LETTERS TO PROGRESS IN PHYSICS

On the Equation which Governs Cavity Radiation

Pierre-Marie Robitaille
Department of Radiology, The Ohio State University, 395 W. 12th Ave, Columbus, Ohio 43210, USA
E-mail: robitaille.1@osu.edu

In this work, the equation which properly governs cavity radiation is presented. Given thermal equilibrium, the radiation contained within an arbitrary cavity depends upon the nature of its walls, in addition to its temperature and its frequency of observation. With this realization, the universality of cavity radiation collapses. The constants of Planck and Boltzmann can no longer be viewed as universal.

Science enhances the moral value of life, because it furthers a love of truth and reverence...

Max Planck, Where is Science Going? 1932 [1]

When Max Planck formulated his law [2, 3], he insisted that cavity radiation must always be black or normal [3, Eqs. 27, 42], as first proposed by Gustav Robert Kirchhoff [4, 5]. The laws of thermal emission [2–7] were considered universal in nature. Based on Kirchhoff's law [4, 5], cavity radiation was said to be independent of the nature of the walls and determined solely by temperature and frequency. Provided that the cavity walls were opaque, the radiation which it contained was always of the same nature [2–5]. All cavities, even those made from arbitrary materials, were endowed with this property.

Cavity radiation gained an almost mystical quality and Planck subsequently insisted that his equation had overarching consequences throughout physics. The constants contained within his formulation, those of Planck and Boltzmann (h and k), became fundamental to all of physics, leading to the development of Planck length, Planck mass, Planck time, and Planck temperature [3, p. 175].

However, it can be demonstrated that cavity radiation is not universal, but depends on the nature of the cavity itself [8–15]. As such, the proper equation governing cavity radiation is hereby presented.

It is appropriate to begin this treatment by considering Kirchhoff's law [3, Eqs. 27, 42]:

$$\frac{\epsilon_\nu}{\kappa_\nu} = f(T, \nu)\,, \tag{1}$$

where $f(T, \nu)$ is the function presented by Max Planck [3, Eq. 300].* In order to avoid confusion, Eq. 1 can be expressed by using the currently accepted symbols for emissive power, E, and absorptivity, κ_ν:

$$\frac{E_\nu}{\kappa_\nu} = f(T, \nu)\,. \tag{2}$$

As Eq. 1 was hypothesized to be applicable to all cavities, we can adopt the limits of two extremes, namely the "perfect absorber" and the "perfect reflector".†

First, the condition under which Kirchhoff's law is often presented, the "perfectly absorbing" cavity, can be considered (emissivity $(\epsilon_\nu) = 1$, absorptivity $(\kappa_\nu) = 1$, reflectivity $(\rho_\nu) = 0$; at the frequency of interest, ν). In setting κ_ν to 1, it is apparent that the mathematical form of the Eq. 1 remains valid. Second, if a "perfectly reflecting" cavity is utilized ($\epsilon_\nu = 0$, $\kappa_\nu = 0$, $\rho_\nu = 1$), it is immediately observed that, in setting κ_ν to 0, Eqs. 1 and 2 become undefined. Max Planck also recognized this problem, but chose to ignore its consequences (see § 48, 49). Yet, this simple mathematical test indicates that arbitrary cavities cannot be black, as Kirchhoff's law cannot be valid over all conditions.

It is also possible to invoke Stewart's law of thermal emission [16] which states that, under conditions of thermal equilibrium, the emissivity and absorptivity are equal:

$$\epsilon_\nu = \kappa_\nu\,. \tag{3}$$

Therefore, Eq. 2 can be expressed as follows:

$$E_\nu = \epsilon_\nu \cdot f(T, \nu)\,. \tag{4}$$

Once again, this expression never states that all cavities contain black radiation. Rather, at thermal equilibrium, cavities contain raditation which will be reduced in intensity from the Planck function by an amount which manifests the lower

*Note that Max Planck refers to ϵ_ν as the "emissionskoeffizienten" [3, §26], which M. Masius translates as the "coefficient of emission". Today, the emission coefficient is also known as the emissivity of a material. Unfortunately, it is also referred to by the symbol ϵ_ν and this can lead to unintended errors in addressing the law of emission. In Eq. 1, dimentional analysis (see [3, Eq. 300]) reveals that Max Planck is referring to the emissive power, denoted by E, and not to emissivity, usually denoted by ϵ_ν. Still, at other points within "The Theory of Heat Radiation" (e.g. see §49) he utilizes the symbol, E, to refer to emissive power or "the radiation emitted". To further complicate the question, in his Eq. 27, Max Planck refers to κ_ν as the "absorptionskoeffizienten" which M. Masius translates at the "coefficient of absorption". In this case, dimentional analysis reveals that he is indeed referring to absortivity, κ_ν, and not to the absorptive power, A, of the medium.

†Perfectly absorbing or reflecting cavities do not exist in nature. Nonetheless, they are hypothesized to exist in mathematical treatments of blackbody radiation (see [3]).

emissivity of the material involved. It is evident that a lower emissivity is tied to a higher reflectivity, but the effect of reflection has not been properly included in Kirchhoff's law.

For any material, the sum of the emissivity and reflectivity is always equal to 1. This constitutes another formulation of Stewart's law [10, 16] which can also be expressed in terms of emissivity or absorptivity:

$$\epsilon_\nu + \rho_\nu = \kappa_\nu + \rho_\nu = 1\,. \tag{5}$$

With simple rearrangement, it is well known that absorptivity, κ_ν, and emissivity, ϵ_ν, can be expressed as:

$$\epsilon_\nu = \kappa_\nu = 1 - \rho_\nu. \tag{6}$$

As such, let us substitute these relations into Eq. 2:

$$\frac{E_\nu}{(1-\rho_\nu)} = f(T,\nu)\,. \tag{7}$$

With simple rearrangement, the law for arbitrary cavity radiation under conditions of thermal equilibrium, arises:

$$E_\nu = f(T,\nu) - \rho_\nu \cdot f(T,\nu)\,. \tag{8}$$

This law is now properly dependent on the nature of the cavity walls, because it includes the reflectivity observed in real materials.

Note that this expression is well known. Planck, for instance, presents it in a slightly modified form [3, § 49]. However, he choses to dismiss its consequences. Still, it is evident that when a cavity is constructed from a material which is "perfectly absorbing", the second term in Eq. 8 makes no contribution ($\rho_\nu \cdot f(T,\nu) = 0$) and the emissive power is simply determined by the Planck function. If the cavity walls are "perfectly reflecting", Eq. 8, unlike Eq. 1 and 2, does not become undefined, but rather, equal to 0. For all other situations, the radiation contained within a cavity will be dependent on the manner in which the reflection term is driven. This will be discussed seperately.

Dedication

This work is dedicated to our mothers on whose knees we learn the most important lesson: love.

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