x = x_N$. If the momentum of a particle is measured over an interval k_N, its position cannot be resolved with a resolution better than sampling rate δx as given by (35) with $k = k_N$.

Upper-bound limit: If the position of a particle is sampled at a rate δx, wave numbers up to k_N can be resolved, while wave numbers larger than k_N cannot be resolved as given by (35). If the momentum of a particle is sampled at a rate δk, lengths up to x_N can be resolved, while lengths longer than x_N cannot be resolved as given by (32).

The lower-bound limit is similar to how the Heisenberg Uncertainty Principle is usually expressed when it is used as a measurement principle, although it is not strictly equivalent. The Nyquist-Shannon Sampling Theorem provides the proper formulation and limitations of this type of measurement.

The upper-bound limit suggests a different type of quantum measurement: regular sampling of a particle's position or momentum. In this case, one can obtain as accurate a measurement of the Fourier transform variable as desired, up to the Nyquist-Shannon Sampling limit of $h/2$ (i.e. in the interval $[0, h/2]$).

An example of this phenomenon occurs in Solid State Physics where the translational symmetry of atoms in a solid resulting from the regular lattice spacing, is equivalent to an effective sampling of the atoms of the solid and gives rise to the Brillouin zone for which the valid values of k are governed by (35). Setting $\delta x = a$, the lattice spacing, and extending by symmetry the k values to include the symmetric negative values, one obtains [19, see p. 34], [20, see p. 100], [10, see p. 21]:

$$-\pi/a \leqslant k \leqslant \pi/a \tag{37}$$

or alternatively

$$k \leqslant |\pi/a|. \tag{38}$$

This is called the reduced zone scheme and π/a is called the Brillouin zone boundary [21, see p. 307]. The Brillouin zones of Solid State Physics are thus a manifestation of the Nyquist-Shannon Sampling theorem at the quantum level.

In essence, this is a theory of measurement for variables that are Fourier transform pairs. The resolution of our measurements is governed by limitations that arise from the Nyquist-Shannon Sampling theorem. Equations (32) and (35) are recognized as measurement relationships for quantum-mechanical conjugate variables. Currently, Quantum Mechanics only considers the Uncertainty Theorem but not the Sampling Theorem. The two theorems are applicable to Quantum Mechanics and have different interpretations: the Uncertainty Theorem defines a relationship between the widths of conjugate variables, while the Sampling Theorem establishes sampling and truncation measurement relationships for conjugate variables.

The value δx is a sampled measurement and as a result can resolve values of p up to its Nyquist value p_N given by the Nyquist-Shannon Sampling theorem, (36). This is a surprising result as the momentum can be resolved up to its Nyquist value, in apparent contradiction to the Heisenberg Uncertainty Principle. Yet this result is known to be correct as demonstrated by the Brillouin zones formulation of Solid State Physics. Physically this result can be understood from the sampling measurement operation which builds up the momentum information during the sampling process, up to the Nyquist limit p_N. It must be remembered that the Nyquist limit depends on the sampling rate δx as per the Nyquist-Shannon Sampling theorem, (36). The Nyquist value must also satisfy (26) to avoid loss of information in the sampling process, due to aliasing.

This improved understanding of the Heisenberg Uncertainty Principle and its sampling counterpart allows us to clarify its interpretation. This is based on our understanding of the behavior of the Uncertainty Theorem and the Nyquist-Shannon Sampling Theorem in other applications such as, for example, Digital Signal Processing.

6 Measurement limitations and inherent limitations

It is important to differentiate between the measurement limitations that arise from the properties of Fourier transform pairs previously considered, and any inherent limitations that may or may not exist for those same variables independently of the measurement process. Quantum theory currently assumes that the inherent limitations are the same as the measurement limitations. This assumption needs to be re-examined based on the improved understanding obtained from the effect of the Uncertainty and Sampling Theorems in other applications.

The properties of Fourier transform pairs considered in the previous sections do not mean that the underlying quantities we are measuring are inherently limited by our measurement limitations. On the contrary, we know from experience in other applications that our measurement limitations do not represent an inherent limitation on the measured quantities in Fourier Transform theory: for example, in Digital Signal Processing, a signal is continuous even though our measurement of the signal results in discrete and aliased values of limited resolution subject to the Nyquist-Shannon Sampling Theorem (analog and digital representation of the signal). The effective width of the signal and its transform are related by the Uncertainty theorem. Even though the time and frequency evolution of a signal that we measure is limited by our measurement limitations, the time domain and frequency domain signals are both continuous, independently of how we measure them.

The measurement limitations apply equally to the macroscopic level and to the quantum level as they are derived from the properties of Fourier transform pairs of variables which are the same at all scales. However, at the quantum level, contrary to our macroscopic environment, we cannot perceive the underlying quantities other than by instrumented measurements. Hence during a measurement process, the quantum level is limited by our measurement limitations. However, assuming that these measurement limitations represent inherent limitations and form a basic characteristic of the quantum level is an assumption that is not justified based on the preceding considerations. Indeed, the Nyquist-Shannon Sampling Theorem of Fourier Transform theory shows that the range of values of variables below the Heisenberg Uncertainty Principle value of $h/2$ is accessible under sampling measurement conditions, as demonstrated by the Brillouin zones formulation of Solid State Physics.

7 Overlap of the Heisenberg Uncertainty Principle and the Nyquist-Shannon Sampling Theorem

Brillouin zone analysis in Solid State Physics demonstrates that one can arbitrarily measure k from 0 up to its Nyquist limit, as long as the variable x is sampled at a constant rate (rather than performing a single x measurement). The Nyquist-Shannon Sampling Theorem can thus be considered to cover the range that the Heisenberg Uncertainty Principle excludes.

However, one should recognize that the coverage results from two disparate theorems, and one should be careful not to try to tie the two Theorems at their value of overlap π. The reason is that one expression involves the widths of conjugate variables as determined by (1) to (3), while the other involves sampling a variable and truncating its conjugate, or vice versa as determined by (32) and (35). The equations are not continuous at the point of overlap π. Indeed, any relation obtained would apply only at the overlap π and would have no applicability or physical validity on either side of the overlap.

8 Discussion and conclusion

In this paper, we have shown that a consistent application of Fourier Transform theory to the derivation of the Heisenberg Uncertainty Principle requires that the R.H.S. of the Heisenberg inequality be $h/2$, not $\hbar/2$. This is confirmed when extending the analysis to the Brillouin zones formulation of Solid State Physics.

We have noted that the Heisenberg Uncertainty Principle, obtained from the Uncertainty Theorem of Fourier Transform theory, arises because of the dependency of momentum on wave number that exists at the quantum level. Quantum mechanical conjugate variables are Fourier Transform pairs of variables.

We have shown from Fourier Transform theory that the Nyquist-Shannon Sampling Theorem affects the nature ofmeasurements of quantum mechanical conjugate variables. We have shown that Brillouin zones in Solid State Physics are a manifestation of the Nyquist-Shannon Sampling Theorem at the quantum level.

We have noted that both the Sampling Theorem and the Uncertainty Theorem are required to fully describe quantum mechanical conjugate variables. The Nyquist-Shannon Sampling Theorem complements the Heisenberg Uncertainty Principle. The overlap of these Theorems at the $h/2$ equality value is a mathematical artifact and has no physical significance.

We have noted that the Uncertainty Theorem and the Nyquist-Shannon Sampling Theorem apply to Fourier Transform pairs of variables independently of the level at which the theorems are applied (macroscopic or microscopic). Conjugate variable measurement limitations due to these Theorems affect how we perceive quantum level events as these can only be perceived by instrumented measurements at that level. However, based on our analysis, quantum measurement limitations affect our perception of the quantum environment only, and are not inherent limitations of the quantum level, as demonstrated by the Brillouin zones formulation of Solid State Physics.

The application of the Nyquist-Shannon Sampling Theorem to the quantum level offers the possibility of investigating new experimental conditions beyond the Brillouin zone example from Solid State Physics considered in this paper, allowing a unique vista into a range of variable values previously considered unreachable due to the Heisenberg Uncertainty Principle. Regular sampling of position allows us to determine momentum below its Nyquist limit, and similarly the regular sampling of momentum will allow us to determine position below its Nyquist limit.

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