Reexamination of Nuclear Shape Transitions in Gadolinium and Dysprosium Isotopes Chains by Using the Geometric Collective Model

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The geometric collective model proposed in a previous paper in examined to describe the nuclear shape transitions for Gd and Dy isotopes chains. The optimized model parameters for each nucleus have been adjusted by fitting procedure using a computer simulated search program in order to reproduce the excitation energies ($2_1^+, 4_1^+, 6_1^+, 8_1^+, 0_2^+, 2_3^+, 4_3^+, 2_2^+, 3_1^+$ and 4_2^+) and the two neutron separation energies in all nuclei in each isotopic chain. Calculated potential energy surface (PES'S) describing all deformation effects of each nucleus have been extracted. Our systematic studies on Gd / Dy chains have revealed a shape transition from spherical vibrator to axially deformed rotor when moving from the lighter to heavier isotopes.

1 Introduction

Recent developments in nuclear structure have brought considerable focusing on the problems of shape phase transition and shape coexistence phenomena [1]. For instance, several isotopes have been found to undergo shape phase evolution of first order from spherical vibrator to deformed axially symmetric rotor [2–6] and phase transition of second order from spherical vibrator to deformed γ - soft [7–9]. The study of shape phase transitions in nuclei was best done by using the interacting boson model (IBM) [10]. The original version of IBM (IBM-1) includes s and d bosons, it defines six-dimensional space and this leads to a description in terms of a unitary group U(6). Three dynamical symmetries in the IBM-1 were shown [11]: the U(5) symmetry corresponding to spherical oscillator, the SU(3) symmetry corresponding to deformed axially rotor and the O(6) symmetry corresponding to the γ - soft asymmetric rotor shapes. These three symmetry limits from a triangle known as a Casten triangle that represents the nuclear phase diagram [12]. The X(5) critical point symmetry [13] has been found to correspond to the first order transition between U(5) and SU(3), while the E(5) critical point symmetry [14] has been found to correspond to the second order transition between U(5) and O(6).

In the previous paper [3], we used the flexible and powerful geometric collective model (GCM) [3, 15–18] to describe the quantum phase transition between spherical and deformed shapes for doubly even nuclei in lanthanide and actinide isotopes chains. The potential energy surfaces (PES'S) describing all deformed effects of each nucleus were extracted in terms of the intrinsic shape parameters β and γ. The parameter β is related to the axial deformation of the nucleus, while γ measure the deviation from axial symmetry. In the present work, it is of interest to examine the GCM in investigating the shape transition from spherical vibrator to axially deformed rotor for Gd and Dy isotopic chains by analyzing the PES'S. In section 2, we construct the GCM Hamiltonian and its eigenfunction. In section 3, we generated the PES'S to classify shape phase transitions and to decide if a nucleus is close to criticality. In section 4, we applied our model to the rare earth Gd / Dy isotopic chains which evolve a rapid structural charges from spherical to well-deformed nuclei when moving from lighter to the heavier isotopes.

2 The GCM Hamiltonian and eigenstates

In GCM, the Hamiltonian of the nucleus, in appropriate units, can be expressed as a series expansion in terms of the surface deformation coordinates α and the conjugate momenta π as [3]:

$$\begin{aligned} H = & \frac{1}{2B_2}[\pi\times\pi]^{(0)} + C_2[\alpha\times\alpha]^{(2)} \\ & + C_3[[\alpha\times\alpha]^{(2)}\times\alpha]^{(0)} \\ & + C_4[\alpha\times\alpha]^{(0)}[\alpha\times\alpha]^{(0)} \end{aligned} \tag{1}$$

The eigenstates of the the Hamiltonian 1 associated with the number ν of quanta and definite seniority λ, angular momentum L and projection M can be denoted by the Ket

$$|\nu\lambda\mu LM\rangle = F_\ell^\lambda(\beta)\sum_k \varphi_k^{\lambda\mu L}(\gamma)D_{Mk}^{L_*}(\omega_i) \tag{2}$$

where

$$\ell = \frac{1}{2}(\nu-\lambda) \tag{3}$$

and μ indicates the remaining quantum numbers required to fully characterize the states of the Hamiltonian 1. ω_i are the Euler angles, β and γ are the intrinsic coordinates. $D_{Mk}^{L_*}(\omega_i)$ are the Wigner functions that are the irreducible representation of the O(3) group.

In equation 2 $F_\ell^\lambda(\beta)$ are functions of β associated with the radial part of a five-dimensional oscillator

$$\begin{aligned} F_\ell^\lambda(\beta) = & \left[\frac{2(n_i)}{\Gamma(n+\lambda+\frac{5}{2})}\right]^{1/2}\left(\frac{C_2}{\hbar\omega}\right)^{\frac{5}{4}+\frac{\lambda}{2}}\beta^\lambda \\ & \cdot L_n^{\lambda+\frac{3}{2}}\cdot\left(\left(\frac{C_2}{\hbar\omega}\right)\beta^2\right)e^{-\frac{1}{2}\frac{C_2}{\hbar\omega}\beta^2} \end{aligned} \tag{4}$$

where $L_n^{\lambda+\frac{3}{2}}$ are the well-known Laguerre polynomials and the function is normalized for the volume element $\beta 4 d\beta$. The γ-dependent functions $\varphi_k^{\lambda\mu L}$ satisfy the differential equation

$$\Lambda^2 \varphi_k^{\lambda\mu L} = \lambda(\lambda+3)\varphi_k^{\lambda\mu L} \tag{5}$$

where Λ^2 is the seniority operator (Casimir operator of O(5)) which has the form

$$\Lambda^2 = -\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma} + \sum_{k=1}^{3} I_k^{-1}\grave{L}_k^2(\omega_i) \tag{6}$$

with

$$I_k = 4B_2 \sin^2\left(\gamma - \frac{2\pi}{3}k\right) \tag{7}$$

I_k are the moments of inertia with respect to the principle axes. For arbitrary angular momentum L and λ , $\varphi(\gamma)$ reads

$$\varphi_{\grave{k}}^{\lambda+2,\mu,L+2}(\gamma) = \sum_k \varphi_{k,\bar{k}}^{\lambda LL+2}\left(\gamma, \frac{\partial}{\partial\gamma}\right)\varphi_k^{\lambda\mu L}(\gamma) \tag{8}$$

$$\varphi_{\bar{k}}^{\lambda+2,\mu,L+2}(\gamma) =$$

$$= \sum_{\bar{L}\bar{k}k}\left(\sqrt{35(2\bar{L}+1)}W(L,L+2,2,2,2\bar{L}) \times \right. \tag{9}$$

$$\left. Q_{\bar{k},\grave{k}}^{\lambda+1,\bar{L},L+2}\left(\gamma, \frac{\partial}{\partial\gamma}\right) Q_{k,\bar{k}}^{\lambda,L,\bar{L}}\left(\gamma, \frac{\partial}{\partial\gamma}\right)\varphi_k^{\lambda\mu L}(\gamma)\right)$$

where W is a Racah coefficient and $Q_{k,\bar{k}}^{\lambda,L,\bar{L}}(\gamma, \frac{d}{d\gamma})$ is an operator function of γ and $\frac{d}{d\gamma}$.

3 Potential energy surfaces (PES'S) and critical point symmetries

The PES depends only upon the shape of the nucleus not it orientation in space, and can thus be expressed purely in terms of the shape coordinates β and γ as [3]:

$$V(\beta,\gamma) = C_2\frac{1}{\sqrt{5}}\beta^2 - C_3\sqrt{\frac{2}{35}}\beta^3\cos 3\gamma + C_4\frac{1}{5}\beta^4 \tag{10}$$

where $\beta \in [0,\infty]$ and $\gamma \in [0, 2\pi/3]$

The equilibrium shape associated to the GCM Hamiltonian can be obtained by determining the minimum of energy surface with respect to the geometric variables β and γ, i.e the first derivative vanish. Since the parameter C_2 controls the steepness of the potential, and therefore, the dynamical fluctuations in γ, it strongly affects the energies of excited intrinsic states. The parameter $C_3 = 0$ gives a γ-flat potential and an increase of C_3 introduces a γ-dependence the potential with a minimum at $\gamma = 0$. Changing C_3 will indeed induce a γ-unstable to symmetric rotor transition, it is best to simultaneously vary C_2 and C_4 as well. The shape transition from vibrator to rotor is achieved by starting from the vibrator limit, lowering C_2 from positive to negative value, increasing C_4 to large positive value, which gradually increasing C_3 (lowering C_2 from positive to negative value, introducing a large positive C_4 and a positive C_3).

4 Numerical results applied to Gd and Dy isotopes chains

The N = 90 isotones ^{154}Gd [15, 16] and ^{156}Dy [17, 18] were seen to provide good example to transition from spherical to axially deformed. In our calculation we will examine and systematically study the lanthanide $^{148-162}$Gd and $^{150-164}$Dy isotopes because of the richness of available experimental data indicating a transition of nuclear shapes from spherical to deformed form. The ground band levels are shown in Figure (1). We can see that the energy values for each spin states in lanthanide change almost linearly for $N \leq 88$ and become quite flat for N ≥ 90. This is consistent with the onset of the Z = 64 sub-shell effect. For actinide the energy levels become flat for N ≥ 144. The optimized model parameters for each nucleus was adjusted by fitting procedure using a computer simulated search program in order to describe the gradual change in the structure as neutron number varied and to reproduce the properties of the selected reliable state of positive parity excitation ($2_1^+, 4_1^+, 6_1^+, 8_1^+, 0_2^+, 2_3^+, 4_3^+, 2_2^+, 3_1^+$ and 4_2^+) and the two neutron separation energies of all isotopes in each isotopic chain. The resulting parameters are listed explicitly in Tables (1).

For the isotopic chains investigated here, the collective properties are illustrated by represented the calculated potential energy surface (PES) describing all deformation effects of the nucleus. We investigated the change of nuclear structure

Table 1: The GCM parameters as derived in fitting procedure used in the calculation of the Gd and Dy isotopes.

Nucleus	C_2	C_3	C_4
^{148}Gd	16.53067	1.48970	-34.76151
^{150}Gd	9.79566	11.28331	-5.21603
^{152}Gd	-26.55250	53.24420	138.12500
^{154}Gd	-71.41529	104.21630	313.83380
^{156}Gd	-91.19133	127.81150	392.95380
^{158}Gd	-101.97220	141.63350	437.50440
^{160}Gd	-111.19320	153.76500	476.06680
^{162}Gd	-120.17800	165.64110	513.72330
^{150}Dy	18.56558	1.70251	-38.99710
^{152}Dy	10.69898	12.69373	-5.14990
^{154}Dy	-29.90650	59.16022	154.37500
^{156}Dy	-79.02660	114.63790	346.26770
^{158}Dy	-99.93424	139.43080	429.68950
^{160}Dy	-110.88850	153.43620	474.89930
^{162}Dy	-120.13350	165.59310	513.55260
^{164}Dy	-129.12150	177.47260	551.221306

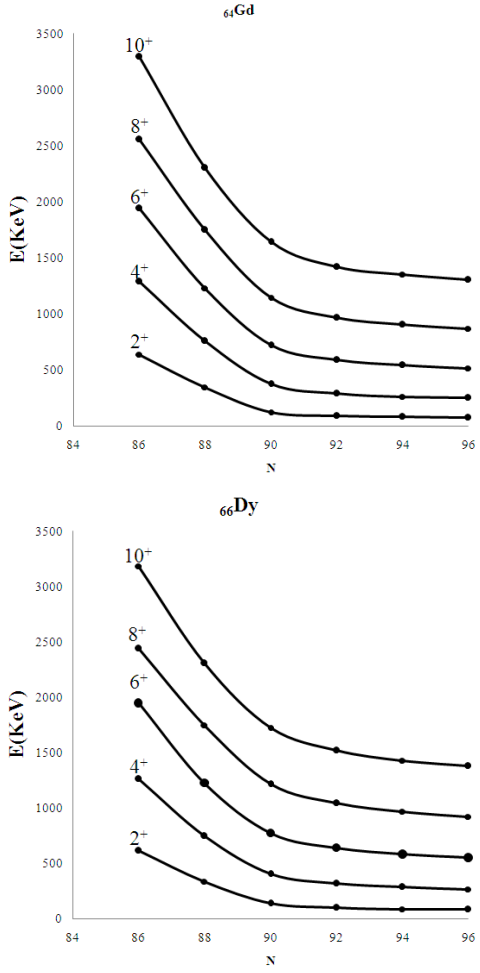

Fig. 1: Systematics of low-lying yrast level energies in even-even lanthanides Gd/Dy isotopes. The $2^+, 4^+, ...10^+$ level energies are plotted. The states are labeled by I^π.

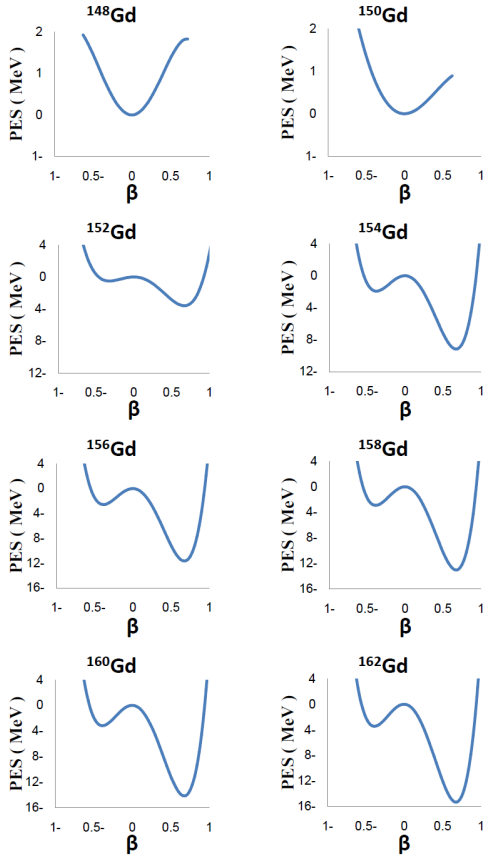

Fig. 2: Potential energy surface (PES) calculated with GCM as a function of the shape parameter β for shape phase transition from spherical to prolate deformed for Gadolinium isotope chain $^{148-162}_{64}$Gd.

within these chains as illustrated in Figures (2, 3). The PES's versus the deformation parameter β for lanthanide isotopic chains of nuclei evolving from spherical to axially symmetric well deformed nuclei. We remark that for all mentioned nuclei, the PES is not flat, exhibiting a deeper minimum in the prolate ($\beta > 0$) region and a shallower minimum in the oblate ($\beta < 0$) region. Relatively flat PES occur for the N = 86 nuclei ^{150}Gd and ^{152}Dy. A first order shape phase transition with change in number of neutrons when moving from the lighter to heavier isotopes, i.e U(5) - SU(3) transitional region are observed.

The present result for ^{154}Gd is in good agreement with Nilsson-Strutinsky BCS calculations [18]. However, the existence of a bump in the PES is related to the success of the confined γ-soft (BCS) rotor model [19], employing an infinite square well potential displaced from zero, as well as to the relevance of Davidson potentials [20–22]. It also be related to the significant five-dimensional centrifugal effect [22–25].

5 Conclusion

In the present paper exact numerical results of GCM Hamiltonian along the shape phase transition line from harmonic spherical vibrator shape to axially deformed rotor shape are obtained. A systematic study of even-even $^{148-162}$Gd and $^{150-164}$Dy isotopes chains in the lanthanide region is presented. For each nucleus the GCM parameters C_2, C_3, C_4 were optimized to fit the energy ratios between selected low-lying states. The geometric character of the nuclei has been visualized by plotting the PES'S obtained from the GCM Hamiltonian. In these chains, nuclei evolve from spherical to prolate axially deformed rotor when moving from the lighter to the heavier isotopes. Also we have analyzed the critical points of the shape phase transition in the space of the GCM parameters C_2, C_3 and C_4.

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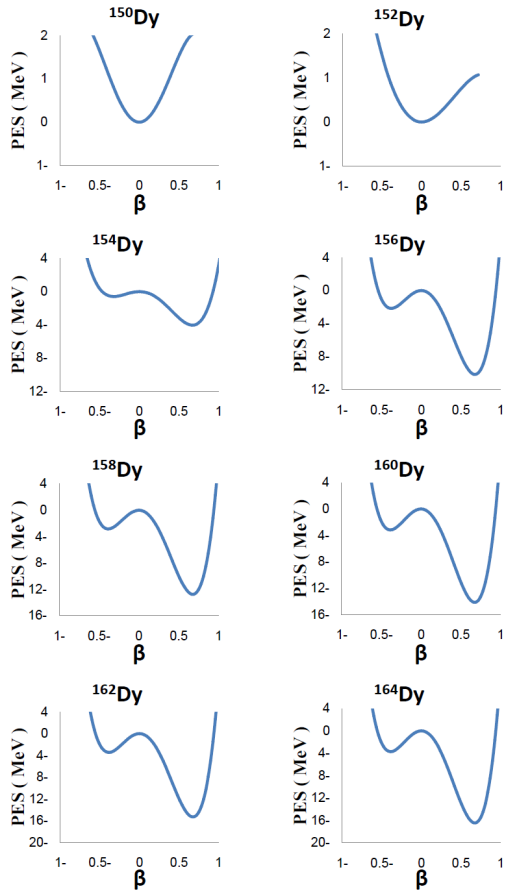

Fig. 3: Potential energy surface (PES) calculated with GCM as a function of the shape parameter β for shape phase transition from spherical to prolate deformed for Dysprosium isotope chain $^{150-164}_{66}$Dy.

References

1. Iachello F. and Zamfir N.V. Quantum Phase Transitions in Mesoscopic Systems. *Physical Review Letters*, 2004, v. 92(3), 212501–212505.
2. Khalaf A.M. and Awwad T.M. A Theoretical Description of U(5)-SU(3) Nuclear Shape Transitions in the Interacting Boson Model. *Progress in Physics*,2013, v. 1, 7–11.
3. Khalaf A.M. and Ismail A.M. The Nuclear Shape Phase Transitions Studied Within the Geometric Collective Model. *Progress in Physics*, 2013, v. 2, 51–55.
4. Khalaf A.M. and Ismail A.M. Structure Shape Evolution in Lanthanide and Actinide Nuclei. *Progress in Physics*, 2013, v. 2, 98-104.
5. Rosensteel G and Rowe D.J. Phase Transitions and Quasidynamical Symmetry in Nuclear Collective Models, III: The U(5) to SU(3) Phase Transition in the IBM. *Nuclear Physics*, 2005, v. A759, 92–128.
6. Liu Y.X., Mu L.Z. and Wei H. Approach to The Rotation Driven Vibrational to Axially Rotational Shape Phase Transition Along The Yrast Line of a Nucleus. *Physics Letters*, 2006, v. B633, 49–53.
7. Heinze S. et al. Evolution of spectral properties along the O(6)-U(5) transition in the interacting boson model. I. Level dynamics. *Physical Review*, 2006, v. C76, 014306–014316.
8. Zhang Y., Hau Z. and Liu Y.X. Distinguishing a First Order From a Second Order Nuclear Shape Phase Transition in The Interacting Boson Model. *Physical Review*, 2007, v. C76, 011305R–011308R.
9. Casten R.F. and Zamfir N.V. Evidence for a Possible E(5) Symmetry in 134Ba. *Physical Review Letters*, 2000, v. 85, 3584–3586, Casten R.F. and Zamfir N.V. Empirical Realization of a Critical Point Description in Atomic Nuclei. *Physical Review Letters*, 2001, v. 87, 052503–052507.
10. Iachello F. and Arima A. The Interacting Boson Model. Cambridge University Press, Cambridge, England, 1987.
11. Arima A. and Iachello F. Interacting Boson Model of Collective States I. The Vibrational Limit. *Annals of Physics*, 1976,(N.Y.) v. 99, 253–317, Arima A. and Iachello F. Interacting Boson Model of Collective States II. The Rotational Limit. *Annals of Physics*, 1978,(N.Y.) v. 111, 201–238.
12. Casten R.F. Nuclear Structure from a Simple Prespective. Oxford University, Oxford, 1990.
13. Iachello F. Analytic Prescription of Critical Point Nuclei in a Spherical Axially Deformed Shape Phase Transtion. *Physical Review Letters*, 2001, v. 87, 052502–052506.
14. Iachello F. Dynamic Symmetries at The Critical Point. *Physical Review Letters*, 2000, v. 85 3580–3583.
15. Tonev D. et al. Transition Probabilities in 154Gd: Evidence for X(5) Critical Point Symmetry. *Physical Review*, 2004, v. C69, 034334–034339.
16. Dewald A. et al. Shape changes and test of the critical-point symmetry X(5) in N = 90 nuclei. *The European Physical Journal*, 2004, v. A20, 173–178.
17. Caprio M.A. et al. Low-Spin Structure of 156Dy Through -ray Spectroscopy. *Physical Review*, 2002, v. C66, 054310–054328.
18. Zhang J.Y. et al. Phase/shape coexistence in 152Sm in the geometric collective model. *Physical Review*, 1999, v. C60, 061304R–061308R.
19. Turken N. and Inci I. Comparing some predictions between Davidson-like potentials and interacting boson model: X(5) behavior of even-even 128-140Nd isotopes. *Physics of Atomic Nuclei*, 2008, v. 71, 1918–1925.
20. Bonatsos D. et al. Ground state bands of the E(5) and X(5) critical symmetries obtained from Davidson potentials through a variational procedure. *Physics Letters*, 2004, v. B584, 40–47.
21. Bonatsos D. et al. E(5) and X(5) Critical Point Symmetries Obtained From Davidson Potentials Through a Variational Procedure. *Physical Review*, 2004, v. C70, 024305–024314.
22. Caprio M.A. Effects of $\beta - \gamma$ coupling in transitional nuclei and the validity of the approximate separation of variables. *Physical Review*, 2005, v. C72, 054323–054333.
23. Rowe D.J. A computationally tractable version of the collective model. *Nuclear Physics*, 2004, v. A735, 372–392.
24. Rowe D.J., Turner P.S. and Repka J. Spherical harmonics and basic coupling coefficients for the group SO(5) in an SO(3) basis. *Journal of Mathematical Physics*, 2004, v. 45, 2761–2784.
25. Rowe D.J. and Turner P.S. The Algebraic Collective Model. *Nuclear Physics*, 2005, v. A753, 94–105.