Staggering Phenomenon in Gamma Transitional Energies over Spin for Negative Parity States of Octupole Vibrational Nuclear Bands

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The negative parity states of octupole vibrational bands in Tungsten and Osmium nuclei have perturbed structure. To explore the $\Delta I = 1$ staggering, we plotted the gamma transitional energy over spin (EGOS) versus I^2. Such a plot exhibit large deviation from a linear $I(I+1)$ dependence $E(I) = A[I(I+1)]+B[I(I+1)]^2$ and effectively splits into two different curves for odd and even spin states and a staggering pattern is found. The odd-spin members $I^\pi = 1^-, 3^-, 5^-, \dots$ were displaced relatively to the even-spin members $I^\pi = 2^-, 4^-, 6^-, \dots$ i.e. the odd levels do not lie at the energies predicted by the pure rotator fit to the even levels, but all of them lie systematically above or all of them lie systematically below the predicted energies because the odd-spin states can be aligned completely, while the even-spin states can only be aligned partially. Also the $\Delta I = 1$ staggering effect has been clearly investigated by examining the usual backbeding plot.

1 Introduction

The properties of nuclear rotational bands built on octupole degrees of freedom have been extensively studied within various microscopic as will as macroscopic model approaches in nuclear structure [1–6]. It is will known that heavy nuclei have low-lying $K^\pi = 0^-$ octupole deformed bands [7, 8]. Theoretical works of such bands have been presented in framework of cranked random phase approximation (RPA) [9, 10], the collective model [5], the interacting boson model (IBM) [3, 11], the variable moment of inertia (VMI) model [12] and the alpha particle cluster model [4, 13]. The IBM and the exotic cluster models address the existence of negative parity bands with $K^\pi \neq 0^-$.

Several staggering effects are known in nuclear spectroscopy. The $\Delta I = 2$ staggering has been observed and interpreted in superdeformed (SD) nuclei [14–22], where the levels with $I = I_0 + 2, I_0 + 6, I_0 + 10, \dots$ are displaced relatively to the levels with $I = I_0, I_0 + 4, I_0 + 8, \dots$, i.e. the level with angular momentum I is displaced relatively to its neighbors with angular momentum $I \pm 2$. There is another kind of staggering happening in SD odd-A nuclei, the $\Delta I = 1$ signature splitting in signature partners pairs [23].

The $\Delta I = 1$ Staggering in odd normal deformed (ND) nuclei is familiar for a long time [24–28], where the rotational bands with $K = 1/2$ separate into signature partners, i.e. the levels with $I = 3/2, 7/2, 11/2, \dots$ are displaced relatively to the levels with $I = 1/2, 5/2, 9/2, \dots$. In this paper, we will investigate another type of $\Delta I = 1$ energy staggering occurring in the negative parity octupole bands of even-even nuclei, where the levels with odd spin $I^\pi = 1^-, 3^-, 5^-, \dots$ are displaced relatively to the levels with even spin $I^\pi = 2^-, 4^-, 6^-, \dots$. This is more strikingly revealed when one makes the usual backbending plot of the energies in which the kinematic moment of inertia is plotted against the square of rotational frequency. The negative parity octupole band breaks into even and odd-spin bands with, however, very little backbending tendency.

2 Outline of the Theory of ΔI = 1 Energy Staggering

To analyze the $\Delta I = 1$ energy staggering in collective bands, several tests have been considered in the literature. In our analysis, the basic staggering parameter is the gamma transitional energy over spin (EGOS=$E_\gamma(I)/I$) of the transitional energies in a $\Delta I = 1$, where E(I) is the energy of the state of the spin I, and $E_\gamma(I)$ denotes the dipole transition energy

$$E_\gamma(I) = E(I) - E(I-1). \tag{1}$$

The level energies in a band can be more realistic parameterize by two-term rotational formula as a reference

$$E(I) = A[I(I+1)] + B[I(I+1)]^2. \tag{2}$$

The first two-term represents the perfect purely collective rigid rotational energy, where A denotes the inertial parameter $A = \hbar/2J$ (where J is the kinematic moment of inertia). The introduction of the second term is based on the assumption that, on rotation, the moment of inertia of the nucleus increases as does the quadratic function of the square of the angular velocity of rotation of the nucleus.

It is interesting to discuss the energy levels by plotting EGOS against spin. This is not helpful to identify the structure of the nucleus, but also to see clearly changes as a function of spin. For pure rotator, the energies of the yrast states are:

$$E(I) = A[I(I+1)]. \tag{3}$$

Then the $E2$ γ-ray energies are given by

$$E_\gamma(I) = A[4I-2] \tag{4}$$

which yield

$$EGOS = A\left(4 - \frac{2}{I}\right). \tag{5}$$

Table 1: The adopted best model parameters A and B for our selected octupole vibrational bands.

	^{178}W	^{180}W	^{176}Os	^{178}Os	^{180}Os	^{182}Os
A (keV)	13.637	13.027	9.665	10.083	11.796	9.491
B (eV)	-13.821	-8.517	-2.223	-3.032	-8.607	0.140

In units of A, EGOS evolves from 3 for $I = 2$ up to 4 for high I, and so gradually increasing and asymptotic function of I.

EGOS for our proposed reference formula (2) is given by

$$EGOS = 2A + 4BI^2. \qquad (6)$$

The EGOS when plotted against I^2, it represent a straight line of intercept 2A and slope 4B. Practically, the plot splits into two different curves for the odd and even spin states respectively. To see fine variation in the plot (EGOS & I^2), we use the staggering parameter

$$e(I) = EGOS - \left(2A + 4BI^2\right)_{\text{ref}} \qquad (7)$$

where the unknown A and B are determined by minimizing the function F

$$F(I, A, B) = \sum_I |e(I)|^2. \qquad (8)$$

The summation over spin in equation (8) is taken in step of $\Delta I = 1$. The function F has a minimum value when all its partial derivatives with respect to A and B vanish ($\partial F/\partial A = 0, \partial F/\partial B = 0$), this leads to

$$2nA + 4\sum_I I^2 B = \sum_I EGOS(I) \qquad (9)$$

$$2\sum_I I^2 A + 4\sum_I I^4 B = \sum_I I^2 EGOS(I) \qquad (10)$$

where n is the number of data points.

The behavior of the octupole band is most clearly illustrated by a conventional backbending plot. For each $\Delta I = 2$ value, the effective nuclear kinematic moment of inertia is plotted versus the square of the rotational frequency. If we consider the variation of the kinematic moment of inertia $J^{(1)}$ with angular momentum I, we can write

$$\frac{2J^{(1)}}{\hbar^2} = \frac{4I - 2}{E(I) - E(I-2)}. \qquad (11)$$

Lets us define the rotational frequency $\hbar\omega$ as a derivative of the energy E(I) with respect to the angular momentum $[I(I+1)]^{1/2}$,

$$\hbar\omega = \frac{dE}{d[I(I+1)]^{1/2}} \qquad (12)$$

usually we adopt the relation

$$(\hbar\omega)^2 = \frac{4(I^2 - I + 1)}{(2J^{(1)}/\hbar^2)^2}. \qquad (13)$$

3 Numerical Calculation and Discusion

Our selected octupole bands are namely: ^{178}W, ^{180}W, ^{176}Os, ^{178}Os, ^{180}Os and ^{182}Os. The optimized model parameters A and B for each nucleus have been adjusted by using a compuer simulation search program to fit the calculated theoretical energies $E^{cal}(I_i)$, with the corresponding experimental ones $E^{exp}(I_i)$. The procedure of fitting is repeated for several trail values A and B to minimize the standard quantity χ which represent the root mean square deviation

$$\chi = \left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{E^{exp}(I_i) - E^{Cal}(I_i)}{\Delta E^{exp}(I_i)}\right)^2\right]^{1/2}$$

where N is the number of data points and $\Delta E^{exp}(I_i)$ are the experimental errors. The best optimized parameters are listed in table (1). The negative parity octupole bands have several interesting characteristics, the most obvious of which is the staggering effect. In this paper the $\Delta I = 1$ staggering is evident on a plot of staggering parameter $e(I)$ against I^2 and illustrated in figure (1), the band effectively splits into an odd- and even-spin sequence with a slight favoring in energy for the odd-spin states. In terms of an alignment of the angular momentum of the octupole vibration, the odd energy favoring can be understood since the odd-spin states can be aligned completely ($I \sim R + 3$, where $R = 0, 2, 4, \ldots$ is the collective rotation), while the even spins can only be aligned partially ($I \sim R + 2$). As expected from a good rotor model, the γ-ray transition energy $E_\gamma(I)$ increases with increasing the angular momentum I. It is found in some rotational deformed nuclei that the transition energy decreases with increasing I, this anomalous behavior is called nuclear backbending. In order to represent this backbending, one prefers to plot twice the kinematic moment of inertia $2J^{(1)}/\hbar^2$ versus the square of the rotational frequency $(\hbar\omega)^2$. Figure (2) shows the backbending plot for our selected octupole bands. It is seen that the bands are essentially separate into odd and even spin sequences which shows the effects of rotation alignment. The increase in Coriolis effects is due to the lowering of the Fermi level, then these effects depress the odd spin states relative to the even spin states. When the Coriolis effects are large compered with the octupole correlations effected through the residual interaction, it becomes inappropriate to identify these bands as octupole bands (decoupled two quasiparticle bands). These are bands in which the intrinsic spin has been aligned with the rotational spin through the decoupling action of the Coriolis force.

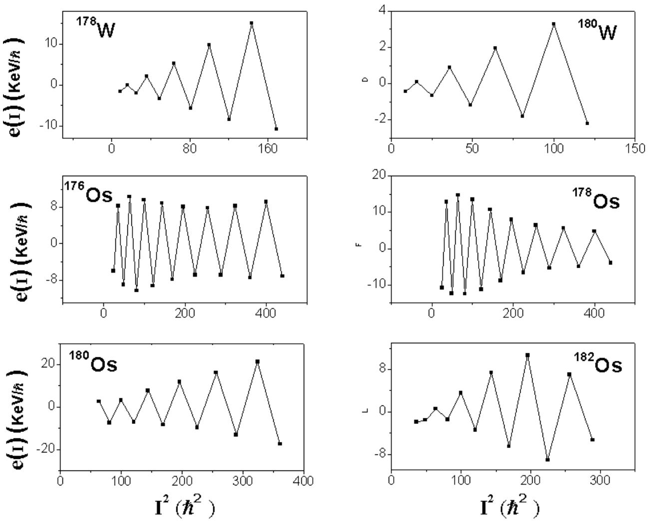

Fig. 1: The odd-even $\Delta I = 1$ energy staggering parameters $e(I)$ versus I^2 for negative parity states of octupole vibrational bands in doubly even nuclei ^{178,180}W and ^{176,178,180,182}Os.

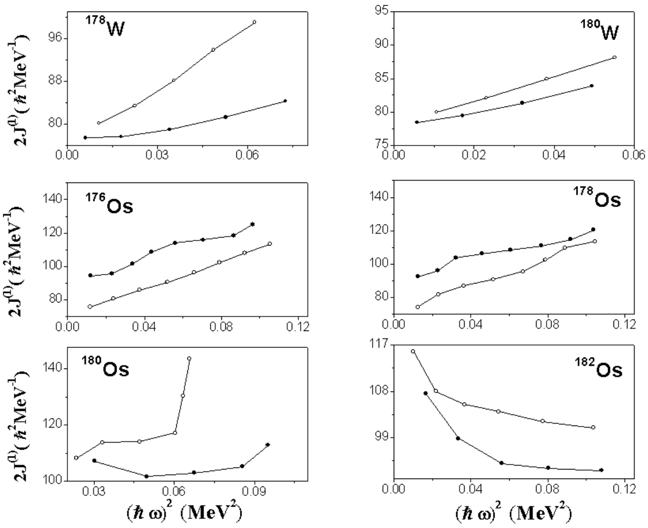

Fig. 2: Plot of twice Kinematic moment of inertia $2J^{(1)}$ against the square of the rotational frequency $(\hbar\omega)^2$ for the negative parity bands in ^{178,180}W and ^{176,178,180,182}Os isotopes.

4 Conclusion

In negative parity octupole bands of even-even W/Os nuclei, the levels with odd spins $I^\pi = 1^-, 3^-, 5^-, ...$ are displaced relatively to the levels with even spins $I^\pi = 2^-, 4^-, 6^-, ...$ The effect is called $\Delta I = 1$ staggering and its magnitude is clearly larger than the experimental errors. The phase and amplitude of the splitting is due to rotation particle Corialis coupling. Our proposed two terms formula provided us with information about the effective moment of inertia.

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