Properties of Nuclear Superdeformed Rotational Bands in A ∼ 190 Mass Region

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Two-parameters formula based on the conventional collective rotational model is applied to describe superdeformed rotational bands (SDRB's) in nuclei in the $A \sim 190$ mass region, namely the five SDRB's ^{192}Hg(SD1), ^{194}Hg(SD1), ^{194}Hg(SD2), ^{194}Pb(SD1) and ^{194}Pb(SD2). The bandhead spins of the observed levels have been extracted by first and second-hand estimation corresponding to pure rotator and our proposed formula respectively by plotting the E-Gamma Over Spin (EGOS) versus spin. A computer simulated search program is used to extract the model parameters in order to obtain a minimum root mean square (rms) deviation between the calculated and the experimental transition energies The values of spins resulting from second estimation method are excellent consistent with spin assignment of other models. The calculated transition energies, level spins, rotational frequencies, kinematic and dynamic moments of inertia are systematically examined. The difference in γ- ray transition energies ΔE_γ between transitions in the two isotones ^{192}Hg(SD1) and ^{194}Pb(SD1) were small and constant up to rotational frequency $\hbar\omega \sim 0.25$ MeV. Therefore, these two bands have been considered as identical bands. The $\Delta I = 2$ energy staggering observed in ^{194}Hg(SD1) and ^{194}Hg(SD2) of our selected SDRB's are also described from a smooth reference representing the finite difference approximation to the fourth order derivative of the transition energies at a given spin.

1 Introduction

Superdeformed (SD) nuclei were observed in a wide range of nuclear chart, and a wealth of experimental data on the resulting superdeformed rotational bands (SDRR's) was accumulated in recent years [1, 2]. These bands consists of long cascades of regularly spaced quadruple γ-ray transitions, which reveal a high degree of collectivity in a strongly deformed prolate nucleus. Lifetime measurements lead to very large values for the quadrupole moments of $Q_0 \sim 15-20$ eb which indeed correspond to an elongated ellipsoid with an axis ratio close to 2:1.

The superdeformation at high angular momentum remains one of the most interesting and challenging topics of nuclear structure. At present, although a general understanding of the properties of such SD nuclei has been achieved, there are still many open un expected problems. One of the outstanding experimental problems in the study of SD nuclei concerns their decay to the ground state. After a rapid decay out occurs over 2-4 states, and transitions linking the SD band to known levels in the first well are unobserved. As a result, the excitation energy, spin and parity of the levels in the first well are unobserved. As a result several theoretical approaches to predict the spins of SD bands were suggested [3–14].

To date, SD spectroscopy has given us much information concerning the behavior of moment of inertia in SD nuclei. For example it was shown [15] that for SD nuclei near $A \sim 150$, the variation in the dynamical moment of inertia $J^{(2)}$ with rotational frequency $\hbar\omega$ is dependant on the proton and neutron occupation of high-N intruder orbitals. For most SD bands in even-even and odd-A nuclei in the $A \sim 190$, $J^{(2)}$ exhibits a smooth gradual increase with increasing $\hbar\omega$ [16], which is due to the gradual alignment of quasinucleons occupying high -N intruder orbitals (originating from the $i_{13/2}$ proton and $j_{15/2}$ neutron subshells)in the presence of the pair correlations, while in the odd-odd nuclei, quite a good part of the moments of inertia for SD bands keep constant.

An unexpected discovery was the existence of identical bands (IB's) [17–21]. IB's are two bands in different nuclei, which have essentially identical transition energies within 2 keV, and thus essentially identical dynamical moment of inertia.

It was found that some SDRB's in different mass regions show an unexpected $\Delta I = 2$ staggering effects in the γ-ray energies [22–25]. The effect is best seen in long rotational sequences, where the expected regular behavior of the energy levels with respect to spin or to rotational frequency, is perturbed. The result is that the rotational sequences is split into two parts with states separated by $\Delta I = 4$ (bifurcation) shifting up in energy and the intermediate states shifting down in energy. The curve found by smoothly interpolating the band energy of the spin sequence $I, I + 4, I + 8, ...,$ is some what displaced from the corresponding curve of the sequence $I+2, I+6, I+10, ...$ The magnitude of the displacement in the gamma transition energy is found to be in the range of some hundred eV to a few keV. The $\Delta I = 2$ staggering effect has attracted considerable interest in the nuclear structure community. A few theoretical proposal for the possible explanation of this $\Delta I = 2$ staggering have already been made [26–31].

Calculations using the cranked Nilsson-Strutinsky method [32], and the Hartee-Fock method [33] suggest that nuclei with $N = 112$ and $Z = 80$ or 82 should be particularly stable, due to the existence of SD gaps in the single particle spectrum. As a result ^{192}Hg and ^{194}Pb are considered as doubly magic SD nuclei. Excited SD bands in these two nuclei are therefore expected to exist a somewhat higher excitation energies, and consequently to be populated with lower intensity than excited SD bands in other nuclei in this region.

In this paper, we shall present a theoretical study for Hg and Pb nuclei, our results are in framework of collective rotational formula including two parameters, obtained by adopted best fit method. We need first and second estimation to predict the spins for the studied SDRB's, and the best fitted parameters have been used to evaluate the E2 transition γ-ray energies, rotational frequencies, kinematic and dynamic moments of inertia. The appearance of identical bands (IB's) and the occurrance of a $\Delta I = 2$ staggering effect have been examined.

2 Parametrization of SDRB's by Two-Parameter Collective Rotational Formula

For the description of normally deformed (ND) bands, some useful expressions were presented. Bohr and Mottelson [34] pointed out that, under the adiabatic approximation, the rotational energy of an axially symmetric even-even nucleus may be expanded as (for $k = 0$, where k is the projection of the angular momentum I onto the symmetric axis) a power series in terms of of I^2=I(I+1):

$$E(I) = A[I(I+1)] + B[I(I+1)]^2 \tag{1}$$

with common constants A and B. We will adopt the energy of the SD state with spin I by equation (1).

For SD bands, gamma-ray transition energies are unfortunately, the only spectroscopic information universally available. The gamma-ray transition energy between levels differing by two units of angular momentum $\Delta I = 2$ are:

$$\begin{aligned} E_\gamma(I) &= E(I) - E(I-2) \\ &= (I - 1/2)\left[4A + 8B(I^2 - I + 1)\right]. \end{aligned} \tag{2}$$

3 Spin Assignment of SDRB's in A ∼ 190 Mass Region

In the method used, the energies of the SD nuclear rotational bands are firstly expressed by pure rotator as a first estimation of bandhead spin

$$E(I) = AI(I+1). \tag{3}$$

Thus

$$E_\gamma(I) = 4A(I - 1/2). \tag{4}$$

If I_0 represent the bandhead spin, then

$$\frac{E_\gamma(I_0+4)}{E_\gamma(I_0+2)} = \frac{4I_0+14}{4I_0+6}. \tag{5}$$

Therefore,

$$I_0 = \frac{1}{4}\left[\frac{8E_\gamma(I_0+2)}{E_\gamma(I_0+4) - E_\gamma(I_0+2)} - 6\right]. \tag{6}$$

The ratio $E_\gamma(I)$ over spin I (E-Gamma Over Spin(EGOS)) is given by

$$EGOS = \frac{E_\gamma(I)}{I - 1/2} = 4A \tag{7}$$

when EGOS plotted against spin, it gives horizontal line.

For second estimation of bandhead spin, our proposed formula equation (1) is used, thus EGOS becomes

$$EGOS = 4A + 8B(I^2 - I + 1) \tag{8}$$

which decrease hyperbolically.

4 Rotational Frequency and Moments of Inertia

In the framework of nuclear collective rotational model with $k = 0$, the rotational frequency $\hbar\omega$ for the expression (1) is given by

$$\begin{aligned} \hbar\omega(I) &= \frac{dE(I)}{d\sqrt{I(I+1)}} \\ &= 2A\,[I(I+1)]^{\frac{1}{2}} + 4B\,[I(I+1)]^{\frac{3}{2}}. \end{aligned} \tag{9}$$

The kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia for the expression(1) are:

$$\begin{aligned} \frac{J^{(1)}}{\hbar^2} &= \frac{1}{\sqrt{I(I+1)}}\left[\frac{dE(I)}{d\sqrt{I(I+1)}}\right]^{-1} \\ &= J_0 - \frac{B}{A^2}[I(I+1)] \\ &+ \frac{2B^2}{A^3}[I(I+1)]^2 - 4\frac{B^3}{A^4}[I(I+1)]^3 \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{J^{(2)}}{\hbar^2} &= \left[\frac{d^2E(I)}{d[I(I+1)]^2}\right]^{-1} \\ &= J_0 - 3\frac{B}{A^2}[I(I+1)] \\ &+ 18\frac{B^2}{A^3}[I(I+1)^2] - 108\frac{B^3}{A^4}[I(I+1)]^3 \end{aligned} \tag{11}$$

where J_0 is refereed to as the bandhead moment of inertia

$$J_0 = \frac{1}{2A}. \tag{12}$$

The two moments of inertia are obviously dependent.

One has

$$J^{(2)} = J^{(1)} + \omega\frac{dJ^{(1)}}{d\omega}. \tag{13}$$

Table 1: Bandhead spin for ^{194}Hg(SD1) derived from EGOS for first estimation A=5.2902 keV, I_0 = 10.5

I	E_γ	EGOS^{cal}(keV/$\hbar$)			EGOS^{exp}(keV/$\hbar$)		
$\hbar$	(keV)	$I_0 - 2$	I_0	$I_0 + 2$	$I_0 - 2$	I_0	$I_0 + 2$
12.5	253.929	25.392	21.16	18.137	25.393	21.160	18.137
14.5	296.2512	24.687	21.16	18.515	24.665	21.142	18.499
16.5	338.572	24.183	21.16	18.809	24.084	21.073	18.732
18.5	380.894	23.805	21.16	19.044	23.586	20.966	18.869
20.5	423.216	23.512	21.16	19.237	23.144	20.830	18.936
22.5	465.537	23.376	21.16	19.397	22.738	20.670	18.948
24.5	507.859	23.084	21.16	19.533	22.357	20.494	18.917
26.5	550.180	22.924	21.16	19.649	21.995	20.303	18.852
28.5	592.502	22.788	21.16	19.750	21.650	20.104	18.764
30.5	634.824	22.672	21.16	19.838	21.316	19.895	18.652
32.5	677.145	22.571	21.16	19.916	20.997	19.685	18.527
34.5	719.467	22.483	21.16	19.985	20.689	19.472	18.390
36.5	761.788	22.405	21.16	20.047	20.394	19.261	18.247
38.5	804.110	22.336	21.16	20.102	20.108	19.050	18.097
40.5	846.432	22.274	21.16	20.153	19.840	18.848	17.950
42.5	888.753	22.218	21.16	20.198	19.591	18.658	17.810
44.5	931.075	22.168	21.16	20.240	19.360	18.480	17.676
46.5	973.396	22.122	21.16	20.279	19.148	18.316	17.553

The dynamical moment of inertia varies often in a very sensitive way with rotational frequency $\hbar\omega$. In particular for rigid rotor, we shall obtain:

$$J^{(2)} = J^{(1)} = J_{rigid}. \tag{14}$$

Experimentally, for SDRB's, the gamma-ray transition energies are the only spectroscopic information universally available. Therefore, to compare the structure of the SD bands, information about their gamma-ray transition energies are commonly translated into values of rotational frequency $\hbar\omega$ and moments of inertia:

$$\hbar\omega = \frac{1}{4}[E_\gamma(I) + E_\gamma(I+2)] \qquad \text{(MeV)} \tag{15}$$

$$J^{(1)}(I-1) = \frac{2I-1}{E_\gamma(I)} \qquad (\hbar^2\text{MeV}^{-1}) \tag{16}$$

$$J^{(2)}(I) = \frac{4}{\Delta E_\gamma(I)} \qquad (\hbar^2\text{MeV}^{-1}) \tag{17}$$

where $\Delta E_\gamma(I)$=$E_\gamma(I+2)$-$E_\gamma(I)$ is the difference between two consecutive transition energies. Therefore, the dynamical moment of inertia $J^{(2)}$ which is linked to the second derivatives of energy, does not depend on the knowledge of the spin I but only on the measured transition energies. Theoretically, the $J^{(2)}$ moment of inertia reflects the curvature of the single-particle orbitals, while experimentally it is simply extracted from the measured γ-ray energies. In terms of A and B, yield directly:

$$J^{(1)} = \frac{1}{2[A + 2B(I^2 - 2I + 1)]}, \tag{18}$$

$$J^{(2)} = \frac{1}{2[A + B(6I^2 + 6I + 5)]}. \tag{19}$$

5 Identical Bands in SDRB's

Since the experimental discovery of SD bands in rapidly rotating nuclei, many unexpected features of these highly excited configurations were observed. One of the most striking feature is the existence of identical bands (IB's) or twin bands, that is identical transition energies E_γ in bands belonging to neighboring nuclei with different mass numbers. To determine whether a pair of bands is identical or not, one must calculate the difference between their gamma-transition energies of the two bands 1 and 2, ΔE_γ=$E_\gamma(1)$-$E_\gamma(2)$.

6 ΔI = 2 Staggering Effect in Transition Energies

To explore more clearly the $\Delta I = 2$ staggering, for each band the deviation of transition energies from a smooth reference is determined by calculating the finite difference approximation to the fourth derivative of the γ-ray energies at a given spin d^4E_γ/dI^4. This smooth reference is given by

$$\begin{aligned}\Delta^4 E_\gamma(I) &= \tfrac{1}{16}[E_\gamma(I-4) \\ &\quad -4E_\gamma(I-2) + 6E_\gamma(I) \\ &\quad -4E_\gamma(I+2) + E_\gamma(I+4)].\end{aligned} \tag{20}$$

This formula includes five consecutive transition energies and is denoted by five-point formula. For equation (1), we can easily notice that in this case $\Delta^4E_\gamma(I)$ vanishes.

Table 2: The same as in Table (1) but for second estimation $A = 5.5904$ keV and B= -3.395×10^{-4} keV, $I_0 = 10$

I	E_γ	EGOS^{cal}(keV/$\hbar$)			EGOS^{exp}(keV/$\hbar$)		
$\hbar$	(keV)	$I_0 - 2$	I_0	$I_0 + 2$	$I_0 - 2$	I_0	$I_0 + 2$
12	253.102	26.642	22.008	18.748	26.729	22.080	18.809
14	295.399	25.686	21.881	19.058	25.738	21.925	19.096
16	336.884	24.954	21.734	19.250	24.976	21.753	19.267
18	377.513	24.355	21.572	19.359	24.347	21.565	19.353
20	417.181	23.838	21.393	19.403	23.805	21.364	19.376
22	455.830	23.375	21.201	19.397	23.321	21.151	19.351
24	493.408	22.949	20.996	19.349	22.877	20.930	19.288
26	529.873	22.547	20.779	19.268	22.462	20.701	19.195
28	565.185	22.164	20.552	19.158	22.075	20.469	19.082
30	599.324	21.793	20.316	19.026	21.704	20.232	18.948
32	632.266	21.432	20.071	18.873	21.353	19.997	18.803
34	664.012	21.079	19.821	18.704	21.018	19.763	18.649
36	694.573	20.733	19.565	18.521	20.698	19.532	18.490
38	723.943	20.392	19.305	18.327	20.391	19.304	18.326
40	752.145	20.057	19.041	18.123	20.104	19.086	18.166
42	779.183	19.726	18.775	17.912	19.839	18.883	18.015
44	805.102	19.400	18.508	17.694	19.593	18.692	17.870
46	829.924	19.078	18.240	17.472	19.368	18.517	17.737

7 Results and Discussion

The SDRB's ^{192}Hg(SD1), ^{194}Hg(SD1), ^{194}Hg(SD2), ^{194}Pb(SD1) and ^{196}Pb(SD1) in $A \sim 190$ mass region are considered. For each nucleus the optimized two parameters A,B of the model in question are fitted to reproduce the observed experimental γ-ray transition energies $E_\gamma^{exp}(I)$. The procedure is repeated for several trial values A,B by using a computer simulation search programm. The best parameters lead to minimize the root mean square (rms) deviation

$$\chi = \left[\frac{1}{N} \sum_{i=1}^{N} \left[\frac{E_\gamma^{exp}(I_i) - E_\gamma^{cal}(I_i)}{E_\gamma^{exp}(I_i)} \right]^2 \right]^{\frac{1}{2}} \tag{21}$$

where N is the total number of experimental points considered in fitting procedure. The experimental data are taken from reference [1,2]. The bandhead spins of the observed levels have been extracted by applying the first and second-hand estimations corresponding to pure rotator and our proposed formula respectively by plotting EGOS versus spin.

The EGOS is a horizontal line for the exact I_0 and will shift to parabola when $I_0 \pm 2$ is assigned to I_0 for pure rotator (first estimation) and three parabola curves for our proposed model (second estimation). As an example, this procedure illustrated in Figure (1) for ^{194}Hg(SD1) for bandheads $I_0 + 2$, I_0, $I_0 - 2$. The closed circles represents the experimental values while the solid curves the calculated ones. The numerical values are presented in Tables (1,2).

The resulting best parameters A,B of the model and the values of the lowest bandhead spins I_0 and the bandhead moment of inertia J_0 for our selected SDRB's are listed in Table (3).

In framework of the applied theoretical model, the dynamic $J^{(2)}$ and kinematic $J^{(1)}$ moments of inertia corresponding to the calculated spins have been extracted. The comparison between the experimental γ-ray transition energies and our calculations using the values of the model parameters given in Table(1) for the SD bands of our selected nuclei is illustrated in Figure(2).

Figure (3) illustrates the calculated kinematic $J^{(1)}$ (open circle) and dynamic $J^{(2)}$ (closed circle) moments of inertia as a function of rotational frequency $\hbar\omega$. Both the moments of inertia $J^{(1)}$ and $J^{(2)}$ exhibits a smooth increase with increasing rotational frequency, the $J^{(2)}$ is significantly larger than $J^{(1)}$ over a large rotational frequency range.

Investigating the tables and figures, we know that the γ-ray transition energies, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia of the SD states can be quantitatively described excellently with our two-parameters collective rotational formula. The $J^{(2)}$ values for both ^{192}Hg(SD1) and ^{194}Pb(SD1) are very close over the entire frequency range $\hbar\omega < 0.25$ MeV. However, at higher frequencies the differences in transition energies are no longer constant.

Moreover, the SD band of ^{194}Pb(SD1) is populated at lower spin values $I_0 = 6\hbar$ than that of ^{192}Hg(SD1), $I_0 = 10\hbar$. The difference in γ-ray energies ΔE_γ between transitions in ^{192}Hg(SD1) and ^{194}Pb(SD1) are plotted in Figure (4). Up to $\hbar\omega \sim 0.25$ MeV, the ΔE_γ values are small and constant. Therefore, these two bands have been considered as identical bands (IB), however at higher frequency the difference in transition energies are no longer constant. also the difference

Table 3: The adapted model parameters A,B obtained by fitting procedure, the suggested bandhead spins I_0 and the bandhead moments of inertia J_0. The SDRB's are identified by the lowest observed E_γ.

SDRB	A(keV)	$B(10^{-4}$keV)	$I_0(\hbar)$	$J_0(\hbar^2$MeV$^{-1})$	E_γ(keV)
^{192}Hg(SD1)	5.6470	-3.5087	8	88.5425	214.4
^{194}Hg(SD1)	5.5904	-3.3951	10	89.4390	253.93
^{194}Hg(SD2)	5.3154	-2.2537	8	94.0662	200.79
^{194}Pb(SD1)	5.6637	-1.5590	4	88.2815	124.9
^{196}Pb(SD1)	5.7282	-3.1319	6	87.2874	171.5

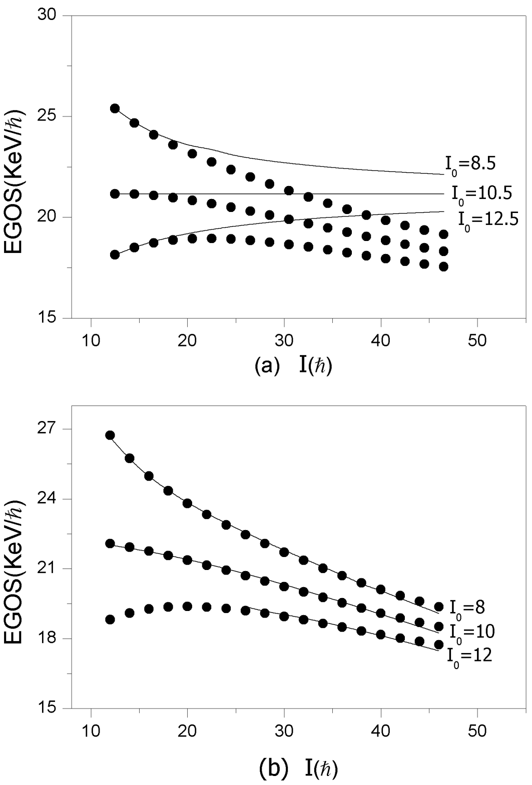

Fig. 1: EGOS versus spin to determine the band head spin for ^{194}Hg(SD-1) (a) for first estimation (b) for second estimation.

ΔE_γ between ^{194}Hg(SD1) and ^{192}Hg(SD1) is approximately 4 keV at low frequency (see Figure (4)) are too longer to consider these two bands as identical ones.

Another result of the present work is the observation of a $\Delta I = 2$ staggering effects in the transition energies for ^{194}Hg(SD1) and ^{194}Hg(SD2). For each band, the deviation of the γ-ray transition energies from a smooth reference representing the finite difference approximation to the fourth derivative of the γ-ray transition energies in a $\Delta I = 2$ band is calculated. Figure (5) show the resulting values of $\Delta^4 E_\gamma(I)$ against rotational frequency $\hbar\omega$ for the two SD bands. A significant staggering has been observed for ^{194}Hg(SD2) in frequency range $\hbar\omega \sim 0.3$ MeV.

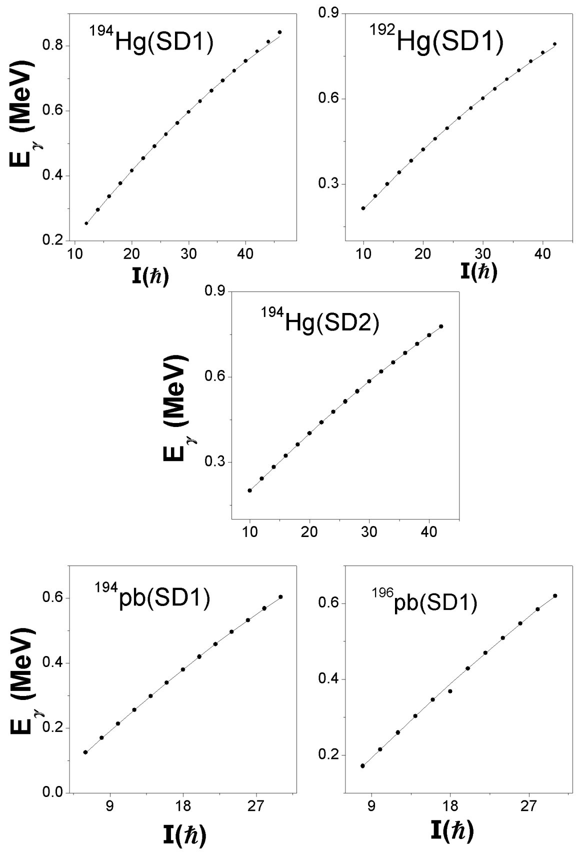

Fig. 2: Theoretical (solid curve) and experimental (closed circles) gamma-ray transition energies E_γ of the SD bands observed in even-even Hg and Pb nuclei. The theoretical values are calculated with the corresponding parameters taken from Table (3).

8 Conclusion

We studied in a simple version of two parameters collective model the five SDRB's ^{192}Hg(SD1), ^{194}Hg(SD1,SD2), ^{194}Pb(SD1) and ^{196}Pb(SD1) in the mass region 190. Transition energies, rotational frequencies, dynamic and kinematic

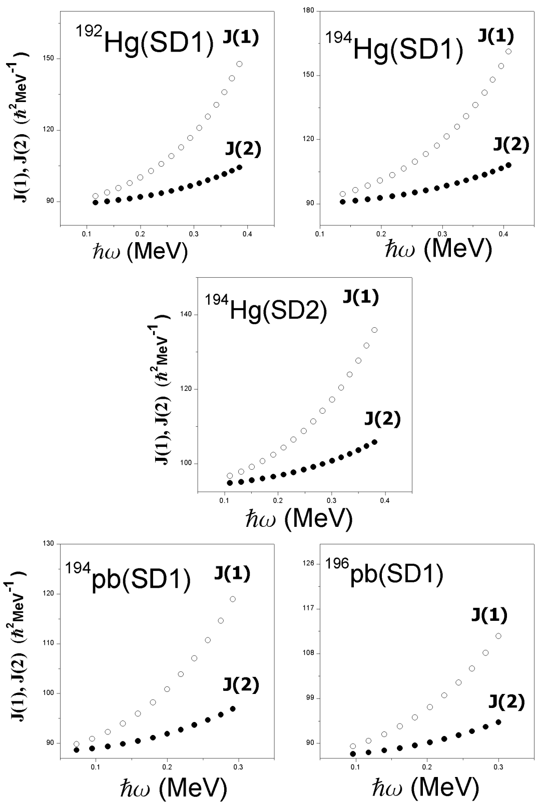

Fig. 3: The calculated results of kinematic $J^{(1)}$ (open circles) and dynematic $J^{(2)}$ (closed circles) moments of inertia plotted as a function of the rotational frequency $\hbar\omega$ for the studied SDRB's.

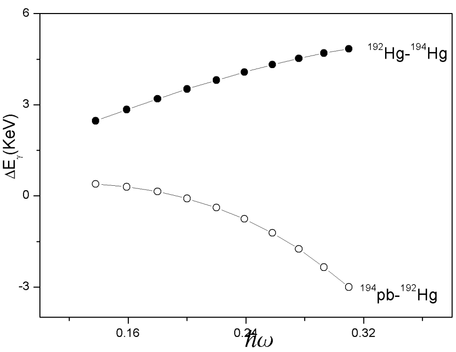

Fig. 4: Differences in the calculated γ-ray transition energies between ^{192}Hg(SD-1)-^{194}Pb(SD-1) and between ^{192}Hg(SD-1)-^{194}Hg(SD-1).

moments of inertia have been calculated. An excellent agreement with the experimental data justifies the application of

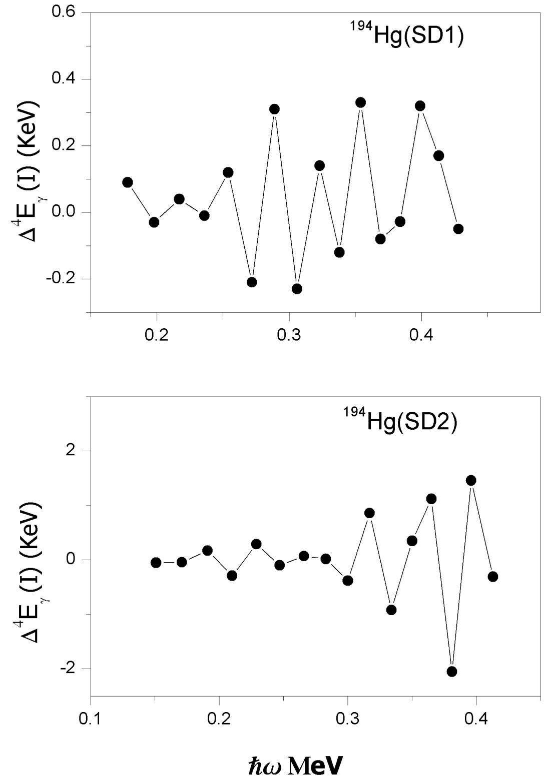

Fig. 5: The $\Delta = 2$ staggering obtained by the five points formula $\Delta^4 E_\gamma(I)$ as a function of rotational frequency $\hbar\omega$ for ^{194}Hg(SD-1, SD-2).

this version of the model. For the first-hand estimation of the bandhead spin I_0 of each SD band we have used the simple rigid rotator to extrapolated the experimentally transition energies, and from the ratio between two consecutive transition energies $\frac{E_\gamma(I_0+4)}{E_\gamma(I_0+2)}$, the spin value of the bandhead has been calculated. For second hand estimation of I_0, the EGOS versus spin for our model are plotted, the plot gives three parabola curves for I_0 and $I_0 \pm 2$. The existence of identical bands in the isotones ^{192}Hg(SD1) and ^{194}Pb(SD1) are investigated. The $\Delta I = 2$ staggering has been examined in the notation of Cedercwall [23]. The staggering plot has been extracted and investigated.

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