The Self-Gravity Model of the Longitudinal Span of the Neptune Arc Fraternité

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According to recent work [13, 14], the Neptune Adams ring main arc Fraternité is regarded as captured by the corotation elliptic resonance (CER) potential of Galatea. The minor arcs Egalité (2,1), Liberté, and Courage are located at positions where the time averaged forces, due to the 42-43 corotation-Lindblad resonances under the central field of Neptune, vanish. With adequately chosen Fraternité mass and Galatea eccentricity, this model gives minor arc locations compatible to observed positions, and allows a dynamic transport of materials among arcs. To complement this model, the effect of self-gravity of Fraternité, with a distributed mass, is evaluated together with the CER potential to account for its 10° longitudinal span. Although self-gravity is the collective action of all the particles in the arc, each individual particle will see the self-potential with a central maximum as an external potential generated by other particles.

1 Introduction

From the very first observations of the Neptune Adams ring arcs [6, 12], plus the subsequent observations [2, 11], the Adams arcs seemed to change in arc locations and in brightness. More recently, these dynamic natures of the arcs, Fraternité, Egalité (2,1), Liberté, and Courage, have been confirmed beyond any doubt in another ground observation [1]. Measuring from the center of the main arc Fraternité, they extend a total of about 40° ahead of Fraternité. Occasionally, some arcs flare up and others fade away. Furthermore, the arc configuration appears to be changing in time as well. The leading arc Courage appears to have leaped over to another CER site recently [1]. Although the twin arc Egalité (2,1) is small, it is a very bright arc. According to de Pater et al [1], its relative intensity to Fraternité varied from 17 percent higher in 2002 to seven percent lower in 2003 totaling a 24 percent relative change over a short period of time. The angular span of the twin arc Egalité appeared to be 30 percent larger in 2005 and 1999 publications than in 1989 Voyager 2 results. This widening of Egalité was accompanied by a corresponding narrowing of Fraternité, which indicated a likely exchange of material between the two. As for Liberté, 1999 data showed it was about 3° ahead of its position in Voyager 2 pictures. For the 2005 results, the 2002 data appeared to show Liberté as a twin arc separated by about 4.5° with the leading twin at the original Voyager 1989 location, while in 2003 it returned again as one single arc at the Voyager location. With respect to the normally low intensity arc Courage, it flared in intensity to become as bright as Liberté in 1998 indicating a possible exchange of material between the two arcs. Most interestingly, it was observed in the 2005 data that Courage has moved 8° ahead from 31.2° to 39.7° [1].

According to the prevailing theories, based on the restricted three-body framework (Neptune-Galatea-arcs) with a conservative disturbing potential, these arcs are radially and longitudinally confined by the corotation resonance potential of the inner moon Galatea. In order to account for these arcs, the 84/86 corotation resonance due to the inclination of Galatea (CIR) had been invoked to give a potential site of 4.18° [4]. Later on, because of its eccentricity (CER), the 42/43 resonance was considered giving a resonant site of 8.37° on the Adams ring arcs [3, 5, 10]. The arc particles librate about the potential maximum imposed by the corotational resonance satellite Galatea. Dissipated energy of the particle is replenished by the Lindblad resonance. Nevertheless, well established as it is, there are several difficulties. Firstly, with Fraternité centered at the potential maximum spanning approximately 5° on each side, it crosses two unstable potential points which ought to reduce the angular spread. Secondly, the minor arcs leading ahead of Fraternité are mislocated with the CIR or CER potential maxima. Furthermore, should the arcs were confined by the corotation potential, there ought to be arcs in other locations along the Adams ring distributed randomly instead of clustered near Fraternité.

2 Time-dependent arcs

Recently, there is a model that considers Fraternité as being captured by the CER potential of Galatea. With Fraternité having a finite mass, the minor arcs are clustered at locations along the Adams ring where the time averaged force vanishes under the corotation-Lindblad resonances [13, 14]. The finite mass of Fraternité has been suggested by Namouni [9] and Porco [10] to pull on the pericenter precession of Galatea to account for the mismatch between the CER pattern speed and the mean motion of the arcs. The arc locations are determined by the Lindblad resonance reaction of the arc itself. Because the force vanishes only on a time averaged base, as comparing to the stationary CER potential in the rotating frame, the arc material could migrate on a long time scale from one site to another leading to flaring of some arcs and fading of others. This could also generate twin arcs (Egalité, Liberté) and displace Courage from 31.2° to 39.7° (resonant jump) [1], as required by observations. Although there are only arcs in the leading positions ahead, arcs in the trailing positions behind could be allowed in this model. According to this Lindblad reaction model, only Fraternité f is confined by the externally imposed CER potential of Galatea x which reads

$$\Phi_c = \frac{Gm_x}{a_x}\frac{1}{2}\left(2n + a_x\frac{\partial}{\partial a_x}\right)\frac{1}{a_x}\, b^{(n)}_{1/2}(\alpha)\, e_x \cos\phi_{fx}, \quad (1)$$

where $\vec{r}_x = (r_x, \theta_x)$ and $\vec{r} = (r, \theta)$ are the position vectors of Galatea x and Fraternité mass distribution, a_x and a are the respective semi-major axes, ϕ_x and e_x are the arguments of perihelion and eccentricity of x, $\phi_{fx} = (n\theta - (n-1)\theta_x - \phi_x)$ is the corotation resonance variable, $b^{(n)}_{1/2}(\alpha)$ is the Laplace coefficient, $\alpha = a_x/a < 1$, and $n = 43$. With $a_x = 61952.60$ km, $a = 62932.85$ km, and $\alpha = 0.98444$ [2, 11], the CER potential is

$$\Phi_c = \frac{Gm_x}{a_x}\, 34\, e_x \cos\phi_{fx}. \quad (2)$$

To complement this model, we consider the self-gravity of Fraternité, which has a distributed mass, on the CER potential to account for its longitudinal 10° arc span. We first consider a qualitative spherical self-gravity physical model to grasp the 10° arc span. We begin with the Gauss law of the gravitational field

$$\nabla\cdot\vec{g}(\vec{r}) = -4\pi G\rho(\vec{r}), \quad (3)$$

$$\vec{g} = +\nabla\Phi. \quad (4)$$

Under a qualitative physical model of arc span, we take a spherical uniform mass distribution of radius r_0. Solving for the potential $\Phi(r_*)$ inside the sphere with $\rho(\vec{r}) = \rho_0$ and outside the sphere with $\rho(\vec{r}) = 0$ respectively, where r_* is measured from the center of Fraternité, and matching the potential and the gravitational field across the boundary, we get

$$\Phi_f = -\frac{1}{2}\frac{Gm_f}{r_0}\left(\frac{r_*}{r_0}\right)^2 + \frac{3}{2}\frac{Gm_f}{r_0}, \quad 0 < r_* < r_0, \quad (5)$$

$$\Phi_f = +\frac{Gm_f}{r_*}, \quad r_0 < r_* < \infty. \quad (6)$$

This potential shows a normal $1/r_*$ decaying form for $r_0 < r_*$, but a r_*^2 form for $r_* < r_0$. Writing in terms of a_x and m_x, we have for $0 < r_* < r_0$, $\delta\theta < \delta\theta_0$,

$$\begin{aligned}\Phi_f &= -\frac{1}{2}\frac{Gm_f}{a_x}\frac{a_x}{r_0}\left(\frac{r_*}{r_0}\right)^2 + \frac{3}{2}\frac{Gm_f}{r_0}\\ &= -\frac{1}{2}\frac{Gm_x}{a_x}\frac{m_f}{m_x}\frac{a_x}{r_0}\left(\frac{a}{r_0}\right)^2(\delta\theta)^2 + \frac{3}{2}\frac{Gm_x}{a_x}\frac{m_f}{m_x}\frac{a_x}{r_0},\end{aligned} \quad (7)$$

and for $r_0 < r_* < \infty$, $\delta\theta_0 < \delta\theta$,

$$\Phi_f = +\frac{Gm_f}{a_x}\frac{a_x}{r_*} = +\frac{Gm_x}{a_x}\frac{m_f}{m_x}\frac{1}{\delta\theta}, \quad (8)$$

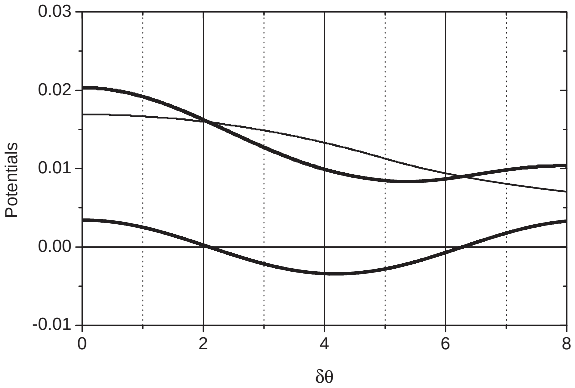

Fig. 1: The CER sinusoidal potential of Galatea in thick line, the self-potential of Fraternité with spherical model in thin line, and the sum of the two in thick line are plotted in units of Gm_x/a_x.

where r_* is now taken on the longitudinal direction along the arc, so that we can write $r_* = a\delta\theta$ and $r_0 = a\delta\theta_0$ with $\delta\theta$ as the angular span in radian. Taking $m_f/m_x = 10^{-3}$, $e_x = 10^{-4}$, and $\delta\theta_0 = 5° = 0.087$ rad, which are within the estimates of the arc parameters [9], we have plotted in Fig. 1 the sinusoidal CER potential in thick line with a minimum around $\delta\theta = 4°$ and the self-potential in thin line in units of Gm_x/a_x. The superposition of the two in thick line is also shown in the same figure. The superimposed potential has a maximum at the center and a minimum around $\delta\theta = 5°$. Although self-gravity is resulted from all the particles of the arc, each individual particle will see the self-potential as an external potential. The particles will girate in stable orbit about the central maximum of the superpositioned CER potential and self-potential.

3 Self-gravity

We now present an elongated ellipsoid model of self-gravity. For an ellipsoidal mass distribution with uniform density ρ_0 over a volume

$$\left(\frac{x}{a_1}\right)^2 + \left(\frac{y}{a_2}\right)^2 + \left(\frac{z}{a_3}\right)^2 = 1\,, \quad (9)$$

where $a_1 > a_2 > a_3$, the potential in space for the gravitational field $\vec{g}(\vec{r})$ have been addressed in honorable treatises such as Kellogg [7] and Landau and Lifshitz [8]. Here, we follow the celebrated original work of Kellogg [7] especially in Section 6 of Chapter 7. The potential in space of this homogeneous ellipsoid is given by

$$\begin{aligned}&\Phi_f(x,y,z) = G\rho_0\,\pi\, a_1a_2a_3\,\times\\ &\int_0^\infty \frac{d\lambda}{\sigma^{1/2}(\lambda)}\left[1 - \frac{x^2}{a_1^2+\lambda} - \frac{y^2}{a_2^2+\lambda} - \frac{z^2}{a_3^2+\lambda}\right],\end{aligned} \quad (10)$$

where

$$\sigma(\lambda) = \left(a_1^2+\lambda\right)\left(a_2^2+\lambda\right)\left(a_3^2+\lambda\right), \quad (11)$$

and where λ parameterizes a family of ellipsoids. Consider a prolate ellipsoid with $a_1 > a_2 = a_3$. This ellipsoid has a circular cross section on the y-z plane and an axis of symmetry in x. The y-z plane of $x = 0$ is the equatorial plane. In this prolate case, the self-potential inside and outside the ellipsoid is given respectively by [7, Exercise 6, p.196]

$$\Phi_f(x,r) = G\rho_0 \frac{4\pi}{3} a_1 a_2 a_3 \frac{1}{f^2} \times \left[\left(4x^2 - 2r^2 - f^2\right) \frac{1}{2f} \ln\left(\frac{2a_1 - f}{2a_1 + f}\right)^{1/2} + \right. \left. + \frac{4a_1^2\left(2x^2 - r^2\right) - 2f^2x^2}{2a_1\left(4a_1^2 - f^2\right)}\right], \tag{12}$$

$$\Phi_f(x,r) = G\rho_0 \frac{4\pi}{3} a_1 a_2 a_3 \frac{1}{f^2} \times \left[\left(4x^2 - 2r^2 - f^2\right) \frac{1}{2f} \ln\left(\frac{s - f}{s + f}\right)^{1/2} + \right. \left. + \frac{s^2\left(2x^2 - r^2\right) - 2f^2x^2}{s\,(s^2 - f^2)}\right], \tag{13}$$

where

$$\left(\frac{f}{2}\right)^2 = a_1^2 - a_2^2,$$
$$r^2 = y^2 + z^2,$$

f is the distance between the two foci, r is the perpendicular distance to the axis of symmetry, s is the sum of distances from the two foci to the point of interest $\vec{r}$. The inside potential can be obtained from the outside potential by using $s = 2a_1$. To evaluate the potential on the axis of symmetry, we take $r = 0$. Denoting $m_f = \rho_0(4\pi/3)a_1a_2a_3$ and considering $a_1 \gg a_2$, we get

$$\Phi_f(x) = -\frac{Gm_f}{a_x} \frac{a_x}{4a_1} \left[\ln\left(\frac{2a_1}{a_2}\right) - 1\right] \left(\frac{\delta\theta}{\delta\theta_0}\right)^2 + + \frac{Gm_f}{a_x} \frac{a_x}{4a_1} \ln\left(\frac{2a_1}{a_2}\right), \quad \delta\theta < \delta\theta_0, \tag{14}$$

$$\Phi_f(x) = -\frac{Gm_f}{a_x} \frac{a_x}{4a_1} \left[\frac{1}{2} \ln\left(\frac{x + f/2}{x - f/2}\right) \left(\frac{\delta\theta}{\delta\theta_0}\right)^2 - \right. \left. - \left(\frac{\delta\theta}{\delta\theta_0}\right)\right] + \frac{Gm_f}{a_x} \frac{a_x}{4a_1} \frac{1}{2} \ln\left(\frac{x + f/2}{x - f/2}\right), \quad \delta\theta_0 < \delta\theta, \tag{15}$$

for the self-potential inside and outside the ellipsoid respectively. Taking again $m_f/m_x = 10^{-3}$, with $a_x = 61952.60$ km for Galatea, and semi-major axes $a_1 = 5500$ km and $a_2 = 55$ km, the CER potenial, the self-potential, and the superposition of the two with a minimum around $\delta\theta = 5^{\circ}$ are shown in Fig. 2.

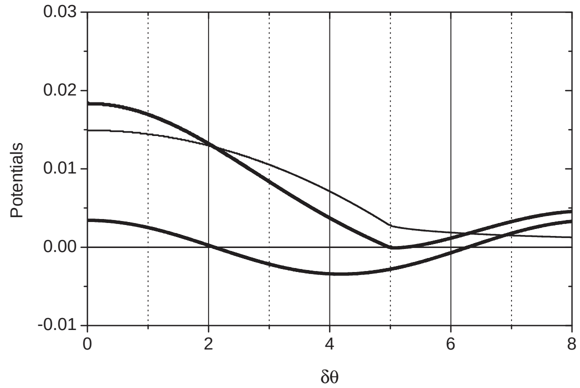

Fig. 2: The CER sinusoidal potential of Galatea in thick line, the self-potential of Fraternité with ellipsoidal model in thin line, and the sum of the two in thick line are plotted in units of Gm_x/a_x.

With Fraternité 1×10^{-3} of the mass of Galatea, the self-potential actually exceeds the CER potential in magnitude, as shown in Fig. 2. Each test mass would be librating around the potential maximum, dominated by the self-gravity of the collective mass distribution. Should Fraternité be elongated further while maintaining the total mass, it would increase the semi-major axis a_1 of the ellipsoid. This would reduce the amplitude of the self-potential of (14) through the $(a_x/4a_1)$ factor in the constant term, and weaken the self-potential. The elongation would feed the minor arcs. With this self-gravity model, not just the minor arcs are dynamically changing [1], the main arc Fraternité could be under a dynamical process as well.

4 Conclusions

In order to explain the 10° arc span of Fraternité, we draw attention to the fact that Fraternité, as an arc, has a significant mass. This mass is a distributed mass, instead of a point-like mass, such that its self-gravity should be taken into considerations to account for its angular span. We have used two models to evaluate the self-potential in the longitudinal direction. First is the tutorial spherical model, as a proof of principle study, with a uniform mass distribution over a sphere of radius r_0. Second is the elongated ellipsoidal model for a more realistic evaluation. Using the accepted range of Fraternité parameters, the ellipsoid model shows that the self-potential of the arc could be the cause of its angular span. For a longer arc, the ellipsoid gets longer and the ratio a_1/a_2 becomes larger. Eventually, for a complete ring, the ellipsoid is infinitely long and the self-potential in the longitudinal direction becomes constant. The effects of self-gravity are felt only in the transverse direction for a planetary ring.

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