The Dark Side Revealed: A Complete Relativity Theory Predicts the Content of the Universe

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Dark energy and dark matter constitute about 95% of the Universe. Nonetheless, not much is known about them. Existing theories, including General Relativity, fail to provide plausible definitions of the two entities, or to predict their amounts in the Universe. The present paper proposes a new special relativity theory, called *Complete Relativity theory (CR)* that is anchored in Galileo's relativity, but without the notion of a preferred frame. The theory results are consistent with Newtonian and Quantum mechanics. More importantly, the theory yields natural definitions of dark energy and dark matter and predicts the content of the Universe with high accuracy.

1 Introduction

1.1 Dark energy

The nature of dark energy ranks among the very most compelling of all outstanding problems in physical science [1, 2]. Conclusive evidence from supernovas and other observations show that, despite gravitation, the Universe is expanding with acceleration [3–6]. No existing theory is capable of explaining what dark energy is, but it is widely believed that it is some unknown substance with an enormous antigravitational force, which drives the galaxies of our Universe apart. It is also well established that at our time the Universe is comprised of $\approx 4.6\%$ atoms, $\approx 72\%$ dark energy and $\approx 23\%$ dark matter (see e.g., [1]). One explanation for dark energy is founded on Einstein's Cosmological Constant (λ), despite the fact that Einstein himself abandoned his constant, calling it his biggest mistake. According to this explanation the Universe is permeated by an energy density, constant in time and uniform in space. The big problem with this explanation is that for $\lambda \neq 0$ it requires that the magnitude of λ be $\approx 10^{120}$ (!) times the measured ratio of pressure to energy density [1].

An alternative explanation argues that dark energy is an unknown dynamical fluid, i.e., one with a state equation that is dynamic in time. This type of explanation is represented by theories and models which differ in their assumptions regarding the nature of the state equation dynamics [7–9]. This explanation is no less problematic since it entails the prediction of new particles with masses thirty-five orders of magnitude smaller than the electron mass, which might imply the existence of new forces in addition to gravity and electromagnetism [1]. At present there is no persuasive theoretical explanation for the existence, dynamics and magnitude of dark energy and its resulting acceleration of the Universe.

1.2 Dark matter

Dark matter is more of an enigma than dark energy. Scientists are more certain about what dark matter is not, than about what it is. Some contend that it could be Baryonic matter tied up in brown dwarfs or in chunks of massive compact halo objects "or MACHOs" [10, 11], but the common prejudice is that dark matter is not baryonic, and that it is comprised of particles that are not part of the "standard model" of particle physics. Candidates that were considered include very light axions and Weakly Interacting Massive Particles (WIMPs) which are believed to constitute a major fraction of the Universe's dark matter [2, 12–14].

Given the frustrating lack of knowledge about the nature of dark energy and dark matter, most experts contend that understanding the content of the Universe and its cosmic acceleration requires nothing less than "discovering a new physics" [14]. As example, the Dark Energy Task Force (DETF), summarized its 2006 comprehensive report on dark energy by stating that there is consensus among most physicists that "nothing short of a revolution in our understanding of fundamental physics will be required to achieve a full understanding of the cosmic acceleration" [1, see p. 6]. This statement includes the possibility of reconsidering Einstein's Special and General Relativity altogether.

The present paper meets the challenge by proposing a new relativity theory. The proposed theory, *which I term Complete Relativity Theory (or CR)*, is anchored in Galileo's relativity, but without the notion of a preferred frame. Alternatively, the theory could be seen as a generalization of the Doppler Formula [15, 16] to account for the relative dynamics of moving objects of mass. The theory's results are consistent with Newtonian mechanics and with Quantum mechanics. More importantly, the theory yields relativistic definitions of dark energy and dark matter, describes their dynamics and predicts the content of the Universe with impressive accuracy.

The following sections describe the theory for the special case of zero forces, resulting in constant relative velocities. I derive its time, distance, density, and energy transformations (sections 2.1–2.3) and compare the derived energy-term with Newton's and Einstein's *Special Relativity* terms. Section 3, which constitutes the core of this paper, puts forward a relativistic definition of dark energy and dark matter, describes

their dynamics as function of the relative velocity $\beta = \mathrm{v}/\mathrm{c}$, and calculates the present content of the Universe. Section 4 concludes with a brief discussion.

2 Complete Relativity (*CR*) theory postulates and transformations

CR theory rests on two postulates:

1. The magnitudes of ***all*** physical entities, as measured by an observer, depend on the relative motion of the observer with respect to the rest frame of the measured entities.
2. All translations of information from one frame of reference to another are carried by light or electromagnetic waves of equal velocity.

Note that postulate 1 applies to all measured entities, including the velocity of light. Thus, *CR* treats the velocity of light as a relativistic quantity and not as an invariant one as postulated by Einstein's *SR*.

2.1 Time transformation

The derivation of the time transformation of *CR* is similar to the derivation of the Doppler Formula, except that *CR* treats the relative time of a moving object with constant velocity, instead of the frequency of a traveling wave.

Consider the two frames of reference F and F' shown in Figure 1. Assume that the two frames are moving away from each other at a constant velocity v. Assume further that at time t_1 in F (and t'_1 in F') a body starts moving in the $+x$ direction from point x_1 (x'_1 in F') to point x_2 (x'_2 in F'), and that its arrival is signaled by a light pulse, which emits exactly when the body arrives at its destination. Denote the times of arrivals in F and F' by t_2 and t'_2, respectively. Finally, assume that the start times in F and F' are synchronized. Without loss of generality, we can set $t_1 = t' = 0$ and $x_1 = x'_1 = 0$.

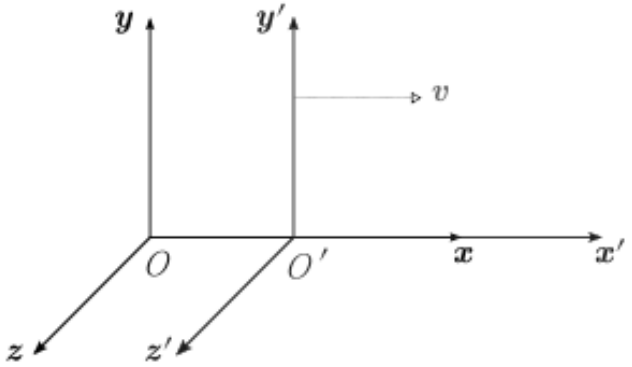

Fig. 1: Two observers in two reference frames moving with velocity v with respect to each other

The end time t_2, measured in F, equals the end time t'_2 plus the time δt which takes the light beam signaling the body's arrival at x_2 to reach the observer in F, or: $t_2 = t'_2 + \delta t$. But $\delta t = d/c$ where d is the distance (measured in F) travelled by F' relative to F, and ***c* is the velocity of light as measured in *F***. But $d = vt_2$, thus we can write:

$$t_2 = t'_2 + \frac{vt_2}{c} = t'_2 + \beta t_2 \,, \tag{1}$$

where $\beta = \frac{v}{c}$. Defining $t_2 = t$, $t_2 = t'$ and $\hat{t} = t/t'$, we get:

$$\hat{t} = \frac{t}{t'} = \frac{1}{1-\beta} \,. \tag{2}$$

Equation (2) is identical to the Doppler Formula, except that the Doppler Effect describes red- and blue-shifts of waves propagating from a departing or approaching wave source, whereas the result above describes the time transformation of moving objects. Note that $1/(1-\beta)$ is *positive* if F and F' *depart* from each other, and *negative* if they *approach* each other.

For the *round trip* from F and back, synchronization of the start time is not required. For this case the total relative time is given by (See Appendix, section1):

$$\hat{t} = \frac{t}{t'} = \frac{2}{1-\beta^2} \,. \tag{3}$$

For the one-way trip and a *departing* F' at velocity β ($0 \leqslant \beta \leqslant 1$), the proposed theory (*CR*) and Einstein's Special Relativity (*SR*) yield similar predictions, although the time dilation predicted by *CR* is larger than that predicted by *SR* (see Fig. 1Aa in the Appendix). Conversely, for an *approaching* F' ($\beta < 0$), *CR* predicts that the internal time measured at F will be *shorter* than that measured at F'. For the round trip the results of *CR* and *SR* (in $-1 \leqslant \beta \leqslant 1$) are qualitatively similar, except that the time dilation predicted by *CR* is larger than that predicted by *SR* (see Fig. 1Ab in the Appendix). For small β values the two theories yield almost identical results.

Note that the assumption that information is translated by light should not be considered a limitation of the theory, since its results are directly applicable to physical systems which use different transporters of information between two reference frames.

2.2 Distance transformation

The time duration, in frame F, of the event described above is equal to:

$$t_2 = \frac{x_2 - x_1}{c} = \frac{x_2}{c} \,, \tag{4}$$

where c is the velocity of light as measured in F. Similarly, the time duration of the event in F' could be written as:

$$t'_2 = \frac{x'_2 - x'_1}{c'} = \frac{x'_2}{c'} \,, \tag{5}$$

where c' is the velocity of light as measured in F'. From equations (4) and (5) we obtain:

$$\frac{x_2}{x'_2} = \frac{c'}{c}\frac{t_2}{t'_2} = \frac{c+v}{c}\frac{t_2}{t'_2} = \left(1 + \frac{v}{c}\right)\frac{t_2}{t'_2} = (1+\beta)\frac{t_2}{t'_2} \,. \tag{6}$$

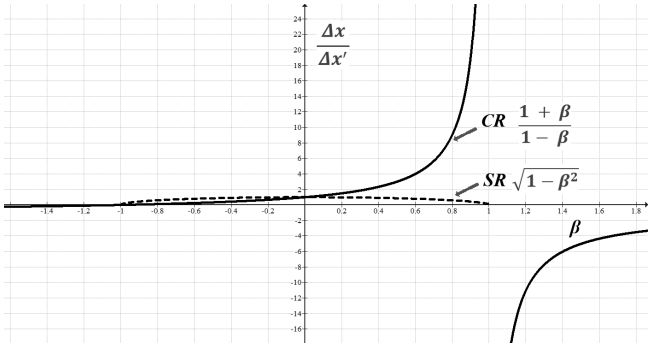

Fig. 2: Distance transformation.

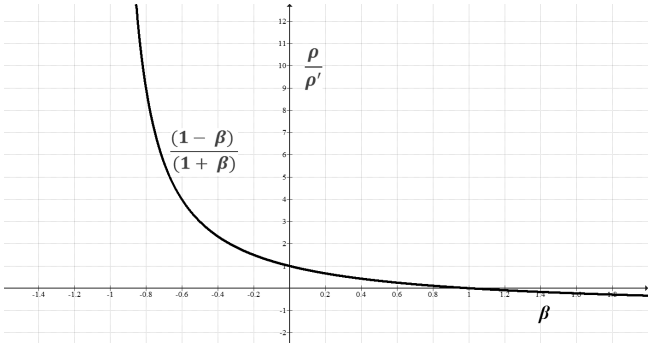

Fig. 3a: Density.

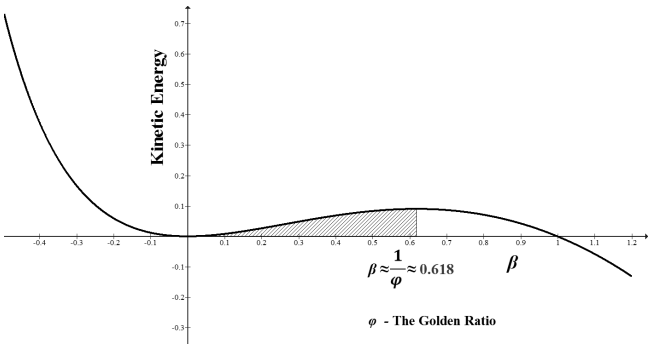

Fig. 3b: Energy.

Fig. 3: Density and energy as functions of velocity.

Substituting t_2/t_2' from (2) in (6) and denoting $x_2 = x$, $x_2' = x'$ and $\hat{x} = x_2/x_2'$ we get:

$$\hat{x} = \frac{x_2}{x_2'} = \frac{1+\beta}{1-\beta}. \tag{7}$$

The relative distance $\hat{x} = \Delta x \,/\, \Delta x' = (x_2 - x_1) \,/\, (x_2' - x_1')$ as a function of β, together with the respective relative distance according to *SR* (in dashed black) are shown in Figure 2. As shown by the figure, while *SR* prescribes that irrespective of direction, objects moving relative to an internal frame will contract, *CR* predicts that a moving object will contract or expand, depending on whether it approaches the internal frame or departs from it. For relative velocities exceeding the velocity of light ($\beta > 1$), *CR* predicts that $\hat{x}$ will become negative. Since $\Delta x'$ is positive, this implies that for bodies departing from an internal frame with a velocity higher than the velocity of light, the length of a rod of rest-length l_0, placed along the x axis, will be negative.

2.3 Density and energy transformations

Similar analyses for the density and kinetic energy (see Appendix, section 2) yield the following transformations:

Density:

$$\hat{\rho} = \frac{\rho}{\rho'} = \frac{(1-\beta)}{(1+\beta)} \tag{8}$$

and energy:

$$E = \frac{1}{2} m_0 c^2 \beta^2 \frac{(1-\beta)}{(1+\beta)}, \tag{9}$$

where m_0 is the rest mass in F'. Note that for $\beta \to 0$ (or $v \ll c$) *CR* reduces to Newton's mechanics ($\hat{t} = \hat{x} = \hat{\rho} = 1$, $E = \frac{1}{2} m v^2$). Figures 3 (a & b) depict the density and energy as functions of the velocity β. As shown by the figure the density of departing bodies relative to an observer in F is predicted to *decrease* with β, reaching zero for velocity equaling the speed of light. For bodies approaching the observer ($\beta < 1$) *CR*, similar to *SR*, predicts that the relative density will increase nonlinearly, from $\rho = \rho' = \rho_0$ at $\beta = 0$, to infinitely high values as β approaches -1. For $\beta < -1$ and $\beta > 1$, *CR* predicts that the relative density, as measured in F, will be negative.

The kinetic energy displays a non-monotonic behavior with two maxima: one at negative β values (approaching bodies) and the other at positive β values (departing bodies). The points of maxima (see Appendix, section 2) are $\beta_1 = \varphi - 1 \approx \approx 0.618$, and $\beta_2 = -\varphi \approx -1.618$, where φ is the Golden Ratio defined as $\varphi = \frac{\sqrt{5}+1}{2} \approx 1.618$ (see derivation in Appendix, section 2). The predicted decline in kinetic energy at velocities above $\beta \approx 0.618$ (see Fig. 3b), despite the decrease in velocity, suggests that mass and energy transform gradually from normal mass and energy to unobservable (dark) mass and energy.

The maximal kinetic energy at $\beta \approx 0.618$ is equal to:

$$\begin{aligned} E_{max} &= \frac{1}{2} m_0 c^2 (\varphi - 1)^2 \frac{1 - (\varphi - 1)}{1 + (\varphi - 1)} = \\ &= \frac{1}{2} m_0 c^2 (\varphi - 1)^2 \frac{(2 - \varphi)}{\varphi}. \end{aligned} \tag{10}$$

Since $\varphi - 1 = \frac{1}{\varphi}$ (See Appendix, section 2), Eq. 10 could be rewritten as:

$$E_{max} = \frac{1}{2} m_0 c^2 \frac{(2-\varphi)}{\varphi^3}. \tag{11}$$

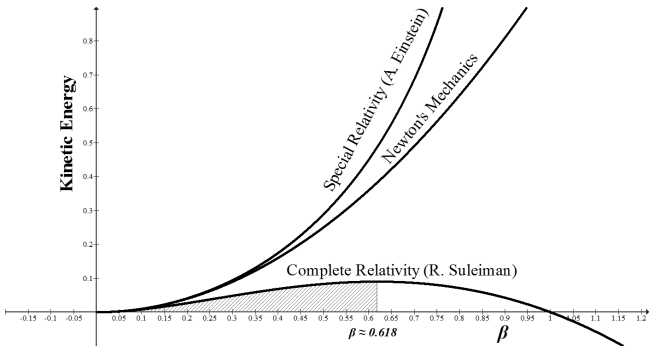

Fig. 4: Energy as a function of velocity according to three theories.

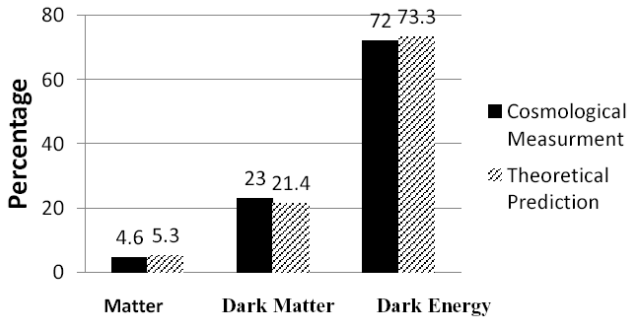

Fig. 5: Comparison between *CR*'s prediction of the content of the Universe and cosmological measurements

Substituting $\varphi = \frac{\sqrt{5}+1}{2}$ we obtain:

$$E_{max} \approx 0.04508497 m_0 c^2. \quad (12)$$

Notably, the energy-mass equivalent according to Eq. 12 is only $\approx 4.51\%$ of the amount predicted by the Einstein's famous equation $E = mc^2$. The above result is consistent with cosmological findings indicating that the percentage of Baryonic matter in the Universe is $\approx 4.6\%$. No less important the mass/energy conversion ratio (≈ 0.04508497) is precisely half of L. Hardy's probability of entanglement (0.09016994) [17–19]. This result confirms with a recent experimental finding [20], which demonstrated that applying a magnetic field at right angles to an aligned chain of cobalt niobate atoms, makes the cobalt enter a quantum critical state, in which the ratio between the frequencies of the first two notes of the resonance equals the Golden Ratio; the highest-order *E8* symmetry group discovered in mathematics [21].

For positive β values (departing objects) Figure 4 depicts *CR*'s energy function $E(\beta)$ together with the energy terms of Newton and Einstein's Special Relativity. As could be seen, while the latter theories predict that energy is strictly increasing with velocity, *CR* predicts a non-monotonic relationship with a maximum at $\beta \approx 0.618$ (the Golden ratio). As I shall show in the following section, this non-monotonic nature holds the key for explaining dark matter and dark energy.

3 The content of the Universe

The energy function Eq. 9 suggests that dark energy at a given velocity could be interpreted as the *difference between the energy measured at the internal frame and the energy measured at the external frame*. In other words, ***dark energy is defined as the energy loss due to relativity***. In formal terms, denote the energy at the internal and external frames by E' and E respectively, the kinetic energy measured at the internal and external frames could be expressed as: $E(\beta) = \frac{1}{2} m_0 c^2 \beta^2$ and $E'(\beta) = \frac{1}{2} m_0 c^2 \beta^2 \frac{(1-\beta)}{(1+\beta)}$, respectively, and the amount of dark energy, $DE(\beta)$, could be expressed as:

$$DE(\beta) = E'(\beta) - E(\beta) = \\ = \frac{1}{2} m_0 c^2 \beta^2 \left(1 - \frac{1-\beta}{1+\beta}\right) = m_0 c^2 \frac{\beta^3}{1+\beta}. \quad (13)$$

Similarly, ***dark matter, $m\_(\beta)$, at a given velocity is defined as the relativistic loss of matter at that velocity***. In other words, it equals the difference between the mass of normal matter measured at the internal and external frames. In formal notation: $m\_(\beta) = m_0 - m(\beta)$. Using the density transformation (Eq. 13), dark matter, $m(\beta)$, could be expressed as:

$$m\_(\beta) = m_0 - m(\beta) = m_0 \left(1 - \frac{1-\beta}{1+\beta}\right) = m_0 \left(\frac{2\beta}{1+\beta}\right). \quad (14)$$

The standard cosmological model of the Universe prescribes that it is comprised mainly of dark energy and dark matter (around 72% and 23%, respectively), with only less than 5% normal (Baryonic) matter. To compare matter with energy I use the matter-energy equivalence depicted in Eq. 12, according to which every unit of mass is equivalent to $\approx 0.045\, c^2$ energy units. Figure 5 depicts the dynamics of normal matter, dark matter, and dark energy as functions of β in the range $0 \leqslant \beta \leqslant 1$. Calculating the percentage of each component at $\beta = \varphi - 1 \approx 0.618$, or equivalently at redshift $z \approx 0.382$ (see Appendix, section 3) (yields $\approx 5.3\%$ Baryonic matter, $\approx 21.4\%$ dark matter, and $\approx 73.3\%$ dark energy, which is in excellent fit with current cosmological observations (See Fig. 6).

Statistical comparisons between the empirical and theoretical distributions of matter, dark matter, and dark energy, show that the difference is not significant ($p > 0.699$, Kolmogorov-Smirnov test). For velocities higher than $\beta = \frac{v}{c} \approx 0.618$ we get slightly different compositions. For example, for $\beta = 0.9$ (redshift $z \approx 0.474$) we get $\approx 89.4\%$ dark energy, $\approx 10\%$ dark matter and $\approx 0.6\%$ Baryonic matter. The average proportions in the range $0 \leqslant \beta \leqslant 1$ are about 85.80% dark energy, 12.35% dark matter and 1.85% Baryonic matter.

4 Concluding remarks

The biggest challenge of standard cosmology nowadays is to find a natural and more fundamental way to explain the detected presence of dark energy and dark matter. Most physicists agree that if this challenge is not met in the near future, then nothing less than "discovering a new physics" [14] and "a revolution in our understanding of fundamental physics" [2] will be required.

The present paper responds to the challenge by proposing a new relativity theory that is based on Galileo's relativity, but without the notion of a preferred frame. The analyses reveal that for low velocities the theory confirms with Newtonian mechanics and for high velocities it confirms with main predictions of quantum mechanics. More important for the present context, the proposed theory puts forward, for the first time, plausible definitions of dark matter and dark energy. The two entities are defined simply as the unobserved (dark) side of the matter-energy in the Universe. This definition yields formal expressions for the two entities which enable to predict the present content of the Universe with high accuracy. Two additional important results emerge from the analysis, each deserving a comprehensive treatment, are mentioned here very briefly:

1. For *departing* objects relative to the laboratory the mass-energy equivalence derived by the theory, is found to be $0.04508497 m_0 c^2$, which is exactly half Hardy's quantum coupling constant
2. The theory suggests a novel perspective of quantum phenomena, according to which the observed wave property of matter at high energies could be interpreted as a gradual transition of normal matter and normal energy to dark matter and dark energy. Such interpretation enables a long sought-after unification between Quantum Theory, and Newtonian mechanics, without leaving 95% of the Universe completely in the dark side of our knowledge.

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Appendix

1. The time transformation for the round-trip
2. Derivation of the density and energy transformations
3. The relationship between velocity (β) and redshift (z)
4. References

1 The time transformation for the round-trip

$$t = t_{Depart} + t_{Arrive} = \left(\frac{1}{1-\beta} + \frac{1}{1+\beta}\right)t' = \left(\frac{2}{1-\beta^2}\right)t', \quad (A1)$$

or,

$$\hat{t} = \frac{t}{t'} = \frac{2}{1-\beta^2}\,. \quad (A2)$$

Figure A1 depicts the relative time $\hat{t}$ as a function of β for the one-way and round trip. The dashed lines depict the corresponding predictions of *SR*.

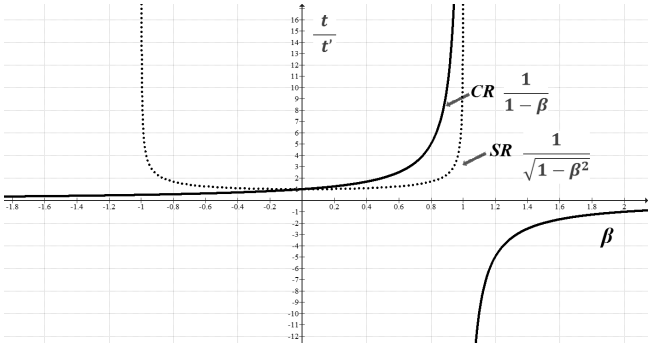

Fig. A1a

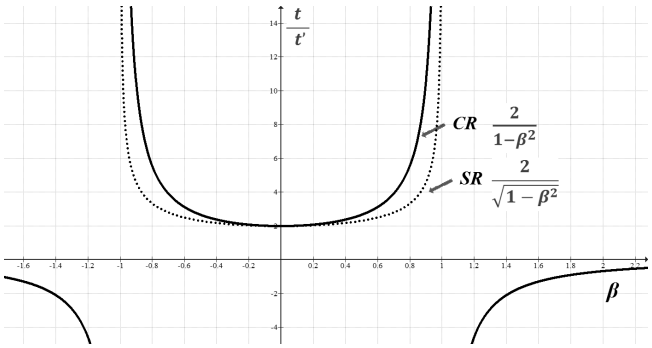

Fig. A1b

Fig. A1: Time transformations for the one-way (Fig. A1a) and round trip (Fig. A1b). The dashed lines depict the corresponding *SR* results.

2 Derivation of the density and energy transformations

To derive the density and kinetic energy transformation, consider the two frames of reference F and F' shown in Figure A2. Suppose that the two frames are moving relative to each other at a constant velocity v.

Consider a uniform cylindrical body of rest mass $m' = m_0$ and length $l' = l_0$ placed in F' along its travel direction. Suppose that at time t_1 the body leaves point x_1 (x'_1 in F') and moves with constant velocity v in the the $+x$ direction, until it reaches point x_2 (x'_2 in F') in time t_2 (t'_2 in F').

The body's density in the internal frame F' is given by:

$$\rho' = \frac{m_0}{Al_0}\,, \tag{A4}$$

where A is the area of the body's cross section, perpendicular to the direction of movement. In F the density is given by $\rho = \frac{m_0}{Al}$, where l is the object's length in F. Using the distance transformation l could be written as:

$$l = l_0\frac{1+\beta}{1-\beta}\,, \tag{A5}$$

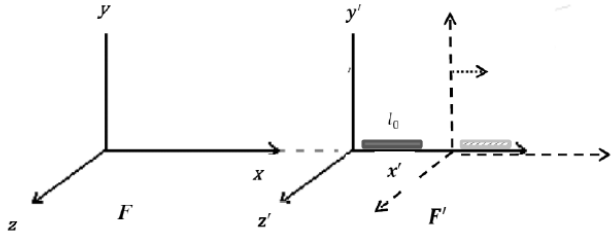

Fig. A2: Two observers in two reference frames, moving with velocity v with respect to each other

which yields:

$$\rho = \frac{m_0}{Al} = \frac{m_0}{Al_0}\frac{1-\beta}{1+\beta} = \rho'\frac{1-\beta}{1+\beta}\,,$$

or:

$$\frac{\rho}{\rho'} = \frac{1-\beta}{1+\beta}\,. \tag{A6}$$

Since the radius of the moving cylinder is perpendicular to the direction of motion, an observer at the internal frame F will measure a cylinder radius of $\Delta r = \Delta r_0$. The kinetic energy of a *unit of volume* is given by:

$$E = \frac{1}{2}\rho v^2 = \frac{1}{2}\rho_0\frac{1-\beta}{1+\beta}v^2,$$

or:

$$E = \frac{1}{2}\rho_0 c^2\beta^2\frac{1-\beta}{1+\beta}\,. \tag{A7}$$

And the energy for a departing particle of rest mass m_0 is given by:

$$E = \frac{1}{2}m_0 c^2\beta^2\frac{1-\beta}{1+\beta}\,. \tag{A8}$$

To calculate the value $\beta = \beta_{cr.}$ which satisfies $E = E_{max}$ we derive $\beta^2\frac{1-\beta}{1+\beta}$ with respect to β and equate the derivative to zero. This yields:

$$\begin{aligned}\frac{d}{d\beta}\left(\beta^2\frac{1-\beta}{1+\beta}\right) &= 2\beta\frac{1-\beta}{1+\beta} + \\ &+ \beta^2\frac{[(1+\beta)(-1)-(1-\beta)(1)]}{(1+\beta)^2} = \\ &= 2\beta\frac{(1-\beta^2-\beta)}{(1+\beta)^2} = 0\end{aligned} \tag{A9}$$

for $\beta \neq 0$ and we get:

$$\beta^2 + \beta - 1 = 0\,, \tag{A10}$$

which yields:

$$\beta_1 = -\varphi = -\frac{\sqrt{5}+1}{2} \approx -1.618 \tag{A11}$$

and

$$\beta_2 = \varphi - 1 = \frac{1}{\varphi} = \frac{\sqrt{5}-1}{2} \approx 0.618\,, \qquad (A12)$$

where φ is the Golden Ratio defined as: $\varphi = \frac{\sqrt{5}+1}{2}$ [A1-A3]. This is a striking result given the properties of this phenomenal number, due to its importance, together with the Fibonacci numbers, in mathematics, aesthetics, art, music, and more and its key role in nature, including the structure of plants, animals, the human body, human DNA [A1-A8] and brain waves [A9-A12] and in physics [A13]. The maximal kinetic energy at $\beta \approx 0.618$ is equal to:

$$E_{max} = \frac{1}{2} m_0 c^2 (\varphi - 1)^2 \frac{1-(\varphi-1)}{1+(\varphi-1)} = \\ = \frac{1}{2} m_0 c^2 (\varphi - 1)^2 \frac{2-\varphi}{\varphi}\,. \qquad (A13)$$

The term $\varphi - 1$ could be written as: $\varphi - 1 = \frac{\sqrt{5}+1}{2} - 1 = \\ = \frac{(\sqrt{5}+1)-2}{2} = \frac{\sqrt{5}-1}{2}$ Multiplying the numerator and denominator by $\frac{\sqrt{5}+1}{\sqrt{5}+1}$ yields:

$$\varphi - 1 = \frac{\sqrt{5}-1}{2}\frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{5-1}{2\sqrt{5}+1} = \\ = \frac{2}{\sqrt{5}+1} = \frac{1}{\frac{\sqrt{5}+1}{2}} = \frac{1}{\varphi}\,. \qquad (A14)$$

Eq. (A14) could be rewritten as:

$$E_{max} = \frac{1}{2} m_0 c^2 \frac{(2-\varphi)}{\varphi^3}\,. \qquad (A15)$$

Substituting $\varphi = \frac{\sqrt{5}+1}{2}$ we obtain:

$$E_{max} \approx 0.04508497 m_0 c^2. \qquad (A16)$$

3 The relationship between velocity and redshift

Redshift could be described as the relative difference between the observed and emitted wavelengths (or frequency). Let λ represents wavelength and f represents frequency ($\lambda f = c$ where c is the speed of light), then the redshift z is given by:

$$z = \frac{\lambda_r - \lambda_s}{\lambda_s} \qquad (\text{or} \quad z = \frac{f_s - f_r}{f_r})\,, \qquad (A17)$$

where $\lambda_s(f_s)$ is the wavelength (frequency) measured at the source and $\lambda_r(f_r)$ is the wavelength (frequency) measured at the receiver's laboratory.

Substituting $f_s = \frac{1}{t_s}$ and $f_r = \frac{1}{t_r}$ in (A17) above we obtain

$$z = \frac{f_s - f_r}{f_r} = \frac{\frac{1}{t_s} - \frac{1}{t_r}}{\frac{1}{t_r}} = \frac{t_r - t_s}{t_s} = \frac{t_r}{t_s} - 1. \qquad (A18)$$

But from Eq. 2 we have:

$$\frac{t_r}{t_s} = \frac{1}{1-\beta}\,. \qquad (A19)$$

Thus:

$$z = \frac{1}{1-\beta} - 1 = \frac{\beta}{1-\beta} \qquad (A20)$$

and

$$\beta = \frac{v}{c} = \frac{z}{1+z}\,. \qquad (A21)$$

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