The Electron-Vacuum Coupling Force in the Dirac Electron Theory and its Relation to the Zitterbewegung

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From the perspective of the Planck vacuum theory, this paper argues that the standard estimate of the onset radius for electron-positron pair production as the Dirac electron is approached (in its rest frame) is significantly overestimated. The standard value is taken to be the electron Compton radius, while the estimate derived here from the coupling force is over four times smaller. The resulting separation of the Compton radius from the onset radius leads to a clear explanation of the zitterbewegung in terms of vacuum dynamics, making the zitterbewegung a relevant part of the electron theory.

1 Dirac Electron

The size of the electron has been a long debated question. In classical physics the idea that the electron radius r_0 is purely electromagnetic leads to the calculation

$$r_0 = \frac{e^2}{mc^2} = 2.82 \times 10^{-13} \tag{1}$$

centimeters, while the electron's Compton radius

$$r_c = \frac{e^2}{\alpha mc^2} = \frac{e_*^2}{mc^2} = 3.86 \times 10^{-11} \tag{2}$$

is larger by the factor $1/\alpha$ (≈ 137), where α ($= e^2/e_*^2$) is the fine structure constant. The standard caveat at this point in the calculations is that, for any radius smaller than r_c (like r_0), classical considerations are irrelevant due to the possible appearance of electron-positron pairs. So the onset radius for electron-position pair production is an important parameter in the Dirac theory of the electron. What follows takes a detailed look at the structure of the second ratio in (2) and suggests that an onset radius derived from the coupling force the Dirac electron (De) exerts on the vacuum state produces a better estimate of that radius.

In the Planck vacuum (PV) theory [1] the product $e_*^2 = (-e_*)(-e_*)$ in (2) consists of two distinctively different charges. One of the bare charges belongs to the De (a massive point charge $(-e_*, m)$ that obeys the Dirac equation and that is coupled to the Dirac vacuum [2]), and the other to the separate Planck particles constituting the PV negative-energy state. In addition, it can be argued [3] that the force

$$\frac{e_*^2}{r^2} \tag{3}$$

is a polarization-distortion force that the free-space De exerts on the omnipresent PV state. Since this force exists between the electron charge and the individual Planck-particle charges within the PV, a potential

$$V(r) = -\int_{r_1}^{r} \frac{e_*^2}{r^2} dr = \left(\frac{1}{r} - \frac{1}{r_1}\right) e_*^2 \tag{4}$$

can be defined for the De-PV system, except for the difficulty in determining the integration constant r_1.

The massive point charge $(-e_*, m)$ has two parts, its charge $(-e_*)$ and its mass m. Thus, in addition to the polarization force (3), the De distorts the PV due to a gravitational-like attraction between its mass and the individual masses of the Planck particles in the PV. This curvature force is given by [3]

$$-\frac{mc^2}{r} = -\frac{mc^2 G}{r\,G} = -\frac{mm_* G}{r_* r} \tag{5}$$

where m_* and r_* are the mass and Compton radius of the individual Planck particles and G is Newton's gravitational constant. ($G = e_*^2/m_*^2$ and $e_*^2 = r_* m_* c^2$ are used in deriving the final ratio in (5).) This force is the force of attraction the massive point charge at $\langle r \rangle \approx 0$ exerts on the negative-energy Planck particle at a radius r from that charge. Now the total De distortion force becomes

$$\frac{e_*^2}{r^2} - \frac{mc^2}{r} \tag{6}$$

and, as seen in the next section, the r_1-problem of the previous paragraph disappears. Part of the response to the De force (6) acting on the PV is hidden in the Dirac equation as the zitterbewegung.

[The average $\langle r \rangle \approx 0$ signifies a small, but unknown, radius encircling the massive point charge $(-e_*, m)$ and in which the electron mass is created (see the Appendix). This average is more properly expressed as $\sqrt{\langle r^2 \rangle} \ll r_c$.]

2 Dirac Equation

The force difference in (6) vanishes at the De's Compton radius

$$r_c = \frac{e_*^2}{mc^2} \tag{7}$$

which is that radius where the polarization and curvature forces have the same magnitude. This is a central parameter in the theory of the electron-positron system, for the free-particle Dirac equation can be expressed as (using $c\hbar = e_*^2$) [4, p. 74]

$$ie_*^2\left(\frac{\partial}{c\partial t}+\boldsymbol{\alpha}\cdot\nabla\right)\psi = mc^2\beta\psi \qquad \text{or}$$

$$ir_c\left(\frac{\partial}{c\partial t}+\boldsymbol{\alpha}\cdot\nabla\right)\psi = \beta\psi \tag{8}$$

where, in the rest frame of the De, the parameter r_c represents the radius of an imaginary sphere surrounding the massive point charge and on which the PV is undistorted (where (9) and (10) vanish).

Now the De-PV coupling force

$$F(r) = \frac{e_*^2}{r^2} - \frac{mc^2}{r} \tag{9}$$

leads, in place of (4), to the potential

$$V(r) = -\int_{r_c}^{r} F(r)dr = \left(\frac{1}{r}-\frac{1}{r_c}\right)e_*^2$$
$$-mc^2\ln\frac{r_c}{r} \qquad (r \leqslant r_c) \tag{10}$$

with no undetermined constants.

Recalling that any sufficiently strong positive potential acting on the vacuum state enables electron-positron pair production to take place in free space (see any relativistic discussion of the Klein Paradox, e.g. [4, p. 131]), it is reasonable to conclude that the point at which pairs may begin to show up as the De is approached is where $V(r) = 2mc^2$ since the positive energy in free space and negative energy of the PV begin to overlap at this potential. Then solving (10) for r yields the quadrature formulas

$$\frac{r_c}{r} - \ln\frac{r_c}{r} = 3 \qquad \text{or} \qquad \frac{\exp(r_c/r)}{r_c/r} = e^3 \tag{11}$$

either one of which produces $r \approx r_c/4.5$. This pair-production onset radius is significantly smaller than the standard estimate ($r \sim r_c$) because the curvature-force term in (9) compresses the PV state, countering the polarization force that expands that state and exposes its energies to free space. This important result implies that, for any $r > r_c/4.5$, there can be no exchange of free electrons with electrons from electron-positron pairs associated with the PV state.

3 QED Comparison

The standard estimate of the onset radius is based on virtual electron-positron transitions and the time-energy uncertainty relation [5, p. 323]

$$\Delta t\,\Delta E \sim \hbar \quad \longrightarrow \quad c\Delta t \sim \frac{c\hbar}{\Delta E} = \frac{e_*^2}{2mc^2} = \frac{r_c}{2} \tag{12}$$

where the original free electron jumps into the positron hole and the electron from the pair becomes the new free electron. As this process takes place at a high rate, the resulting cloud of "hide-and-seek" electrons is perceived as a spread-out point electron with a radius $r \sim r_c/2$. This radius is usually rounded off to $r \sim r_c$. It is interesting that arbitrarily replacing r_1 in (4) by r_c leads to the estimate $r = r_c/3$.

Whatever the true magnitude of the onset radius, it is worth noting the following quantum electrodynamic conclusions [5, pp. 402–403]: the interaction of the De with the quantum vacuum spreads out the point-like nature of the De and leads to a natural scale r_c for the model; the De in some respects behaves as though it increases in size from a point particle to a particle with a radius of about one r_c; it is improbable that the electron has "structure"; and the apparent spread of the De does not alter the fact that the electron in QED is still regarded as a pure point particle. In addition to these conclusions, high-energy scattering experiments probing small distances indicate that the electron, if not a point particle, is certainly not larger than about 10^{-15} cm ($r_c/39,000$).

Except for the magnitude of the onset and spread radii, the calculations in Sections 1 and 2 are mostly in agreement with the spirit of the QED conclusions of the previous paragraph. Also the earlier assumption at the end of Section 1, that $\langle r\rangle \approx 0$, is in line with the experimental result ($r_c/39,000$) at the end of the previous paragraph.

Since the onset radius is an important concept in the electron model, a definitive calculation of this radius is crucial to understanding the electron — indeed, contrary to the standard view, it is shown in the present paper that the Compton radius r_c and the pair-creation onset radius $r_c/4.5$ are two *distinctly different* parameters, the first referring to the vanishing-coupling-force sphere centered on the point electron (in its rest frame), and the second to the possible onset of electron-positron pairs. This separation of the Compton and onset radii leads to a believable zitterbewegung model.

4 Zitterbewegung

The zitterbewegung (a highly oscillatory, microscopic motion with velocity c) has been a long-time mathematical conundrum. Barut and Bracken [6, p. 2458] reexamine the Schrödinger calculations leading to the zitterbewegung and replace his "microscopic momentum" vector with a "relative momentum" vector in the rest frame of the particle. Of interest here are the two resulting commutator brackets ($\hbar = r_c mc$ and $c\hbar = e_*^2$ are used)

$$[Q_j, H_r] = ir_c cP_j \qquad \text{and}$$

$$[P_j, H_r] = -4i\frac{mce_*^2}{r_c^2}\,Q_j \tag{13}$$

from the theory, where ($j = 1,2,3$) and $H_r = mc^2\beta$ is the Dirac Hamiltonian in the rest frame.

Applying the Heisenberg-picture time derivative

$$\dot{A} = \frac{i}{\hbar}[H_r, A] \tag{14}$$

to the commutators in (13) leads to the "relative momentum"

$$P_j = m\dot{Q}_j \qquad \text{and} \qquad \dot{P}_j = -4\left(\frac{e_*^2}{r_c^3}\right)Q_j \tag{15}$$

which describes the dynamics of a harmonic oscillator with angular frequency

$$\omega = \left(\frac{4 \cdot e_*^2}{mr_c^3}\right)^{1/2} = \left(\frac{4 \cdot r_c mc^2}{mr_c^3}\right)^{1/2} = \frac{2c}{r_c}\,. \tag{16}$$

Since the Compton relation derives from the equality of the polarization- and curvature-force magnitudes on the r_c-sphere surrounding the massive point charge, the oscillator dynamics must be due to a reaction of the PV to the De perturbing force $e_*^2/r^2 - mc^2/r$, *not* to a direct dynamical involvement of the massive point charge itself. This latter conclusion is supported by the fact that the eigenvalues of the $\dot{Q}_j$ operator are $\pm c$, outlawing the involvement of a massive particle whose velocity must be less that c.

The "spring constant", $4(e_*^2/r_c^3)$, in (15) is easily shown to be related to the r_c-sphere, for $r = r_c + \Delta r$ in (9) leads to

$$F(r_c + \Delta r) = \frac{e_*^2}{(r_c + \Delta r)^2} - \frac{mc^2}{r_c + \Delta r}$$

$$= -\frac{(e_*^2/r_c^3)\Delta r}{(1 + \Delta r/r_c)^2} \approx -\left(\frac{e_*^2}{r_c^3}\right)\Delta r \tag{17}$$

where $F(r_c) = 0$, and $\Delta r \ll r_c$ in the final ratio.

The Schrödinger "microscopic coordinate"

$$\boldsymbol{\xi} = \left[\boldsymbol{\alpha}(0) - \frac{mc^2}{H}\frac{\widehat{\mathbf{p}}}{mc}\right] \cdot \frac{ir_c}{2}\frac{mc^2}{H}\exp\left[-i\frac{2c}{r_c}\frac{H}{mc^2}t\right] \tag{18}$$

is retained in the Barut-Bracken analysis [6, eqn. 19]. The first part of this operator equation corresponds to the macroscopic motion of the massive point charge and the second part to the high-frequency zitterbewegung superimposed on the macroscopic motion. In the rest frame of the massive charge (18) reduces to [6, eqn. 34]

$$Q_j(t) = [\boldsymbol{\xi}_r(t)]_j = \alpha_j(0) \cdot \frac{ir_c}{2}\beta\exp\left[-i\frac{2c}{r_c}\beta t\right] \neq 0 \tag{19}$$

the nonvanishing of which emphasizes again that the zitterbewegung is not fundamentally associated with the motion of the particle, as the particle leading to (19) is at rest. (The rest frame operators $H_r = mc^2\beta$ and $H_r^{-1} = \beta/mc^2$ are used in (19)).

5 Comments and Summary

The preceding calculations have separated the Compton radius (r_c) from the onset radius ($r_c/4.5$), with the result that the Compton radius is no longer associated with electron-positron pair production, being outside the onset radius. Thus the zitterbewegung is not related to the pair-production characteristic of an over-stressed ($V(r) \geqslant 2mc^2$) PV state. Instead, the zitterbewegung is seen to be the consequence of a PV-resonance phenomenon (with the resonant frequency $2c/r_c$) associated with the r_c-sphere. Also, most of the confusion surrounding the zitterbewegung is the result of attempting to attribute the phenomenon directly to the dynamics of the electron particle rather than the dynamics of the vacuum state. Finally, the zitterbewegung can now be seen, not as a mathematical curiosity, but as an integral part of the Dirac electron theory.

The following picture of the Dirac electron emerges: centered at the origin of the rest frame is the massive point charge with an effective volumetric radius $\langle r\rangle \approx 0$; surrounding this charge is a hypothetical sphere of radius $r_c/4.5$ within which the positive energy of the free electron and the negative energy of the PV overlap, allowing electron-positron pairs to be excited; surrounding this combination is a spherical annulus of radius $r_c/4.5 < r \leqslant r_c$, where pair production does not occur; and beyond the r_c-sphere ($r \geqslant r_c$) is a region of diminishing PV stress, a compression that decreases with increasing r according to the force difference (9).

Appendix: Electron Mass

The massless point charge is denoted by $(-e_*)$ and the massive point charge by $(-e_*, m)$, where m is the electron mass. In the PV theory this mass is an acquired property of the electron, resulting from the point charge being driven by the random electromagnetic zero-point background field [7, 8]. Furthermore, the energy absorbed by the charge from the field is re-radiated back into free space in a detailed-balance manner, leaving the isotropy and spectral density of the zero-point background unchanged [9].

The derived mass is

$$m = \frac{4r_c e_*^2}{9r_*^2 c^2}\frac{\left\langle (dr'/dt)^2\right\rangle}{c^2} \tag{A1}$$

where $e_* r'$ is the dipole moment of the point charge $(-e_*)$ about $r' = 0$ as it is being driven by the zero point field. The relative root-mean-square velocity of the charge within $\langle r\rangle \approx 0$ is [7]

$$\left(\frac{\left\langle (dr'/dt)^2\right\rangle}{c^2}\right)^{1/2} = \frac{3}{2}\frac{r_*}{r_c} \sim 10^{-22} \tag{A2}$$

which is vanishingly small because of the large density ($\sim 1/r_*^3$) of Planck particles in the PV contributing simultaneously to the zero-point background field; endowing the corresponding field spectrum with frequencies as high as $\sim c/r_*$, where r_* is the Planck length. It is predominately the high frequencies in the spectrum that define the mass and prevent the r-m-s velocity from significantly increasing in magnitude [10].

The squared charge e_*^2 in (A1) comes from squaring the time derivative of the dipole moment $e_* r'$. Thus (A1) implies that *the center-of-mass and the center-of-charge are the same*. The question of centers often comes up in the discussion of the zitterbewegung [11, pp. 62–64] and is a reflection of the fact that the zitterbewegung is being explained in terms of the massive-charge motion rather than the $2c/r_c$ resonance associated with the r_c-sphere and the vacuum state.

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