Wave-Particle Duality in the Elastodynamics of the Spacetime Continuum (STCED)

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We examine the nature of the wave-particle duality in the Elastodynamics of the Spacetime Continuum (STCED), due to the propagation of deformations in the STC by longitudinal dilatation and transverse distortion wave displacements. We first consider the special case of Electromagnetism which consists of transverse waves only, and use the photon wavefunction to demonstrate that $|\Psi|^2$ represents a physical energy density, not a probability density. However, normalization by the system energy allows use of the probabilistic formulation of quantum theory. In the STCED longitudinal and transverse wave equations, the transverse wave is the source of the interference pattern in double slit experiments, influencing the location of the longitudinal wave, as observed experimentally. We note the similarity of STCED wave-particle duality and Louis de Broglie's "double solution".

1 Introduction

As shown previously, in the Elastodynamics of the Spacetime Continuum (*STCED*) [1–6], energy propagates in the STC (spacetime continuum) as wave-like deformations which can be decomposed into *dilatations* and *distortions*.

Dilatations include an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation. The rest-mass energy density of this longitudinal mode is given by [1, see Eq.(32)]

$$\rho c^2 = 4\bar{\kappa}_0 \varepsilon \tag{1}$$

where ρ is the dilatation rest-mass density, c is the speed of light, $\bar{\kappa}_0$ is the bulk modulus of the *STC* (the resistance of the spacetime continuum to *dilatations*), and ε is the volume dilatation. On the other hand, *distortions* correspond to a change of shape (shearing stress) of the spacetime continuum without a change in volume and are thus massless.

Thus deformations propagate in the spacetime continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements. This provides a natural explanation for wave-particle duality, with the transverse mode corresponding to the wave aspects of the deformations and the longitudinal mode corresponding to the particle aspects of the deformations.

2 Wave-particle duality in Electromagnetism

In Electromagnetism, as shown in [1, see (121)], the volume dilatation is $\varepsilon = 0$. Hence, the photon is massless and there is no longitudinal mode of propagation. Electromagnetic waves are massless transverse distortion waves.

The photons correspond to an energy flow along the direction of propagation in 3-space resulting from the Poynting vector. This longitudinal electromagnetic energy flux is massless as it is due to distortion, not dilatation, of the spacetime continuum. However, because this energy flux is along the direction of propagation, it gives rise to the particle aspect of the electromagnetic field, the photon. We should note however that the modern understanding of photons is that they are massless excitations of the quantized electromagnetic field, not particles *per se*. Thus in this case, the kinetic energy in the longitudinal direction is carried by the distortion part of the deformation, while the dilatation part, which carries the rest-mass energy, is not present as the mass is 0.

This situation provides us with an opportunity to investigate the transverse mode of propagation, independently of the longitudinal mode. In general, the transverse propagation of electromagnetic waves is given by sinusoidal waves ψ and the intensity of the waves, corresponding to the energy density, is given by $|\psi|^2$. This is equivalent to the modulus squared of the wavefunction used in Quantum Mechanics as a probability density. A full analysis requires that we investigate further the Quantum Mechanics of the photon, and in particular, the photon wavefunction.

2.1 Photon wavefunction

The photon wavefunction is a first quantization description of the electromagnetic field [7,8]. Historically, this development was not done, as second quantization of the electromagnetic field was first developed. As a result, photon wave mechanics is not fully accepted in the scientific community, mainly because of the differences between particle and photon dynamics. As opposed to a particle, the photon has zero rest-mass and propagates at the speed of light. In addition, the position operator cannot be defined for a photon, only the momentum operator (photon localization problem).

Bialynicki-Birula [8–12], Sipe [13], and more recently Mohr [14], Raymer and Smith [15–17] and others have derived and promoted the use of the photon wavefunction. Bialynicki-Birula defines the photon wavefunction as "a complex vector-function of space coordinates r and time t that adequately describes the quantum state of a single photon" [8].

He sees three advantages to introducing a photon wavefunction [11]: it provides 1) a unified description of both massive and massless particles both in first quantization and second quantization; 2) an easier description of photon dynamics without having to resort to second quantization; 3) new methods of describing photons.

As pointed out in [7] and references therein, the photon wave equation is now used to study the propagation of photons in media, the quantum properties of electromagnetic waves in structured media, and the scattering of electromagnetic waves in both isotropic and anisotropic inhomogeneous media. Raymer and Smith [16, 17] have extended the use of the photon wavefunction to the analysis of multi-photon states and coherence theory. To the above list, in this paper, we add an additional benefit of the photon wavefunction: the clarification of the physical interpretation of the quantum mechanical wavefunction.

The photon wavefunction is derived from the description of the electromagnetic field based on the complex form of the Maxwell equations first used by Riemann, Silberstein and Bateman [8] (the Riemann–Silberstein vector). As summarized by Bialynicki-Birula [12], "[t]he Riemann–Silberstein vector on the one hand contains full information about the state of the classical electromagnetic field and on the other hand it may serve as the photon wave function in the quantum theory". The Maxwell equations are then written as [8]

$$\begin{aligned} i\,\partial_t \mathbf{F}(\mathbf{r},t) &= c\,\nabla\times\mathbf{F}(\mathbf{r},t) \\ \nabla\cdot\mathbf{F}(\mathbf{r},t) &= 0 \end{aligned} \tag{2}$$

where

$$\mathbf{F}(\mathbf{r},t) = \left(\frac{\mathbf{D}(\mathbf{r},t)}{\sqrt{2\epsilon_0}} + i\,\frac{\mathbf{B}(\mathbf{r},t)}{\sqrt{2\mu_0}}\right) \tag{3}$$

and where $\mathbf{D}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ have their usual significance.

Then the dynamical quantities like the energy density and the Poynting vector are given by [8]

$$\begin{aligned} E &= \int \mathbf{F}^*\cdot\mathbf{F}\ \mathrm{d}^3r \\ \mathbf{S} &= \frac{1}{2ic}\int \mathbf{F}^*\times\mathbf{F}\ \mathrm{d}^3r \end{aligned} \tag{4}$$

where $\mathbf{F}^*$ denotes the complex conjugate. The sign selected in (3) reflects positive helicity (projection of the spin on the direction of momentum) corresponding to left-handed circular polarization. Photons of negative helicity corresponding to right-handed circular polarization are represented by changing the sign from i to $-i$ in (3). Hence (3) can be written as

$$\mathbf{F}_\pm(\mathbf{r},t) = \left(\frac{\mathbf{D}(\mathbf{r},t)}{\sqrt{2\epsilon_0}} \pm i\,\frac{\mathbf{B}(\mathbf{r},t)}{\sqrt{2\mu_0}}\right) \tag{5}$$

to represent both photon polarization states.

A photon of arbitrary polarization is thus represented by a combination of left- and right-handed circular polarization states. The photon wavefunction is then given by the six-component vector

$$\Psi(\mathbf{r},t) = \begin{pmatrix} \mathbf{F}_+(\mathbf{r},t) \\ \mathbf{F}_-(\mathbf{r},t) \end{pmatrix}. \tag{6}$$

The corresponding photon wave equation is discussed in [11].

2.2 Physical interpretation of the photon wavefunction

From (6) and (5), we calculate the modulus squared of the photon wavefunction to obtain [7]

$$|\Psi(\mathbf{r},t)|^2 = \left(\frac{\epsilon_0|\mathbf{E}|^2}{2} + \frac{|\mathbf{B}|^2}{2\mu_0}\right). \tag{7}$$

The modulus squared of the photon wavefunction $\Psi(\mathbf{r},t)$ gives the electromagnetic energy density at a given position and time. This is the physical interpretation of the quantum mechanical $|\Psi(\mathbf{r},t)|^2$ for electromagnetic transverse waves in the absence of longitudinal waves.

Bialynicki-Birula proposes to convert $|\Psi(\mathbf{r},t)|^2$ to a probability density as required by the accepted quantum mechanical probabilistic interpretation [11]. This he achieves by dividing the modulus squared of the photon wavefunction by the expectation value of the energy $<E>$ [11, see his equation (44)]. In this way, it is made to describe in probabilistic terms the energy distribution in space associated with a photon.

Thus the probabilistic formulation of quantum theory is preserved, while the physical interpretation of $|\Psi|^2$ is shown to correspond to an energy density. Raymer and Smith [17] state that "[a] strong argument in favour of the energy-density wave function form of PWM [Photon Wave Mechanics] is that it bears strong connections to other, well-established theories—both quantum and classical—such as photodetection theory, classical and quantum optical coherence theory, and the biphoton amplitude, which is used in most discussions of spontaneous parametric down conversion".

Hence, we have to conclude that the appropriate physical interpretation of $|\Psi|^2$ is that it represents a physical energy density, not a probability density. However, the energy density can be converted to a probability density once it is normalized with the system energy (as done by Bialynicki-Birula for the photon wavefunction). In this way, *STCED* does not replace the probabilistic formulation of quantum theory, it just helps to understand the physics of quantum theory. The two formulations are equivalent, which explains the success of the probabilistic formulation of quantum theory. In actual practice, the quantum mechanical probability formulation can be used as is, as it gives the same results as the physical energy density formulation of *STCED*. However, the physical intensity waves of *STCED* help us understand the physics of the quantum mechanical wavefunction and the physics of wave-particle duality.

It is important to note that the energy density physical interpretation of $|\Psi|^2$ applies just as much to systems as to single particles, as for the probability density interpretation.

3 Wave-particle duality in STCED

In *STCED*, the displacement u^ν of a deformation from its undeformed state can be decomposed into a longitudinal (dilatation) component $u_\parallel^\nu$ and a transverse (distortion) component $u_\perp^\nu$. The volume dilatation ε is given by the relation [1, see (44)]

$$\varepsilon = u_\parallel{}^\mu{}_{;\mu}. \tag{8}$$

The longitudinal displacement wave equation and the transverse displacement wave equation of a deformation are given respectively by [1, see (196)]

$$\begin{aligned} \nabla^2 u_\parallel^\nu &= -\frac{\bar{\mu}_0 + \bar{\lambda}_0}{\bar{\mu}_0}\varepsilon^{;\nu} \\ \nabla^2 u_\perp^\nu + \frac{\bar{k}_0}{\bar{\mu}_0}\,\varepsilon\,(x^\mu)\; u_\perp^\nu &= 0 \end{aligned} \tag{9}$$

where ∇^2 is the 4-D operator, $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the spacetime continuum and $\bar{k}_0$ is the elastic force constant of the spacetime continuum. The constant $\bar{\mu}_0$ is the shear modulus (the resistance of the continuum to *distortions*) and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus (as in (1) in Section 1) according to

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \bar{\mu}_0/2 \tag{10}$$

in a four-dimensional continuum. The wave equation for $u_\parallel^\nu$ describes the propagation of longitudinal displacements, while the wave equation for $u_\perp^\nu$ describes the propagation of transverse displacements in the spacetime continuum. The *STCED* deformation wave displacements solution is similar to Louis de Broglie's "double solution" [18, 19].

3.1 Wave propagation in STCED

The electromagnetic case, as seen in Section 2, provides a physical interpretation of the wavefunction for transverse wave displacements. This interpretation should apply in general to any wavefunction Ψ. In *STCED*, in the general case, every deformation can be decomposed into a combination of a transverse mode corresponding to the wave aspect of the deformation, and a longitudinal mode corresponding to the particle aspect of the deformation [2]. Thus the physical interpretation of Section 2.2 applies to the general *STCED* transverse wave displacements, not only to the electromagnetic ones.

Hence, $|\Psi|^2$ represents the physical intensity (energy density) of the transverse (*distortion*) wave, rather than the probability density of quantum theory. It corresponds to the transverse field energy of the deformation. It is not the same as the particle, which corresponds to the longitudinal (*dilatation*) wave displacement and is localized within the deformation via the massive volume dilatation, as discussed in the next Section 3.2. However, $|\Psi|^2$ can be normalized with the system energy and converted into a probability density, thus allowing the use of the existing probabilistic formulation of quantum theory. Additionally, the physical intensity waves of *STCED* help us understand the physics of wave-particle duality and resolve the paradoxes of quantum theory.

3.2 Particle propagation in STCED

Particles propagate in the spacetime continuum as longitudinal wave displacements. Mass is proportional to the volume dilatation ε of the longitudinal mode of the deformation as per (1). This longitudinal mode displacement satisfies a wave equation for ε, different from the transverse mode displacement wave equation for Ψ. This longitudinal dilatation wave equation for ε is given by [1, see (204)]

$$\nabla^2\varepsilon = -\frac{\bar{k}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\, u_\perp^\nu \varepsilon_{;\nu}\,. \tag{11}$$

It is important to note that the inhomogeneous term on the R.H.S. includes a dot product coupling between the transverse displacement $u_\perp^\nu$ and the gradient of the volume dilatation $\varepsilon_{;\nu}$ for the solution of the longitudinal dilatation wave equation for ε. This explains the behavior of electrons in the double slit interference experiment.

The transverse distortion wave equation for $\omega^{\mu\nu}$ [1, see (210)]

$$\nabla^2\omega^{\mu\nu} + \frac{\bar{k}_0}{\bar{\mu}_0}\,\varepsilon\,(x^\mu)\;\omega^{\mu\nu} = \frac{1}{2}\frac{\bar{k}_0}{\bar{\mu}_0}\,(\varepsilon^{;\mu}u_\perp^\nu - \varepsilon^{;\nu}u_\perp^\mu) \tag{12}$$

shows a R.H.S. cross product coupling between the transverse displacement $u_\perp^\nu$ and the gradient of the volume dilatation $\varepsilon^{;\mu}$ for the solution of the transverse distortion wave equation for $\omega^{\mu\nu}$. The transverse distortion wave $\omega^{\mu\nu}$ corresponds to a multi-component wavefunction Ψ.

A deformation propagating in the spacetime continuum consists of a combination of a transverse and a longitudinal wave. The transverse wave is the source of the interference pattern in double slit experiments, which impacts the location of the associated longitudinal wave of the individual particle in generating the interference pattern. The longitudinal dilatation wave behaves as a particle and goes through one of the slits, even as it follows the interference pattern dictated by the transverse distortion wave, as observed experimentally [20, see in particular Figure 4] and as seen in the coupling between $\varepsilon_{;\nu}$ and $u_\perp^\nu$ in (11) and (12) above.

These results are in agreement with the results of the Jánossy-Naray, Clauser, and Dagenais and Mandel experiments on the self-interference of photons and the neutron interferometry experiments performed by Bonse and Rauch [21, see pp. 73-81]. The transverse distortion wave generates the interference pattern, while the longitudinal wave's dilatation (particle) follows a specific action, with its final location guided by the transverse wave's interference pattern.

The longitudinal wave is similar to the de Broglie "singularity-wave function" [18]. However, in *STCED* the particle is not a singularity of the wave, but is instead characterized by its mass which arises from the volume dilatation propagating as part of the longitudinal wave. There is no need for the collapse of the wavefunction Ψ, as the particle resides in the longitudinal wave, not the transverse one. A measurement of a particle's position is a measurement of the longitudinal wave, not the transverse wave.

4 Discussion and conclusion

In this paper, we have examined the nature of the wave-particle duality that comes out of the Elastodynamics of the Spacetime Continuum (*STCED*). We have noted that deformations propagate in the spacetime continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements, which provides a natural explanation for wave-particle duality, with the transverse mode corresponding to the wave aspects of the deformations and the longitudinal mode corresponding to the particle aspects of the deformations.

We have considered the special case of Electromagnetism, which is characterized by a transverse mode (the electromagnetic radiation), but no longitudinal mode (as the photon is massless), to help in the clarification of the physical interpretation of the quantum mechanical wavefunction. To that purpose, we have considered the photon wavefunction, and have demonstrated that the physical interpretation of $|\Psi|^2$ represents an energy density, not a probability density. However, it can be normalized with the system energy to be converted to a probability density and allow the use of the probabilistic formulation of quantum theory. We have also noted that the energy density physical interpretation of $|\Psi|^2$ applies just as much to systems as to single particles.

We have then looked at the general *STCED* case, where every deformation can be decomposed into a combination of a transverse mode corresponding to the wave aspect of the deformation, and a longitudinal mode corresponding to the particle aspect of the deformation, and concluded that the physical interpretation of the photon wavefunction applies to the general *STCED* transverse wave displacements, not only to the electromagnetic ones.

We have reviewed the *STCED* longitudinal dilatation wave equation for ε corresponding to the mass component (particle) and the transverse distortion wave equation for $\omega^{\mu\nu}$ corresponding to a multi-component wavefunction Ψ. We have noted the coupling on the R.H.S. of both equations between $\varepsilon^{;\mu}$ and $u^{\nu}_{\perp}$, showing that even though the transverse wave is the source of the interference pattern in double slit experiments as for the photon wavefunction, and the longitudinal dilatation wave behaves as a particle, the latter follows the interference pattern dictated by the transverse distortion wave as observed experimentally.

We have also noted the similarity of *STCED* wave-particle duality to Louis de Broglie's "double solution" and "singularity-wave function", even though in *STCED* the particle is not a singularity of the wave, but is instead characterized by its mass which arises from the volume dilatation propagating as part of the longitudinal wave.

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