Understanding the Dirac Equation and the Electron-Vacuum System

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It has been close to a century since the Dirac equation first appeared, but it has yet to be understood on an intuitive, fundamental level. The reason for this lack of understanding is twofold: the equation is expressed in terms of the *secondary* constant $\hbar$; and the vacuum state and its coupling to the electron particle have not been developed as part of the electron model. What follows briefly reviews the vacuum coupling and illustrates it by deriving the Schrödinger and Pauli equations as derivatives of the Dirac equation, and by explaining the zitterbewegung response that is a vacuum dynamic associated with the coupling force. It is argued that the fields of quantum electrodynamics have as their origin the degenerate vacuum state.

1 Introduction

The Dirac electron defined here is a massive "point" charge $(-e_*, m)$ that obeys the Dirac equation and is coupled to the negative-energy Planck vacuum (PV) continuum via the two-term coupling force [1]

$$\frac{e_*^2}{r^2} - \frac{mc^2}{r} \tag{1}$$

the massive charge exerts on the PV. The electron Compton radius r_c $(= e_*^2/mc^2)$ is that radius from the center of the massive charge (in its rest frame) to the radius r_c where the coupling force vanishes. The bare charge $(-e_*)$ itself is massless, while the electron mass m results from the bare charge being driven by the zero-point electromagnetic field [2] [3]; corresponding to which is a vanishingly small sphere containing the driven charge whose center defines the center of both the driven charge and its derived mass. It is from the center of this small sphere that the position operator $\mathbf{r}$ for the massive charge and the electron-vacuum complex is defined and from which the radius r in (1) emerges.

The PV model of the complete electron consists of two interdependent dynamics, the dynamics of the massive charge in the previous paragraph and the dynamics of the PV continuum to which the massive charge is coupled. An example of the latter dynamic is the (properly interpreted) zitterbewegung [4] [1] that represents a harmonic-oscillator-type excitation taking place at the $r = r_c$ sphere surrounding the massive point charge, an oscillation resulting from the vacuum response to the vanishing of (1) at r_c. The point-like nature of the massive charge, in conjunction with the continuum nature of the PV, are what give the electron its so-called wave-particle-duality. Mathematically, the electron's wave nature is apparent from the fact that the spinor solutions to the Dirac equation are spinor *fields*, and it is upon these fields that the covariant gradient operator

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{c\partial t}, \nabla\right) \tag{2}$$

operates. Thus the spinors are associated with PV distortion — with no distortion the gradients vanish, resulting in null spinors and the dissolution of (3).

The free-particle Dirac equation can be expressed in the form (from (A10) in Appendix A)

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi\\ \chi\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot ir_c\nabla\chi\\ \vec{\sigma}\cdot ir_c\nabla\phi\end{pmatrix} = \begin{pmatrix}\phi\\ -\chi\end{pmatrix} \tag{3}$$

in terms of the single constant r_c, a constant that normalizes the operator in (2). The free-space particle solution ϕ, and the negative-energy vacuum solution χ, for this electron-vacuum system are $2{\times}1$ spinors and $\vec{\sigma}$ is the Pauli $2{\times}2$ vector matrix. The spinor solutions from the two simultaneous equations in (3) are strongly coupled by the inverted χ-ϕ spinor configuration of the second term, showing the vacuum state to be an integral part of the electron phenomenon. (It will be seen that this coupling is even present in the nonrelativistic Schrödinger equation.) The negative spinor $(-\chi)$ on the right is a manifestation of the negative-energy nature of the vacuum. Equation (3) expresses the Dirac equation in terms of the normalized PV gradients on the left of the equal sign.

What follows illustrates the previous ideas by reiterating the standard development of the free-particle Schrödinger equation and the minimal coupling substitution leading to the Pauli equation.

2 Schrödinger equation

The Dirac-to-Schrödinger reduction [5, p. 79] begins with eliminating the high-frequency components from (3) by assuming

$$\begin{pmatrix}\phi\\ \chi\end{pmatrix} = \begin{pmatrix}\phi_0\\ \chi_0\end{pmatrix} e^{-imc^2t/\hbar} = \begin{pmatrix}\phi_0\\ \chi_0\end{pmatrix} e^{-ict/r_c} \tag{4}$$

where ϕ_0 and χ_0 are slowly varying functions of time compared to the exponentials. Inserting (4) into (3) gives

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi_0\\ \chi_0\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot ir_c\nabla\chi_0\\ \vec{\sigma}\cdot ir_c\nabla\phi_0\end{pmatrix} = \begin{pmatrix}0\\ -2\chi_0\end{pmatrix} \tag{5}$$

where the 0 on the right is a 2×1 null spinor. This zero spinor indicates that the mass energy of the free particle is being ignored, while the effective negative-mass energy of the "vacuum particle" has been doubled. In effect, mass energy for the particle-vacuum system has been conserved by shifting the mass energy of the free particle to the vacuum particle.

The lower of the two simultaneous equations in (5) can be reduced from three to two terms by the assumption

$$\left| ir_c \frac{\partial \chi_0}{c\partial t} \right| \ll |-2\chi_0| \tag{6}$$

if the kinetic energy (from the first equation in (A2)) of the vacuum particle is significantly less than its effective mass energy. Inserting (6) into (5) yields

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi_0\\0\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot ir_c\nabla\chi_0\\ \vec{\sigma}\cdot ir_c\nabla\phi_0\end{pmatrix} = \begin{pmatrix}0\\-2\chi_0\end{pmatrix} \tag{7}$$

as the nonrelativistic version of (3). The mass energy of the free particle, and the kinetic energy of the vacuum particle (associated with the lower-left null spinor), are discarded in the Schrödinger approximation.

Separating the two equations in (7) produces

$$ir_c \frac{\partial \phi_0}{c\partial t} + \vec{\sigma}\cdot ir_c\nabla\chi_0 = 0 \tag{8}$$

and

$$\vec{\sigma}\cdot ir_c\nabla\phi_0 = -2\chi_0 \tag{9}$$

and inserting (9) into (8) leads to

$$ir_c \frac{\partial \phi_0}{c\partial t} - \frac{(\vec{\sigma}\cdot ir_c\nabla)^2}{2}\phi_0 = 0\,. \tag{10}$$

Finally, inserting the Pauli-matrix identity (A12)

$$(\vec{\sigma}\cdot ir_c\nabla)^2 = I\,(ir_c\nabla)^2 \tag{11}$$

into (10) yields the free-particle Schrödinger equation

$$ir_c \frac{\partial \phi_0}{c\partial t} = \frac{(ir_c\nabla)^2}{2}\,\phi_0 \quad \text{or} \quad i\hbar\frac{\partial \phi_0}{\partial t} = \frac{(i\hbar\nabla)^2}{2m}\,\phi_0 \tag{12}$$

where the two spin components in ϕ_0 are ignored in this approximation. The scalar harmonic function

$$\phi_0 \longrightarrow \exp\left[-i(Et - \mathbf{p}\cdot\mathbf{r})/\hbar)\right] \tag{13}$$

satisfies both equations as it should, and leads to the nonrelativistic energy-momentum relation $E = p^2/2m$, where $\mathbf{p} = m\mathbf{v}$. The equation on the left in (12) expresses the Schrödinger equation in terms of PV gradients.

The vacuum property implied by (11), and the fact that ϕ_0 is a spinor field, show that the vacuum state is a significant (but hidden) part of the nonrelativistic Schrödinger equation. The Dirac-to-Pauli reduction leads to the same conclusion.

3 Minimal coupling

By itself the coupling force (1) is insufficient to split the two-fold degeneracy of the spinors in the free-particle Dirac (3) and Schrödinger (12) equations. It takes an external field to effect the split and create the well-known 1/2-spin electron states. The following illustrates this conclusion for the case of the minimal coupling substitution.

The minimal coupling substitution [5, p.78] is

$$p^\mu \longrightarrow p^\mu - eA^\mu/c \tag{14}$$

where e is the magnitude of the observed electron charge, $p^\mu = (E/c, \mathbf{p})$ is the 4-momentum, and $A^\mu = (A_0, \mathbf{A})$ is the electromagnetic 4-potential. Inserting (14) with (A1) and (A2) into the Dirac equation (A3) leads to

$$\left(i\hbar\frac{\partial}{\partial t} - eA_0\right)\psi - c\boldsymbol{\alpha}\cdot\left(\widehat{\mathbf{p}} - \frac{e\mathbf{A}}{c}\right)\psi = mc^2\beta\psi \tag{15}$$

which can be expressed as

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi\\ \chi\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot(ir_c\nabla + \mathbf{a})\chi\\ \vec{\sigma}\cdot(ir_c\nabla + \mathbf{a})\phi\end{pmatrix} = a_0\begin{pmatrix}\phi\\ \chi\end{pmatrix} + \begin{pmatrix}\phi\\ -\chi\end{pmatrix} \tag{16}$$

in the 2×1 spinor formulation, where $a_0 \equiv eA_0/mc^2$ and $\mathbf{a} \equiv e\mathbf{A}/mc^2$. Then proceeding as in Section 2 produces

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi_0\\ 0\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot(ir_c\nabla + \mathbf{a})\chi_0\\ \vec{\sigma}\cdot(ir_c\nabla + \mathbf{a})\phi_0\end{pmatrix} = a_0\begin{pmatrix}\phi_0\\ \chi_0\end{pmatrix} + \begin{pmatrix}0\\ -2\chi_0\end{pmatrix}. \tag{17}$$

The Compton radius in (16) and (17) has been accounted for as a gradient normalizer. The remaining constants (e and m) appear only in association with the 4-potential A^μ — if the external potential vanishes, the electron charge and mass are removed ($a_0 = 0$ and $\mathbf{a} = \mathbf{0}$) from the equations, and (16) and (17) reduce to (3) and (7) respectively. Furthermore, the energy eA_0 appears to increase the energy level of the negative-energy PV continuum. This latter conclusion can be appreciated by combining the two terms on the right side of (17):

$$\begin{pmatrix}\left(eA_0/mc^2\right)\phi_0\\ \left(eA_0/mc^2 - 2\right)\chi_0\end{pmatrix} \tag{18}$$

where a_0 has been replaced by its definition. With a constant potential energy $eA_0 = 2mc^2$, the lower parenthesis vanishes and the free-space electron energy and the vacuum-energy spectrum just begin to overlap [1]. This latter result is the phenomenon that leads to the relativistic Klein paradox [5, p. 127].

If it is further assumed that

$$|a_0\chi_0| \ll |-2\chi_0| \tag{19}$$

then (17) becomes

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix}\phi_0\\ 0\end{pmatrix} + \begin{pmatrix}\vec{\sigma}\cdot(ir_c\nabla + \mathbf{a})\chi_0\\ \vec{\sigma}\cdot(ir_c\nabla + \mathbf{a})\phi_0\end{pmatrix} = a_0\begin{pmatrix}\phi_0\\ 0\end{pmatrix} + \begin{pmatrix}0\\ -2\chi_0\end{pmatrix} \tag{20}$$

which is the nonrelativistic version of (16). Then eliminating χ_0 from the two simultaneous equations in (20) leads to the equation

$$ir_c \frac{\partial \phi_0}{c\partial t} = \frac{\vec{\sigma} \cdot (ir_c \nabla + \mathbf{a}) \vec{\sigma} \cdot (ir_c \nabla + \mathbf{a})}{2} \phi_0 + a_0 \phi_0 \qquad (21)$$

for the spinor ϕ_0. Equation (21) then leads to the Pauli equation [5, p.81].

Using (A11) to calculate the square of the numerator in the first term on the right of the equal sign in (21) yields

$$ir_c \frac{\partial \phi_0}{c\partial t} = \frac{(ir_c \nabla + \mathbf{a})^2}{2} \phi_0 + \frac{i\vec{\sigma} \cdot (ir_c \nabla \times \mathbf{a})}{2} \phi_0 + a_0 \phi_0 \qquad (22)$$

remembering that ϕ_0 post-multiplies the square before calculation. The first term in (22) contains the electron's orbital angular momentum; and the second its spin, as manifested in the scaler product of $\vec{\sigma}$ and the curl of the vector potential $\mathbf{A}$. Using (A1), the corresponding spin operator can be expressed as

$$\widehat{s} = \frac{\hbar \vec{\sigma}}{2} = \frac{e_*^2 \vec{\sigma}}{2c} = \frac{(-e_*)(-e_*)\vec{\sigma}}{2c} \qquad (23)$$

where one of the charges $(-e_*)$ in (23) belongs to the massive point charge $(-e_*, m)$ and the other to the separate Planck particles $(-e_*, m_*)$ within the PV. The product e_*^2 suggests that the spin may be related to the interaction of the massive point charge with the PV charges when the vacuum is under the influence of a magnetic field $\mathbf{B}\,(= \nabla \times \mathbf{A})$.

4 Conclusions and comments

The physics of the PV state [1, 6] has provided a simple intuitive explanation for the Dirac, Schrödinger, and Pauli equations in terms of the massive point charge $(-e_*, m)$ and its interaction (1) with the PV. It is the ignorance of this coupling force that has obscured the meaning of the Dirac equation since its inception and, as seen in the next paragraph, the meaning of the zitterbewegung frequency.

The electron Compton relation $r_c m = e_*^2$ in (A1) holds for both combinations $(\mp e_*, \pm m)$; so the vacuum hole $(e_*, -m)$ exerts a coupling force on the vacuum state that is the negative of (1). The combination of the two forces explain why the zitterbewegung frequency ($2c/r_c$ [1] [4]) is twice the angular frequency ($mc^2/\hbar = c/r_c$) associated with the electron mass (from Appendix B).

The purpose of this paper is to illustrate the massive-charge-PV nature of the electron phenomenon; and to reestablish the vacuum state as an essential and necessary part of a complete electron theory, that part that has been superseded by the idea of the quantum field. While the quantum field formalism, like the Green function formalism, is an important tool [5, p. 143] [7], the present author believes that the corresponding quantum field does not constitute an essential physical phenomenon apart from the dynamics of vacuum state (from Appendix C).

Appendix A: Dirac equation

The PV is characterized in part by the two Compton relations [1]

$$r_c mc^2 = r_* m_* c^2 = e_*^2 \;\; (= c\hbar) \qquad (A1)$$

connecting the massive point charge $(-e_*, m)$ of the electron to the individual Planck particles $(-e_*, m_*)$ within the degenerate PV, where r_c and m, and r_* and m_* are the Compton radius and mass of the electron and Planck particles respectively. The bare charge $(-e_*)$ is massless and is related to the observed electronic charge $(-e)$ via the fine structure constant $\alpha = e^2/e_*^2$. From (A1), the energy and momentum operators can be expressed as

$$\widehat{E} = i\hbar \frac{\partial}{\partial t} = mc^2 \left(ir_c \frac{\partial}{c\partial t} \right) \text{ and } \widehat{\mathbf{p}} = -i\hbar \nabla = mc(-ir_c \nabla) \quad (A2)$$

the parenthetical factors implying that the operators, operating on the Dirac spinors, provide a measure of the gradients within the PV continuum. In the present free-electron case, these gradients are caused solely by the coupling force (1) and its negative (Appendix B).

The upper and lower limits to the PV negative-energy spectrum are $-mc^2$ and $-m_*c^2$ respectively, where m_* is the Planck mass. The continuum nature of the vacuum is an approximation that applies down to length intervals as small as ten Planck lengths ($10\, r_*$) or so; that is, as small as $\sim 10^{-32}$cm.

Using (A1) and (A2), the Dirac equation [5, p.74]

$$ie_*^2 \frac{\partial \psi}{c\partial t} + \boldsymbol{\alpha} \cdot ie_*^2 \nabla \psi = mc^2 \beta \psi \qquad (A3)$$

can be expressed as

$$ir_c \frac{\partial \psi}{c\partial t} + \boldsymbol{\alpha} \cdot ir_c \nabla \psi = \beta \psi \qquad (A4)$$

where the 4×4 vector-matrix operator

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \qquad (A5)$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (A6)$$

are the three 2×2 Pauli matrices. The 4×4 matrix operator

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad (A7)$$

where I represents the 2×2 unit matrix and the zeros here and in (A5) are 2×2 null matrices. The covariant gradient operator

$$\frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3} \right) = \left(\frac{\partial}{c\partial t}, \nabla \right) \qquad (A8)$$

is seen in (A2) and (A4) to have its differential coordinates normalized $(\partial x^\mu / r_c)$ by the electron Compton radius.

The 4×1 spinor wavefunction ψ can be expressed as [5, p. 79]

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \tag{A9}$$

where ϕ and χ are the usual 2×1 spinors, and where the two components in each represent two possible spin states. The spinor ϕ is the free-space particle solution and χ is the negative-energy hole solution. Inserting (A9) into (A4), and carrying out the indicated matrix operations, yields the Dirac equation

$$ir_c \frac{\partial}{c\partial t}\begin{pmatrix} \phi \\ \chi \end{pmatrix} + \begin{pmatrix} \vec{\sigma} \cdot ir_c \nabla\chi \\ \vec{\sigma} \cdot ir_c \nabla\phi \end{pmatrix} = \begin{pmatrix} \phi \\ -\chi \end{pmatrix} \tag{A10}$$

in terms of the 2×1 spinors.

The following is an important property of the Pauli matrices, and the PV state (because of $\vec{\sigma}$): the vector Pauli matrix $\vec{\sigma}$ obeys the identity [5, p.12]

$$(\vec{\sigma} \cdot \mathbf{a})(\vec{\sigma} \cdot \mathbf{b}) = I\,\mathbf{a} \cdot \mathbf{b} + i\vec{\sigma} \cdot \mathbf{a} \times \mathbf{b} \tag{A11}$$

where $\mathbf{a}$ and $\mathbf{b}$ both commute with $\vec{\sigma}$, but are otherwise arbitrary three-vectors. Using (A11) (with $\mathbf{a} = \mathbf{b} = r_c\nabla$) leads to

$$(\vec{\sigma} \cdot r_c\nabla)^2 = I\,(r_c\nabla)^2 \tag{A12}$$

which connects the normalized ∇ operator in the relativistic Dirac equation to the same operator in the nonrelativistic Schrödinger equation.

Inserting the operators from (A2) into (A10) and rearranging the result leads to the two simultaneous equations

$$\left(\widehat{E} - mc^2\right)\phi = c\vec{\sigma} \cdot \widehat{p}\,\chi \tag{A13}$$

and

$$\left(\widehat{E} + mc^2\right)\chi = c\vec{\sigma} \cdot \widehat{p}\,\phi\,. \tag{A14}$$

Then, pre-multiplying (A13) by $(\widehat{E} + mc^2)$ and using (A14) and (A11) leads to

$$\left(\widehat{E}^2 - m^2c^4\right)\phi = c^2\widehat{p}^2\,\phi \tag{A15}$$

and, after reversing the process, to an identical equation for χ. Thus both ϕ and χ separately obey the Klein-Gordon equation [5, p.31].

Appendix B: Zitterbewegung frequency

The following rough heuristic argument identifies the two coupling forces that explain why the zitterbewegung frequency [1, 4] is twice the angular frequency ($mc^2/\hbar = c/r_c$) associated with the electron mass energy.

The force the massive point charge $(-e_*, m)$ exerts on the PV is given by equation (1) which, using $r = r_c + \Delta r$ and $r_c = e_*^2/mc^2$, leads to

$$\frac{e_*^2}{(r_c + \Delta r)^2} - \frac{mc^2}{r_c + \Delta r} = -\frac{(e_*^2/r_c^3)\Delta r}{(1 + \Delta r/r_c)^2} \approx -\left(\frac{e_*^2}{r_c^3}\right)\Delta r \tag{B1}$$

for small $\Delta r/r_c$. This yields the harmonic oscillator motion from Newton's second law

$$\frac{d^2\Delta r}{dt^2} = -\left(\frac{e_*^2}{mr_c^3}\right)\Delta r = -\left(\frac{c}{r_c}\right)^2 \Delta r \tag{B2}$$

with the "spring constant" (e_*^2/r_c^3) and oscillator frequency c/r_c. The corresponding motion that is due to the vacuum hole $(e_*, -m)$ (whose charge and mass fields exert a force that is the negative of (1)) is

$$-\frac{d^2\Delta r}{dt^2} = +\left(\frac{c}{r_c}\right)^2 \Delta r \tag{B3}$$

showing that the massive free charge and the vacuum hole cause identical accelerations within the PV continuum.

The total vacuum acceleration is the sum of (B2) and (B3)

$$\frac{d^2\Delta r}{dt^2} = -2\left(\frac{e_*^2}{mr_c^3}\right)\Delta r = -2\left(\frac{c}{r_c}\right)^2 \Delta r \tag{B4}$$

with the corresponding harmonic oscillator frequency

$$\sqrt{\frac{2e_*^2}{mr_c^3}} = \sqrt{2}\,\frac{c}{r_c} \tag{B5}$$

which is $\sqrt{2}$ times the angular frequency associated with the electron mass energy. Given the roughness of the calculations, this result implies that the combined massive-charge forces, acting simultaneously on the PV continuum, are the source of the zitterbewegung with its $2c/r_c$ frequency.

Appendix C: Quantum field

The PV is envisioned as a *degenerate* negative-energy sea of fermionic Planck particles. Because of this degeneracy, the vacuum experiences only small displacements from equilibrium when stressed. Thus the displacements due to the coupling force (1) are small, and so the potential energy corresponding to the stress can be approximated as a quadratic in those displacements. This important result enables the vacuum to support normal mode coordinates and their assumed quantum fields, as explained in the simple demonstration to follow.

The normal mode connection [8, pp. 109–119] to the quantum field can be easily understood by examining a string, stretched between two fixed points in a stationary reference frame, that exhibits small transverse displacements from equilibrium. In this case, the corresponding potential energy can be expressed in terms of quadratic displacements. If the displacements are represented by the function $\phi(t, x)$ at time t and position x along the string, then the quadratic assumption implies that the displacements must obey the wave equation

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial x^2} \tag{C1}$$

where c is a propagation velocity. The string geometry leads to the Fourier series representation

$$\phi(t, x) = \sum_{n=1}^{N} a_n(t) \sin(n\pi x/L) \tag{C2}$$

for the standing wave on the string, where L is the string length. Inserting (C2) into (C1) produces

$$\ddot{a}_n(t) = -\omega_n^2 a_n(t) \qquad \text{where} \qquad \omega_n = n\pi c/L \tag{C3}$$

and where the amplitude $a_n(t)$ is that of a harmonic oscillator.

The constant characterizing the Dirac equation is the Compton radius r_c. So it is reasonable to set the string length $L \sim r_c$ to determine the fundamental frequency $\omega_1 = \pi c/L$ in (C3). Furthermore, the harmonics of ω_1 can have wavelengths of the order of the Planck length r_* (antiparticle excitation is, of course, ignored in this rough argument); so the length L can be subdivided

$$N = \frac{L}{\text{minimum length division of string}} \sim \frac{r_c}{r_*}$$

$$= \frac{3.86 \times 10^{-11}}{1.62 \times 10^{-33}} \sim 10^{22} \tag{C4}$$

times, and ϕ in (C2) can be expressed as an integral if convenient since $r_* \ll r_c$.

The total energy of the vibrating string can thus be expressed as

$$E = \int_0^L \left[\frac{\rho}{2} \left(\frac{\partial\phi}{\partial t} \right)^2 + \frac{\rho}{2} c^2 \left(\frac{\partial\phi}{\partial x} \right)^2 \right] dx \tag{C5}$$

which, inserting (C2) into (C5), results in [8, p.117]

$$E = \frac{L}{2} \sum_{n=1}^{N} \left[\frac{\rho \dot{a}_n^2}{2} + \frac{\rho \omega_n^2 a_n^2}{2} \right] \tag{C6}$$

where the first and second terms in (C5) and (C6) are the kinetic and potential string energies respectively (ρ is the string density).

The crucial significance of (C6) is that it is a sum of independent normal-mode energies, where the $a_n(t)$ are the normal mode coordinates. From this normal mode setting, the quantum field energy

$$E = \sum_{n=1}^{N} \left(n_n + \frac{1}{2} \right) \hbar\omega_n = mc^2 \sum_{n=1}^{N} \left(n_n + \frac{1}{2} \right) r_c k_n \tag{C7}$$

is defined, where n_n is the number of normal modes associated with the wavenumber $k_n = \omega_n/c$. In effect, the integers n_n ($\geqslant 0$) determine the quantized energy level of each normal mode oscillator $a_n(t)$. The $1/2$ component in (C7) is the zero-point energy of the string-vacuum system.

At this point the quantum-field formalism discards the preceding foundation upon which the fields are derived, and assumes that the fields themselves are the primary reality [8, p. 119]. Part of the reason for this assumption is that, in the past, no obvious foundation was available. However, the demonstration here provides such a foundation on the simple, but far-reaching assumption that the vacuum is a degenerate state which can sustain a large stress without a correspondingly large strain.

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